

# **DISTRIBUTIONS OF VARIABILITY IN TRAVEL TIMES ON URBAN ROADS – A LONGITUDINAL STUDY**

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## **ABSTRACT**

Reliability is an important factor in route choice analysis and is a key performance indicator for transport systems. However, the current parameters used to measure travel time variability may be not sufficient to fully represent reliability. Better understanding of the distributions of travel times is needed for the development of improved metrics for reliability. A comprehensive data analysis involving the assessment of longitudinal travel time data for two urban arterial road corridors in Adelaide, Australia demonstrates that the observed distributions are more complex than previously assumed. The data sets demonstrate strong positive skew, very long upper tails, and sometimes bimodality. This paper proposes the use of alternative statistical distributions for travel time variability, with the Burr Type XII distribution emerging as an appropriate model for both links and routes.

## **INTRODUCTION**

Many factors can adversely affect transport network performance. Different types of incidents, either short term (e.g. vehicle breakdowns) or long term (e.g. bridge collapse), or random (e.g. road crashes) or intentional (e.g. road works) can happen at any time and may lead to higher travel time variability and perhaps wider consequences for the community. In addition, the need for more reliable transportation systems and demands for 'just-in-time' services have generated new interest in transportation system reliability, which is thus a major research topic.

Travel time reliability is based on the concept of a travel time that meets travellers' expectations (Small, 1982). Travellers expect their travel times not to exceed a scheduled value, or average travel time plus some acceptable additional time, and hence they can decide on a starting time for the journey. The concept of an acceptable additional time is subjective, and will vary depending on perceptions and individual circumstances. Overly conservative travel time estimates may be unhelpful as these may cause travellers to arrive

too early. This leads to the use of maximum utility models to jointly determine departure time and trip time, as discussed in Fosgerau and Karlstrom (2010).

Acknowledging the appropriate travel time distribution and the probability of travel time 'failure'<sup>1</sup> is thus important for the development of travel time reliability metrics. This is consistent with practice in reliability engineering, which is concerned with measuring the consistency and the persistency of a product under different conditions over a period of time. On the basis of the following considerations:

1. the current parameters used to measure travel time variability may not be sufficient to fully represent travel time reliability, and
2. there is known to be significant variability in individual travel times

a better understanding of the distribution of individual travel times is needed for the development of relevant metrics for assessing travel time reliability.

This paper focuses on the specification of appropriate travel time variability distributions. It first reviews previous research, and then tests different statistical distributions using empirical data. While previous travel time reliability studies have often focused on freeway travel times – usually because of the availability of suitable data sets involving observations of large numbers of individual travel times over a short period of time (hours of the day) – the present study investigates travel time reliability of two urban arterial road corridors. The study used continuous travel time data collected using GPS-equipped probe vehicles travelling along the routes, with repeated runs made over long periods of time (weeks and months) for individual journeys each starting at about the same time of day. This data collection replicates the experiences of an individual traveller making a routine trip, such as the journey to work.

## **TRAVEL TIME RELIABILITY AND TRAVEL TIME VARIABILITY**

Previous studies have considered travel time reliability in two ways (FHWA, 2006). The first approach is the concept of reliability as applied in engineering practice, from which travel time reliability is the level of consistency of transportation services for a mode, trip, route or corridor. The second examines the inherent variability in travel times, and defines travel time reliability in terms of that variability.

The standard deviation and the coefficient of variation have been the usual parameters adopted to describe how travel times vary (Bates et al, 2001), although some alternative metrics of travel time reliability have also been proposed. FHWA (2006) introduced a buffer time ( $BT_i$ ) to represent the additional time above the average travel time ( $\bar{t}$ ) required for on-time arrival. The buffer time is the difference between the 95th percentile travel time ( $t_{95}$ ) and the mean travel time:

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<sup>1</sup> Travel time failure is taken to be excess travel time incurred above some acceptable threshold.

$$BT_t = t_{95} - \bar{t}$$

FHWA (2006) also established a travel time reliability index (Planning Index, PI<sub>t</sub>), which is the ratio of the 95th percentile travel time to the 'ideal' travel time, taken to be the free flow travel time (t<sub>f</sub>):

$$PI_t = t_{95} / t_f$$

In the UK, Black and Chin (2007) developed a model of link and corridor travel time variability. This related the coefficient of variation of travel time variability (CV<sub>t</sub>) to the congestion level in the study area:

$$CV_t = \alpha CI_t^\beta$$

Where  $CI_t$  is a congestion index, defined as  $CI_t = t/t_f$ , and  $\alpha$  and  $\beta$  are estimated parameters. They first considered travel time variability at the link level, and then used standardised link travel times to develop a corridor travel time reliability model:

$$CV_t = 0.16 CI_t^{1.02} D^{-0.39}$$

where  $D$  (km) is the route length and -0.39 is an estimated parameter (the elasticity of  $CV_t$  with respect to distance).

A similar model was developed by Richardson and Taylor (1978), who showed that under certain restrictive conditions the theoretical value of  $\beta$  would be 0.5.

Similarly, Eliasson (2006) developed a model for estimating the standard deviation ( $s$ ) of individual travel times in terms of mean travel time, link length ( $L$ ) and free flow travel time. This model is

$$s = \rho \lambda_{TOD} \lambda_{SPD} L^\kappa \bar{t}^\gamma \left( \frac{\bar{t}}{t_f} - 1 \right)^\omega$$

where  $\lambda_{TOD}$  and  $\lambda_{SPD}$  are dummy variables representing time of day and the speed limit,  $\rho$  is a constant, and  $\kappa$ ,  $\gamma$  and  $\omega$  are estimated parameters.

## TRAVEL TIME VARIABILITY DISTRIBUTIONS

Research on fitting continuous distributions to observed travel time data began many decades ago. While initial belief was that the normal distribution was appropriate, Wardrop (1952) first suggested that travel times followed a skewed distribution. Later, Herman and Lam (1974) analysed urban arterial travel time data collected in Detroit in a longitudinal study

of work trip journey times. They found significant skew in the observed times and proposed either the Gamma or lognormal distributions to represent travel time variability.

