

# **SELF-ORGANIZATION OF PEDESTRIAN FLOW WITH A GAME THEORY BASED MICROSCOPIC SIMULATION**

*Miho ASANO, Dr. Eng. Assistant Professor, Nagoya University*

*Takamasa IRYO, Dr. Eng. Assistant Professor, Kobe University*

*Masao KUWAHARA, Ph.D. Professor, Tohoku University*

## **ABSTRACTS**

This study aims to analyze a self-organization of a pedestrian flow with a game theory based microscopic simulation so as to propose controlling strategies that can realize smoother pedestrian movements. It is known that a pedestrian flow containing different directions of movements can be self-organized under certain conditions and the self-organization can be utilized to improve the level of service of pedestrian facilities. To investigate the self-organization, this study adopts a microscopic simulation that incorporates game theory to describe pedestrian decisions in their movements. It is revealed that, in an intersection with four-direction streams, putting obstacles in a certain alignment can promote the self-organization of pedestrians and improve their average travel times.

Key words: Pedestrian simulation, self organization, game theory

## **INTRODUCTION**

This study aims to analyze a self-organization of pedestrian flow with a game theory based microscopic simulation so as to propose controlling strategies that can realize smoother pedestrian movements. There is an increasing interest to evaluate level of services of walking facilities these days. One of the most important and challenging topics is LOS evaluation and improvement in multidirectional pedestrian flows in quantitative manner.

One important characteristic of pedestrian flow is that people are self-controlled in it. Unlike vehicle traffic systems, pedestrians can avoid collisions without any external controlling measures like traffic signals at road intersections. They can choose their movements so as not to collide with another pedestrian in a crowd. Such a microscopic behaviour made by each pedestrian can cause a 'self-organization' phenomenon of a pedestrian flow. The definition of the 'self-organization' in this paper is that 'pedestrians endogenously form a

multidirectional flow where different directions are properly handled to maintain smooth movements'. There are many existing studies that claim such a phenomenon in pedestrian flow; a most famous case may be a 'stripe pattern' that can be empirically observed in opposed or crossing two-directional flows. Understanding characteristics of the self-organization is important to control pedestrian flow effectively so as to provide smoother movements to the users.

In order to control pedestrian flow by considering self-organization, following items should be quantitatively clarified; (1) how pedestrian flow converges toward self-organization in different geometric constraint as well as different distribution of pedestrian characteristics and (2) how each pattern of self-organization affects capacities of walking facilities. Although some existing studies proposes several effective geometries in multi-directional flow (for example, Helbing et al. (2005)) and analyzes self-organization patterns related to heterogeneity of pedestrians (Campanella et al. (2009)), no research that works on quantitative performance analysis considering self-organization in different geometries has found so far.

The contribution of this paper is to investigate how the self-organization phenomena occur and how far they affect on efficiency of pedestrian flows in different geometric constraint. This knowledge will be useful for determining effective control strategies of pedestrian flows, such as an installation of barriers or small obstacles.

To understand the mechanism of self-organization phenomena, we use a microscopic pedestrian flow model that explicitly considers spatial structures without any discretization like cell-automata models. Among the microscopic pedestrian models proposed by existing studies, we employ a model with game theory (Asano et al, 2009). This model explicitly considers the decision process of pedestrians, especially how pedestrians anticipate other's movements in a congested crowd.

In the next chapter, state-of-the-arts of pedestrian self organization analyses are introduced, followed by the overview of a microscopic pedestrian flow model employed in this study. Then, the calculation results of simulations are shown. Mean travel time, distributions of streams and density are calculated from the results. In addition, the number of pedestrian who walks nearby in the same direction is also calculated to evaluate how far pedestrians going to the same direction are consolidated. Finally, the summary of the all results are shown, followed by the conclusion and future issues are discussed in the last section.

## **LITERATURE REVIEW**

### **Self organization of pedestrian flow**

It is known that a pedestrian flow in certain situations make the self-organization. In corridors with bi-directional opposite streams, people often experience that pedestrians toward different directions tend to make separate lanes so that they reduces the opportunity to get conflict with pedestrians from the opposite direction. A crossing flow also makes the self-organization. Ando et al. (1988) mentioned that bi-directional crossing flow makes a "stripe

formation” from a result of an empirical observation. This stripe formation has also been observed in several experiments (e.g., Hoogendoorn and Bovy (2006), Asano et al. (2007))

Relationships between the self-organization and effectiveness of a flow were firstly discussed by Helbing and Vicsek (1999). They proposed a macroscopic model that describes a bi-directional opposite flow and performed mathematical analyses to explain the mechanism of the self-organization by describing how streams of different directions are apart from each other. They also showed that the self-organized conditions also derive optimal behaviour of pedestrians in terms of smoothness of flow. They also mentioned that a pedestrian flow with more than two directions cannot achieve fixed self-organized formation. Dzubiella et al. (2002) also proved that stripe formation will be generated with two different groups of particles which are forced to move in different directions. As pedestrians can be regarded as particles, this phenomenon can also be applied to a pedestrian flow to explain the mechanism of the phenomena found in empirical observations. In addition, the relationship between self-organization and variations of pedestrian characteristics is discussed by Campanella et al. (2009). By using a simulation model, they concluded that heterogeneity of pedestrian characteristics significantly affects stripe patterns as well as capacities at bottlenecks of a unidirectional flow and corridors with a bi-directional flow.

Based on the studies of the self-organization, Helbing et al. (2005, 2007) proposed several effective design strategies of walking area which increase capacity of a pedestrian flow. For example, they proposed obstacles in front of bottlenecks in order to avoid demand concentration at a bottleneck, obstacles in order to divide opposed flows in corridors, obstacles in order for pedestrians to walk in one direction at an intersection as if there is a roundabout, and so on. These designs are aimed to decrease degree of freedom of pedestrian behaviour by using obstacles and then to make flow commutated. They tested the difference by using the social-force pedestrian model (Helbing and Molnár (1995)).

So far, these literatures discussed characteristics of the self-organization and rather qualitative validation has been done for a pedestrian flow. Proposed designs by Helbing et al. (2005, 2007) still demand detailed analyses in order to be applied to make a concrete design strategy. In the case of roundabouts, parts of designs such as size of the obstacle at a centre of the intersection, how to control inflow and outflow taking into account pedestrian demand and width of corridors are not clearly identified.

## **Pedestrian simulation modelling**

Recently several types of pedestrian behaviour modelling were proposed. One of the most well-known models is the social force model originally proposed by Helbing and Molnár (1995). This model considers each pedestrian as a particle whose movement is described as a law that is similar to Newton’s dynamics. If two pedestrians are near to each other, a strong force is generated between them so as to avoid conflicts.

Although the social model is intuitively acceptable and easy to understand, it should be pointed out that pedestrians are not particles but humans who decide their own movements

so as to maximize utility of their trips. Several models that explicitly describe people's utility maximization behaviour were proposed (e.g., Hoogendoorn and Bovy (2003)). Among them, Asano et al. (2009, 2010) proposed models with game theory. When the density of pedestrians is higher, they should consider other's movements in a near future (say, a few seconds) in order to avoid collisions. In such a case, they should decide their own movements by anticipating others' movements, meaning that decisions of pedestrians interact with each other and will converge to equilibrium point. Game theory is adequate to describe such phenomena. This study adopted the model proposed by Asano et al. (2009) to analyze the self-organization in a congested pedestrian flow.

