OPTIMAL DIFFERENTIATION AND NUMBER OF AIRLINE FARE TYPES[1]

12th World Conference on Transport Research

Lisbon 11-15 July 2010

Christiaan Behrens Eric Pels

VU University, Department of Spatial Economics

De Boelelaan 1105

1081HV Amsterdam

T +31 20 59 82742

F +31 20 59 86004

cbehrens@feweb.vu.nl

DRAFT VERSION, PLEASE DO NOT CITE

Version 25 May

In order to maximize profits and strengthen market position airlines use yield management. Product differentiation in airline economics, offering different fare types, different prices, services and qualities, is an important element of yield management.

In this paper we address the strategic behavior of airlines regarding product differentiation using a non-localized competition random utility model. Here, the airlines are explicitly assumed to differentiate products in terms of price, quality, and number of fare types. The theoretical optimal patterns of product differentiation as well as the effects on airline profits and social welfare in a duopolistic market are analyzed. The purpose of the analysis is twofold: to make trade-offs in product differentiation explicit and subsequently studying the underlying factors of this trade-off. The analysis shows that airlines have an optimal number of variants (fare types) which is dependent on the underlying price-quality game, cost factors, the relative valuation of quality and the interfirm and intrafirm unobserved heterogeneity. Furthermore, we find that the patterns of product differentiation, as encountered in the real world, can only be explained in the random utility framework under strict assumptions about unobserved heterogeneity at the demand side of the model (not earlier indicated in the literature).

[1] We gratefully acknowledge The Netherlands Organization for Scientific Research (NWO) for financial support (Grant number 400-08-012)

Section 1. Introduction

In order to maximize profits and strengthen market position, airlines offer different fare types for the same flight, characterized by different prices, services, restrictions and qualities. In this way, product differentiation has become an integral part of revenue management of airlines. Yield management in transport economics is not only limited to the aviation sector, railway companies also apply product differentiation to maximize revenues. The most extreme result of this product differentiation, as well as real-life evidence of existence of the phenomenon, is the emergence over the last decays of low cost carriers while not all full service carriers left the market. The most classical way to differentiate between fare types is to use an economy and a business class, however, as Table 1 shows, differentiation is not only limited to these two fare types. Table 1 shows, for three major routes, the operating airlines, the number of fare types offered per airline, the price range, and the aspects of (quality) product differentiation. The three routes are among the busiest airline routes in the world in terms of aircraft movements. The price range indication is based on a one-way trip at 21 September 2010 (half year from now), by checking prices for dates further away we avoid the problem of non-availability of certain ticket prices and/or the price effect of the limited availability of seats. In the Sydney-Melbourne market, four airlines are competing, with Qantas being the traditional full service carrier and Tiger Airways the low cost carrier.

Table 1 Overview of product differentiation in selected airline markets

	Number of fare types	Price range (€)	Quality differences between fare types
Airline			**Sydney – Melbourne**
Qantas	5	67-355	Booking changes; Cancelations; FFP program
Virgin Blue	5	60-290	Booking changes; Cancelations; FFP program; Luggage; Inflight personal space
Jetstar Airways	3	26-182	Booking changes; Cancelations; Luggage
Tiger Airways	1	26-53	-
			Barcelona – Madrid
Iberia	5	43-246	Booking changes; Cancelations; Luggage
Spanair	4	29-244	Booking changes; Cancelations; Luggage; Check-in; Inflight personal space; Inflight catering; Lounge access
Air Europa	2	38 – 180	Luggage; Inflight Catering
Vueling	1	34-79	-
			Amsterdam – London
Air France KLM	5	86-310	Booking changes; Cancelations; Lounge access
British Airways	4	77-413	Booking changes; Cancelations; Inflight personal space; Lounge access
EasyJet	1	34-54	-

As Table 1 shows, low cost carriers do not differentiate their product (Vueling and EasyJet) and offer one fare type, with Air Europa being the exemption. Besides the differentiation in marketing – names for the different fare types vary from Red e-Deal to JetFlex and from Euro Traveller to Club World –

the differentiation in quality is remarkable. The fare types differ in restrictions for booking changes, cancelations procedures, luggage handling (weight) and point earnings in the loyalty (FFP) program. Not only high quality inflight services are offered (space, catering and entertainment), also access to better ground facilities (preferred check-in procedure and access to lounge) are used to differentiate the product. It can be concluded, at least for the markets shown in Table 1, that in the oligopolistic airline market the different airlines offer a distinct number of fare types, however the amount of offered fare types seems to have a boundary. In the current paper we theoretically formalize the strategic behavior of airlines which can possibly explain the intrafirm product differentiation as well as the interfirm differences in product diversity.

Product differentiation is a key issue in economic theory, in general three distinctive approaches have been developed over time (see Anderson et al. (1992) for an excellent review): random utility models (e.g. McFadden (1974), Perloff and Salop (1985)), representative consumer models (Dixit and Stiglitz, 1977) and the address models (Hotelling, 1929). Although the three frameworks show overlap, and are therefore not diametrical different, we use here the random utility approach. Using this approach has two major benefits compared to the other two. First, the random utility framework offers the advantage of relative simple, analytical tractable, expressions for demand systems, best reaction functions and equilibrium outcomes. Existence and uniqueness properties of this framework are therefore available in the literature or are relative easy to deduct. Secondly, the random utility framework offers excellent possibilities to combine theoretical insights with empirical estimated models (multinomial logit, nested logit and mixed logit models) and therefore to perform simulation exercises parameterized on real-life data.

Product differentiation occurs if consumers include non-price characteristics of the products in their purchase decision. Therefore, competition in oligopolistic markets with differentiated products cannot result into Bertrand pricing, since the products in the market are not seen as being homogenous by consumers. This notion is very important, because it gives firms in an oligopolistic market an incentive to differentiate their products, in order to increase market power. The step to welfare analysis is therefore not surprising, since there seems to be a trade-off between more products in the market (more choice, higher consumer surplus) and higher costs for society (specific production costs as well as the market power effect). Product differentiation in oligopolistic markets can be regarded as localized (Kaldorian approach, Kaldor (1935)) or non-localized (Chamberlinian approach, Chamberlin (1933)) competition. Since the non-localized approach is better suited to study in the random utility framework, we use here this specific approach. The advantage of localized competition is that consumers are not assumed to be identical but to be distributed in a product characteristics space, analogue to the geographical space. Therefore, consumers are more willing to buy products of the neighboring firm in the product characteristics space compared to products of more distant firms in the product characteristics space. A major drawback is that these types of models are deterministic in the sense that the utility function does not have a probabilistic term; unobserved heterogeneity across consumers is not present. In the non-localized framework observed consumer variation can be incorporated by constructing different consumer groups, latent classes, or applying continuous mixed multinomial logit models, so that specific distributions of the valuation of product characteristics is allowed. In this paper we focus on the non-localized oligopolistic competition applying mainly nested logit models.