Taylor and Richardson (1978) then collected and analysed longitudinal travel time data in Melbourne. They assessed the correlations between travel times on each section of the study route, and developed relationships between the travel time variability and the level of congestion. They concluded that travel times on a link were independent of those on other links along the route, and that the observed travel time variability might be represented by a lognormal distribution.

Using continuous travel time data collected in Chicago, Polus (1979) found that the Gamma distribution was superior to normal or lognormal distributions. More recently, Al Deek and Emam (2006) have used the Weibull distribution to model travel time reliability

## THE BURR DISTRIBUTION

Previous studies have fitted travel time data to normal, lognormal, Gamma and Weibull distributions. However, these distributions do not seem to fit many empirical travel time data sets particularly well, as they are unable to model travel time distributions with strong positive skew and long upper tails. Similar problems have arisen in reliability engineering, where most life-test data is also distributed with positive skew and long tails. Study of the best-fit distributions for product lifetime data are thus of interest. Initial product reliability analyses assumed that the lognormal distribution could be appropriate for life-test data distributions. However, recent research has tended to reject this hypothesis, while the Weibull and Gamma distributions have also proved largely unsuccessful in fitting observed life-test data distributions. Zimmer et al (1998) noted the advantages of the Burr Type XII distribution<sup>2</sup> in modelling observed lifetime data. The Burr distribution is also well known in actuarial theory, where it has found a place in modelling distributions of insurance claims. It was developed by Burr (1942) for the express purpose of fitting a cumulative distribution function (cdf) to a diversity of frequency data forms. In its basic form it has two parameters,  $c$  and  $k$ . The probability density function (pdf)  $f(x, c, k)$  of the Burr distribution is

$$f(x, c, k) = ckx^{c-1} (1 + x^c)^{-(k+1)}$$

where  $x > 0$ ,  $c > 0$  and  $k > 0$ . The cdf  $F(x, c, k)$  is given by

$$F(x, c, k) = 1 - (1 + x^c)^{-k}$$

The distribution has some interesting statistical properties (Tadikamalla, 1980). In the first instance the  $r$ th moment of the distribution ( $E(x^r)$ ) will only exist if  $ck > r$ , in which case

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<sup>2</sup> Subsequently termed the Burr distribution.

$$E(x^r) = \mu'_r = \frac{k\Gamma(k - \frac{r}{c})\Gamma(\frac{r}{c} + 1)}{\Gamma(k + 1)}$$

where  $\Gamma(y)$  is the mathematical Gamma function. In addition, the modal value  $x_m$  is given by

$$x_m = \left[ \frac{c-1}{ck+1} \right]^{1/c}$$

but  $x_m$  will only exist if  $c > 1$ . [If  $c \leq 1$ , then the distribution is L-shaped.]

The Burr distribution thus has a flexible shape and is well behaved algebraically. A number of reliability engineering applications have utilised it to model the product life process (Abdel-Ghaly et al., 1997). The distribution has an algebraic tail that is useful in modelling less frequent failures (Soliman, 2005). As its cdf can be written in closed form, its percentiles are easily computed. It allows a wide variety of shapes in its pdf (Zimmer et al, 1998), making it useful for fitting many types of data and for approximating many different distributions (e.g. lognormal, log-logistic, Weibull, and generalised extreme value).

## EMPIRICAL TRAVEL TIME DATA

Our longitudinal journey to work travel time surveys are being conducted<sup>3</sup> on arterial road routes in the Adelaide metropolitan area, using GPS-equipped probe vehicles. The GPS provides a second-by-second data stream, including location and travel speed continuously recorded as the vehicle moves along the route. The routes, shown in Figure 1, are:

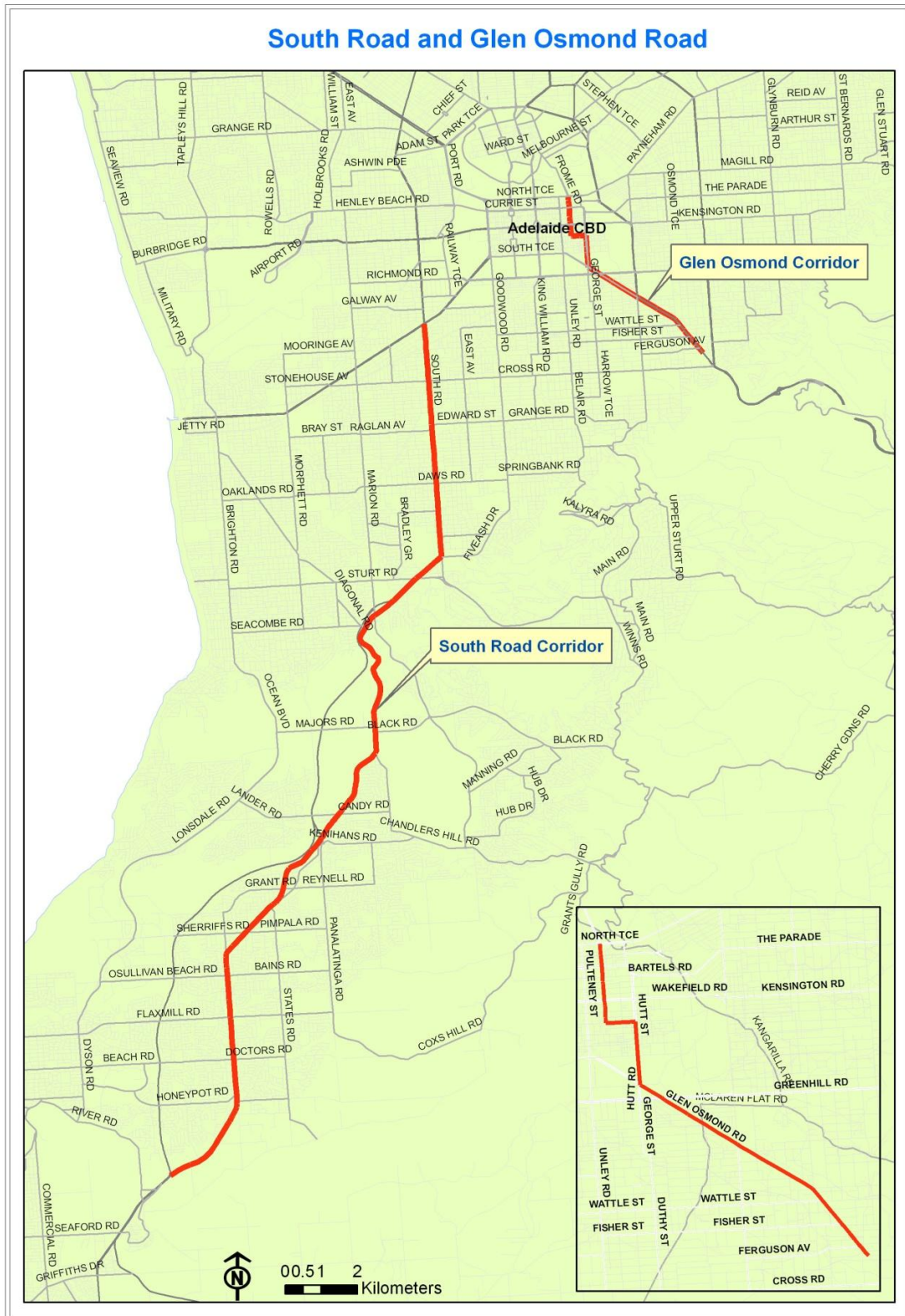
1. Glen Osmond Road, from the eastern suburbs of Adelaide into the CBD. This route comprises 16 links, with link lengths varying from 152 m to 1146 m, and posted speed limits of either 60km/h or 50 km/h
2. the South Road corridor, comprising 22 links. Link lengths vary from 135 m to 4007 m with posted speed limits between 80km/h and 60 km/h.