## **GAME-BASED PEDESTRIAN BEHAVIOUR MODEL**

The model used in this paper is a rolling-horizon-based model proposed by Asano et al. (2009). The important feature of the model is pedestrians' 'game behaviour,' where trajectories they will take are regarded as strategies and walking distance along their desired directions, which is given externally, within a unit of time as payoffs.

It is assumed that pedestrians can anticipate others' movements for a shorter time horizon  $T$  (say, a few seconds) to avoid collisions. To let people do so, this model assumes that each pedestrian exposes his/her plan of the movement to other pedestrians and make negotiations with pedestrians who are walking nearby and may collide to his/her. The plan of movement is referred to as 'intended trajectory'. The intended trajectory is a trajectory from the current time  $t$  to the end of the time horizon  $t+T$ . The model uses a discretised time scheme whose time step is  $\Delta t$ . At each time step, people tell their own intended trajectories to others and negotiate with each other to determine their intended trajectories. Then, they walk along the intended trajectory for one time step. The intended trajectories will be identical to actual trajectories if situations do not change. However, because the model assumes that people's eyesight is limited to a fan-shape whose radius is  $D$  and angle is  $2\phi$ , the situations change actually and pedestrians must update the intended trajectories. This will let the actual trajectories be different from the intended trajectories. Pedestrian's shape is assumed to be a circle whose radius is  $s$ .

The process of the negotiation is modelled by the best response dynamics. The procedure of the model is as follows:

1. Determine the order of the pedestrians randomly. Let  $i = 0$ .
2. Update  $i$ th pedestrian's intended trajectory so as to maximize the walking distance along his/her desired direction and not to collide intended trajectories of other pedestrians.
3. Increment  $i$ . Go back to step 2 until  $i$  reaches the number of all pedestrians in the field.

4. Repeat steps 1 to 3 for  $n_{BP}$  times.

At step 2, each pedestrian selects an intended trajectory that maximizes his/her payoff and therefore repeating steps 1 to 3 for many times can let any pedestrian selects the trajectory that is his/her best response (but may not do so because the convergence of this procedure is not guaranteed). Because the situation where all pedestrians perform the best responses is identical to Nash equilibrium, the procedure shown above can solve Nash equilibrium if people's decisions are based on the intentional trajectories. In this study,  $n_{BP}$  is set to a fixed number and the issue of the convergence is not strictly considered. Because of that, the solutions solved by the actual simulation model may not be an exact solution but an approximated solution of Nash equilibrium. However, Asano *et al.* (2009) concluded by sensitivity analysis that the effect of  $n_{BP}$  on pedestrian behaviour and its travel time is not very significant.

To make a calculation procedure much simpler, it is assumed that the intended trajectory can be just a straight line and pedestrian's speed profile can only contain two modes, i.e., walking in desired speed  $v_f$  and stopping. In addition, the change of direction of the intended trajectory during a unit time is restricted to  $\omega$  to avoid unstable and rapid fluctuations of pedestrians' movements.

Throughout this paper, parameters shown in Table 1 are employed for the calculations.

**Table 1- Parameter settings**

Variables	Value
Time step $\Delta t$	0.05 sec
Radius of pedestrian's body $s$	0.25 m
Searching area $D, \varphi$	$D = 5.0$ m , $\varphi = 60$ degrees
Desired speed $v_f$	$v_f \sim N(1.3, (0.1)_2)$ Values less than 1.0 m/s are replaced by 1.0 m/s and greater than 1.6 m/s are replaced by 1.6 m/s
Change of speed of directions $\omega$	180 degrees/sec
Duration of time horizon $T$	5.00 sec
Number of best response dynamics $n_{BP}$	3 times/ each time step

## **VALIDATION OF FACILITY DESIGN AT CORRIDOR INTERSECTIONS WITH MULTIDIRECCIONAL FLOW**

### **Self-organization in multidirectional flow**

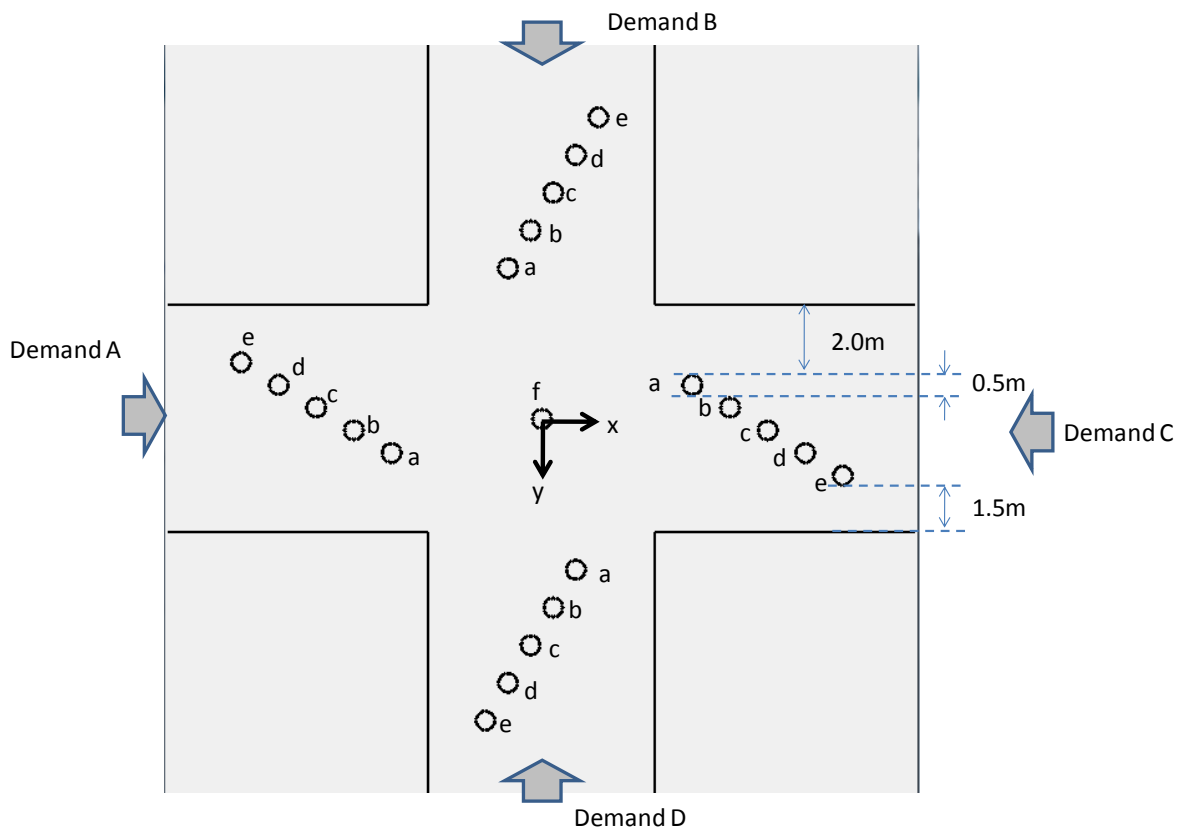
It is known that no stable pattern of the self-organization (such as lane formations of streams going to the opposite directions) can be observed in a multidirectional flow. Instead, a flow pattern that is similar to a car flow in a roundabout can be observed (e.g., Helbing et al. (2001)). Such a roundabout-like stream reduces the number of conflicts between pedestrians; however, because the degree of freedom of direction is large in pedestrian's walking behaviour, the roundabout-like situation cannot be stabilized without any external control. Placing obstacles onto a pedestrian facility at appropriate places can stabilize the roundabout-like situation by forcing pedestrian's movements to be similar to those in a roundabout intersection. To check how this idea works, multidirectional pedestrian movements and their self-organizations at an intersection with and without obstacles are examined in this section.

