

# **ENGINEERING OF CAR2CAR INTERACTIONS BY MEANS OF COLORED PETRI NET ROAD MODELS**

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## **ABSTRACT**

This contribution presents a new approach to traffic flow optimization on highways by means of vehicle sided rule-bases. Regarding present traffic, global traffic dynamics, expressed by state variables as density and flow as well as phenomena as traffic jams, is a direct consequence of the microscopic behavior of the vehicles. The latter depends on the driver's behavior that additionally diverges from vehicle to vehicle. A homogenization shall be a remedy to unintentional behavior of traffic according to Helbing, D. (1997). This may be realized by means of common vehicle sided rule-bases. For this purpose driver's behavior is initially not considered and autonomously driving vehicles are assumed. Every vehicle possesses an exact positioning system, robust longitudinal and lateral control, as well as an ad-hoc network adapter designated to car2car communication.

The major objective in this article is presenting a fundamental concept for the engineering of a common vehicle-sided rule-base. Based on the formal concept of vehicle classes and clusters, which are roughly speaking groups of spatially allocated vehicles, rule bases are developed by implementing the standard consensus algorithm for the coordination of the microscopic variables velocity and longitudinal distance (see Hübner et al. (2009a)). Decision-making is realized by spatial discretization of the highway to permissible positions, which are formally represented by means of Petri nets. Its places refer to permissible positions, whereas its transitions denote possible interactions (see Hübner, M., Lück, T. and Schnieder, E. (2009b)). Due to the introduction of different vehicle classes, it is reasonable to model the vehicle-vehicle-interactions by means of colored Petri Nets with a common places' capacity that equals one.

A desired global behavior of clusters may generally be achieved by choosing the timing and corresponding subset of transitions that shall fire. One approach is to choose the latter manually out of an engineering perspective. From this set of firing transitions it is possible to formally derive a vehicle-sided rule-base. The developed method presented in this paper is based on the reachability analysis of Petri Nets. In addition, by the automatic generation of a truth table a direct coding of the required vehicle-sided rule-base is possible. Several

algorithms, which generate different global behaviors for encountering of clusters and managing passing maneuvers are demonstrated. Those algorithms are compared with respect to a formal definition of traffic safety. The latter depends on the basic states of the present vehicle formation and mean velocity, as well as on the number of interactions per coordination.

The Petri net based formulation of the coordination problem is a quite natural method for the derivation of an optimal vehicle-sided rule-base.

*Keywords: multi agent systems, automatic control of traffic, cooperative control*

## INTRODUCTION

To control traffic towards an optimized global behavior, it is necessary to identify the constituents of the system 'traffic'. As depicted in Figure Fig. 1 any system may be separated into the constituents 'state', 'structure', 'behavior' and 'function' as described by Schnieder, E. (2007).

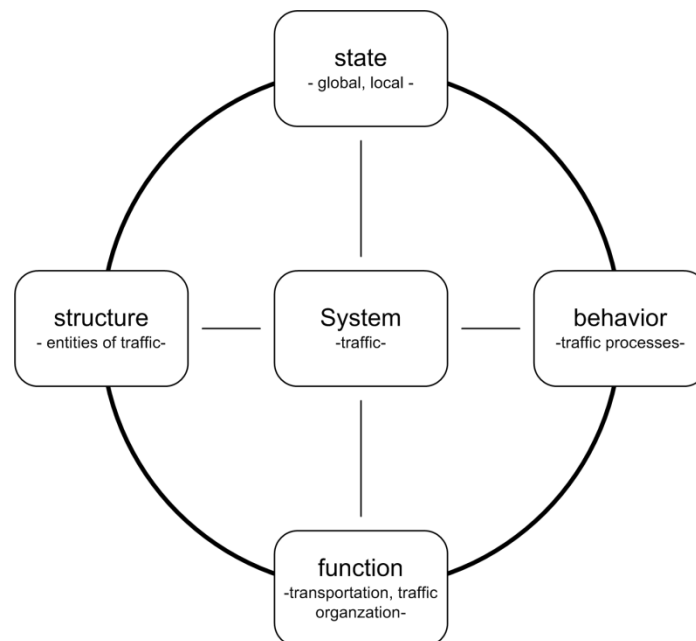


Fig. 1: constituents of a system

All constituents concur so that the attributes of the regarded system result. The structure embraces, defined by the system's boundaries to its environment, all components and their dynamic couplings which generate the observed behavior. The latter is characterized by stationary and quasi-stationary states depending on the system's inputs. According to the overall system's behavior and the corresponding states certain system functions may be assigned, out of an engineering perspective. With respect to these predefined functions the system behavior or states shall be manipulated.

The identification of the system's structure (as lumped or distributed parameter system, or even multi-agent system), means the definition of its boundaries, input and output variables

as well as their mathematical representation and couplings, is essential task of mathematical modeling. In general the latter should ensure that the observed behavior of the real-world system and its states is represented adequately by the model, depending on the intended application. A substantial requirement for this purpose is a sufficient representation of the system structure by means of an appropriate mathematical description and corresponding formalisms.

Regarding road traffic its structure may be classified as numerous driver-vehicle-units which use the same bounded infrastructure. These driver-vehicles-units often differ in the driving behavior, whereas they are able to communicate, however up to now only visually. From systems-theory perspective the question arises, how this structure may be modeled adequately. An answer may be found regarding formalisms of multi-agent-systems. A multi-agent-system is defined by interacting entities which act on or use common resources. An agent has the following intrinsic properties as given by Weiss, G. (2000).