There are 180 runs for route 1 and 67 runs for route 2. Tables 1 and 2 show the mean, standard deviation and coefficient of variation of link travel times for each route. The basic data output by the GPS is a speed-time profile for the journey, from which section travel times and other information (e.g. proportion of stopped time) can be extracted. Figure 2 provides an example speed-time plot.

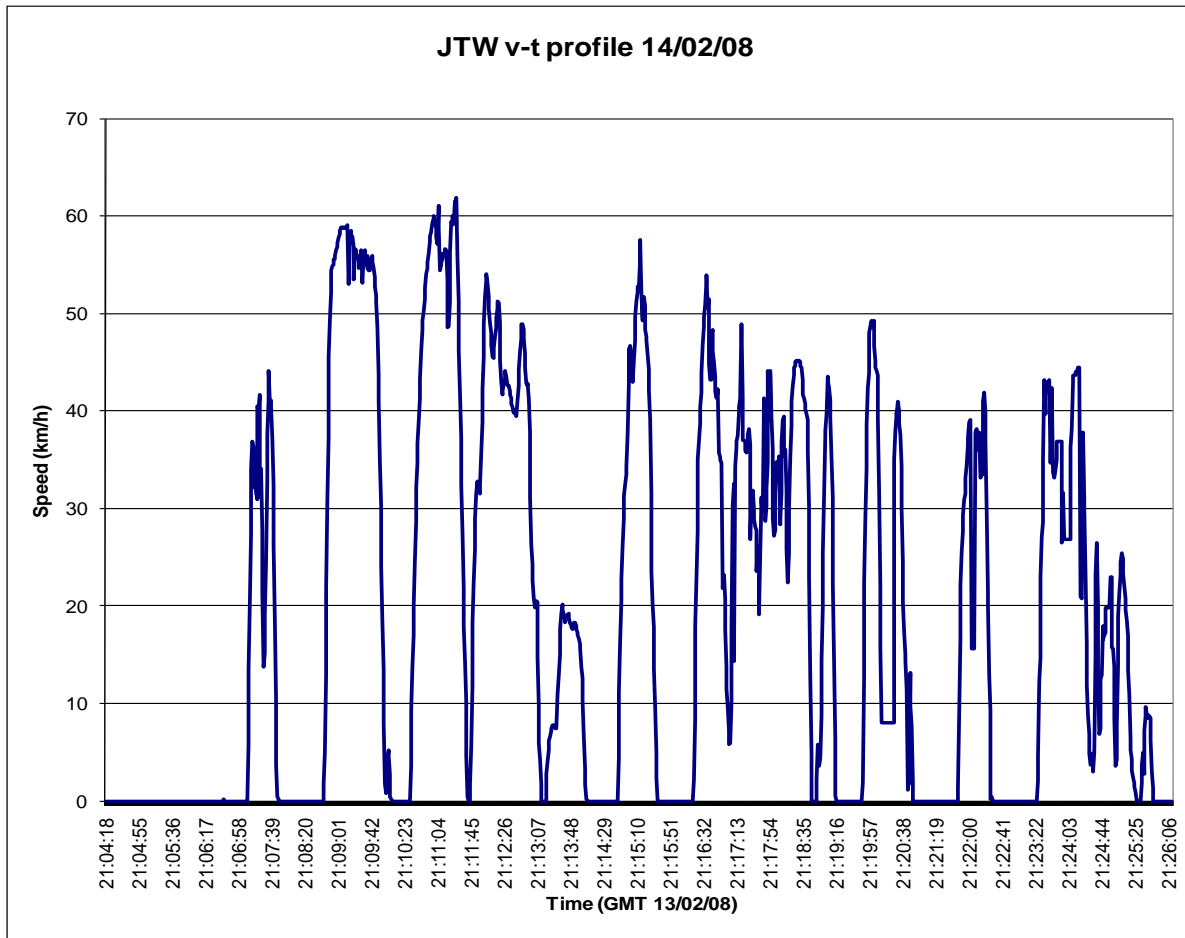
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<sup>3</sup> The longitudinal surveys are ongoing, and the data presented in this paper represent the first 12 months of data collection on two specific routes. Other routes have recently been added to the study.