### **Scenario settings**

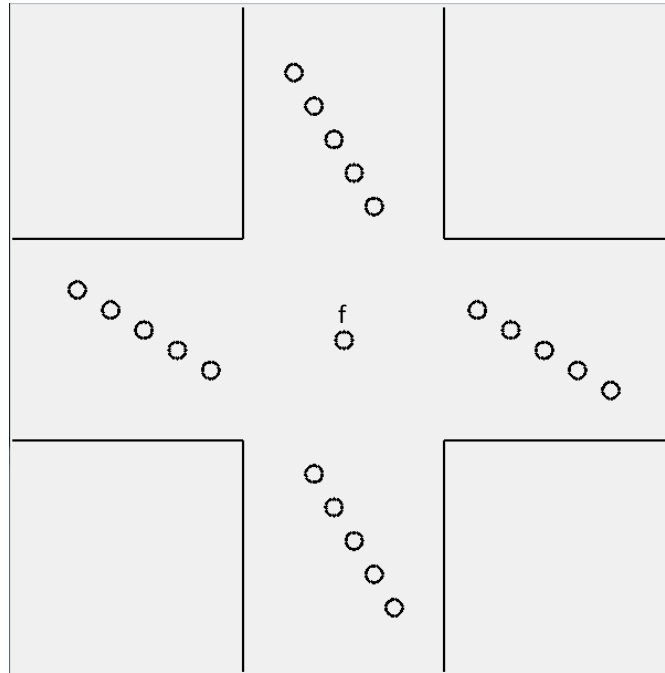
A study intersection is shown in Figure 1. Pedestrian streams coming from four approaches toward the intersection are given. Circles 'a' to 'f' in the figure represents pillars where pedestrians cannot walk through. The pillars in the approach corridor are expected to bias the stream to the right hand side. Then, the biased stream enters into the intersection from each direction and keeping the right hand side. Such movements of the streams can make an anti-clockwise flow that is similar to vehicular movements in a roundabout. The other configuration of the pillars, as depicted in Figure 2, is also investigated. In this configuration, pedestrians entering to the intersection do not make an anti-clockwise movement that is estimated the configuration in Figure 1. The number of pillars varies among the scenarios, as described in Table 2. Different levels of pedestrian demand are also examined in these scenarios. Positions where pedestrians are generated are uniformly distributed along the section at the end of four approaches. For each scenario, the simulations are run 10 times and the duration of each simulation is 300 seconds. All pedestrians are assumed to walk toward the opposite end of the corridor, meaning that they do not intended to turn at the intersection. Because the simulation relies on random numbers, the result calculated by the simulation is stochastic, and sometimes the pedestrians are stuck in the intersection when the demand level is higher. In case 4-2, four stuck situations are found out of 10 trials and therefore these stuck results are removed from the following data analyses.

**Table 2 – Scenario settings**

Scenario number	Pillars placed at intersections	Demand
0	No pillars	For each scenarios, following demand levels are tested 40 ped/min/approach for all directions (referred to as '40 for all') 50 ped/min/approach for all directions ('50 for all') 60 ped/min/approach for all directions ('60 for all') 70 ped/min/approach for all directions ('70 for all')
1-1	Pillar b, c, d	
1-2	Pillar b, c, d, f	
2-1	Pillar a, b, c, d	
2-2	Pillar a, b, c, d, f	
3-1	Pillar a, b, c, d, e	
3-2	Pillar a, b, c, d, e, f	
4-1	As in Figure 2, but without centre pillar <i>f</i>	
4-2	As in Figure 2	



**Figure 1 – Study intersection**



**Figure 2** – Intersection with different pillar settings

## Simulation results

### *Travel time*

Figure 3 shows the average travel time of cases without the centre pillar *f*, that is, cases 0, 1-1, 2-1, 3-1 and 4-1. It is figured out that, when the more pillars are set, travel time in lower demand levels increases slightly but travel time in the highest demand level significantly decreases. It is also shown that settings of pillars shown in Figure 1 reduces travel time more than those in Figure 2 by comparing case 3-1 and 4-1.

Average travel time in cases 0, 3-1 and 3-2 are compared in Figure 4. The centre pillar *f* exists in case 3-1, whereas no centre pillar is set in case 3-2. The case with the centre pillar *f* gives larger travel time than without the pillar. Similar tendency was observed in case 1-2, 2-2 and 4-2 as well, as shown in Figures 5, 6, and 7. Note that the average travel time in case 4-2 is underestimated because of the elimination of stacking conditions occurred in case 4-2.

Applying Welch's t-test, it is confirmed that travel time in case 3-1 with demand 70 ped/min/approach significantly less than that in case 0 (significance level of 1%,  $t = 3.36$ ).



