TWO-DIMENSIONAL INVESTIGATION OF TIME HEADWAYS FOR BETTER REALISM IN MULTILANE TRAFFIC FLOW MODELLING

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ABSTRACT

Existing car following theories assume that each vehicle is influenced directly by the one in front which is true only where lane discipline is extremely ideal and lanes are very wide. Real traffic, however, may pose a more complex picture. The paper, based on preliminary observations, is an attempt to explore the issue in detail for the first time. Below a certain value of same lane time headways, fewer numbers of drivers are willing to keep short headways with the neighbouring lane vehicles. This may mean that except light flow traffic, where the offside lane is primarily used for overtaking, considerable amount of drivers prefer to pass or lag behind the vehicle in the adjacent lane, rather than driving side by side. In addition to its safety concerns, especially in relatively narrow lanes, this issue may have capacity implications. Hence, existing traffic flow models may require further adjustments.

Keywords: lateral psychological friction, lateral pick up effect of headways

INTRODUCTION

Earlier research, through empirical observations and simulation, modelled the phenomenon of low lane discipline at a fundamental level (e.g. Gunay, 2007; Gunay, 2009). Our work now looks at a number of new issues. The first one is the question of whether the traffic lanes work independently from each other in terms of longitudinal spacings (i.e. do the vehicles arrive independently over the lanes?). The initial answer to this question is 'yes, if the lanes

are wide enough'. The Highway Capacity Manual (2010) gives certain values based on the relationship between lane widths and throughput. Garber and Hoel (2009) stated that narrow lanes make it difficult for two vehicles to travel alongside each other. Hence, for microscopic analysis of traffic flow, a clear and defining scrutiny is needed. We treated this issue (by observation) in a number of steps as will be detailed later in the paper.

Existing traffic simulation models (microscopic models in particular) are by in large lane based models. They treat multilane (unidirectional) traffic flow lane by lane. Although models such as VISSIM takes lateral friction into account such as presence of bicycles or large vehicles, the vehicular headway relationships are all same lane based. In other words, the longitudinal location of each vehicle is governed by well known car following theories (as a function of the longitudinal position of the preceding vehicle travelling in the same lane). The presence of neighbouring lane vehicles are not taken into account for these position updates. However, we believe that drivers do not like driving side by side with other vehicles in the adjacent lanes for long periods of times even in moderate or heavy traffic. They either pass the vehicle travelling in the neighbouring lane or lag behind it. There may be two reasons for this behaviour: the physical lateral discomfort effect of the vehicle in the adjacent lane as it narrows down the effective width of the travel corridor and the psychological shy effect of being looked at by the occupants of the neighbouring vehicle. Hence, in modelling an adjustment may be needed for the calculation of the longitudinal positions of vehicles by taking this pick up effect from the adjacent lane vehicles. The present paper is an attempt to survey this issue. The work is not complete yet. Here, we only highlight the matter and show that there may be unique appearance of time headways when these pick up effects are taken into account.

THE DATA

Three sets of data were collected to analyse the time headways between vehicles travelling in the same direction but in neighbouring lanes. The first set of data was collected in Istanbul, Turkey for much broader research purposes (only a small portion of the findings was demonstrated here). The second set was collected in Karlsruhe, Germany and in Newcastle upon Tyne, GB. Both sets used video recording and manual analysis of the video frames. The third set, which formed the main data, was collected at two sites on A55, the South East of Belfast, Northern Ireland (Figure 1), using inductive loop surface detectors, shown in Figure 2 and Figure 3. Both sites were two straight sections of dual carriageways. The third data set was collected by the Roads Service (Northern Ireland) in July 2007 and the readings were passed to us in spreadsheet data format. While the first set represents poor lane discipline, the second and the third sets represent good lane disciplines. More emphasis will be given to the third data set in the paper.