Figure 1 : South Road and Glen Osmond Road study routes



**Figure 2 :** Sample speed-time profile from a GPS run on Glen Osmond Road



**Table 1:** Glen Osmond link travel time mean, standard deviation and coefficient of variation

Link Name	Link no	Link Length (m)	Mean (s)	Standard Deviation (s)	Coefficient of Variation
GOR <sup>a</sup> :Queens Ln-Bevington Rd	1	1146	122.7	54.2	0.442
GOR: Bevington Rd - Fullarton Rd	2	1058	141.1	76.5	0.542
GOR: Fullarton Rd- Young St	3	458	38.5	21.2	0.552
GOR: Young St - Greenhill Rd	4	606	112.1	53.4	0.477
GOR: Greenhill Rd – Hutt Rd	5	331	33.0	21.1	0.640
Hutt Rd: GOR –South Tc	6	405	41.8	13.4	0.320
Hutt St: South Tc – Gilles St	7	165	23.2	13.5	0.583
Hutt St: Gilles St- Halifax St	8	150	16.5	7.3	0.446
Hutt St: Halifax St - Angas St	9	311	31.8	10.6	0.334
Angas St: Hutt St - Frome St	10	337	39.5	10.9	0.277
Frome St: Angas St - Wakefield St	11	165	45.7	26.4	0.578
Frome St: Wakefield St - Flinders St	12	165	26.7	17.2	0.644
Frome St: Flinders St – Pirie St	13	152	26.2	25.7	0.980
Frome St: Pirie St - Grenfell St	14	156	63.6	52.2	0.820
Frome St: Grenfell St - Rundle St	15	153	41.5	47.7	1.148
Frome St: Rundle St – North Tc	16	162	57.6	35.4	0.614

*Distributions of travel time variability on urban roads*  
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**Table 2 :** South Road link travel time mean, standard deviation and coefficient of variation

<b>Link Name</b>	<b>Link no</b>	<b>Link Length (m)</b>	<b>Mean (s)</b>	<b>Standard Deviation (s)</b>	<b>Coefficient of Variation</b>
Southern Expressway II - Penney Hill Rd	1	3019	142.1	3.5	0.025
Penney Hill Rd - Honeypot Rd	2	213	14.5	5.1	0.354
Honeypot Rd - Doctors Rd	3	944	71.6	15.2	0.212
Doctors Rd - Flaxmill Rd	4	1177	100.1	26.7	0.266
Flaxmill Rd - Cannington Rd	5	710	45.2	3.9	0.086
Connington Rd - O'Sullivan Beach Rd	6	442	37.3	13.5	0.362
O'Sullivan Beach Rd - Sheriff Rd	7	1165	117.9	29.5	0.250
Sheriff Rd - Southern Expressway I	8	4008	183.8	20.2	0.110
Panalatinga Road - Lander Road	9	745	45.6	14.5	0.317
Lander Road - Chandlers Hill Road	10	1965	96.4	9.0	0.094
Chandlers Hill Road - Black Road	11	595	60.2	24.5	0.407
Black Road - Majors Road	12	135	4.6	2.9	0.625
Majors Road - Seacombe Road	13	3097	169.5	23.1	0.136
Seacombe Road - Marion Road	14	323	26.9	16.4	0.608
Marion Road - Southern Expressway	15	592	72.0	56.0	0.778
Southern Expressway - Flinders Drive	16	416	43.1	16.8	0.391
Flinders Drive - Sturt Road	17	393	53.5	18.1	0.339
Sturt Road - Ayliffes Road	18	837	69.2	41.7	0.602
Ayliffes Road - Daws Road	19	2037	300.8	158.1	0.525
Daws Road - Edward Street	20	1625	216.4	80.3	0.371
Edward Street - Cross Road	21	1209	105.8	54.1	0.512
Cross Road - Anzac Highway	22	1561	262.3	104.7	0.399



## Normal and lognormal distributions

Normal and lognormal distributions were first fitted to the observed data, using the Kolmogorov-Smirnov (KS) goodness of fit test. The results for the Glen Osmond data set are shown in Table 3. Neither the normal nor lognormal distributions fitted any of link travel time data sets on this route. A slightly different result was found South Road, where the normal distribution fitted four of the 22 links and the lognormal distribution fitted three links (see Table 4).

**Table 1:** Results for the goodness of fit test for Glen Osmond link travel time data

Link Number	Kolmogorov-Smirnov <sup>a</sup>			Kolmogorov-Smirnov <sup>b</sup> (Log data)		
	Statistic	df	Sig.	Statistic	df	Sig.
1	.222	176	.000	.129	176	.000
2	.172	176	.000	.108	176	.000
3	.329	176	.000	.260	176	.000
4	.157	176	.000	.162	176	.000
5	.395	176	.000	.355	176	.000
6	.159	176	.000	.169	176	.000
7	.227	176	.000	.201	176	.000
8	.248	176	.000	.207	176	.000
9	.262	176	.000	.230	176	.000
10	.142	176	.000	.145	176	.000
11	.115	176	.000	.136	176	.000
12	.181	176	.000	.154	176	.000
13	.264	176	.000	.201	176	.000
14	.157	176	.000	.198	176	.000
15	.273	176	.000	.188	176	.000
16	.103	176	.000	.105	176	.000

a. fitting normal distribution to the link travel times

b. fitting normal distribution to the logarithmic values of the link travel times

**Table 2:** Results for the goodness of fit test for South Road link travel time data

Link Number	Kolmogorov-Smirnov <sup>a</sup>			Kolmogorov-Smirnov <sup>b</sup> (Log data)		
	Statistic	df	Sig.	Statistic	df	Sig.
1	.127	47	.057	.132	47	.040
2	.320	47	.000	.302	47	.000
3	.220	47	.000	.189	47	.000
4	.135	47	.031	.125	47	.063
5	.239	47	.000	.203	47	.000
6	.286	47	.000	.245	47	.000
7	.134	47	.033	.124	47	.067
8	.177	47	.001	.162	47	.003
9	.248	47	.000	.208	47	.000
10	.193	47	.000	.181	47	.001
11	.126	47	.061	.065	47	.200 <sup>*</sup>
12	.338	47	.000	.265	47	.000
13	.188	47	.000	.171	47	.001
14	.303	47	.000	.220	47	.000
15	.259	47	.000	.188	47	.000
16	.222	47	.000	.184	47	.000
17	.073	47	.200 <sup>*</sup>	.125	47	.062
18	.349	47	.000	.279	47	.000
19	.091	47	.200 <sup>*</sup>	.133	47	.037
20	.118	47	.102	.103	47	.200 <sup>*</sup>
21	.275	47	.000	.238	47	.000
22	.120	47	.090	.093	47	.200 <sup>*</sup>

