THE OPTIMAL AREA-BASED NETWORK CONGESTION PRICING PROBLEM: DETERMINING OPTIMAL TOLL LEVEL AND CHARGING BOUNDARY

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ABSTRACT

The design of a congestion pricing policy clearly determines its performance. Therefore, many studies have attempted to determine the optimal design of various pricing schemes. Cordon-based and area-based congestion pricing are two well-known pricing policies that are designed to alleviate city-center congestion. In cordon-based pricing (cordon-toll), drivers are charged when they cross the cordon boundary, while in area-based pricing (area-toll), the drivers pay a daily charge to enter or drive within a charged area. For these schemes, the toll level and charging boundary must be determined.

Cordon-toll has been extensively examined (Zhang and Yang, 2004; Sumalee 2004, 2007; and others), but area-toll has received little attention. This study proposes a framework for designing an optimal area-toll policy and investigates the properties of optimal area-toll. The travel costs for area-toll are expressed using a trip-chain-based network equilibrium model proposed by some of the authors (Maruyama and Sumalee 2007). Innovative part of this paper is optimal design (automatic design) of charging boundary of area-based toll, which was determined (manually) by human-brain in past research.

The pricing design model is formulated as a bi-level programming problem and solved using a genetic algorithm. This model was applied to a real-world network in Utsunomiya, Japan. This case study produced several interesting findings: the shapes of the optimal area-toll and cordon-toll areas differ greatly; optimizing the area-toll scheme provides a 50% larger social surplus than the judgmental area. These findings indicate the importance of optimizing the charging boundary and have useful policy implications.

Keywords: Road Pricing, Second-best Pricing, Transportation Network Equilibrium, Genetic Programming, Heuristic Optimization

1. INTRODUCTION

Background and Objective

Several cities around the world, including Singapore, London, and Stockholm, have implemented road-pricing schemes to alleviate traffic congestion in their city centers, while other cities are considering such schemes. Implementing such schemes in a city center requires determining the charging scheme, location of the charging boundary, and the toll level. Cordon-based and area-based pricing are two typical road-pricing schemes for city centers. Under cordon-based pricing (cordon toll for short), drivers are charged when they cross the cordon boundary, while under area-based pricing (area toll), drivers pay a daily charge to enter or drive within a charged area. Examples of actual cordon toll are implemented in Singapore and Stockholm and those of area toll are London's congestion charging system and Ecopass program in Milan, Italy (Rotaris et al., 2010).

Cordon tolls have been extensively examined (Zhang and Yang, 2004; Sumalee, 2004, 2007, among others), but area tolls have received little attention academically. Some studies (Maruyama and Harata, 2006; Maruyama and Sumalee, 2007) have proposed a trip-chain-based network equilibrium model for evaluating area tolls, but these studies only demonstrated numerical investigations of changes in toll levels for predetermined charging boundaries. In other words, they optimized the toll level but not the toll location or charging boundary. These studies applied a judgmental area-toll, which means that the boundary was determined by people and not by systematic procedure using a model or theory. The location of charging boundary was often usually determined by implicit assumptions (e.g. historic urban layout) and we call such toll as judgmental toll (Sumalee et al. 2005).

The current study proposes a network design model that optimizes the toll level and the location of an area toll problem. The model is formulated as a bi-level programming problem, solved using a genetic algorithm, and then applied to a real-world network: Utsunomiya, Japan. Innovative part of this paper is optimal design (automatic design) of charging boundary of area-based toll, which was determined (manually) by human-brain in past research (Maruyama and Sumalee 2007).

The aim of this study was to answer the following research questions.

1. How do the optimal-area and judgmental-area-toll schemes differ?
2. How do the optimal area and optimal cordon toll schemes differ in terms of the charging boundary and toll level?
3. Does an optimal area-toll scheme have a wider or smaller area than an optimal cordon toll scheme?
4. When applying congestion pricing for the first time, which pricing scheme should be introduced first: cordon tolls or area tolls?

The answers to these questions will be useful for city transportation planners worldwide. We'd like to provide such practical and important information using a theoretical and innovative approach.