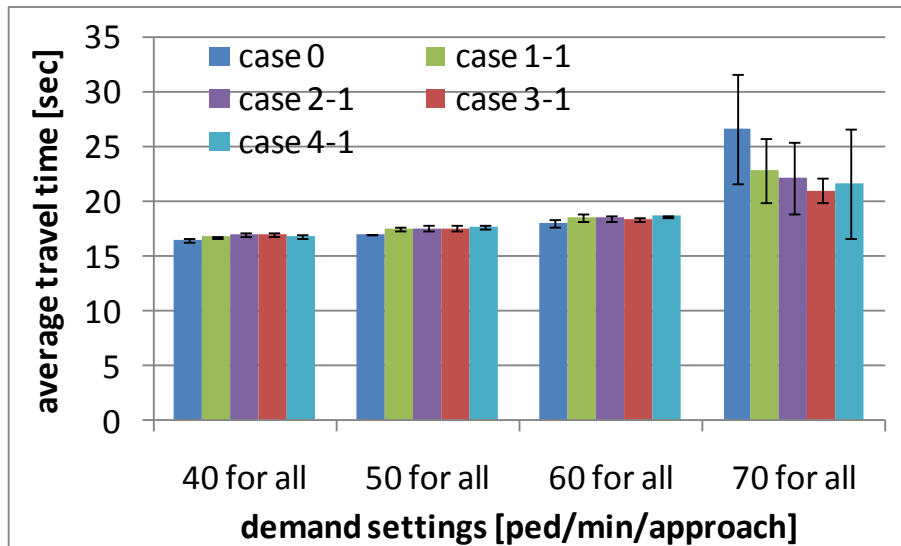


Figure 3 - Average travel time in cases w/o centre pillar f

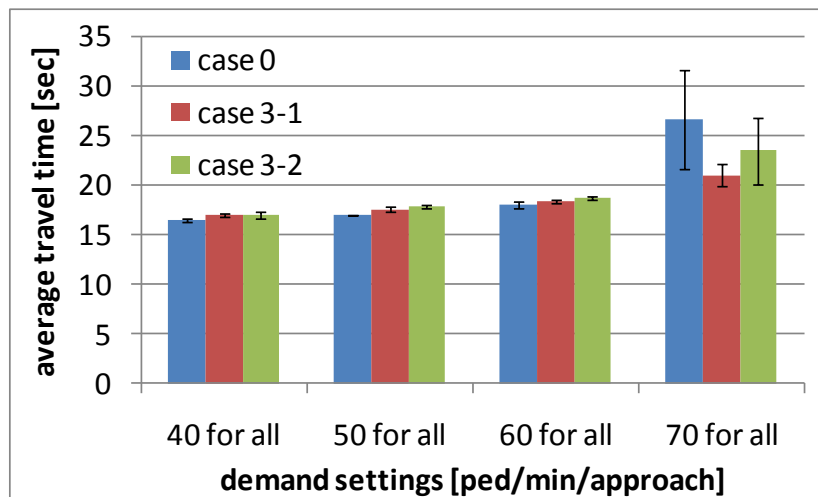


Figure 4 - Average travel time with and without centre pillar f (case 3-x)

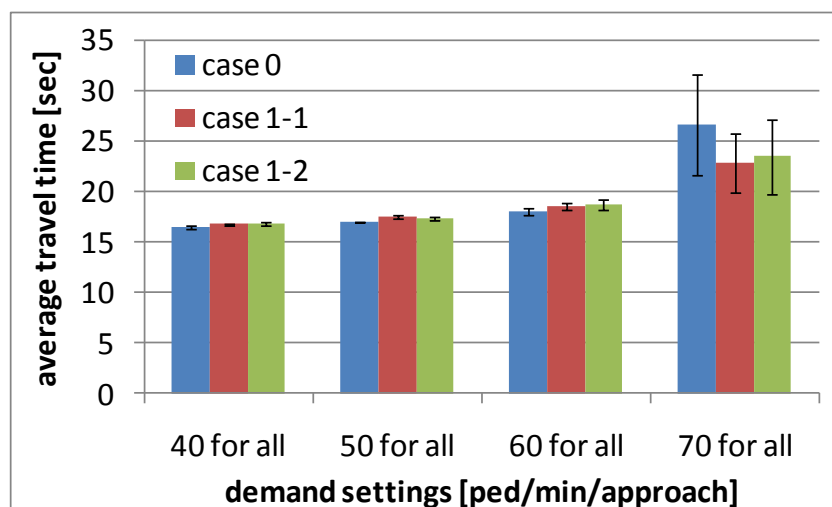


Figure 5 - Average travel time with and without centre pillar f (case 1-x)

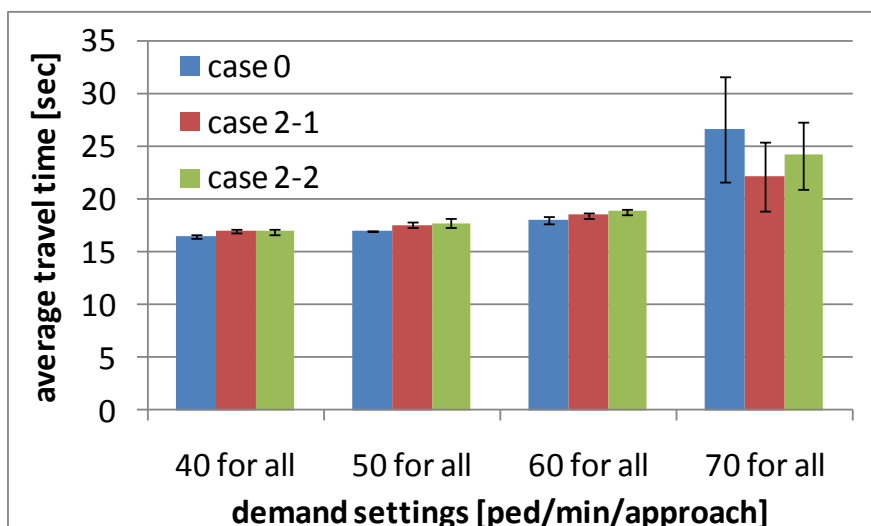


Figure 6 - Average travel time with and without centre pillar f (case 2-x)

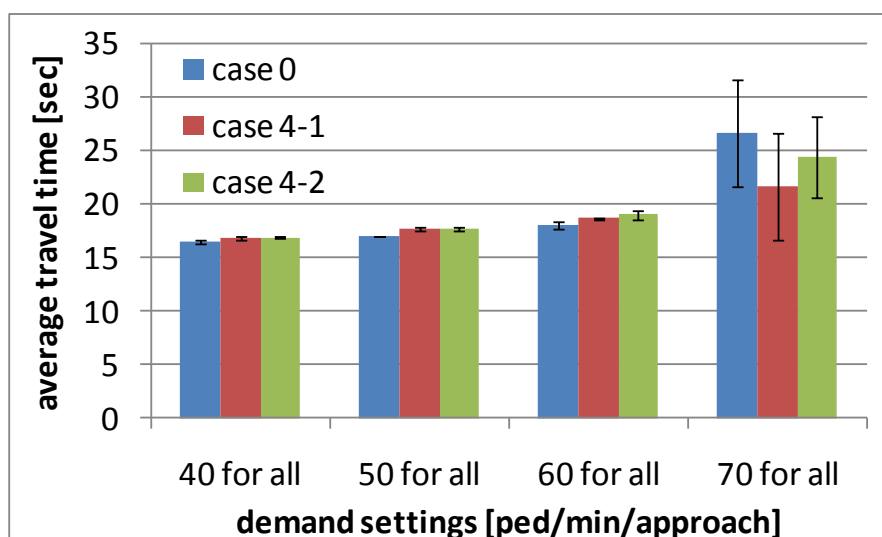
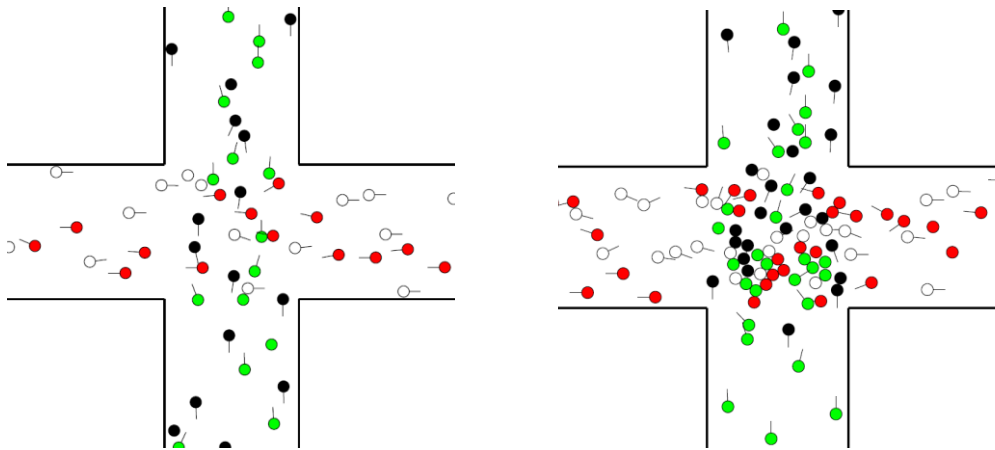