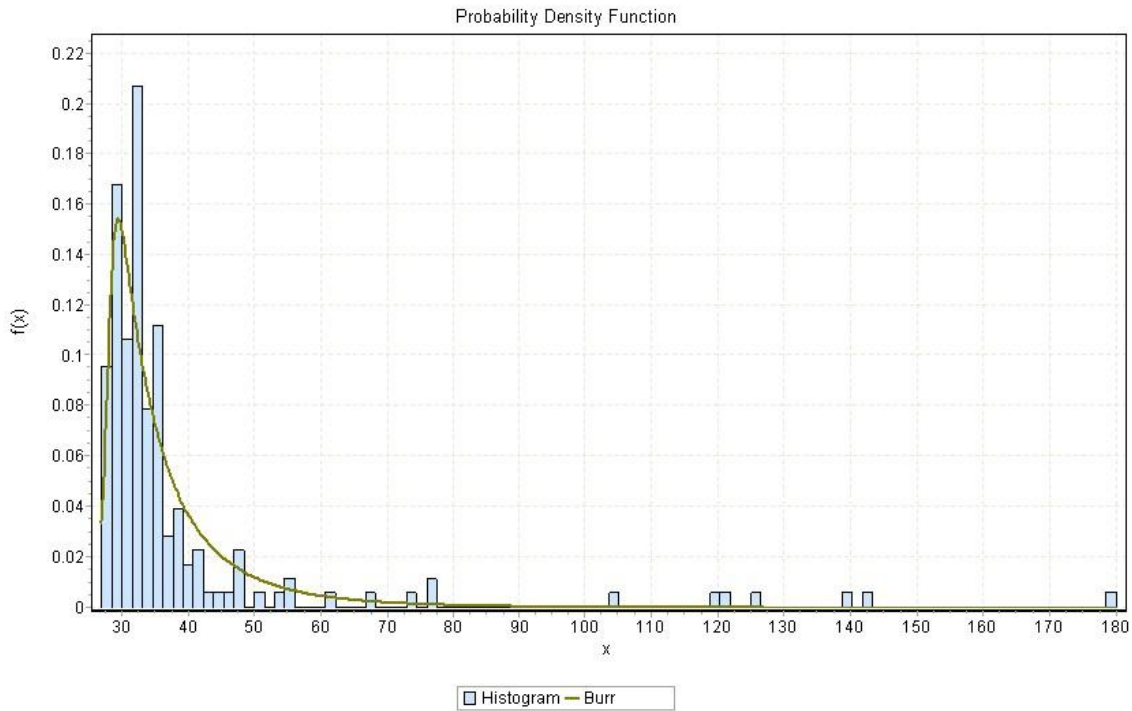
The overall results confirm the inability of either the normal and lognormal distributions to represent the observed data. Other distributions are required.

### Other distributions

Since the travel time distributions were generally right-skewed with long upper tails, the next stage was to test other theoretical distributions that could better represent this phenomenon. This first required exploratory data analysis of the observed distributions, then comparisons with different distribution models. The Burr distribution and the Generalised Pareto distribution were used as candidate models.

In the first step visualisation of the data was important. This was done by drawing histograms of observed link travel times, and superimposing a theoretical pdf on the graph. Some example graphs are provided. Figure 3 shows the histogram for link 3 in the Glen Osmond data. Figure 4 shows the histogram for link 5 in the South Road data. The theoretical curves in these figures are for the Burr distribution, fitted to each observed histogram using maximum likelihood estimation. The two plots show the inherent flexibility of the Burr distribution and its ability to replicate the long tails in the observed data.