Literature Review

A large number of literatures exist on modeling congestion pricing policies. Recently Tsekeris and Voß (2009) extensively surveyed the methodological advances in evaluation models for road pricing. de Palma and Lindsey (2009) categorized current congestion pricing methods and technologies and discussed the choice of pricing method. Yang and Huang (2004) summarize the network equilibrium approach to analyzing road-pricing problems.

Several studies have examined second-best toll schemes. The second-best toll scheme allows only a subset of links to be charged and first-best toll scheme can charge every links. Previous studies optimized either the toll level for a predefined cordon location (Santos, 2004; Akiyama *et al*., 2004) or both the toll level and the location (Shepherd and Sumalee, 2004; Sumalee, 2004; Zhang and Yang, 2004). Due to the complexity of the problem, meta-heuristic algorithms, such as genetic algorithm (GA), are often used (e.g., Akiyama and Okushima, 2006; Ekström *et al*., 2009). Other studies have focused either on designing multiple-cordon schemes (Sumalee 2007) or dealing with constraint and multiple objectives of the scheme design (Sumalee *et al.*, 2009). Other important research topics have included the development of short-cut approaches to designing the cordon (Shepherd *et al.*, 2008) and dynamic modeling (e.g., de Palma *et al.,* 2005a, b). Akiyama and Noiri (2003) proposed using GA to determine the optimal toll levels of multiple zones.

Most existing studies have examined link-based or cordon-based congestion pricing. A few studies have considered the area-toll problem. Although the area toll is similar to the cordon toll in terms of topological structures, its optimal charging boundary and toll level can be substantially changed because it has a different effect on travel behavior due to the different charges imposed upon a traveler. Originality of our research is developing optimal design model of charging boundary of area-based toll. Network flow modeling representing area-toll is same as our past research (Maruyama and Sumalee 2007) but developing optimal toll design model and algorithm using the flow model is our current contribution.

2. OPTIMAL AREA-TOLL DESIGN PROBLEM

Assumption

This paper examines the problem of finding the optimal boundary of an area-toll scheme that consist of connected links, with optimal toll level, designed to maximize or minimize a selected objective function. In this paper, demand model is limited to a single user class and a single mode, and has trip chain-based elastic demand. Users are assumed to follow the trip chain-based user equilibrium (UE) described in the next sub-section. In analyzing the optimal toll design problem, this UE condition must be imposed as a constraint in the optimization problem. The second important constraint is that the selected toll links must form a charging area. This problem is known as a transportation network design problem, a bi-level problem, or Mathematical Programming with Equilibrium Constraints (Sumalee 2004, 2007; Luo et al. 1996).

Zhang and Yang (2004) classified cordon tolls as single-layered, multi-layered or multi-centered. Area-based tolls also can be categorized in the same way. In the case of area tolls, we may also consider multiple areas with a single toll and multi-zone congestion pricing. Such advanced pricing methods are appealing, but this study focuses on the simplest one: the single-layered area-toll. The single-layered area-toll is area-toll with only one charging boundary. The objective of the upper problem of a bi-level problem is to maximize the social surplus from a single-layered area toll. The social surplus is a fundamental economic efficiency measure.

The basic idea of bi-level problem is that policy maker's decision is described in upper problem and user's (driver's) behaviors are expressed in lower problem. Policy maker's decision (pricing scheme) will have impact on the user's behavior and user's behavior will affect the design of policy maker's decision by congestion. The details of upper and lower problem are described below.

Lower Problem: Trip-chain-based Network Model

The lower problem of the current bi-level program is a trip-chain-based network equilibrium model with non-link-additive path costs. Originally proposed by Maruyama and Harata (2006), this model can be formulated as an equivalent convex minimization problem that can then be applied to large networks. The problem minimize an objective function $Z(\mathbf{g,h})$ as shown below.