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Figure 2 – Main data collection sites (inductive lops and the processing unit located in the median)



Figure 3 – Main data collection Site 2

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THE FINDINGS

Using the first data set, vehicular arrivals were observed spatially in that a certain section of the carriageway (15 m) was divided into two sectors on the screen. Each sector consisted of a number of sub-parts which was equal to the number of lanes of that particular carriageway. Each sector was represented by a matrix, each cell of which is set to either '1' or '0', depending on whether the particular part was occupied by a vehicle or not. Then the second sector was also represented by another matrix which is designed in the similar way denoting whether a vehicle arrives on a particular part of the sector or not. The readings were then recorded in the form of occupation and arrival matrices consisted of zeros and ones. Readings were taken three times a minute, by stopping the tape at regular intervals, and hence each row represents a separate reading. Rows were then distributed over a number of groups, the number of which is the all possible permutations of Sector 1. For example, for a there-lane carriageway with a shoulder, there are 15 different possibilities of vehicle occupations in Sector 1. Table 1 and Table 2 are the percent analysis of the arrivals for two different sites without and with shoulder, respectively. It is evident from these two tables that the probability of the next arrival on a lane, which is already occupied by a preceding vehicle, is considerably less than the probability of the next arrival on a non-occupied lane.

	Inside	Middle	Outside Lane	Pre the next	Total		
	Lane	Lane		Inside Lane	Middle Lane	Outside Lane	%
				12.5	46.1	41.4	100
ne	1			2.9	32.4	64.7	100
La		1		21.6	9.8	68.6	100
ent ed			1	30.5	54.2	15.3	100
pie	1	1		0	33.3	66.7	100
រ ភ្ល	1		1	15.4	53.8	30.8	100
ő		1	1	30.8	30.8	38.4	100
	1	1	1	26.3	35.1	38.6	100

Table 1 – Observed lateral arrival pattern on a three-lane highway with no shoulder

Table 2 - Observed lateral arrival pattern on a three-lane highway with shoulder

	Outside	Middle	Inside Lane	Shoulder	Proba	Total			
Lane	Lane	Lane			Outside Lane	Middle Lane	Inside Lane	Shoulder	%
					44.4	33.3	16.7	5.6	100
	1				11.3	58.5	26.4	3.8	100
		1			61.4	7.0	26.3	5.3	100
ne			1		46.2	46.2	2.5	5.1	100
Lai				1	37.5	62.5	0	0	100
ð	1	1			21.6	18.9	48.7	10.8	100
pie	1		1		12.5	87.5	0	0	100
n	1			1	-	-	-	-	
ő		1	1		88.9	0	11.1	0	100
≥		1		1	50	50	0	0	100
ant			1	1	50	50	0	0	100
, E	1	1	1		33.3	33.3	16.7	16.7	100
5	1	1		1	33.3	0	66.7	0	100
	1		1	1	-	-	-	-	
		1	1	1	100	-	-	-	100
	1	1	1	1	-	-	-	-	

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The above findings implied that lane by lane vehicular arrivals are not very independent. The presence of other vehicles in the surrounding may affect the longitudinal position of the vehicle in question. Hence we looked at the second set of data in more detail.

As known very well, a time headway is defined as the time interval between successive vehicles (from a reference point of the first vehicle to the same reference point of the second vehicle) as they pass a point along the lane (Roess et al., 2004). Figure 4 shows the distribution of time headways between vehicles travelling in the same lane (the curves) and between the vehicles travelling in adjacent lanes (the bars).



Figure 4 – Distribution of time headways, Newcastle, GB.

Figure 5 demonstrates the case for a three lane German motorway. In both English and German cases, the shape of the headway distribution (the curves in Figure 4 and Figure 5) for all lanes was in a typical headway distribution shape reported in literature. However, it was the interaction between the neighbouring vehicles (travelling in the same direction but in different lanes) that attracted our attention.





Figure 5 - Distribution of time headways, Karlsruhe, Germany (the analysis of the same lane headways for the inside lane is not complete yet).

We decided to collect further data to scrutinise the case with larger sample sizes. The third set of data contained about 43,838 vehicles (at Site 1) and 42,408 vehicles (at Site 2). Due to the huge size of the data, rather than manual investigations, we developed a piece of software (Figure 6) to carry out the analyses in a more efficient way. Not only it saved time, but also the chance of making human errors in the analyses was eliminated.