While there is a huge pile of literature about oligopolistic competition and differentiated products, the airline specific literature, both theoretically and empirically, is not developed in line. However, yield management in airline economics has been studied in detail, mostly in a non theoretical way. McGill and van Ryzin (1999) provide a good example of such a study. One of the first studies using discrete choice analysis to understand the choice between different air carriers and fare classes is provided by Proussaloglou and Koppelman (1999). Based on stated preference data, they identify tradeoffs between service quality, fare levels, schedule convenience and market presence. Algers and Beser (2001) combine revealed and stated preference data to identify factors underlying the choice for fare category, in particular change and cancellation policies. Their results show that a no change policy makes the specific fare category more undesirable, but that the largest negative effect is caused by a Sunday rule (obliged Sunday night in the trip) dummy. One of the most recent contributions is given by Balcombe et al. (2009); they perform a choice experiment in which they identify the in-flight service and comfort levels. Using a mixed logit model, they estimate in a willingness-to-pay space the effects of price, leg space, seat width, availability of a screen, amenity pack and catering services. Their results indicate that effects are the largest for leg space and seat width, followed by screen availability. Catering services are less valued, while amenity packs have a negative willingness-to-pay.

In the theoretical literature, Botimer and Belobaba (1999) are the first to point out that the airline revenue management literature describes and uses product differentiation models which assume that the demand for each of the fare types is independent of the price levels of the other fare types offered by the same airline or in the market. They come up with a static generalized cost model to include explicitly the trade-off airlines face when selling different types of products; cannibalization effects and the other costs related to offering more than a single product. However, as in Botimer (1996) in which is stated that product differentiation can have beneficial effects for welfare, the authors neither use a random utility framework to model the demand for airline tickets nor include oligopolistic competition. Instead they use a linear demand model in which they assume that a fixed percentage of the expected demand for each fare type will be diverted to the other product types at given prices.

More recent literature discusses the effect of dynamic price competition in airline revenue management. Zhang and Cooper (2009) formulate a model in which a Markov decision process formulation is chosen to represent the pricing behavior of airlines. Their main objective is to test the pooling of heuristics within the Markov decision process under different assumptions about consumer preferences and airline characteristics. The model setting cannot be compared one-to-one to the setting we apply, since they do not take into consideration an oligopolistic market setting and product differentiation is defined as different departure times for the same origin-destination pair. One could argue that a different departure time does not belong to product differentiation because in essence a different trip is offered. Lin and Sibdari (2009) apply a random utility framework and a dynamic duopolistic market setting to show that a Nash pricing equilibrium exists. Contrary to Lin and Sibdari (2009), we allow for more than only the price instrument in the oligopolistic competition. We explicitly address the quality choice per product type of airlines as well as the number of product types, thereby adding more insight into oligopolistic product differentiation. Furthermore we relax the implicit assumption of independence of irrelevant alternatives made by Lin and Sibdari (2009) by using a nested logit model instead of a multinomial logit model. To the best of our knowledge, we are the first to address explicitly the strategic behavior of airlines regarding product differentiation in terms of prices, qualities and number of fare types, in airline revenue management using a non-localized competition random utility framework.

Section 2 discusses the different ways to model the demand side, introduces the cost side of the model, and the manner to solve for the Cournot-Nash equilibrium. In Section 3 we apply the model to a duopoly, showing equilibrium outcomes in prices, qualities and number of fare types. Section 4 provides numerical analyses for the duopoly market setting relaxing the symmetry assumptions used in the preceding sections. Section 5 provides a discussion and conclusions.

Section 2. Model

2.1 Demand

The nested multinomial logit model is used to analyze the behavior of multiproduct firms (see Anderson et al. (1992)). Suppose that in total n firms are present in the market, $i = 1\ldots n$, and that each firm sells m_i products, $k_i = 1\ldots m_i$. The demand for each product depends on the utility derived by person t, $t = 1\ldots T$, from that product. We use the number of restrictions of a specific fare type as a proxy for the quality of the product. The utility depends on the price, p_{ik}, the number of restrictions of a fare type, r_{ik} and an alternative specific constant:

$$V_{ikt} = ASC_{ik} - \theta_t r_{ik}^2 - \alpha_t p_{ik} + \varepsilon_{kt} + \varepsilon_{ikt}. \tag{2.1}$$

In equation (2.1), θ_t is the valuation of the proxy of quality by consumers and α_t is the valuation of price by consumers. In principal these utility parameters can differ across consumers, for now we suppress subscript t. The alternative specific constant captures the unobserved sources of utility of the specific alternative. The product specific and combined error terms (assuming $Var(\varepsilon_i) = 0$) are assumed to be independent and identically Gumbel distributed with scale parameters μ_1 and μ_2 respectively. So, $Var(\varepsilon_{ik}) = \mu_1^2 \pi^2 / 6$ and $Var(\varepsilon_k) = \mu_2^2 \pi^2 / 6$.

Based on *a priori* conjectures about the nesting of alternatives, two approaches are available, see Figure 1 and Figure 2. In the first approach, as followed by Anderson et al. (1992), each firm is associated with a specific nest, while in the second approach each fare type is associated with a specific nest. In this latter approach m_i looses the subscript, becomes m, and represent the total varieties available in the market. The outside alternative is represented by V_{out} and can be handled as a degenerated alternative. In section 3 it becomes clear that the nest structure (as imposed by the modeler) is essential in describing the patterns of production differentiation.