$$\min . Z(\mathbf{g},\mathbf{h}) = \sum_a \int_0^{x_a} t_a(\omega)\, d\omega + \sum_{n,p} \sum_{m \in M_p} \tau_p\, g_n^m - \sum_n \int_0^{h_n} D_n^{-1}(\omega)\, d\omega \tag{1}$$

subject to

$$h_n = \sum_m g_n^m, \quad \forall n \ , \tag{2}$$

$$x_a = \sum_{m,n} \delta_{a,n}^m g_n^m, \quad \forall a \ , \tag{3}$$

$$x_a \geq 0, h_n \geq 0, g_n^m \geq 0 \tag{4}$$

where,

x_a : traffic flow on link a

$t_a(x_a)$: travel cost on link a, which is a function of the traffic flow on the link x_a, and is assumed to be separable and an increasing function of traffic flow

τ_p : toll level of charging pattern p

g_n^m : travel flow on trip-chain-based path m in trip-chain n

M_p : set of trip-chain-based paths with charging pattern p

h_n : travel flow on trip chain n

$D_n^{-1}(.)$: Inverse of the trip-chain based separable demand function associated with trip-chain n

$\delta_{a,n}^{m}$: 1 if link a is on trip-chain-based path m in trip-chain n and 0 otherwise

g: the vector of g_n^m

h: the vector of h_n

This trip-chain-based model is an extension of the trip-based model proposed by Beckmann *et al.,* (1956). (See also Maruyama and Sumalee (2007) for variational inequality formulation and derivation of the same model.) This lower problem can be solved easily by partial linearization of the objective function (Maruyama and Harata, 2006; Maruyama and Sumalee, 2007).

Upper problem

The objective function of the upper problem is the social surplus (SS) shown below.

$$\max \;\; SS = \sum_n \int_0^{h_n} D_n^{-1}(\omega)\,d\omega - \sum_a x_a t_a(x_a) \tag{5}$$

In this paper, we consider only the efficiency of the tolling scheme and neglect the setup and operating costs in the calculation. This is partly to make our results comparable to those of our previous study (Maruyama and Sumalee, 2007). However, different charging-boundary sizes have different setup and operating costs that may affect optimization of the toll design. In addition, such costs are an important consideration for comparison between area-toll and cordon-toll design. This is a topic for future research.

A specific demand function is required to apply the model to a real area. Maruyama and Harata (2006) and Maruyama and Sumalee (2007) used the following exponential-based demand function.

$$D_n(c_n) = D_n^0 \exp\left[\rho\left(1.0 - \frac{c_n}{c_n^0}\right)\right], \tag{6}$$

where

c_n and c_n^0 are the equilibrium and base-case (do-nothing scenario) trip-chain-based travel costs, respectively, on trip chain n, and D_n^0 is the corresponding base-case demand. The demand elasticity with respect to the travel cost is given by $-\rho c_n / c_n^0$, where ρ is regarded as a dimensionless demand elasticity parameter.

Algorithm

Sumalee (2004, 2007) and Zhang and Yang (2004) proposed a similar GA-based optimization method for the cordon-toll design problem, which is how to optimize the charging boundary and toll level while maintaining the closed shape of the cordon. Optimizing an area toll is basically similar to optimizing a cordon toll. In optimal area-toll design, the links inside the charging boundary are also charging links, as well as links that cross the cordon. This is the basic difference between area tolls and cordon tolls. The cordon toll charges travelers per crossing whereas the area toll charges the travelers for an entry permit (e.g. per day). See Maruyama et al. (2010) for an example of graphical representation to demonstrate the difference between two toll schemes. The target of a single-area toll design is to separate the links into toll or no-toll links and ensure that the toll links connect. Designing an optimal multiple-area toll is another interesting topic for future investigation. Note that the designed area sometimes can have amazing shapes, such as a donut-shape. This is possible because the boundary area is closed. Such shapes are not practical, but are interesting in theory.

Conditions of the charging boundary

The charging boundary for road pricing is expressed as a set of links, and the combination of links should be determined by considering the topological conditions of the network. The conditions differ for different charging schemes, and the link set in a charging area should not be separated in our current area-toll problem.

This study considers an area toll with only one charging boundary. This means that every toll link should be connected to another of the toll links. We adopt this single-area toll scheme in this study because it is the simplest and most easily applied type of area toll. Moreover, this single-area toll requires less computational time than other area tolls. If we consider the multiple-area toll, the number of charging pattern p will increases and naturally computational time also increases (Maruyama and Harata, 2005). This single-area toll is easiest in programming code, easiest to understand, easiest for real life application.