**Figure 3:** Histogram and fitted Burr distribution for Link 3 Glen Osmond travel time data



**Figure 4:** Histogram and fitted Burr distribution for Link 5 South Road travel time data

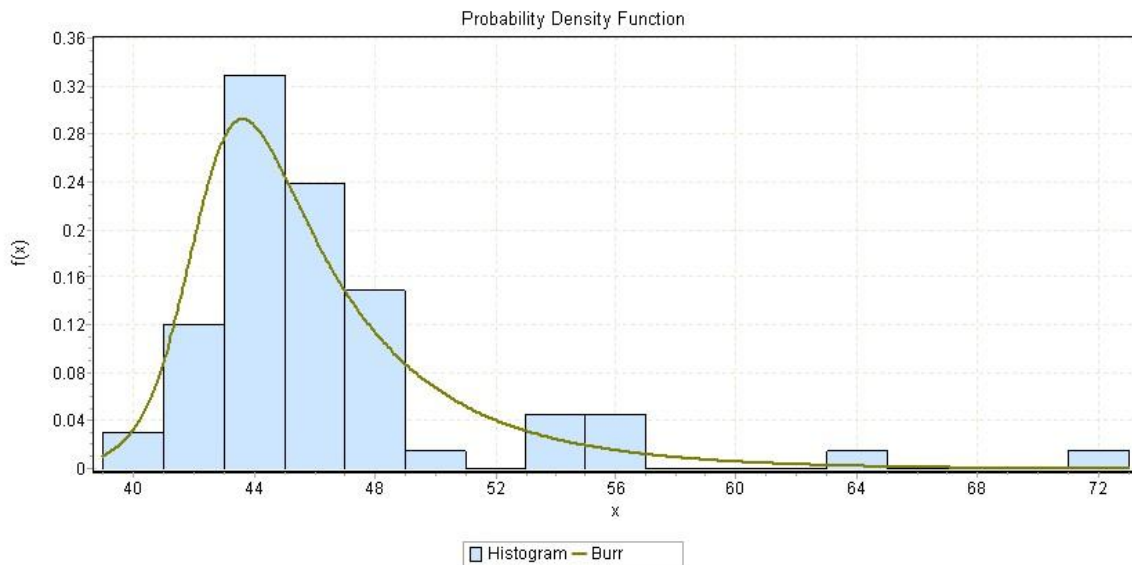
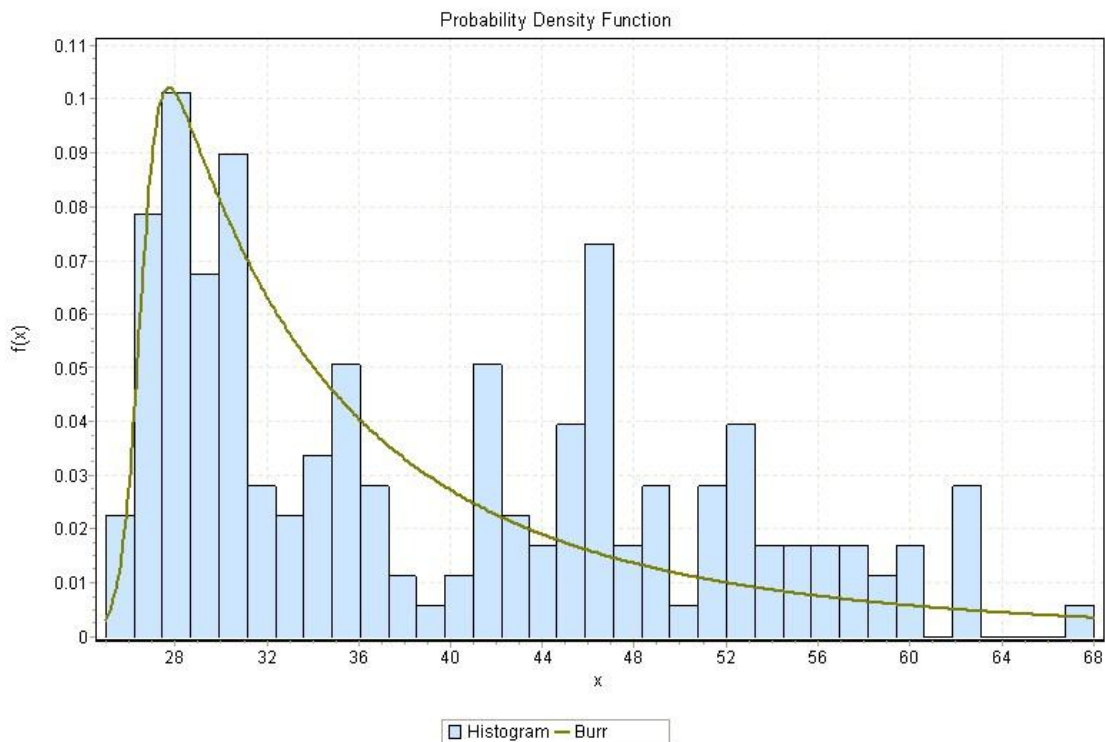


Table 5 summarises the goodness of fit tests for the Glen Osmond data, for the Burr distribution and the Generalised Pareto distribution. The statistical hypothesis that the Generalised Pareto distribution could fit the observed data is rejected for a majority (ten of 16) of the links, and only accepted at 0.05 significance for two of the links. The Burr distribution was rejected for six of the 16 links, and accepted at 0.05 significance for three links and at 0.01 significance for the remaining seven links. It could therefore be a plausible model for the data. A confounding factor is that several of the links on this route showed evidence of bimodality, including links 2, 9, 10, 11 and 12. Figure 5 shows the observed histogram for Glen Osmond link number 10, clearly showing bimodality.

**Table 3:** Kolmogorov-Smirnov goodness of fit test results for the Burr and Generalised Pareto distributions fitted to the Glen Osmond link travel time data

Link number	Glen Osmond	
	Burr	Generalised Pareto
1	Accepted	Rejected
2	Accepted at 0.01	Accepted at 0.01
3	Accepted at 0.01	Rejected
4	Rejected	Rejected
5	Accepted at 0.01	Rejected
6	Accepted at 0.01	Rejected
7	Accepted	Rejected
8	Accepted at 0.01	Rejected
9	Rejected	Rejected
10	Rejected	Accepted at 0.01
11	Rejected	Accepted
12	Rejected	Accepted at 0.01
13	Accepted at 0.01	Accepted at 0.01
14	Rejected	Rejected
15	Accepted at 0.01	Rejected
16	Accepted	Accepted

**Figure 5:** Histogram and fitted Burr distribution for Link 10 Glen Osmond Road travel time data – indication of bimodality?



The results for the South Road data set were more conclusive (see Table 6). The Burr distribution fitted almost all of the links at the 0.05 significance level. Similar results were also

found for the Generalised Pareto distribution. On the basis of the two data sets, it is reasonable to conclude that the Burr distribution can represent longitudinal travel time variability data and may be more useful in this regard than other distributions, such as the Generalised Pareto, and certainly better than the lognormal and normal distributions. The flexible form and attractive mathematical and computational characteristics of the Burr distribution enhance its suitability and therefore likely applications.

**Table 4 :** Goodness of fit test results (Kolmogorov-Smirnov) for the Burr and Generalised Pareto distributions fitted to the South Road link travel time data

Link number	South Road	
	Burr	GP
1	Accepted	Accepted
2	Rejected	Accepted at 0.01
3	Accepted	Accepted
4	Accepted	Accepted
5	Accepted	Accepted
6	Accepted	Accepted
7	Accepted	Accepted
8	Accepted	Accepted
9	Accepted	Accepted
10	Accepted	Accepted
11	Accepted	Accepted
12	Accepted at 0.01	Rejected
13	Accepted	Accepted
14	Accepted	Accepted
15	Accepted	Accepted
16	Accepted	Accepted
17	Accepted	Accepted
18	Accepted	Accepted
19	Accepted	Accepted
20	Accepted	Accepted
21	Accepted	Accepted
22	Accepted	Accepted