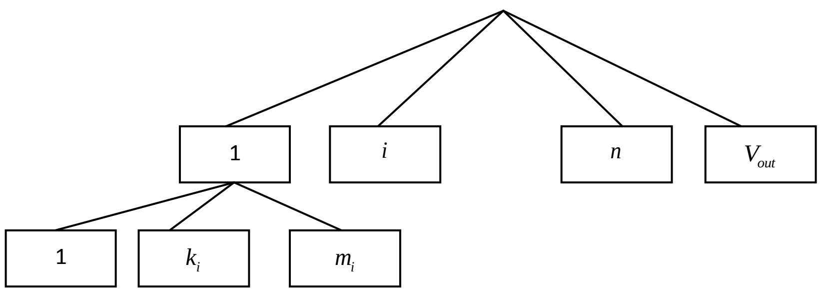

Figure 1 Nested logit set up with firm specific nests

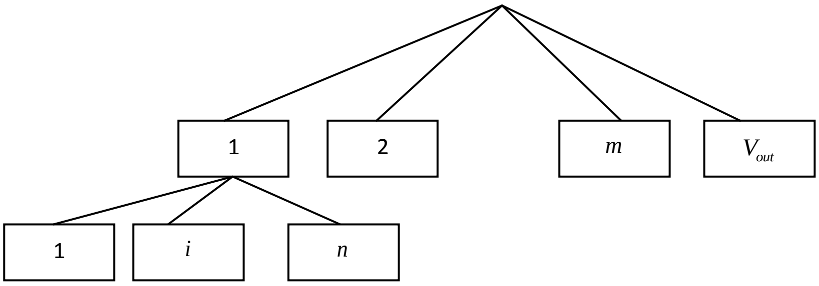

Figure 2 Nested logit set up with fare type specific nests

The demand for variant *k* of firm *i* in the nested logit model as depicted in Figure 1 is as follows:

$$\tilde{X}_{ik}\left(\underline{p};\underline{r};\underline{m}\right)=T\mathrm{P}_i\mathrm{P}_{k|i}, \qquad k=1\ldots m_i, i=1\ldots n, \tag{2.2}$$

with

$$\mathrm{P}_{k|i}=\frac{\exp\left(\dfrac{ASC_{ik}-\theta r_{ik}^2-\alpha p_{ik}}{\mu_2}\right)}{\sum_{h=1}^{m_i}\exp\left(\dfrac{ASC_{ih}-\theta r_{ih}^2-\alpha p_{ih}}{\mu_2}\right)}, \tag{2.3}$$

and

$$\mathrm{P}_i=\frac{\exp\left(S_i/\mu_1\right)}{\sum_{j=1}^{n}\exp\left(S_j/\mu_1\right)+\exp\left(V_{out}/\mu_1\right)}, \tag{2.4}$$

and

$$S_j=\mu_2\ln\left(\sum_{h=1}^{m_j}\exp\left(\frac{ASC_{jh}-\theta r_{jh}^2-\alpha p_{jh}}{\mu_2}\right)\right). \tag{2.5}$$

The probability of choosing a product of a certain firm is the probability that the specific firm is chosen multiplied by the conditional probability that product *k* is chosen. The demand function is a function of $\underline{p}=\left(\underline{p}_1\ldots\underline{p}_n\right)$ with $\underline{p}_i=\left(p_{i1}\ldots p_{im_i}\right)$, and $\underline{r}=\left(\underline{r}_1\ldots\underline{r}_n\right)$ with $\underline{r}_i=\left(r_{i1}\ldots r_{im_i}\right)$ and $\underline{m}=\left(m_1\ldots m_n\right)$. We assume that all p_i, r_i and m_i are larger than zero. The probability that firm *i* is selected, equation (2.4), depends on the expected benefits of choosing firm *i*, which is based on all product varieties offered by firm *i*, the expected benefits of all *n* firms in the market, S_j, see equation (2.5), and the utility derived from the outside alternative, V_{out}. The scale parameters need to be restricted in order to satisfy the requirements of maximizing random utility theory: $\mu_1 \geq \mu_2 \geq 0$. This means that the unobserved heterogeneity of preferences at the higher nest is at least as large as the unobserved heterogeneity at the lower level. According to Anderson et al. (1992) a more specific and behavioral interpretation is possible: μ_1 can be interpreted as measure of interfirm unobserved heterogeneity, whereas μ_2 can be seen as a measure of intrafirm unobserved heterogeneity as well as the preference for diversity. Both interfirm and intrafirm heterogeneity are increasing in μ_1 and μ_2 respectively. Since $\mu_1 \geq \mu_2 \geq 0$ applies, interfirm heterogeneity is larger compared to intrafirm heterogeneity implying that varieties produced by a firm are most similar to each other.

The model as depicted in Figure 2 has a different assumption about the relative magnitude of intrafirm and interfirm hetergoneity:

$$\tilde{X}_{ik}\left(\underline{p};\underline{r};\underline{m}\right) = T\mathrm{P}_k \mathrm{P}_{i|k}, \qquad k = 1\ldots m, i = 1\ldots n, \tag{2.6}$$

$$\mathrm{P}_{i|k} = \frac{\exp\left(\dfrac{ASC_{ik} - \theta r_{ik}^2 - \alpha p_{ik}}{\mu_2}\right)}{\sum_{j=1}^{n} \exp\left(\dfrac{ASC_{jk} - \theta r_{jk}^2 - \alpha p_{jk}}{\mu_2}\right)}, \tag{2.7}$$

$$\mathrm{P}_k = \frac{\exp\left(S_k / \mu_1\right)}{\sum_{h=1}^{m} \exp\left(S_h / \mu_1\right) + \exp\left(V_{out} / \mu_1\right)}, \tag{2.8}$$

$$S_h = \mu_2 \ln\left(\sum_{j=1}^{n} \exp\left(\frac{ASC_{jh} - \theta r_{jh}^2 - \alpha p_{jh}}{\mu_2}\right)\right). \tag{2.9}$$

Here, the probability choosing product ik is the probability of choosing fare type k multiplied by the conditional probability of choosing firm i (given fare type k). From the definition of the nest structure it follows that $\mu_2 \geq \mu_1 \geq 0$ must hold in order to satisfy the random utility maximization theory. So, here the crucial assumption is whether or not the interfirm heterogeneity outweighs the intrafirm heterogeneity, as is the case in Figure 1, or not, as depicted in Figure 2. In essence this is an empirical question, however we will continue in the next section with both models, since both models give diametrical insights in patterns of product differentiation.