Figure 7 - Average travel time with and without centre pillar f (case 4-x)

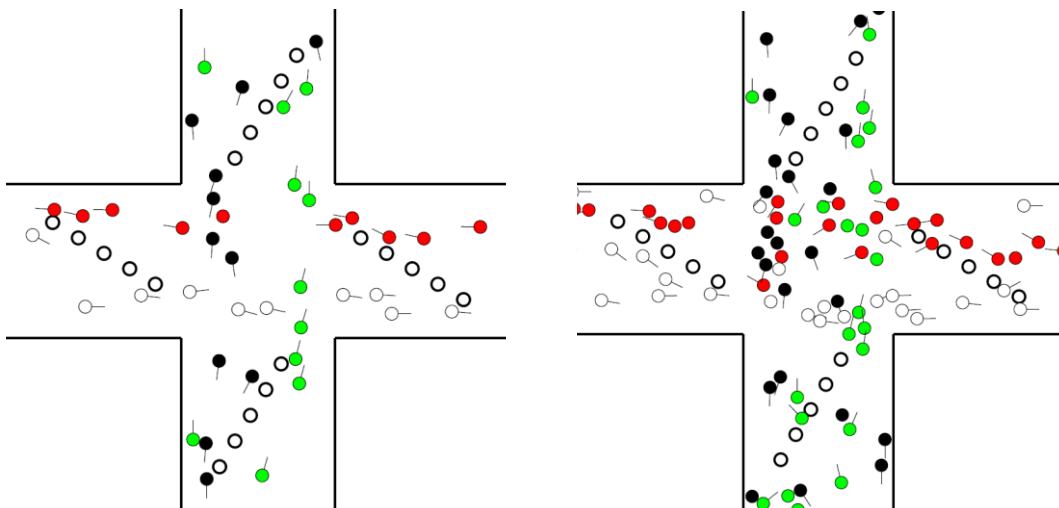
### Pedestrian flow distribution

Figure 8, 9 and 10 show snapshots of the simulation in cases 0, 3-1 and 4-1 respectively. Circles with thin outlines in the figures indicate pedestrians and circles with thick outlines shows pillars. Pedestrians painted with same colours have the same OD pair. In low demand condition of case 0, the self organization of so-called an 'unstable roundabout' is observed. However, when the demand becomes higher, such a roundabout-like self-organization disappears and congestion starts. In case 3-1 with pillars at the entrances of intersection, pedestrians from each approach clearly separated and behave as if they were vehicles at a roundabout. Even in the high demand condition, it seems that this structure sorts pedestrian movements. Similar situation can be seen in case 4-1 both in low and high demand conditions. However, it seems that the pedestrian density is not uniformly distributed like in case 3-1 but concentrated in the upper-right and lower-left corners.



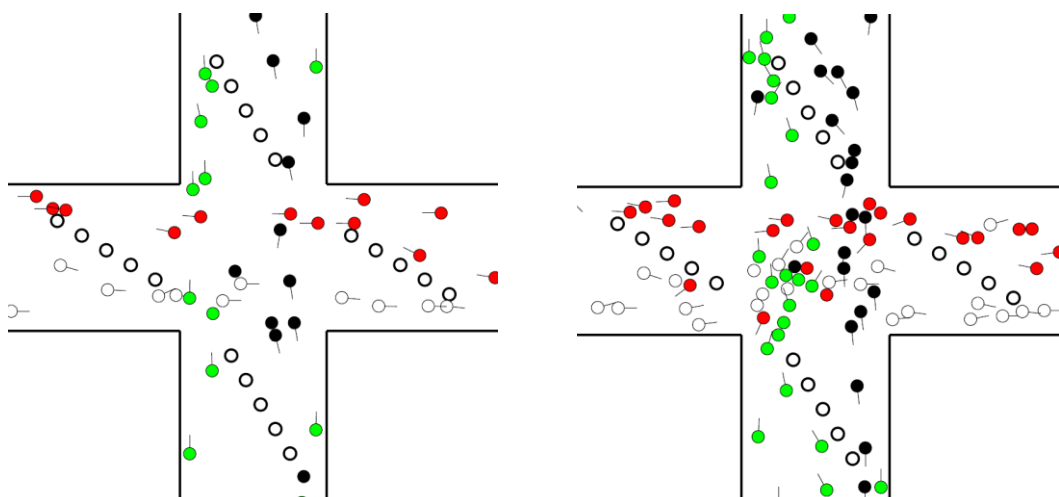
(a) Low demand (40 ped/min for all directions) (b) High demand (70 ped/min for all directions)

**Figure 8** – Distribution of pedestrian flow at intersection without pillars (Case 0)



(a) Low demand (40 ped/min for all directions) (b) High demand (70 ped/min for all directions)

**Figure 9** – Distribution of pedestrian flow at intersection with pillars (Case 3-1)



(a) Low demand (40 ped/min for all directions) (b) High demand (70 ped/min for all directions)