- an agent is acting autonomously
- an agent has a certain rule-base and is target-oriented
- an agent interacts with other agents
- an agent is mobile

Due to the structural system property of interacting dynamic entities which act on common resources, road traffic may be interpreted as multi-agent-system. Basically the present agents of road traffic - means the driver-vehicle-units – indeed have a common objective: traveling to a destination as safe as possible and as quick as possible. Currently these criteria - safety and travel velocity - are often regarded as contradicting by the drivers. This may be traced back to the fact that personal prioritization leads to rather progressive or defensive driving. Because of these and other attributes which differ from driver to driver, present road traffic may be regarded as a heterogeneous multi-agent-system.

Having identified the type of the model next question is how to influence the system. Due to the model structure the system can only be affected at the rule-base, means the decision-making processes of the driver-vehicle-units. On the one hand, presently we may interpret the driving schools as institutions which build up and influence rule-bases of the prospective drivers. On the other, present road traffic is influenced by means of traffic signs, influencing the drivers' behavior discretely. This may be identified as influencing the decision-making by communication which is means for coordinating the interactions between driver-vehicle-units. From these coordinated interactions, which represent the system behavior, certain states are generated, which may be interpreted and modeled from a local perspective (driver-vehicle-unit) as well as from a global perspective (regarding overall traffic behavior of certain road segments). This relation is depicted in Fig. 2.

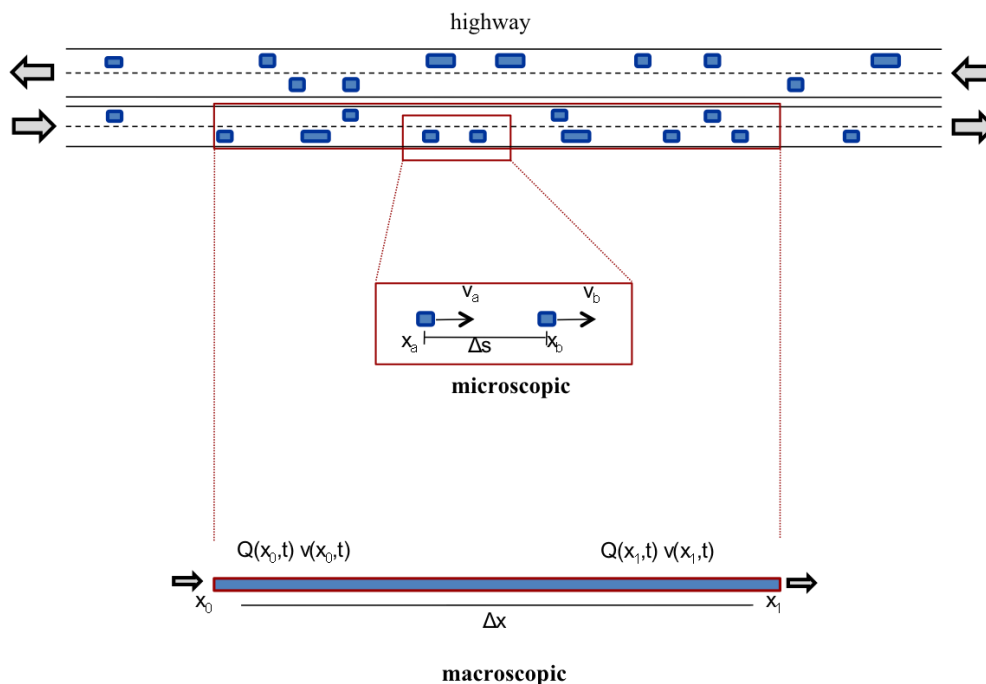


Fig. 2: traffic measures in microscopic and macroscopic view

Here it becomes obvious, that a macroscopic behavior, expressed by certain macroscopic traffic variables, is induced by certain interactions on microscopic level. According to this, present traffic models are designed depending on the objective which relations to represent. Therefore, there exist traffic models which are build only to represent the dynamics of the microscopic traffic variables as the Gipps model. Other traffic models represent the dynamics of the macroscopic traffic variables as density and flow analogously to models in fluid dynamics as in Helbing, D. (1997). However recent road traffic models do not represent microscopic behavior in that way, that it is possible to derive from these microscopic states realistic macroscopic traffic variables (see Detering, S et al. (2009)). Generally the mathematical mapping from microscopic behavior to its macroscopic system dynamic consequences is still matter of research. One possible approach to this problem may be the interpretation of road traffic as a multi-agent-system. Then the basic question to model road traffic is how to express the correct rule-bases which realize the observed microscopic behavior. Here we would have to consider stochastic variation of parameters (e.g. reaction time) that means implementing different driver behaviors and their impacts in the decision-making processes. Following these requirements, a mathematical means of description is to be found which enables modeling the system behavior and the boundary conditions given by the environment. In addition, this modeling technique should provide the possibility to design advanced driver assistance systems (ADAS), which improve besides the microscopic also the global traffic behavior, e.g. by avoiding traffic jams.

Therefore a method is presented how to generate a rule-base for a homogenous multi-agent-system in an unstructured but classified and modeled environment, based on a predefined system specification.

Hereby two basic problems are solved: insufficient communication between the vehicles and high variance in local decision-making and control-procedures. Result of applying the

presented methodology would be an idealized homogenous road traffic, realized by the implementation of a common vehicles' rule-base.

Necessary technological precondition is a full automation of the vehicles by means of robust control. For the improvement of communication it is reasonable to use protocols of car2car-communication as developed in the standards IEEE 802.11p and ETSI TCI ITS. From modeling and specification of the desired behavior the information variables are found which are to be exchanged between the vehicles.

## METHODICAL APPROACH

The methodical approach lies in the modelling of the road network and the vehicle formations as Petrinets, as given in Fig. 3.