It is also possible to write down the demand model assuming individual (or group) specific utility parameters. The random parameters multinomial logit model is best suitable for this purpose (Berry et al., 1995). In contrast to most econometric applications we do not assume that the mixture is continuous, but we assume here a discrete mixture. Therefore we have a latent class model as frequently applied in marketing and consumer behavior sciences (Swait, 1994). The model is in particular useful if G segments are distinguished in the population, each of which has its own choice behavior of preferences. In the case of the aviation markets, airlines could segment the market based upon income, employment, travel purpose etc. The utility of individual t belonging to the latent segment g ($g = 1 \ldots G$) for alternative ik (the firm – fare type combination) is defined as:

$$V_{ikt|g} = \beta_g X_{kt} + \upsilon_{ikt|g}. \tag{2.10}$$

β_g is a row vector with the utility parameters for segment g, X_{ikt} is a column vector of alternative and individual characteristics and $\upsilon_{ik|g}$ is the random component of the utility (IID Gumbel distributed with scale parameter μ_g). So, the demand for variant ik is:

$$\tilde{X}_{ik}\left(\underline{p};\underline{r};\underline{m}\right) = \sum_{t=1}^{T} \mathrm{P}_{ikt|g} \mathrm{P}_{tg}, \qquad k = 1 \ldots m, i = 1 \ldots n, g = 1 \ldots G, \tag{2.11}$$

with

$$\mathrm{P}_{ikt||g} = \frac{\exp\left(\frac{\beta_g X_{ikt}}{\mu_g}\right)}{\sum_{j=1}^{n}\sum_{h=1}^{m} \exp\left(\frac{\beta_g X_{jht}}{\mu_g}\right)}, \tag{2.12}$$

and

$$\mathrm{P}_{tg} = \frac{\exp\left(\frac{\gamma_g Z_t}{\theta}\right)}{\sum_{l=1}^{G} \exp\left(\frac{\gamma_l Z_t}{\theta}\right)}. \tag{2.13}$$

The demand for variant ik by person t is the probability that person t chooses alternative ik conditional on the segment g, equation (2.12), multiplied by the probability that person t belongs to segment g, equation (2.13). The total demand for variant ik is the individual demand summed over all individuals or group of individuals. The scale factor, μ_g, needs to be larger than 0, implying $Var\left(\upsilon_{kt|g}\right)=\mu_g^2\pi^2/6$. The scale parameter θ is larger than zero as well, γ_g and Z_t are the row and column vector representing the utility parameters of belonging to a segment, *g*, and individual characteristics respectively.

2.2 Supply

In order to analyze the pattern of product differentiation, we need to include information about the supply side of the model. Equation (2.14) shows the profit function which we use to describe the supply side.

$$\pi_i\left(\underline{p};\underline{r};\underline{m}\right)=\sum_{k=1}^{m_i}\left(p_{ik}-c_{ik}\left(r_{ik}\right)\right)\tilde{X}_{ik}-F_{ik}-K_i, \qquad i=1\ldots n. \tag{2.14}$$

We assume that $c_{ik}\left(r_{ik}\right)=c-\tau r_{ik}$. So, for each restriction added, the cost per passenger declines. In order to guarantee an equilibrium, the costs are linear in quality (the number of restrictions) while strictly concave in the utility of the alternative. The τ parameter represents the constant marginal cost per restriction. Furthermore, we assume that each airline faces quality-independent fixed costs per variant, F, and fixed (sunk) costs, K.

The airline optimizes profit in two stages. In the first stage, the airline determines the number of products, m_i, to offer, whereas in the second stage the airline determines the quality, r_{ik}, and optimal price, p_{ik}, simultaneously. This two-stage game ensures that each firm takes into account the effect of their actions in the first stage (number of varieties) on the second stage, the price and quality competition stage. We use backward induction to solve this optimization problem, finding optimal prices and qualities for m_i varieties first and use these outcomes to determine the optimal m_i. We use the Cournot-Nash equilibrium concept in both stages of the game. In the second stage airlines optimize their profits by setting prices and quality, given the prices and qualities set by their competitors. While in the first stage the number of varieties is optimized given the number of varieties offered by their competitors. So, given number of varieties $\underline{m}$ in the first stage, the corresponding simultaneous price quality subgame, assuming quality is a continuous variable, is solved by $p_1^*\left(\underline{r};\underline{m}\right)\ldots p_n^*\left(\underline{r};\underline{m}\right)$ and $r_1^*\left(\underline{m}\right)\ldots r_n^*\left(\underline{m}\right)$ ensuring that:

$$\pi_i\left(\underline{p}_i^*,\underline{p}_{-i}^*;\underline{r}_i^*,\underline{r}_{-i}^*;\underline{m}\right)\geq\pi_i\left(\underline{p}_i,\underline{p}_{-i}^*;\underline{r}_i,\underline{r}_{-i}^*;\underline{m}\right). \tag{2.15}$$

$\underline{p}^*\left(\underline{r}^*;\underline{m}\right)$ and $\underline{r}^*\left(\underline{p}^*;\underline{m}\right)$ are the resulting optimal price and quality vectors at which point profits need to be evaluated in the next stage, denote this profit function as $\bar{\pi}_i\left(\underline{m}\right)\equiv\pi_i\left(\underline{p}^*;\underline{r}^*;\underline{m}\right)$. The second-stage game equilibrium is then characterized by $m_1^*\ldots m_n^*$, satisfying the following condition:

$$\hat{\pi}_i\left(m_i^*,m_{-i}^*\right)\geq\hat{\pi}_i\left(m_i,m_{-i}^*\right). \tag{2.16}$$

So, the subgame perfect Nash equilibrium is characterized by $\underline{m}^*$, $\underline{r}^*\left(\underline{p}^*;\underline{m}\right)$ and $\underline{p}^*\left(\underline{r}^*;\underline{m}\right)$. If $\underline{r}^*$ and $\underline{p}^*$ are evaluated at $\left(\underline{m}^*\right)$, we have found the corresponding equilibrium path.