Use of branch-tree concept

Sumalee (2004, 2007) adopted a branch-tree concept to represent the cordon-toll boundary. This concept can be used for analyzing area tolls. Roughly speaking, the leaves in the branch tree represent the cordon and all of the components of the tree correspond to the area boundary. The tree concept itself is a general feature of data structures, and our algorithm can also be considered Genetic Programming (GP). Sumalee (2004, 2007) noted the condition that the branch-tree has meaning of cordon in real network. We adopt similar idea by Sumalee and the details of concepts are skipped here for lack of space.

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3. CASE STUDY IN UTSUNOMIYA AREA

The case study in this paper is identical to the one described in Maruyama and Sumalee (2007) and is based on the network for Utsunomiya City, Japan. Figure 1 shows the arterial road network in the area of Utsunomiya City as well as two of the hypothetical charging boundaries (A and B) around the ring roads. In Maruyama and Sumalee (2007), these boundaries were selected as possible toll areas, and boundary A—a judgmental area—produced the highest social surplus. Utsunomiya is an automobile-dependent city with a typical pattern of traffic congestion. The number of links (1,345) and zones (84), the link cost function settings, the value of time, and demand function (eqn (6),) are identical to those described by Maruyama and Sumalee (2007). Link cost functions are common-used BPR functions, and demand function is demonstrated in eqn (6) and elasticity parameter $\rho = 1.0$. is used. Also, the value of time of 60 JPY/min[1] is used for calculation.

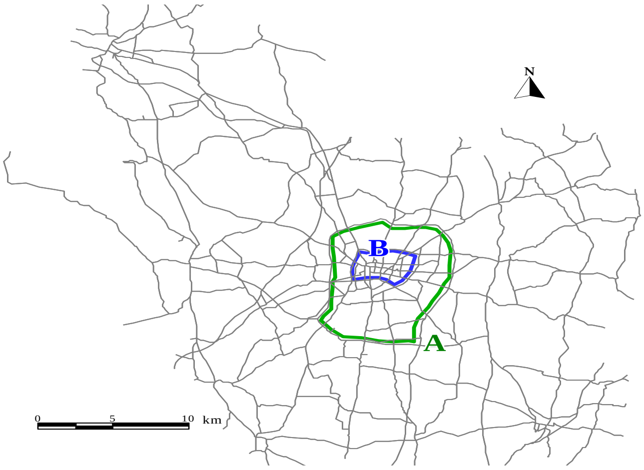

Figure 1. Arterial road network and judgmental charging boundaries A and B in Utsunomiya

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[1] JPY (Japanese Yen): 1US$ ~ 90JPY, 1Euro ~ 110 JPY (as of May 25, 2010)

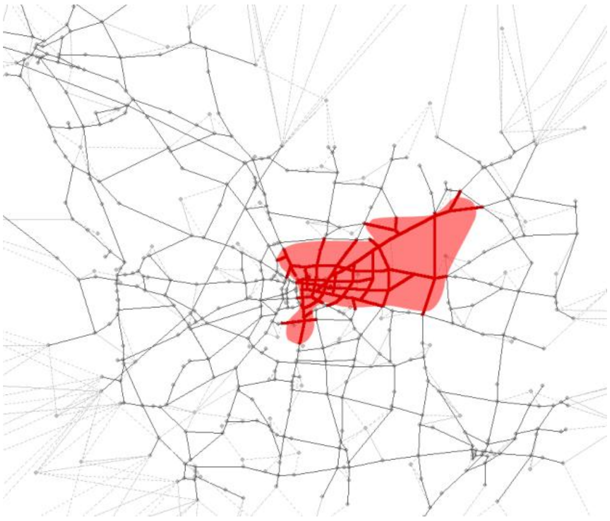

Figure 2. An optimal solution for an area toll boundary

(Toll level = JPY 250)

Figure 2 shows the optimal-area toll boundary generated by our proposed model. It has 149 charging links and an optimal toll level of JPY 250. The boundary is located in the northeastern region of the study area. Figure 3 demonstrates the difference between the traffic volume under a no-toll scenario and that under an optimal area toll scenario. Most of the link traffic volume inside the charging boundary has decreased while the ring-road traffic volume in the southeastern region of the study area has increased. No significant changes were seen outside these regions. In this figure 3, flows on some link increase and the adjacent links decrease. These changes can happen if we consider multiple OD pair and elastic travel demand and is not strange output.