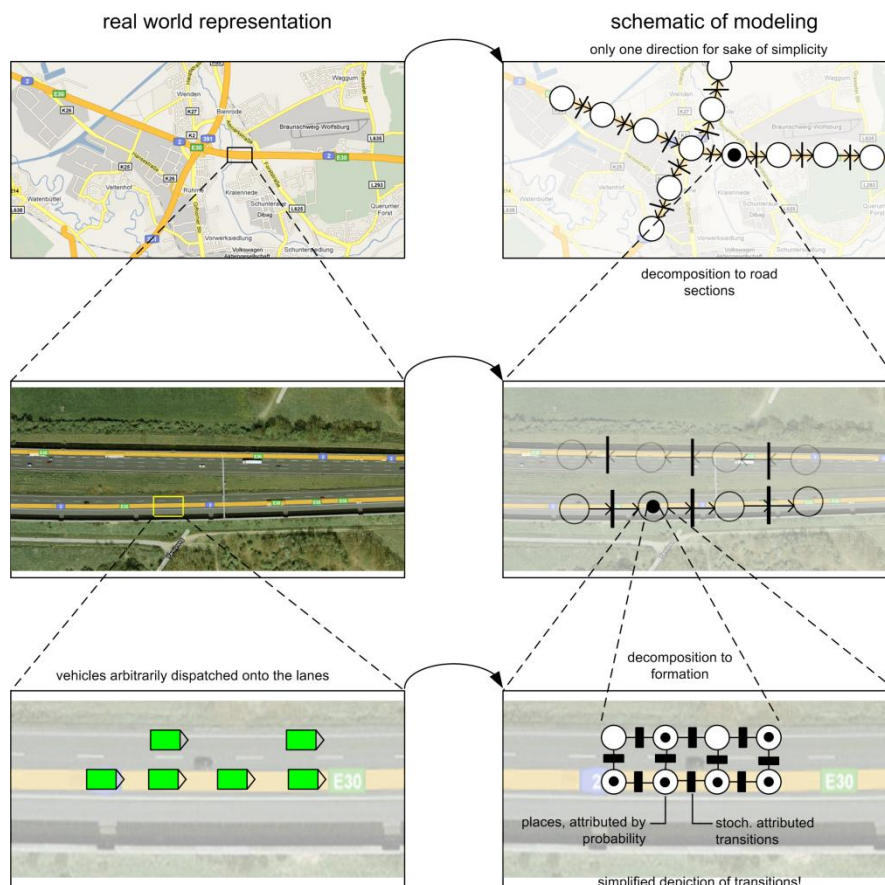


Fig. 3: schematic of the Petri net model

At this approach, on microscopic level, the vehicles are represented directly as tokens of the Petri net, whereas the places mean certain spaces where vehicles may be allocated. For the deduction of the rule-base it is not necessary to know the physical representation in form of longitudinal extend of the cell. For the later implementation of the rule-base, according to the low-level robust control of lateral and longitudinal dynamics, there will exist a mapping from this high-level abstraction to low-level set values. The representation of formations can be

folded as net in a net, which represents the road structure in lanes and segments of the road. Further we concentrate on the formation and the corresponding vehicle-sided rule-base.

Figure 4 shows an overview of the according implementation concept, which is derived from the concept for multi-vehicle control from Ren, W. et al. (2005).

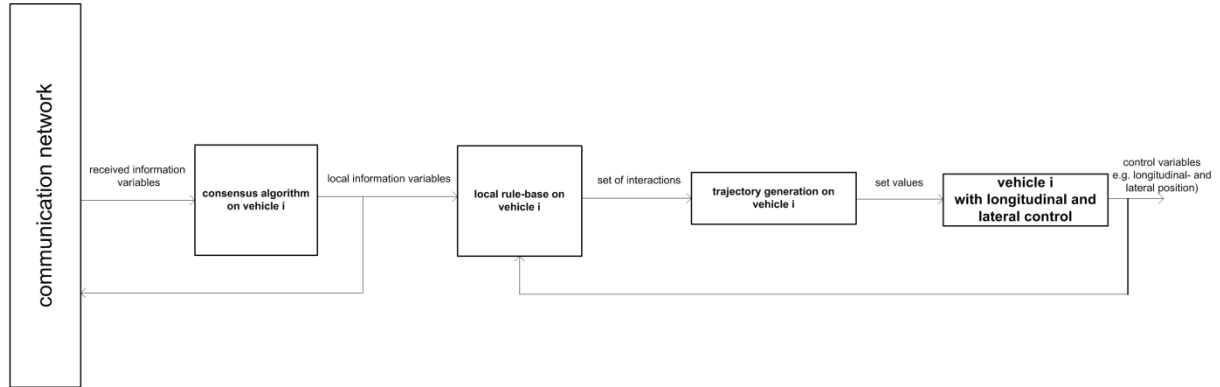


Fig. 4: Implementation Concept

Assuming an ad-hoc communication network, every vehicle has the possibility to get certain information variables of vehicles within a certain communication radius. From these exchanged data, that is to be defined according to the application, the vehicles (agents) shall generate certain common representation of the present situation. This objective may be reached by means of consensus algorithms, which calculate certain local information variables. Besides the local control variables, which are achieved by measurements, these local information variables are used by a vehicle-sided decision-making (rule-base) to generate sets of chronologic interactions between the vehicles. These are modelled in the Petrinet representation as transitions. For the sake of simplification and abstraction, in the application of traffic optimization these interactions are either of the classes 'lane-change' or 'adaption of the longitudinal position'. By nature these event-discrete sets of interactions cannot be fed directly to the time-discrete but event-continuous low-level controls of the vehicle. For this purpose a trajectory generator shall be used which transforms the chronologic interactions to time-discrete set values, means a trajectory. After a brief summary of the consensus principle, this paper's focus lies on the semi-formal derivation of the vehicle-sided rule base using Petrinets.