The nested multinomial logit model as presented in equations (2.2) until (2.5) offers a natural manner to construct a welfare measure. The so-called log-sum, or the expected maximum utility of all alternatives in the market, is an approximation for the consumer surplus in this model, while the combined profits of all firms form the producer surplus. Within the framework of the utilitarian (indirect) welfare function, a component representing aggregate income, $Y=Ny$, should be added. So, total welfare becomes:

$$W=T\mu_1\ln\left(\sum_{j=1}^{n}\exp\left(S_j/\mu_1\right)+\exp\left(V_{out}/\mu_1\right)\right)\frac{1}{\omega}+\omega Y+\sum_{i=1}^{n}\sum_{k=1}^{m_i}\left(p_{ik}-c_{ik}\left(r_{ik}\right)\right)\tilde{X}_{ik}-F_{ik}-K_i. \tag{2.17}$$

Here, ω is the marginal utility of income, so all parts of the welfare functions are in monetary terms. In case we use the nested logit model as depicted in Figure 2 , we replace the first part of equation (2.17) by the natural logarithm of the denominator of equation (2.8) multiplied by $N\mu_1\left(1/\omega\right)$.

Section 3. Duopoly

We consider a duopoly with two firms, i and j. As already indicated in the last section, the airline maximizes profits with respect to number of fare types, prices and qualities. The profit functions in a duopoly market structure are firm specific:

$$\pi_i\left(\underline{p};\underline{r};\underline{m}\right) = \sum_{k=1}^{m_i}\left(p_{ik} - c_{ik} + \tau_{ik} r_{ik}\right)\tilde{X}_{ik} - F_{ik} - K_i, \qquad i = 1, 2. \tag{3.1}$$

With $\tilde{X}_{ik}$ as defined in equations (2.2), (2.6) and (2.11) respectively. Successively we show the first best response functions in prices, qualities and number of fare types for each of the three defined $\tilde{X}_{ik}$.

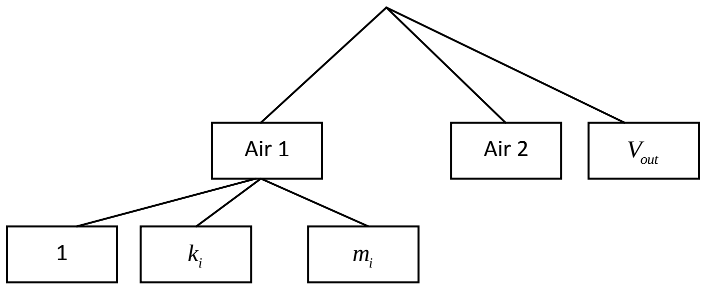

Figure 3 Nested logit set up with firm specific nests in duopolistic market

In the first stage, the best response functions in prices and qualities in the demand model of equation (2.2) and as depicted in Figure 3 are as follows:

$$\left(p_{ik} - c_{ik}\right) = \frac{\dfrac{\mu_2}{\alpha} + \left(1 - \dfrac{\mu_2}{\mu_1}\left(1 - \mathrm{P}_i\right)\right)\displaystyle\sum_{h=1, h\neq k}^{m_i}\left(p_{ih} - c_{ih} + \tau_{ih} r_{ih}\right)\mathrm{P}_{h|i}}{\dfrac{\mu_2}{\mu_1}\left(1 - \mathrm{P}_i\right)\mathrm{P}_{k|i} + \left(1 - \mathrm{P}_{k|i}\right)} - \tau_{ik} r_{ik}, \tag{3.2}$$

And applying equation (3.2)

$$r_{ik} = \frac{\alpha \tau_{ik} \mu_2}{2\theta\left(\alpha \displaystyle\sum_{h=1, h\neq k}^{m_i}\left(p_{ih} - c_{ih} + \tau_{ih} r_{ih}\right)\mathrm{P}_{h|i}\left(1 - \dfrac{\mu_2}{\mu_1} - \mathrm{P}_i\left(1 - \dfrac{\mu_2}{\mu_1}\left(2 - \mathrm{P}_i\right)\right)\right) + \mu_2\right)} \tag{3.3}$$

The absolute markup, as given in equation (3.2), is the same for each of the m varieties. Furthermore, since $\mu_1 > \mu_2$, the absolute markup is necessarily positive. Note that both the price and quality of a fare type depend on all other $m_i - 1$ varieties. Anderson et al. (1992) observe that the absolute markup is constant across all variants supplied by one firm and that this property still holds when the model allows for different (exogenous) costs and qualities of the product. This implies a major drawback of the model as depicted in Figure 3. If this model is applied to determine the simultaneous price and quality setting of a firm, an obvious solution to the set of first order conditions is the symmetric one in which the firm sets multiple products in the market which are identical in observed effects (and the instruments used by the firm) but still non-identical in the unobserved effects. In equilibrium, different prices and qualities can only be obtained via exogenous changes in the cost parameters c_{ik} and τ_{ik}. So, this model can only predict product differentiation in which the products are not differentiated by endogenous characteristics. However, it is still attractive to offer multiple (identical products) because individuals have a preference for variety (μ_2) and it is easy to see that P_i increases in the number of varieties in a nest. In determining the number of fare types, the fixed costs per variant play a major role[2].

Introducing symmetry, $p_{ik} = p_{ih} = p_i$, $r_{ik} = r_{ih} = r_i$, $c_{ik} = c_{ih} = c_i$, $\tau_{ik} = \tau_{ih} = \tau_i$ and $\mathrm{P}_{k|i} = \mathrm{P}_{h|i} = \dfrac{1}{m_i}$, the optimal prices and qualities for both firm *i* and *j* become:

$$p_i^* = c + \frac{\mu_1}{\alpha(1-P_i)} - \tau r_i^*, \qquad i = 1,2, \tag{3.4}$$

$$r_i^* = \frac{\tau_i \mu_2 m_i}{2\theta(2\mu_2 + m_i - 2\mathrm{P}_i\mu_2 - 2)}\alpha(1-\mathrm{P}_i), \qquad i = 1,2. \tag{3.5}$$

So, the profit function in the symmetric case becomes:

$$\pi_i = \frac{\mu_1}{\alpha(1-\mathrm{P}_i)} T\mathrm{P}_i - m_i F_i - K_i. \tag{3.6}$$

Given (3.6) we can determine the first stage outcome of the game as $\max\left[\pi_i(m_i = 1)\ldots\pi_i(m_i = 2)\ldots\pi_i(m_i = \infty)\right]$. We need to define the derivative of the profit function with respect to m_i to find the optimal number of varieties, taking m_i as a continuous variable. Appendix A shows in an intuitive way that the equilibrium in m^* exists and is unique, taking m as a

[2] Based on simulation. These simulations exercises are not included in this paper neither for the nested logit model as depicted in Figure 3 nor for the random parameter model, since both models do not overcome the constant markup property. However, the simulation results are available upon request. The numerical analysis in section 4 is totally devoted to the nested logit model as depicted in Figure 4.

continuous variable. Unfortunately the derivative cannot be written down in an analytical expression for m_i, due to implicit definitions of the best response functions in prices and the non-linearity of the demand model.