**Figure 10** – Distribution of pedestrian flow at intersection with pillars (Case 4-1)

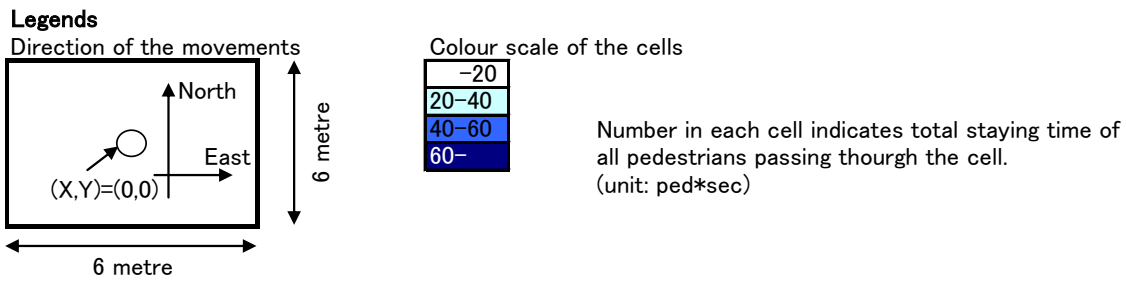
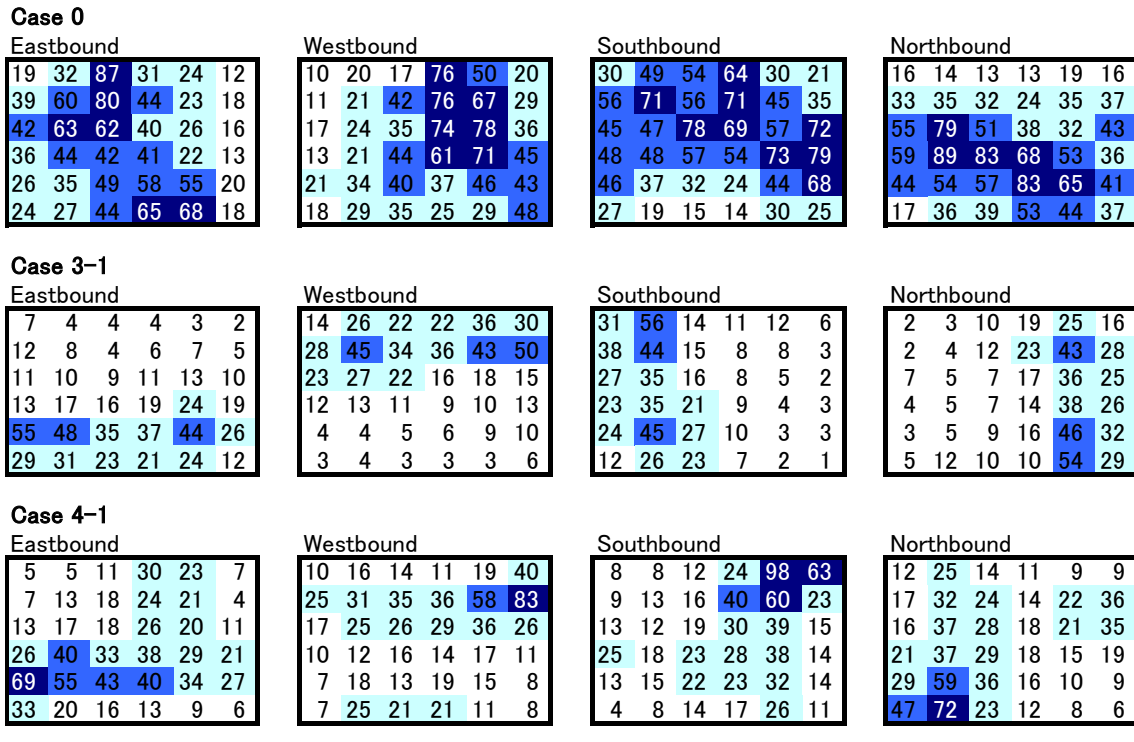


Figure 11 – Distribution of total staying time (Case 0, 3-1 and 4-1)

Figure 10 shows the snapshots at one moment and cannot conclude significance of differences throughout the simulation time, while Figure 11 supports the above discussion. This figure shows distributions of total staying time of all pedestrians in the high demand cases (cases 0, 3-1 and 4-1). The 6 x 6 m intersection area is divided by 1 x 1 m cells and counted the total staying time. First of all, it can be seen that pedestrians passing through the intersection in case 0 tend to stay longer in the first half of the intersection (for example, western side for eastbound pedestrians, northern side for southbound pedestrians). This result implies that the pedestrians must stay longer at the entrance of the intersection and take delays to get through it. By comparing different cases, it is obvious that total staying time is much smaller in cases with pillars (case 3-1 and 4-1) than that in the case without pillars (case 0). In case 3-1, pedestrians in each direction are clearly separated and effective movements are realized. Total travel time is also decreased in case 4-1, although larger staying time is observed at south-west and north-east corners where two entering streams collide with each other.

Figure 12 shows the distributions of pedestrians' lateral positions at each section of the entrance of the intersection in high demand conditions. Case 0, where there are no pillars,

entering positions of pedestrians are uniformly distributed. When the more pillars are placed, pedestrians tend to be concentrated to one side. This result indicates that the installation of the pillars makes separate the streams away from each other.

Figure 13 shows the pedestrians' lateral distributions at the exit sections of the intersection. The amount of change in lateral positions while the pedestrians walk through the intersection can be checked by comparing the results depicted in Figures 12 and 13. Biases of pedestrian streams at each entrance become weaker but still remain at the exit of the intersection in these cases. Especially in case 3-1 and 4-1, 70-80 % of pedestrians enter from one side and remain there until they exit from the intersection. These biases may help pedestrian to keep the roundabout-like self organization.

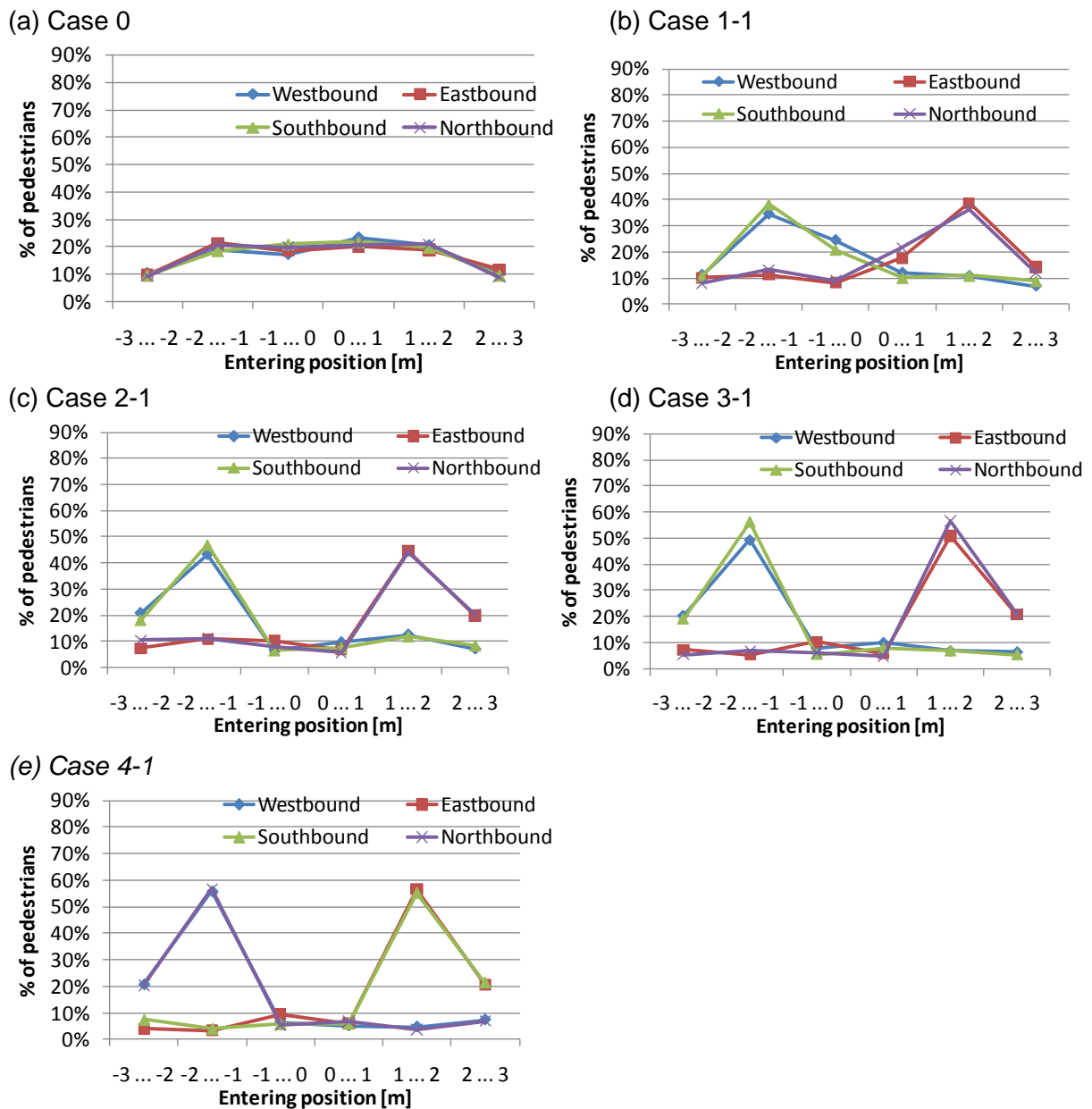


Figure 12 - Distribution of pedestrians' entering position (70 ped/min for all directions)