### Principle of Consensus-Algorithms

Assuming that the vehicles are connected via a wireless network, we have to consider the basic problems of data-transfer: latency, data-drop-out and changing network topology due to the vehicles' dynamic positions. But when we use distributed-consensus algorithms which are based on Laplacian concatenated dynamics, as the given standard-consensus in equation (1), we are able to cope with these problems.

$$\dot{x}_i = -\sum_j a_{ij} \cdot (x_i - x_j) \quad (1)$$

Here,  $x_i$  denotes an arbitrary information variable, that is exchanged between the vehicles, and  $a_{ij}$  represents the entries of the associated adjacency matrix, corresponding to the (time-variant) communication network. At the application of road traffic natural information variables would be velocity and relative longitudinal position. Concatenating the information variables of each vehicle leads to the expression of state-space dynamics of all vehicles, given by equation (2).

$$\dot{\underline{x}} = -\underline{L} \cdot \underline{x} \quad (2)$$

$\underline{L}$  denotes the so-called Laplacian matrix which is defined by equation (3):

$$\underline{L} = \underline{D} - \underline{A} \quad (3)$$

The matrix  $\underline{D}$  is given in the corresponding literature as degree matrix and  $\underline{A}$  denotes the adjacency matrix of the communication network.

The assumption of a bidirectional communication between the vehicles induces an undirected graph topology. Vehicles that are not within the communication distance of the group of  $p$  vehicles are not meant to be considered here, so the graph is always connected.

Average consensus means that  $x_i(t) \rightarrow \frac{1}{p} \sum_j x_j(0)$  for  $t \rightarrow \infty$  and is achieved iff a directed

topology is connected and balanced. Concerning situations of a connected topology, average consensus is always reached.

The changing communication topology, which is due to the movements of the vehicles may be modelled as an infinite number of communication topologies which have certain dwell-times. This leads to piecewise constant adjacency and Laplacian matrices. It may be assumed that vehicles stay in communication radius, so that for each possible communication topology connectedness is given. For this constellation it can be shown that consensus is reached also at changing topologies (see Ren, W. (2005)).

The challenge of stochastic latency ( $\sigma$ ) in information exchange may be modelled by equation (4).

$$\dot{x}_i = -\sum_j a_{ij}(t) [x_i(t) - x_j(t - \sigma)] \quad (4)$$

Following Xiao, F. et al. (2006) latency does not affect consensus at time-invariant undirected communication topology. Therefore above consensus algorithm is core of a Petri net based decision-making for formation-flocking. It ensures the convergence of certain state-variables to desired values even at latency and changing topology. That means, the formation reaches the coordination objective, which is to be formulated as desired state after the vehicles' interactions.

## Engineering Approach for Derivation of Vehicle-Sided Rule-Base

Using upper explained consensus concept, the basic question arises, what information variables are to be exchanged. Basically these depend on the application and, regarding the implementation concept of Fig. 4, on the vehicle-sided decision-making. So first, we have to develop a method for the generation of a rule-base for a homogenous multi agent system. The authors' proposition for this procedure is given in Fig. 5.

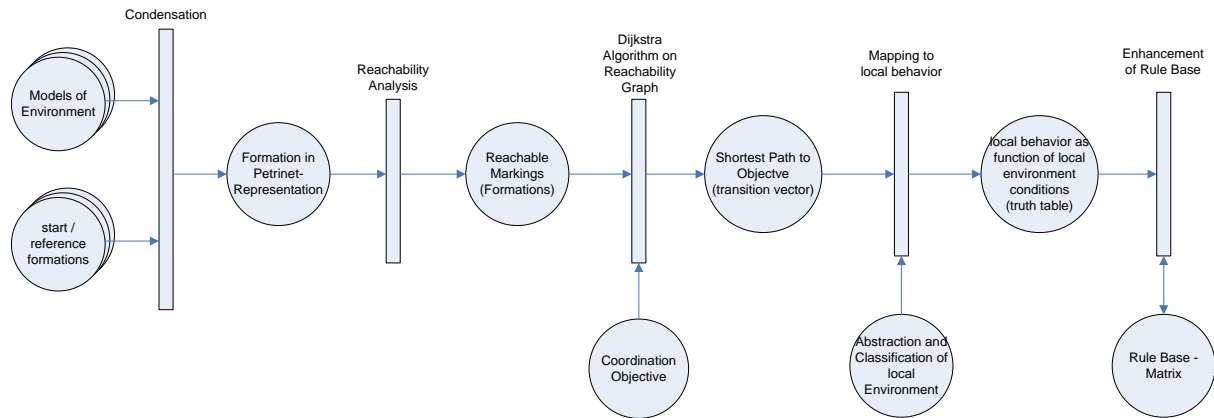


Fig. 5: Concept of generation a Vehicle-Sided Rule-Base for a Multi Agent System

A rule-base is a framework that decides a subset of actions of an agent (here: vehicle) depending on environmental parameters. So a formal representation of (several) environmental conditions is needed. Starting with models of the environment and certain start or reference formations we are able to formulate each combination as a vehicle formation in Petrinet representation. In fact, at the application of road traffic the models of the environment will include different types of road conditions e.g. off-ramps or on-ramps. Due to changing number of vehicles and their distribution to lanes, there will also exist several reference formations that have to be analyzed.

For each of these Petrinet-formations we are able to calculate the reachability graph, which represents the whole state-space. So it is also an engineering challenge to model the environment conditions as simple as possible with respect to the calculation effort.

Based on the determined reachable states (markings) and with a formal coordination objective we may calculate the shortest path to a marking that complies our specification. Implicitly, the Dijkstra algorithm minimizes the number of vehicle interactions, which may be regarded as an improvement to traffic safety: The less interactions, the less possible conflicts may exist.