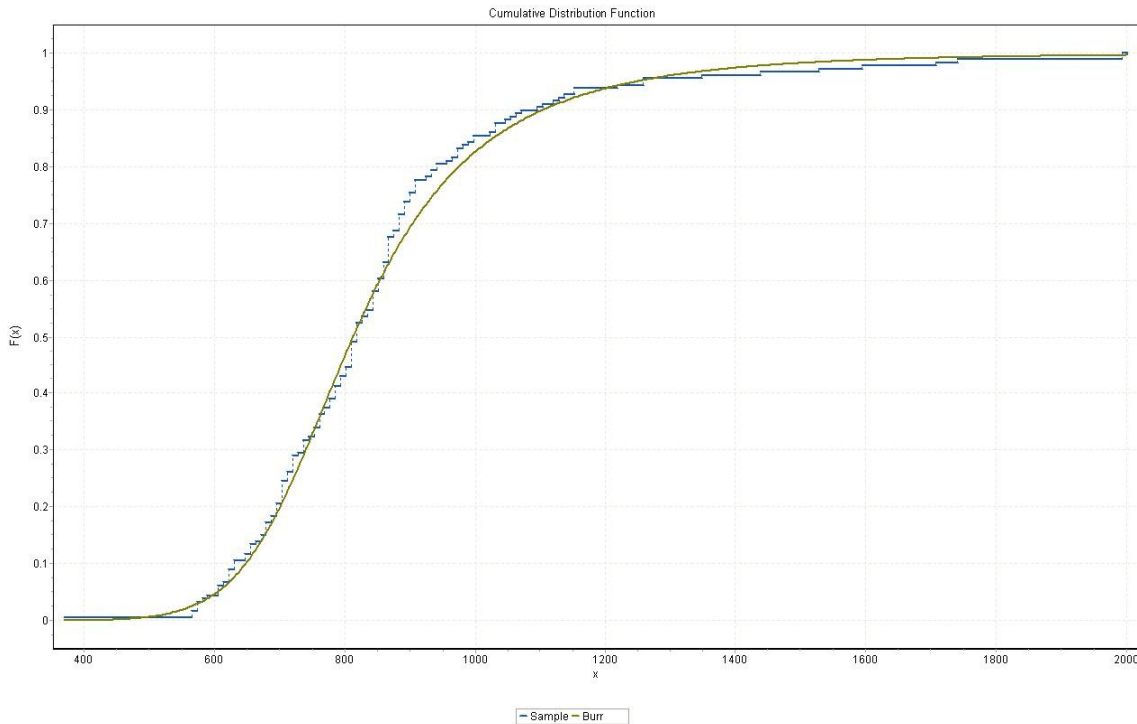
The analysis presented above is for individual links in the route. Similar results were also found in the assessment of travel time variability at the overall route level. In this case the Weibull, Gamma, Burr and Generalised Pareto distributions were fitted to the overall route travel times. Goodness of fit tests were conducted and the test results are shown in Table 7. Three distributions were quite successful in representing the observed route travel time distributions particularly in relation to positive values and long tails. However, the goodness of fit tests indicated that only the Burr and Generalised Pareto distribution results are promising. The Weibull and the Gamma distributions did not fit the Glen Osmond corridor at all. However, the Weibull and Gamma distributions do fit the South Road data set. The Burr

Distribution fits both data sets well, further supporting the notion that this distribution could be a useful model of travel time variability. Figure 6 shows the Burr cdf and the observed cdf for the Glen Osmond route. Figure 7 shows the corresponding plots for South Road. The ability of the Burr distribution to model the long upper tails of the observed distributions is evident in these graphs.

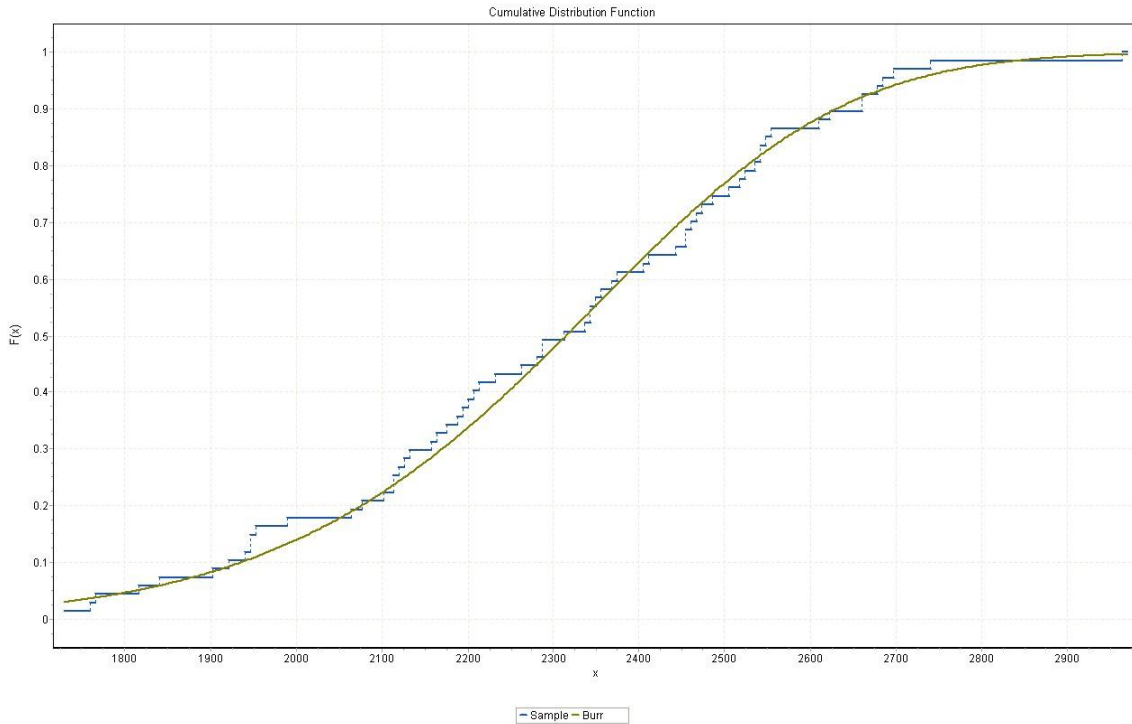
**Table 5:** Goodness of fit tests for overall travel times on the two routes