As already mentioned in the last section, the latent class discrete choice model can be used to check whether or not the constant markup property still holds when the preferences for quality and prices are distributed across (groups of) individuals. Again, in the first stage the airline maximizes profits by optimizing the profit function with respect to the price and number of restrictions:

$$\pi_i = \sum_{k=1}^{m_i}\sum_{t=1}^{T}\sum_{g=1}^{G}\left(p_{ik} - c_{ik}\right)\left(\frac{\exp\left(\frac{\beta_g X_{ikt}}{\mu_g}\right)}{\sum_{j=1}^{n}\sum_{h=1}^{m}\exp\left(\frac{\beta_g X_{jht}}{\mu_g}\right)}\right)\left(\frac{\exp\left(\frac{\gamma_g Z_t}{\theta}\right)}{\sum_{l=1}^{G}\exp\left(\frac{\gamma_l Z_t}{\theta}\right)}\right) - F_{ik} - K_i, \quad i = 1,2. \tag{3.7}$$

While analytically it is not that easy to see whether or not the random parameter model overcomes the constant markup property, a very simple numerical analysis shows that the property still holds. Assume that T (groups of) individuals are distinguished, each with a different income, and that two latent classes are defined by the airline (researcher): business and leisure traveler. The utility function of the conditional probability is defined as in equation (2.10). The scale parameters as well as the utility parameters in the conditional choice probability differ over the two defined classes. Simulation in Matlab shows that, using numerical values for all unknown parameters, independent of the number of varieties per firm supplied, each variety of the same firm has the same (constant) markup and the same number of restrictions. So, the assumption about different elasticities of demand, the elasticity of the market share of the outside alternative with respect to a change in the price of the outside alternative is calculate for each (group of) individual(s), across individuals does not play a role in the pattern of product differentiation.

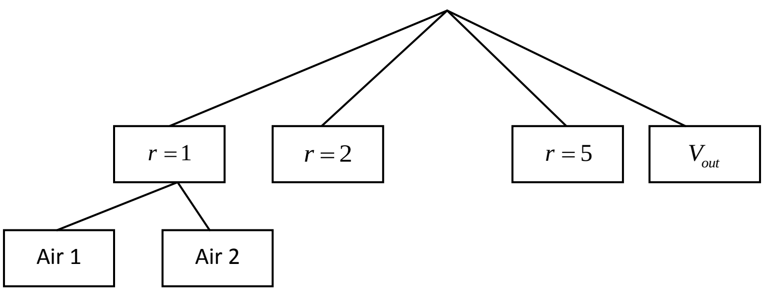

Figure 4 Nested logit set up with fare type specific nests in duopolistic markets, r=1..5.

As noted earlier, the assumption that unobserved intrafirm is smaller compared to unobserved interfirm heterogeneity is the major reason that there is a pattern of product differentiation which is predicted to have similar prices and qualities over all products of the same firm. The nested logit model as introduced in the equations (2.6) until (2.9) and depicted in Figure 4 provides a different assumption about the heterogeneity and therefore, as will become clear now, a different pattern of product differentiation.

In the first stage, the best response function in prices is as follows:

$$\left(p_{ik}-c_{ik}\right)=\frac{\dfrac{\mu_k}{\alpha}-\mu_k\sum_{h=1,h\neq k}^{m_i}\left(p_{ih}-c_{ih}+\tau_{ih}r_{ih}\right)\tilde{X}_{h|i}}{\dfrac{\mu_k}{\mu_1}\left(1-\mathrm{P}_i\right)\mathrm{P}_{k|i}+\left(1-\mathrm{P}_{k|i}\right)}-\tau_{ik}r_{ik}, \tag{3.8}$$

In contrast to equation (3.2), it can be now readily verified that the markup is not constant anymore across the varieties of the same firm. The first order condition for each variety is different because the interfirm heterogeneity (or intra fare type heterogeneity), μ_k, is different for the different varieties. The way to determine the optimal number of restrictions per variety differs compared to the former approach. We assume that the range of restrictions (and therefore possible nests) is limited, in this case $r=1\ldots5$, as depicted in Figure 4. The airlines determine in which of the nests (with accompanying number of restrictions / service level) they position their product(s) with the constraint that an airline can just position one product in each nest. So, both airlines choose the strategy defined as:

$$\max\left[\pi_i\left(m_i=1,r=1\right)\ldots\pi_i\left(m_i=1,r=2\right)\ldots\pi_i\left(m_i=1,r=5\right)\ldots\pi_i\left(m_i=5,r=[1,2,3,4,5]\right)\right] \tag{3.9}$$

Without loose of generality we can also assume that both m_i and r_i are less restricted and that they are in the range $\left[1\ldots\infty\right]$, however that would increase the complexity of the model without increasing the insights in the model. In the next section we show a numerical example to illustrate the pattern of product differentiation implied by this model.

Section 4. Numerical analysis

Here we show the simplest, and very restrictive, set-up needed for a numerical analysis to get a better understanding of the model as depicted in Figure 4. In later versions of this paper, the numerical analysis will be extended.

Here we only show that if the number of nests (and number of varieties supplied) are exogenous determined, product differentiation in prices occurs, this in contrast to the nested logit model as presented in Figure 3. We assume that there are two nests, one with a fare type with only one restriction (high quality), and the other nest with a fare type of four restrictions (low quality).