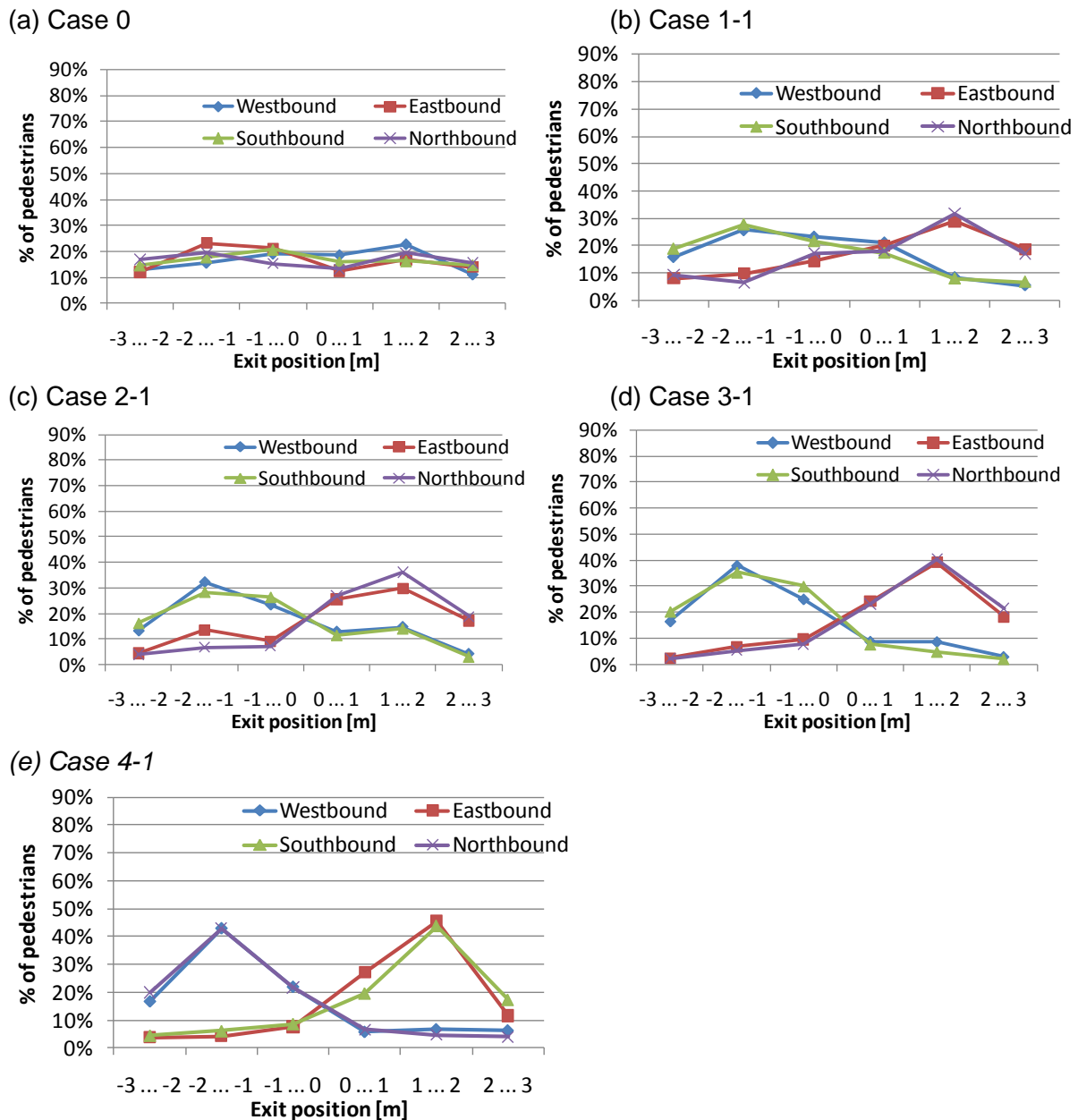


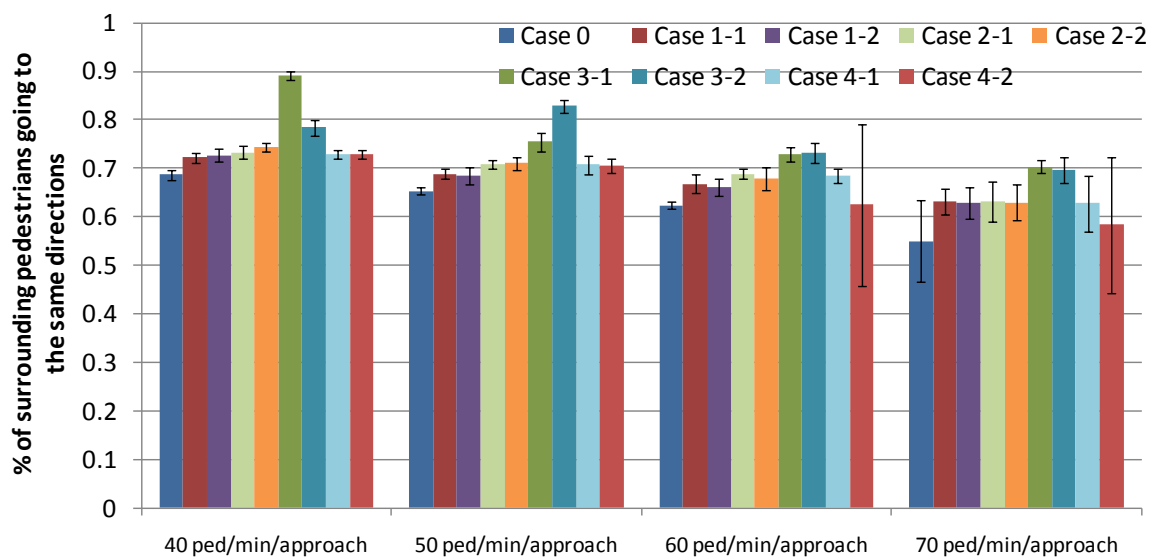
Figure 13 - Distribution of pedestrians' exit position (70 ped/min for all directions)

### Self organization of pedestrians

This section analyses whether pedestrians going toward the same direction are getting together in a flow to estimate how far they perform the self organization. When pedestrian flow is self-organized, each individual pedestrian has less number of conflict opportunities to the other pedestrians. As conflict often occurs between pedestrians walking toward different directions, it may be reasonable to consider that the self-organized pedestrians tend to have clusters with other pedestrians walking in the same directions. This section uses an index that explains the degree of the self organization by accumulating individual pedestrians' circumstances at each time step. The index used is percentage of surrounding pedestrians going to the same directions out of total number of pedestrian surrounding the subject

pedestrian. If this percentage is higher, pedestrians are considered to make groups so that they have less conflict with other pedestrians. Surrounding pedestrians at one moment are defined as pedestrians whose distance to the subject pedestrian at this moment is equal to or less than 1m in this study.