An important of output of our model is social surplus defined in eqn (5). Its absolute has no meaning but relative change has meaning. As a result of our new algorithm that optimizes both the charging boundary and the toll level, the social surplus generated by the optimal area toll increased to around JPY $1.3785*10^9$. The judgmental area toll reported in Maruyama and Sumalee (2007) produced a social surplus of around JPY $1.3780*10^9$. Maruyama and Sumalee optimized the toll level for each charging boundary but not the boundary shape. Sumalee, May, and Shepherd (2005) reported that an optimal single-cordon toll generates twice the welfare benefits of the best judgmentally designed cordon toll.

In this case, the no-toll scenario produces a social surplus of JPY $1.3770 *10^9$. The optimal area toll and the judgmental area toll produce social surpluses of almost JPY 1.5 million and JPY 1 million, respectively. In this case, optimizing the charging boundary of the area toll increases the social surplus by 50%. Note that the first-best pricing (marginal cost pricing on every link) for this network produces a social surplus of JPY $1.3850 *10^9$.

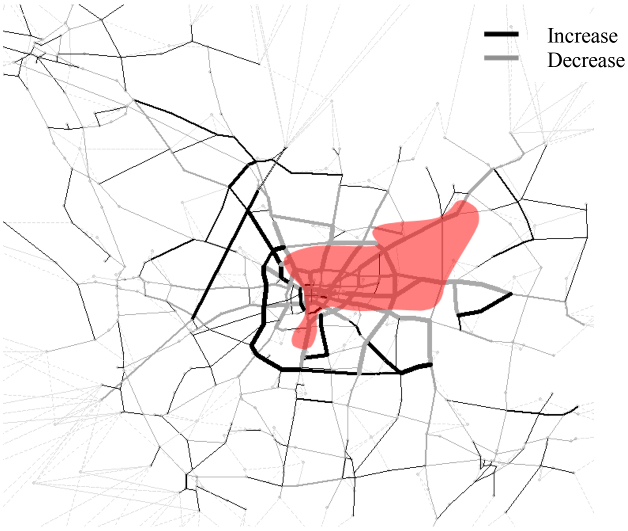

Figure 3. Change in traffic volume resulting from optimal area toll

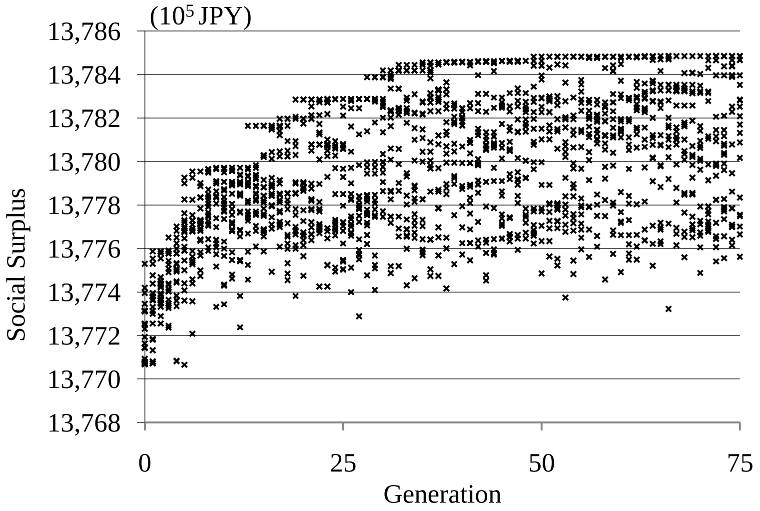

Figure 4. Evolution of social surplus by area-toll by proposed algorithm

Figure 4 shows the change of social surplus by optimization process of GA. Each plot is social surplus values for a generation (iteration number) in GA. We computed 150 generations, but the social surplus stabilized after around 70 generations.