Having consecutive transitions to a desired formation, next step is the mapping of these to vehicle-local behavior by means of analysis and classification of the vehicle-local environment. Hence, a kind of truth table may be generated, representing at what environmental conditions certain interactions shall be performed. The rule-base is a kind of matrix that encodes those relations. For each environment abstraction and reference formation, which have to be defined well previously, a truth table is generated and put to the rule-base matrix. We have to remark at this point, that this method may work fine at well-structured environments as on highways.

According to this, in the following subchapters this methodical approach is concretized by each step of Fig. 5 at highways with an arbitrary number of lanes and vehicles.



## RULE-BASE GENERATION FOR FLOCKING ON HIGHWAYS

### Condensation of environment and reference formation to a Petrinet representation and Reachability Analysis

Hübner M. et al. (2009a) already presented an approach to model highways by Petrinets. For an automatic rule-base generation it is furthermore necessary to have a formalism of a direct representation of each Petrinet by its incidence matrix in direct dependence of lane numbers and number of vehicles.

Regarding an exemplary Petrinet with a given reference formation as depicted in Fig. 6, we have a compact description of possibilities of interactions for each place where a vehicle may be allocated. For sake of simplicity, means state-space reduction, the latter are just adapting longitudinal relative position and lane change.

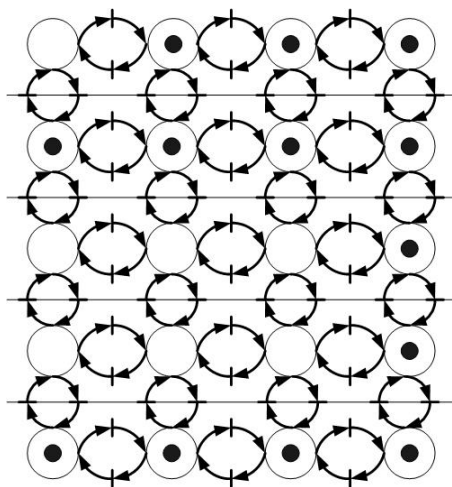


Fig. 6: Example of the reference-formation Petrinet – 5 lanes and 13 vehicles

Representing this “orthogonal” structure of places’ relations analytically is necessary for automatic generation of the corresponding incidence matrix, which is the mapping from places to transitions.

First step to this incidence matrix is the calculation of the places’ adjacency matrix. It just says which place is connected with another. Due to the symmetry of the net structure this matrix may be composed of two types of sublevel places’ adjacency matrixes: a lane-adjacency describing the connections of places in a single lane and a neighboring-lane-adjacency describing the connection of places from one lane to a directly neighboring one. These two types of adjacencies may describe the overall places’ adjacency.

Let us denote the lane-adjacency by  $A_{lane} \in \mathbb{R}^{n \times n}$ , whereas the weighting of every connection is limited to one.  $n$  equals the maximum number of vehicles in a single lane. Due to the inherent net structure one may identify that this matrix is of a special type: it is a Toeplitz matrix. This is a matrix in which each descending diagonal from left to right is constant, here equal to one. So this matrix may be constructed just by knowing the first row and the first column of the matrix. Due to the fact, that the first place in a lane is only connected to the

second and vice versa, the first row vector and the first column vector is given by equations (5) and (6).

$$a_{firstRow} = [0, 1, 0^T] \quad (5)$$

$$a_{firstColumn} = a_{firstRow}^T \quad (6)$$

The neighboring-lane-adjacency shall be assigned by  $A_{neighborLane} \in \mathbb{R}^{n \times n}$ . Due to the fact, that each place is only connected with its direct neighbor on the neighboring lane, one may easily realize that it is always an identity matrix (equation (7)).

$$A_{neighborLane} = \mathbb{1}^{n \times n} \quad (7)$$

Depending on the number of lanes, it is now possible to generate the hole places' adjacency by using these block matrixes and thinking the adjacency constructed as mappings from one lane to another. So, the first row of blocks is the mapping from the first lane to all others (including itself). Due to the fact that there exist no connections to lanes, which are not neighboring the respective one, these mappings are described by zero-matrix blocks.

For example for three lanes the adjacency is constructed by equation (8):

$$A = \begin{bmatrix} A_{lane} & A_{neighborLane} & 0_{n \times n} \\ A_{neighborLane} & A_{lane} & 0_{n \times n} \\ 0_{n \times n} & A_{neighborLane} & A_{lane} \end{bmatrix} \quad (8)$$

It may be recognized that even the overall places' adjacency is of a block Toeplitz structure. This may be interpreted as an instantiation of a self-affine structure, which is given by definition of the possible interactions.

In this incidence matrix every entry represents a single transition, so that we may assign each transition by numbering from one to the total number of transitions.

Reading the adjacency row-wise shall furthermore be interpreted as moving a token to another place which is denoted by the respective column-number, that means these corresponding entries in the incidence matrix are equal to +1. Reading the adjacency column-wise shall be interpreted as taking a token from the place denoted by the respective row-number, means these corresponding entries in the incidence matrix are equal to -1.

As an example let us assume a generated places' adjacency as given by equation (9). This small example represents two lanes, with at maximum three vehicles per lane.