Table 2 and Table 3 show the supply and demand parameters used as input for this small numerical analysis. The constant part of the marginal costs per passenger, c_m, is in the range as reported by Brueckner and Spiller (1994). The fixed costs per variant, F , are chosen in such a way that combined with the number of potential passengers in the market, T , the optimal number of products are between zero and five.[3] Since we are here not interested in the optimal number of firms, the fixed costs per firm K can be set at zero without affecting the relative profit and welfare results. The demand parameters as stated in Table 3 are based on literature, in particular Pels et al. (2000), Algers and Beser (2001), Balcombe et al. (2009) and Adler et al. (2010). Since parameter estimates cannot be directly compared over studies, we calculate in our numerical analysis the accompanying price elasticities of market share and compare these with the values of the elasticities as reported in the literature. Note that we distinguish two airlines, airline 1 and airline 2, in which airline 1 is a conventional carrier (higher average cost, less cost advantage from more restrictions, but lower fixed costs per variety) and airline 2 is a low cost carrier. Furthermore, we assume that the products with more restrictions are more similar (higher competition) than fare types with less restrictions.

Table 2 Cost parameters

Parameter	F_1	F_2	c_1	c_2	τ_1	τ_2	K
Value	30000	60000	150	140	10	15	0

Table 3 Demand parameters

Parameter	θ	α	μ_1	$\mu_{2,nest1}$	$\mu_{2,nest2}$	ASC	V_{out}	T	Y
Value	1.2	0.04	1	0.6	0.8	10	1	10000	0

[3] Based on simulation exercises, not shown here.

For this scenario, the pattern of product differentiation for both airlines is characterized by Figure 5:

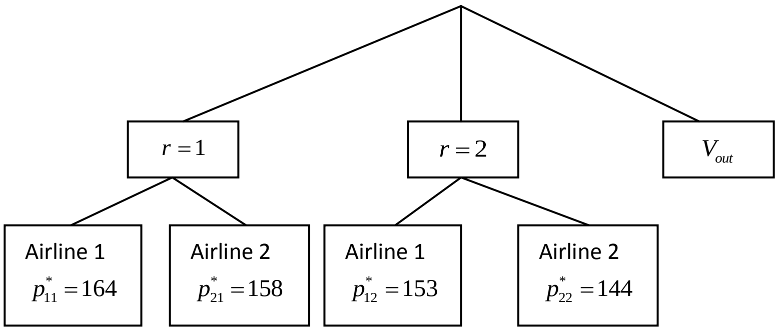

Figure 5 Simulation results for nested logit model with exogenous number of fare types and restrictions

As can be noticed from the figure, the way the nested logit model now is defined results in intrafirm product differentiation. As expected, the higher costs of airline 1 result in higher prices, and therefore lower market share. Further (numerical) research should, using this set up, focus on taking both the nest structure as well as the number of restrictions and fare types as endogenous. Furthermore, the way how to deal here with degenerated alternatives should be clarified.

5. Conclusion

A quick scan of airline pricing nowadays, shows that airlines use product differentiation in their yield management policies, resulting in different airlines offering a different number of products with accompanying prices and qualities. In the current paper we studied three different models to study this strategic behavior of airlines. The model we use can be described as a non-localized competition random utility model, to the best of our knowledge we are the first to study yield management in this framework, explicitly addressing prices, qualities and number of products as instruments of yield management.

The theoretical model shows that, using the nested multinomial logit model with fixed consumer preferences for price and quality, and applying non-cooperative game theory, the equilibrium, given a certain parameterization, in prices, qualities and product varieties is existent and unique. Conditional on the assumptions about the unobserved heterogeneity, we find that the markup for each product variety is constant, even under different marginal costs and qualities. This property results in non realistic prediction of the pattern of product differentiation: each firm produces multiple products with the same price and quality characteristics. This seems to be contradicting with what we observe in reality. This somewhat unexpected results, forced to rethink the model in order to avoid the constant markup property as is captured in the multinominal logit model and the nested logit model as advocated by Anderson et al. (1992).

A quick simulation exercise shows that this constant markup cannot be avoided using a random parameter logit (latent class) model allowing consumer preferences to be distributed across consumers. The reversed nested logit model, with fare types as nests and airlines as elemental alternatives, turned out to be the only model circumventing the constant markup property.

Further research should be devoted to two main topics. First, the applied cost functions used for the numerical analyses need to be improved and made more realistic. Secondly, and more important, a next step to circumvent (or to understand) the reasoning behind the constant markup property should be in the research agenda.

Appendix A

In order to show that the symmetric simultaneous price quality equilibrium exists, two conditions need to hold[4]:

$$\frac{\partial^2 \pi_i}{\partial^2 p} = \frac{\alpha N}{\mu_1}\left(\mathrm{P}_i\left(\mathrm{P}_i - 1\right) - 1\right) < 0, \tag{A.1}$$

and

$$\frac{\partial^2 \pi_i}{\partial^2 p_k}\frac{\partial^2 \pi_i}{\partial^2 q_k} - \left(\frac{\partial^2 \pi_i}{\partial p_k, q_k}\right)^2 = -\frac{\alpha N^2}{\theta \tau^2}\left(\left(\mathrm{P}\left(\mathrm{P}-1\right)-1\right)\Phi + \alpha\theta\left(\Omega\right)\right) > 0, \tag{A.2}$$

where $\Phi = \left(\alpha\left(\theta\left(\mathrm{P}-1\right)\left(1-2\tau^2\right)+1\right) + \mu_2\theta\tau\mathrm{P}\right)$ and $\Omega = \left(\mathrm{P}\left(\mathrm{P}-2\right) + \tau^2\left(1-\mathrm{P}\right)\right)^2$.

In words, the second order derivative of the profit function with respect to prices needs to be negative, equation (A.1), while the product of the second order derivative with respect to prices and qualities minus the square of the second order partial derivative to prices and qualities, equation (A.2), needs to be positive. In the first case, it is obvious that the condition as mentioned in equation (A.1) holds. In the latter condition it is less obvious, however it is observed that if $\tau \geq 1$, $\Phi > 0$ and the absolute value of $\left(\mathrm{P}\left(\mathrm{P}-1\right)-1\right)\Phi$ should be larger than $\alpha\theta\Omega$ for the condition in equation (A.2) to hold. There is no strict expression for this condition, although we can infer that the condition holds as long as P is sufficiently large (not near zero) and τ is not too large.

As it is shown that the symmetric price quality equilibrium exists, we conjecture that the asymmetric equilibrium is existent and unique as well. Unfortunately it is unwieldy to write down the appropriate second order conditions and the resulting Hessian of the determinants in case of asymmetric equilibriums. Therefore, in our numerical analysis we calculate the determinants of the Hessian, in order to be sure that the outcomes represent an equilibrium.