The current bi-level problem is non-convex problem and may have multiple optimal solutions. (Sumalee 2004, 2007; Luo et al. 1996) The algorithm is then computed for several conditions (with several starting points and parameters) to produce another optimal area toll solution (Figure 5). This optimal area toll produces a social surplus similar to the highest surplus previously reported optimal area toll (Figure 2). Both of these two optimal area tolls are located in the eastern region of the study area. However, the new optimal area toll is wider in area than the previously reported area and its optimal toll level is lower.

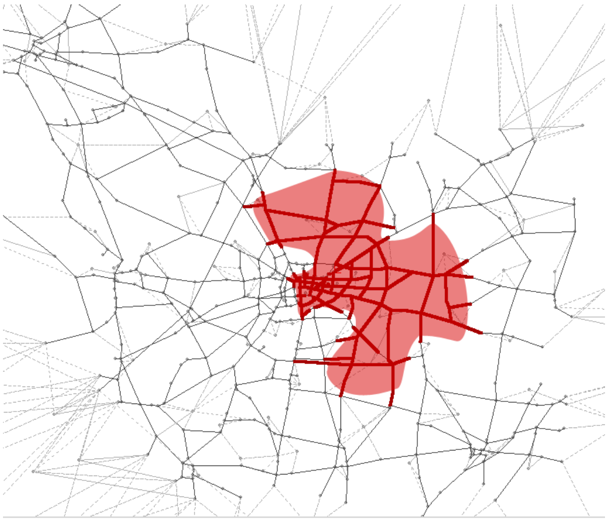

Figure 5. A boundary for optimal area toll (Toll level: JPY 150)

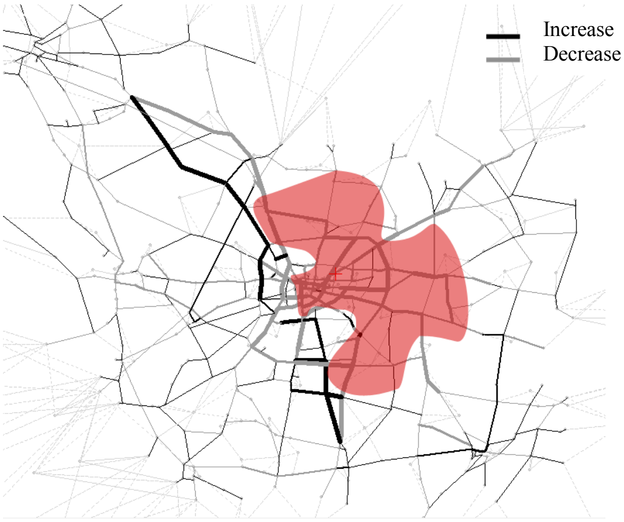

Figure 6. Change in traffic volume resulting from an optimal area toll scheme

The change in traffic volume resulting from the new optimal area toll (Figure 6) differs from that of the old optimal area toll (Figure 3). The choice of the two optimal area tolls is another interesting research topic. Although the social surpluses of the two are similar, their revenues, impacts on user, and equity effects differ. We can consider the extended problem based upon these other measures.

Comparison with Optimal Cordon Toll

The area toll design method in this study was originally proposed for designing an optimal cordon toll scheme. We can compute the optimal cordon and compare the optimal area and cordon. Figure 7 shows the boundary of an optimal cordon toll scheme. It has 110 links inside the cordon and a JPY 100 toll. The charging boundary is located in the northwestern region of the study area.

Figure 8 shows the change in traffic volume resulting from optimal cordon toll. The pattern is not very straightforward, but the traffic flows around the downtown area generally have decreased and those around the ring road have increased.

The optimal social surplus generated by the optimal cordon toll is around JPY 1.3790*10^9, which is higher than that generated by an optimal area toll scheme. In Maruyama and Sumalee (2007), the social surplus for a judgmental cordon toll was JPY 1.3780*10^9. Optimizing the area toll increases the social surplus by around 50%, as noted above, while optimizing the cordon toll increases the social surplus by around 100%.