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \quad (9)$$

Denoting the transition-assigned adjacency as  $A^\xi$ , it results by numbering each transition to equation (10).

$$A^\xi = \begin{bmatrix} 0 & 1 & 0 & 2 & 0 & 0 \\ 3 & 0 & 4 & 0 & 5 & 0 \\ 0 & 6 & 0 & 0 & 0 & 7 \\ 8 & 0 & 0 & 0 & 9 & 0 \\ 0 & 10 & 0 & 11 & 0 & 12 \\ 0 & 0 & 13 & 0 & 14 & 0 \end{bmatrix} \quad (10)$$

So the incidence matrix, mapping places to transitions is given by equation (11).

$$C = \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 1 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 1 & 1 \end{bmatrix} \quad (11)$$

This may describe the token-flow in the net described by the standard equation of Condition-Event-Petrinets, where the places' capacity is limited to one (equation (12)).

$$m_{k+1} = C \cdot t + m_k \quad (12)$$

$m$  always denotes the marking of the Petri net at the different timesteps and  $t$  is the respective transition vector. To generate a consecutive transition-vector for the coordination to the objective single-firing rule is applied, which means only one transition may fire at each time step.

By these transformations a Petri net representation may be generated with a certain reference-formation given by  $m_0$ . The equation (12) is furthermore used to calculate the reachability graph and all reachable markings, exemplary depicted at Fig. 7. At this point we have to admit that for reduction of calculation effort the transitions, which represent a forward movement, were neglected in consideration with the optimization objective.

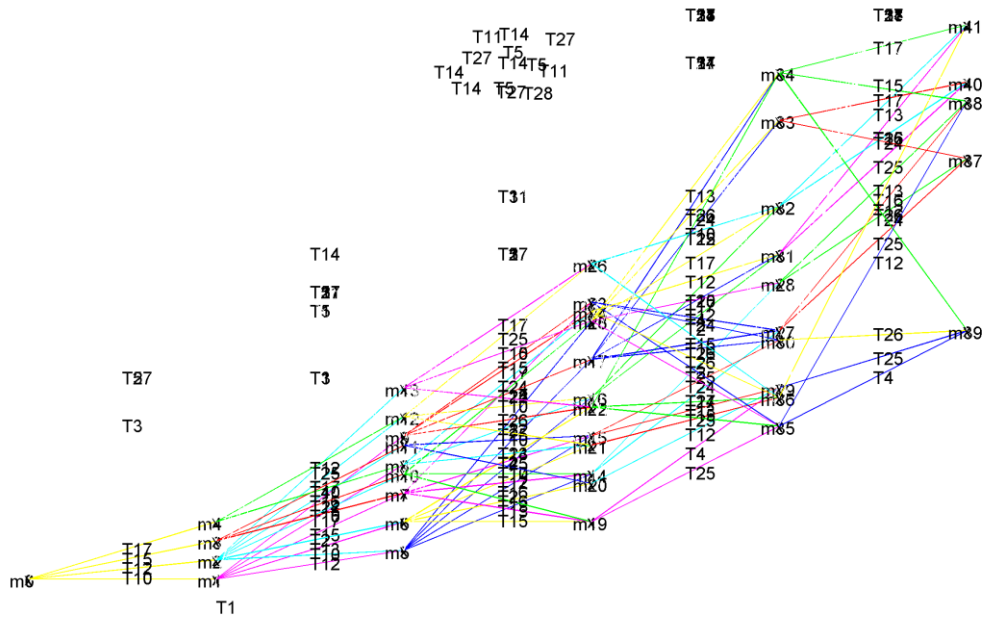


Fig. 7: Exemplary Reachability Graph for 3 lanes, maximum 5 vehicles per lane and 10 vehicles in total

### Calculation of shortest Path to Coordination Objective

The former reachability graph may be interpreted as an adjacency mapping the reachable markings. According to this, it can be fed to the Dijkstra algorithm which is able to calculate the shortest path to a marking that fulfils the coordination objective.

The Dijkstra algorithm is standard in informatics, so we go on with the definition of the coordination objective.

As coordination objective for road traffic we define best utilization of road capacity that means we want to generate a formation, which has a maximum density. In (Hübner M. et al. (2009a)) the authors have shown conditions under which this proposition holds.

At the implementation of the concept of Fig. 5 a subset of reachable markings is chosen that fulfil the coordination objective. Due to the fact that the places' capacity is limited to one we are able to check the density-condition at each reachable marking by bit-operations.

Let us denote the marking of the Petrinet by  $m$ . Then, knowing the maximum number of vehicles per lane ( $n$ ), the marking per lane is given by  $m(1:n)$ ,  $m(n+1:2n)$ , etc. that shall be assigned by  $m_1$ ,  $m_2$ , etc. The sum of vehicles per lane shall be given by the variables  $S_1$ ,  $S_2$ ,...

Maximum density is achieved (Hübner M. et al. (2009a)) when all the differences between the lanes are less or equal to one. This is the first subset of all reachable markings. From this subset we have to choose all markings that fulfil the condition that there does not exist any free place between occupied places.

The mathematical formalization is given as follows: Let us denote the set of markings fulfilling the high-density condition as  $H_{Dens}$ ,  $q$  the number of lanes and  $p$  the total number of vehicles.

The operator  $\&$  means bitwise 'and',  $|$  bitwise 'or',  $\%$  means modulo operation.

A marking  $m$  belongs to this set  $H_{Dens}$  iff

$$m_1 \& m_2 \& m_3 \& \dots \& m_q = [0,0,\dots,0,1,\dots,1] \quad \text{if} \quad p \% q = 0 \quad (12)$$

$$\begin{aligned}
 & m_1 \& m_2 \& m_3 \& \dots \& m_q = [0,0,\dots,0,1,\dots,1] \\
 \wedge & \\
 & [m_1 | m_2 | m_3 | \dots | m_q] - [m_1 \& m_2 \& m_3 \& \dots \& m_q] = [0,0,\dots,0,1,0,0,\dots,0]
 \end{aligned}
 \tag{13}$$

The upper equations may be interpreted as formalization of the coordination objective.

### **Abstraction of local environment variables and rule-base generation**

For each transition, means movement of a vehicle, the vehicle-local environment conditions have to be recognized for a systematic rule-base generation.