Figure 14 shows the average percentage of surrounding pedestrians walking to the same direction. High percentages are achieved in cases 3-1 and 3-2, implying that the pedestrians going to the same direction effectively make groups. On the other hand, the indices in cases 4-1 and 4-2 are relatively smaller than cases 3-1 and 3-2, although they have strong biases of entering and exit positions as shown in Figure 12 and 13. This index supports the assumption that the self-organization strongly affects on the efficiency of a flow at the intersection.



**Figure 14 - Percentages of surrounding pedestrians going to the same directions**

## SUMMARY OF THE RESULTS

From the results of the numerical tests shown above, following results can be stated.

1. Placing the pillars in the study area decreases both average travel time and density in the intersection.
2. Density of pedestrians going to the each direction in the intersection is biased when the pillars are placed.
3. Both incoming and outgoing flows are biased by placing the pillars. The amount of the bias is greater when the number of pillars is more.
4. Pedestrians tend to be surrounded by other pedestrians going to the same direction when the pillars are placed.

All of the results 2, 3, and 4 support that the self-organization of the pedestrian flow is produced by placing the pillars. Combined with these results and result 1, it can be concluded that, at least, the self-organization has a relationship to the improvement of travel time of pedestrians passing through the intersection.

Geometrical configuration of the pillars seems to affect the results. In the study cases, placing more pillars tends to encourage the self organization and decrease average travel time, whereas placing the centre pillar tends to increase average travel time. Placing pillars in a different configuration as depicted in Figure 2 decreases the effect of the self organization and make the area where density is higher.

## **DISCUSSIONS AND CONCLUSION REMARKS**

This study analyzed the self organization of a multidirectional pedestrian flow by using a microscopic simulation model. The results supported that the installation of obstacles such as pillars can sort out pedestrian streams going towards different directions and encourage pedestrians to keep self organization. The idea of the effective facility design at intersections originally comes from Helbing et al. (2005). Taking their idea into account, this paper quantitatively analysed the self organization phenomena in the multidirectional flow with various scenarios. The results implied that placing obstacles can significantly improve the efficiency of pedestrian flows. Especially the obstacles upstream of the intersection have a function to make pedestrian flows biased. Once pedestrian flows become biased, they tend to keep the biases as it achieves less number of conflicts for each pedestrian. In this condition, pedestrians are self-organized and travel time of pedestrians passing through the intersection has significantly improved.

If pedestrian flow can keep the roundabout-like self organization condition, interactions between pedestrians become limited. Similar to actual roundabouts, pedestrians need not to consider the conflict with streams going toward two or more directions. In this case, pedestrian streams having three or more directions seems to be divided into the sub-groups where no multi-conflict (i.e., conflicts by three or more directions). This phenomenon might reduce loads of avoiding behaviour for pedestrians and increase level of services. This feature also implies possibility to simplify the multidirectional flow analysis into a combination of bidirectional flow analysis.

There are several important situations that have not been analyzed in this study but should be made in future researches. First, pedestrians making turns at intersections (e.g., coming from the north end and going to the west end) are not considered. The turning movement may make flows inside the intersections more complicated. It should be tested how turning movements affects the self-organization and performance of intersections. Another issue is that the results of multidirectional flows are not validated by using empirical data. In reality, heterogeneity of pedestrians or other factors may affect on variation of pedestrian behaviour,



and as a result, travel time and capacity may also vary. Comparison to the actual data is needed to make the knowledge more persuasive.

## ACKNOWLEDGEMENT

This research is supported by Obayashi Foundation.

## REFERENCES

- Ando, K.; Oto, H.; Aoki, T. (1988). Forecasting the flow of people, *Railway Research Review*, Vol.45, pp.8-14.
- Asano, M.; Kuwahara, M.; Tanaka, S. (2007). Multi-directional pedestrian flow model based on empirical data, *Proceedings of the 11th World Conference on Transport Research*, CD-ROM.
- Asano, M.; Iryo, T.; Kuwahara, M. (2009). A Pedestrian Model Considering Anticipatory Behaviour for Capacity Evaluation, in *Transportation and Traffic Theory 2009: Golden Jubilee* (W. H. K. Lam, S. C. Wong and H. K. Lo, Eds.), pp. 559-581. Springer, New York.
- Asano, M.; Iryo, T.; Kuwahara, M. (2010). Microscopic Pedestrian Simulation Model Combined with a Tactical Model for Route Choice Behaviour, *Transportation Research Part C* (Accepted).
- Campanella, M.; Hoogendoorn, S. P.; Daamen, W. (2009). The effects of heterogeneity in self-organized pedestrian flows, *Proceedings of Transportation Research Board 2009 Annual Meetings*, CD-ROM.
- Dzubiella, J.; Loewen, H. (2002). Pattern formation in driven colloidal-mixtures: tilted driving forces and re-entrant crystal freezing, *Journal of Physics: Condensed Matter*, Vol.14, pp. 9383-9395.
- Helbing, D. and Molnár, P. (1995). Social Force Model for Pedestrian Dynamics. *Physical Review E*, 51(5), 4282.
- Helbing, D.; Vicsek, T. (1999). Optimal self-organization, *New Journal of Physics*, Vol. 1, pp.13.1-13.17.
- Helbing, D.; Buzna, L.; Johansson, A.; Werner, T. (2005) Self-Organized Pedestrian Crowd Dynamics: Experiments, Simulations, and Design Solutions, *Transportation Science*, Vol.39, No.1, pp.1-24.
- Helbing, D.; Johansson, A.; Lammer, S. (2007) Self-Organization and Optimization of Pedestrian and Vehicle Traffic in Urban Environments, in *The Dynamics of Complex Urban Systems: An Interdisciplinary Approach* (S. Albeverio, D. Andrey and P. Giordano, Eds.), pp. 287-308, Physica Verlag.
- Hoogendoorn, S. P.; Bovy, P. H. L. (2003). Simulation of Pedestrian Flows by Optimal Control and Differential Games. *Optimal Control Applications and Methods*, 24(3), 153-172.

*Self-organization of pedestrian flow with a game theory based microscopic simulation*  
*ASANO, Miho; IRYO, Takamasa; KUWAHARA, Masao*

Hoogendoorn, S. P.; Bovy, P.H.L. (2006). Experiments and Theory of Self-organization in Pedestrian Flow, Proceedings of Transportation Research Board Annual Meeting, CD-ROM.