Route	Significance Level	Significance Value	Computed KS Statistic			
			Weibull	Gamma	Gen Pareto	Burr
Glen Osmond	0.05	0.10150	0.16771	0.14573	0.09107	0.05778
	0.01	0.11346	0.16771	0.14573	0.09107	0.05778
			<b>Rejected</b>	<b>Rejected</b>	<b>Accepted</b>	<b>Accepted</b>
South Road	0.05	0.16322	0.06134	0.07707	0.09000	0.05666
	0.01	0.18252	0.06134	0.07707	0.09000	0.05666
			<b>Accepted</b>	<b>Accepted</b>	<b>Accepted</b>	<b>Accepted</b>

**Figure 6:** Burr distribution and observed cumulative density functions for the Glen Osmond route travel times



**Figure 7:** Burr distribution and observed cumulative density functions for the South Road route travel times



## Bimodality

The issue of bimodality as suggested in Figure 5 is of interest. If  $f(x)$  is the pdf of a bimodal distribution comprising two component unimodal distributions  $f_1(x)$  and  $f_2(x)$  then it can be described mathematically as

$$f(x) = \phi_1 f_1(x) + (1 - \phi_1) f_2(x)$$

where  $\phi_1$  is the proportion of the overall distribution belonging to  $f_1(x)$ . The corresponding cdf  $F(x)$  is given by

$$F(x) = \phi_1 F_1(x) + (1 - \phi_1) F_2(x)$$

Determination of the split of observed values of  $x$  between the two component populations, of the value of  $\phi_1$ , and the resulting values of the parameters describing distributions  $f_1(x)$  and  $f_2(x)$  is a major issue. One approach to test for bimodality is the Hartigan dip test (Hartigan and Hartigan, 1984). The dip statistic measures the maximum difference between empirical cdf and the unimodal cdf that minimises that maximum difference. It produces a probability that the observed data could come from that unimodal distribution and can be used to test the null hypothesis that the data are unimodal. The test was applied to each link in each of the two travel time data sets. It indicated that eight of the 16 links in the Glen Osmond data showed statistical evidence of bimodality at the 5% significance level (see Table 8). Only one of the 19 links in the South Road data showed bimodality (see Table 9).

These tables show the probability of rejection of the null hypothesis, and the estimated means, standard deviations and proportions of the component distributions.

**Table 6:** Bimodality results for Glen Osmond link travel times, including dip statistic, means and standard deviations of component populations, and proportions

Link number	DIP statistic	Probability	Component population 1			Component population 2		
			Mean1 (s)	StDev1 (s)	Proportion $\phi_1$	Mean2 (s)	StDev2 (s)	Proportion $\phi_2 = 1-\phi_1$
2	0.0693	0.999	126.0	41.5	0.91	294.5	148.2	0.09
3	0.0531	0.95	32.4	3.4	0.86	74.9	39.3	0.14
6	0.0478	0.99	30.4	1.2	0.37	48.6	12.7	0.63
7	0.0646	0.999	14.1	1.9	0.56	34.8	13.1	0.44
9	0.0618	0.99	26.9	2.7	0.77	47.5	11.5	0.23
10	0.0393	0.95	28.5	1.4	0.33	44.9	9.4	0.67
13	0.0562	0.99	14.1	2.6	0.64	47.5	33.1	0.36
15	0.0393	0.95	17.0	3.9	0.61	79.3	58.2	0.39

**Table 7:** Bimodality results for South Road link travel times, including dip statistic, means and standard deviations of component populations, and proportions

Link number	DIP statistic	Probability	Component population 1			Component population 2		
			Mean1 (s)	StDev1 (s)	Proportion $\phi_1$	Mean2 (s)	StDev2 (s)	Proportion $\phi_2 = 1-\phi_1$
5	0.0759	0.99	44.5	2.2	0.86	54.7	8.0	0.14

Statistical evidence is useful, but an explanation of the phenomenon of bimodality in travel time variability distributions is also required. For the case of urban arterial roads, the influence of delays at traffic signal may provide an explanation. For instance, experiencing two or more red signal phases at an intersection may substantially increase link travel times. On already congested sections or routes, the queuing delay then experienced by drivers could be similar to or even exceed the running time needed to traverse the link, so doubling or even tripling the total link travel time. On the other hand, experiencing less queuing at signalised intersections will substantially reduce the total travel time. This result was found by Davis and Xiong (2007), who also observed bimodality in travel time distributions, and were able to ascribe this to signal performance. We suspect similar factors to apply in the Adelaide data sets. Future research using new methods to interrogate historical data from urban traffic control systems (Zhang et al, 2007) coupled with the continuous (time stamped) data from the GPS runs will address this issue.

## CONCLUSIONS

This paper has approached the question of travel time reliability by considering two separate sets of longitudinal travel time data sets from arterial road routes. The search is for a tractable model that can reasonably represent observed variations in day to day travel times and thus provide a statistical model for the analysis of travel time reliability. The observed travel time distributions are characterised by very long upper tails and strong positive skew. Analysis of these data sets led to the conclusion that the lognormal distribution, although having the characteristics of positive skew and a reasonably long upper tail, was unable to



fully represent the observed data. Therefore the research focused on other continuous distributions that can accommodate those patterns. The Burr distribution was considered as a leading candidate for travel time variability distribution, with some other distributions also suggested. The Burr and Generalised Pareto distributions emerged as reasonable models, for both links and routes. However, in terms of overall performance the Generalised Pareto was less able to represent the characteristics of the observed travel time distributions. The Burr distribution was able to provide good overall representation of the observed data. Given the attractive features of this distribution in terms of its mathematical tractability and its flexibility, this distribution can be proposed as a useful model of variations in travel times.

This is not the end of the story, however. Further research is required to develop appropriate general Burr parameters that can characterise the variability of urban arterial road travel times, and to relate those parameters to environmental and operational factors for road corridors. There is also the intrigue of the observations of bimodal distributions of travel times on some links. Further study is required of the incidence of bimodality and the factors that may lead to this, especially the relationships with signal-based delays. There will then be the issue of the development of suitable models for bimodal travel time variability distributions, most likely using mixture models.

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