---

[4] Note that we here specify quality as a continuous variable increasing in q, in order to follow Anderson et al. (1992) more closely. This is the mirror case compared to the definition used in section 3, however, without loose of generality, the reasoning about existence and uniqueness of the equilibrium are the same.

We now use the above analysis and the arguments as provided by Anderson et al. (1992) to show that the equilibrium in prices, qualities and number of products exists. There exists a markup $\left(\hat{p}_k - c_k - \tau\hat{q}_k\right)$ for product ik which is large enough compared to the markup of all other $j = 1\ldots m$ products to ensure that the derivative with respect to prices becomes negative, even accounting for the fact that the probability $\mathrm{P}_{k|i}$ goes to zero. So, if $\left(\hat{p}_k - c_k - \tau\hat{q}_k\right)$ is large enough:

$$\left.\frac{\partial\pi}{\partial p_k}\right|_{p_k=\hat{p}} < 0. \tag{A.3}$$

We know that profit is a function of the *m* dimensional vectors, $\underline{p}_h$ and $\underline{q}_h$ with the components $p_j,\ j = 1...m$ and $q_j,\ j = 1...m$. Furthermore, the profit function, $\pi\left(\underline{p}_h; \underline{q}_h; \underline{m}\right)$, has a global maximizer $\left(\underline{p}_h^M; \underline{q}_h^M\right)$ in the compact set $\left[c + \tau\hat{q}_k, \left(\hat{p}; \hat{q}\right)\right]^m$.

Suppose that $\left(\underline{p}_h^M; \underline{q}_h^M\right)$ is a boundary solution, than at least one component of $\left(\underline{p}_h^M; \underline{q}_h^M\right)$ (one of the product prices) needs to be equal to $c + \tau\hat{q}_k$ or $\hat{p}$. It can be shown that (Anderson et al., 1992):

$$\left.\frac{\partial\pi}{\partial p_k}\right|_{p_k=c+\tau\hat{q}_k} > 0,\ k = 1\ldots m. \tag{A.4}$$

So, equations (A.3) and (A.4) show that at the boundary solutions the derivate with respect to prices is not equal to zero, so both of the boundaries cannot be a profit maximizing equilibrium. Hence $\left(\underline{p}_h^M; \underline{q}_h^M\right)$ must belong to $\left[c + \tau\hat{q}_k, \left(\hat{p}; \hat{q}\right)\right]^m$ and is therefore the existent and unique solution to the first order conditions with respect to prices $p_j,\ j = 1...m$

Adler, N., Pels, E. & Nash, C. (2010) High-Speed Rail and Air Transport Competition: Game Engineering As Tool for Cost-Benefit Analysis, *Transportation Research Part B: Methodological*, *forthcoming*.

Algers, S. & Beser, M. (2001) Modelling Choice of Flight and Booking Class - a Study Using Stated Preference and Revealed Preference Data, *International Journal of Services Technology and Management*, *2*(1/2), 28-45.

Anderson, S.P., De Palma, A. & Thisse, J.F. (1992) *Discrete Choice Theory of Product Differentiation*. Cambridge, MA: The MIT Press.

Balcombe, K., Fraser, I. & Harris, L. (2009a) Consumer Willingness to Pay for in-Flight Service and Comfort Levels: A Choice Experiment, *Journal of Air Transport Management*, *15*(5), 221-226.

Balcombe, K., Fraser, I. & Harris, L. (2009b) Consumer Willingness to Pay for in-Flight Service and Comfort Levels: A Choice Experiment, *Journal of Air Transport Management*, *15*(5), 221-226.

Berry, S., Levinsohn, J. & Pakes, A. (1995) Automobile Prices in Market Equilibrium, *Econometrica*, *63*(4), 841-890.

Botimer, T. C. & Belobaba, P. P. (/11) Airline Pricing and Fare Product Differentiation: A New Theoretical Framework, *The Journal of the Operational Research Society*, *50*(11), 1085-1097.

Botimer, T. C. (1996) Efficiency Considerations in Airline Pricing and Yield Management, *Transportation Research Part A: Policy and Practice*, *30*(4), 307-317.

Brueckner, J. K. & Spiller, P. T. (1994) Economies of Traffic Density in the Deregulated Airline Industry, *Journal of Law and Economics*, *37*(2), 379-416.

Chamberlin, E. (1933) *The Theory of Monpolistic Competition*. Cambridge: Harvard University Press.

Dixit, A. K. & Stiglitz, J. E. (1977) Monopolistic Competition and Optimum Product Diversity, *American Economic Review*, *67*(3), 297-308.

Hotelling, H. (1929) Stability in Competition, *Economic Journal*, *39*, 41-57.

Kaldor, N. (1935) Market Imperfection and Excess Capacity, *Economica*, *2*, 35-50.

Lin, K. Y. & Sibdari, S. Y. (2009) Dynamic Price Competition With Discrete Customer Choices, *European Journal of Operational Research*, *197*(3), 969-980.

McFadden, D. (1974) *Conditional Logit Analysis of Qualitative Choice Behavior*. New York: Academic Press.

McGill, J. I. & van Ryzin, G. J. (1999) Revenue Management: Research Overview and Prospects, *Transportation Science*, *33*(2), 233-256.

Pels, E., Nijkamp, P. & Rietveld, P. (2000) Airport and Airline Competition for Passengers Departing From a Large Metropolitan Area, *Journal of Urban Economics*, *48*(1), 29-45.

Perloff, J. M. & Salop, S. C. (1985) Equilibrium With Product Differentiation, *Review of Economic Studies*, *52*(1), 107-120.

Proussaloglou, K. & Koppelman, F. S. (1999) The Choice of Air Carrier, Flight, and Fare Class, *Journal of Air Transport Management*, *5*(4), 193-201.

Swait, J. (1994) A Structural Equation Model of Latent Segmentation and Product Choice for Cross-Sectional Revealed Preference Choice Data, *Journal of Retailing and Consumer Services*, *1*(2), 77-89.

Zhang, D. & Cooper, W. L. (2009) Pricing Substitutable Flights in Airline Revenue Management, *European Journal of Operational Research*, *197*(3), 848-861.