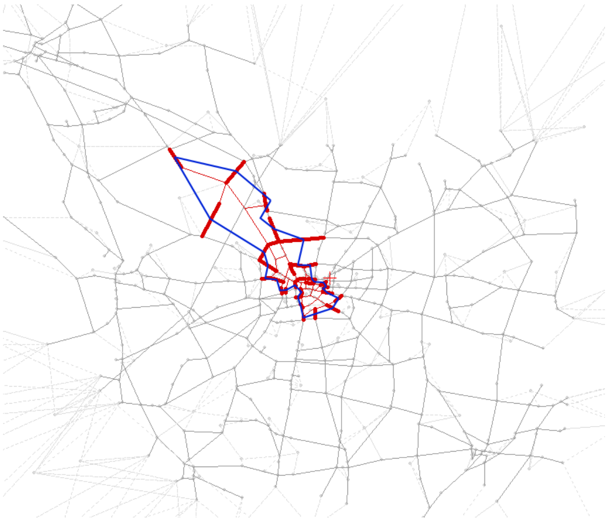

Figure 7. An optimal boundary cordon toll scheme

Figure 8. Change in traffic volume resulting from optimal cordon toll

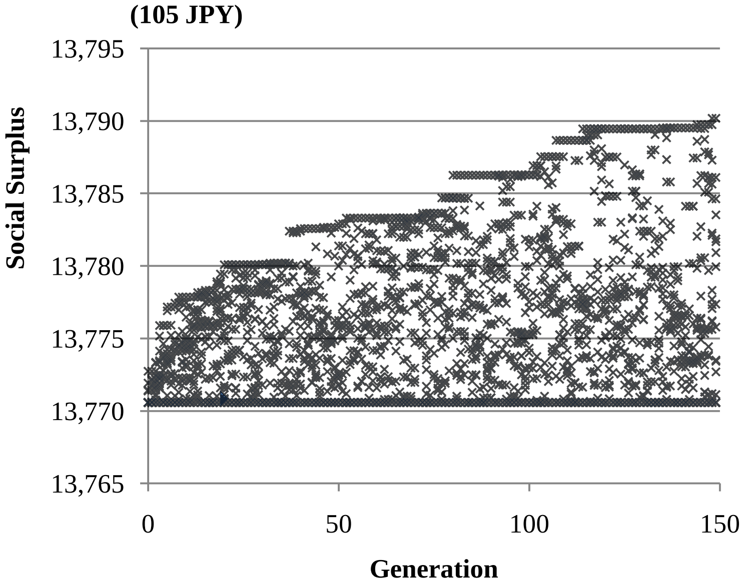

Figure 9. Evolution of social surplus by cordon-toll

Figure 9 shows the evolution of social surplus by cordon-toll in this application. Compared with the area toll (Figure 4), the frontier line (upper limit of data region) of the cordon toll is higher, but the other points are not very high and the density of these dots is higher in the region of lower surplus. This implies that a carefully designed cordon toll could produce a higher social surplus, but an area toll would be safer and more stable and would produce a moderate but reliable increase in the social surplus. Therefore, careful consideration of the charging boundary for the cordon toll is important, perhaps more important than the charging boundary of an area toll.

Analyzing the size and shape of a charging boundary using the random search method

In this section, the toll level is fixed and only the size and shape of the charging boundary are changed. A regeneration process generates a random shape and size for the charging boundary. The effects of an area toll and a cordon toll on the size and shape of the charging boundary are analyzed. Figures 10 and 11 show the results.

The area toll scheme and the cordon toll scheme produce quite different variations in the social surplus resulting from a change in the boundary shape. For a certain number of links in a charging boundary, an area toll produces a small variation in the social surplus (Figure 10), while a cordon toll produces a large variation (Figure 11). In the case of an area toll (Figure 10), as the number of toll links increases, the social surplus from the different shapes changes little. Even if every link in the study area is within the charging boundary, the social surplus is very high. In the case of a cordon toll (Figure 11), the social surplus changes dramatically as the shape and size of the tolling boundary changes. The social surplus from a no-toll scenario is JPY 1.3770 $*10^9$. Many cordon toll scenarios produce lower social surpluses.