These conditions are application dependent and can be classified at highways as follows:

- attributes of the lane, that the vehicle is driving *on*, before transition
  - lane number
  - number of vehicles on the lane
    - maximum number of vehicles over all lanes?
    - minimum number of vehicles over all lanes?
- attributes of the lane, that the vehicle is driving *to*, before transition
  - lane number
  - number of vehicles on the lane
    - maximum number of vehicles over all lanes?
    - minimum number of vehicles over all lanes?
- attributes of the lane, that the vehicle is driving *on*, after transition
  - lane number
  - number of vehicles on lane
    - maximum number of vehicles over all lanes?
    - minimum number of vehicles over all lanes?
- attributes of the lane, that the vehicle is driving *to*, after transition
  - lane number
  - number of vehicles on the lane
    - maximum number of vehicles over all lanes?
    - minimum number of vehicles over all lanes?
- attributes of the places nearby the vehicle that is supposed to perform the movement
  - place in front occupied?
  - place in front free?
  - place behind occupied?
  - place behind free?

For each simulated transition these values may be calculated and entered into a matrix. From this matrix the engineer sees that same conditions (vector of environment variables) always the same action of the respective vehicle is performed. These actions are limited by definition of the Petrinet representation to lane change (left/right), longitudinal change of position.

For the coordination from a reference formation to an objective formation of maximum density, the rule-base may be visualized by Fig. 8.

First step of the vehicle-sided rule-base is the identification of the lanes with maximum and minimum number of vehicles. If the ego-vehicle is last vehicle in one of the lanes with maximum number of vehicles, it shall move to the nearest lane with minimum number of vehicles. Afterwards the respective vehicles are supposed to adapt their longitudinal distances to the preceding vehicles to the desired one.

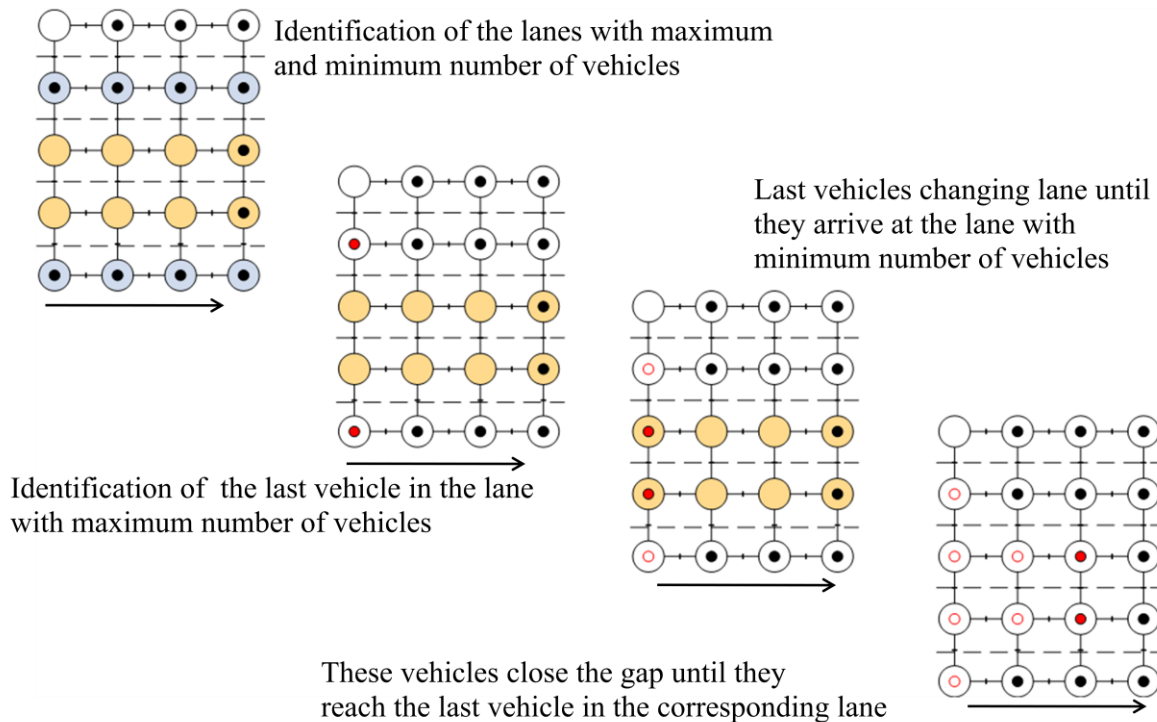


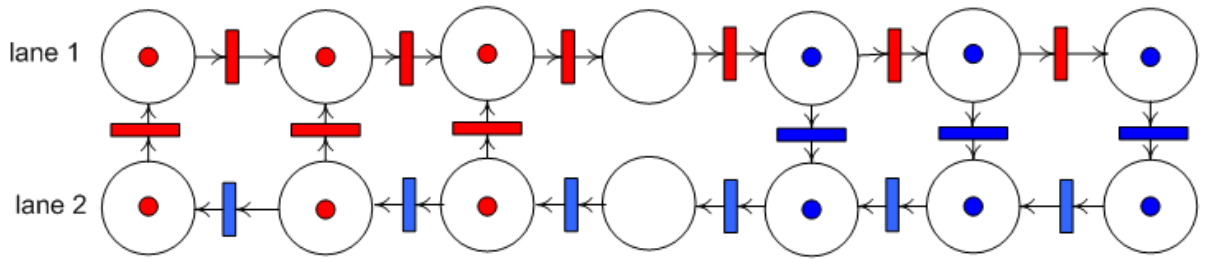
Fig. 8: Exemplary visualization of the Rule-Base for Target Flocking

### Enhancements to vehicle interactions of different vehicle-classes

The previously described principle of rule-base generation can also be adapted to vehicle formations that are supposed to pass each other when driving at different velocities. In road traffic it may occur, that two clusters of vehicles as given in Fig. 8 are encountering whereas they are driving at different velocities. Then an ordered passing algorithm is besides the automatic flocking to the target formation is necessary. Hübner, M. et al (2009b) have presented an algorithm for this purpose. The main conceptual change at this type of problems is that we need at least two different kinds of transitions that fire depending on the cluster the respective vehicle belongs to. According to the Petrinet representation the latter may be formally described by the introduction of colored Petrinets, which is an enhancement to the Condition-Event-Nets used in the previous sections. Transitions are only enabled for a certain color a token has.