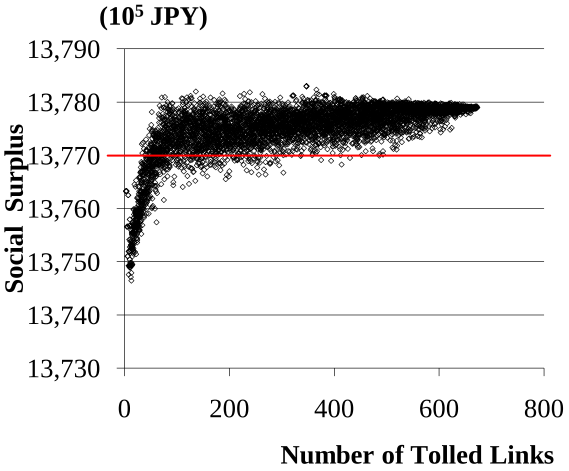

Figure 10. Change in social surplus resulting from an area toll with a fixed toll level (JPY 200)

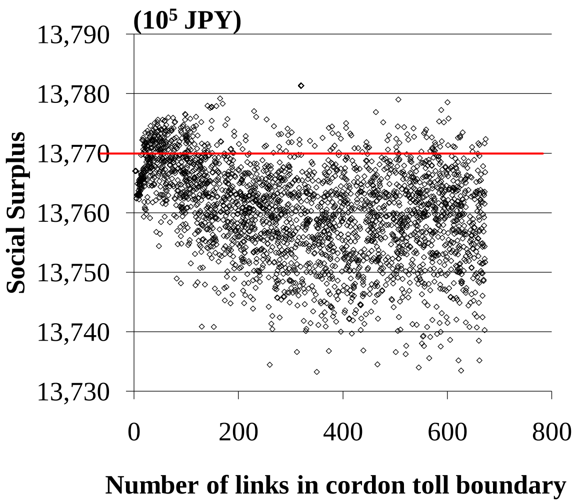

Figure 11. Change in social surplus resulting from a cordon toll with a fixed toll level (JPY 50)

4. CONCLUSION

In this study we propose an optimal area toll design model and apply the methods to a real-world network. A model similar to an existing optimal cordon-toll design model was developed for designing an optimal area toll scheme. Innovative part of this paper is optimal design (automatic design) of charging boundary of area-based toll, which was determined (manually) by human-brain in past research.

The shapes of the optimal boundaries for area toll and cordon toll schemes differ significantly. Optimizing the area toll increases the social surplus by 50% over that generated by a judgmental area toll, while optimizing the cordon toll increases the social surplus by 100%. An optimal area toll has a wider area than an optimal cordon toll. A cordon toll has a more sensible boundary shape than an area toll. Then, an area toll may be a better first option because we don't know the demand information accurately in the city before introducing some toll policy.

Expanding our model produces several policy evaluation frameworks. Maruyama *et al*., (2010), provided a similar modeling framework for investigating bi-directional cordon, price-cap cordon and area-based tolls with destination credits, in addition to the area and cordon tolls described in this study. Applying their model to the lower problem in this study is easy and allows us to consider the effect on the optimal boundary of a bi-directional cordon and other policies. A price cap and a refunding parameter (Maruyama *et al*., 2010) are other decision variables for policy makers.

Implementation on larger networks, however, requires a faster algorithm. The current study adopts a traditional partial linearization (or Evans algorithm) to solve the lower problem of the optimal toll design problem. Recently, many advances have been made in solving the traditional static user equilibrium assignment with fixed demand (Bar-Gera, 2002, in press; Dial, 2006; Nie, 2010; and others). These methods definitely will be useful for our trip chain-based equilibrium model with variable demand. A parallel computing technique could one option for practical application on large-scale network.

Our model is a basic static model but the lower problem of the bi-level problem can be any advanced model (such as activity-based dynamic models). Such models may be useful for improving the accuracy of the representation of reality. These models can be very time-consuming to run, but they can analyze more sophisticated policies, such as that of a time-varying cordon toll.

Currently our case study covers only one city, and the results and implications could be city-dependent. Applying the model to other cities definitely will be useful. In addition, real-world networks can sometimes produce results that are difficult to understand, so studying a hypothetical radial-ring road network (e.g., de Palma et al., 2005) also may be useful.

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