As depicted in Fig. 9, when two cluster of different velocities encounter, vehicles of the faster cluster have the tendency to move to the upper lane, whereas vehicles of the slower one are supposed to move to the lower one. So an ordered passing is realized. For fast vehicles being on the upper lane the relative movement is 'forward', whereas for slow vehicles being on the lower lane the relative movement is 'backward' (Fig. 10). Having passed one another, the meaning of the lane-changing transition changes (Fig. 11). The slower vehicles are then supposed to move back to lane 1 and the faster ones vice versa, so that the reallocation to the target formation of highest density is possible.

faster cluster, vehicles shall change to upper lane



slower cluster, vehicles shall change to lower lane

Fig. 9: Encountering of Clusters, red: faster cluster, blue: slower one

ordered passing

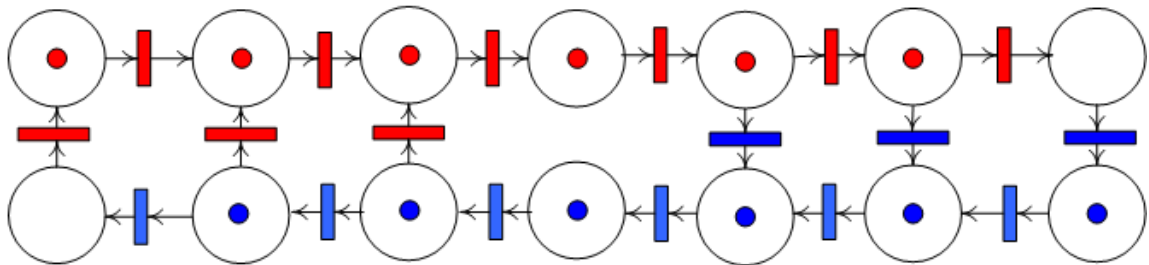


Fig. 10: Ordered passing of clusters

reallocation

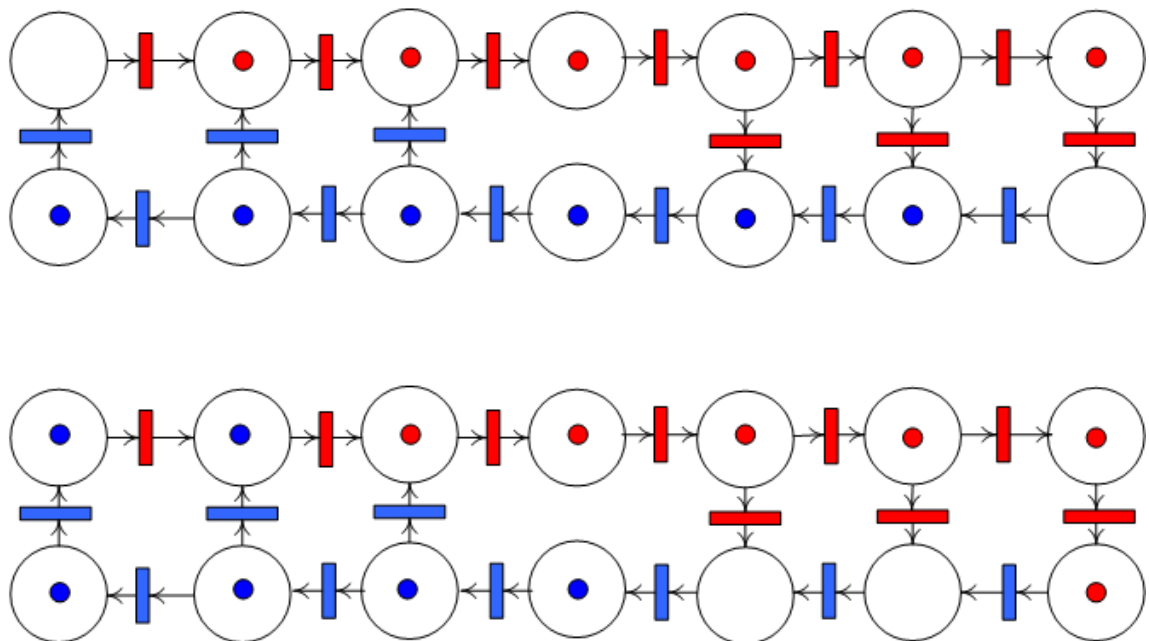


Fig. 11: Reallocation to target formation

## CONCLUSION AND OUTLOOK

This contribution has presented a concept for the rule-base generation of multi-agent systems in well-structured environments based on Petrinets. The application area has been set to road traffic for the prospective realization of an optimized traffic behavior by fully automated vehicles. Watching the general concept at Fig. 5, the trajectory generation for the vehicle movement is still to be developed, whereas vehicle-local sublevel control algorithms have already been tested as given in Hübner, M. et al. (2008) and Ganzelmeier, et al. (2001). Next steps are the integration of the rule base to a network model with a consensus algorithm using these sublevel control algorithms to have a testbed for road-traffic multi-vehicle control.

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