											Tuesday, 07:57:28	X-0
					Gap b						Tuesday, 07:57:30 Tuesday, 07:57:31	0-X
	0≤b<0.5	0.5≤b<1.5	1.5≤b<2.5	2.5≤b<3.5	3.5≤b<4.5	4.5≤b<5.5	5.5≤b<6.5	6.5≤b<7.5	7.5≤b		Tuesday, 07:57:32	0-X
0≤a<0,	205	39	14	8	0	1	0	1	10		Tuesday, 07:57:33 Tuesday, 07:57:34	X-0 0-X
0,5≤a<1	5 2014	1085	503	269	162	117	75	49	336		Tuesday. 07:57:35	X-0
1,5≤a<2	5 2283	1462	660	404	251	186	95	88	612		Tuesday, 07:57:38 Tuesday, 07:57:39	0-X 0-X
2,5≤a<3	5 1121	643	363	218	121	119	88	52	510		Tuesday, 07:57:40	X-0
Gap a 3,5≤a<4	5 529	333	183	148	74	64	45	44	358		Tuesday, 07:57:41 Tuesday, 07:57:42	U-X X-0
4,5≤a<5	5 287	178	111	78	62	48	41	26	283		Tuesday, 07:57:45	X-0
5,5≤a<6	5 174	112	66	45	36	27	35	20	217		Tuesday, 07:57:48 Tuesday, 07:57:50	X-0
6,5≾a<7	5 115	69	47	30	28	21	21	23	202		Tuesday, 07:57:52	x - x
7,5≾a	616	389	211	162	100	74	88	52	3528		Tuesday, 07:57:55 Tuesday, 07:57:56	X-0
					Grad					-	Tuesday, 07:58:02 Tuesday, 07:58:01 Tuesday, 07:59:03 Tuesday, 07:59:03 Tuesday, 07:59:05 Tuesday, 07:59:06	0-X 0-X X-0 X-X 0-X
	0≤d<0.5	0.5≤d<1.5	1.5≤d<2.5	2.5≤d<3.5	Gap d 3.5≤d≪4.5	4.5≤d<5.5	5.5≤d≪6.5	6.5≤d<7.5	i 7.5≤d		Tuesday, 07:58:02 Tuesday, 07:58:01 Tuesday, 07:59:02 Tuesday, 07:59:02 Tuesday, 07:59:05 Tuesday, 07:59:05 Tuesday, 07:59:08 Tuesday, 07:59:08 Tuesday, 07:59:09	0-X 0-X 0-X 0-X 0-X 0-X 0-X 0-X
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Figure 6 – A screenshot of our data analysis tool GAPPER.

First of all we looked at the time headways ('b' and 'd' in Figure 7 and Figure 8) between the neighbouring vehicles without taking the same lane leading vehicle's position into account. In other words, the relationship between 'a' and 'b' headways, and between 'c' and 'd' headways were not considered. When these two figures are compared with the German and English counterparts (Figure 4 and Figure 5), the findings did not show strong similarities especially for the < 0.5 second interval, although the bars for the first category in Figure 8 did not exceed 25% (unlike Site 1). This showed that further scrutiny is needed. It may be a good idea to remind the reader here that the lower the percentages for the first category (i.e., the <0.5 s interval), the stronger the argument we put forward become, that is the interaction between the neighbouring lanes leading to the need of adjustments in microscopic simulation models.



Figure 7 – A55 Site 1



Figure 8 – A55 Site 2 (the legend is the same as Figure 7)

However, when we examine the data in more detail (with the inclusion of the longitudinal position of the leading vehicle travelling in the same lane as the follower), interesting results were discovered. When the gap between the two same lane vehicles is small (0 < a < 1.5 seconds), the number of vehicles (travelling in the faster lane) with short neighbouring headways of (0 < a < 1.5 seconds) are smaller than the next interval (1.5 < a < 2.5 seconds) as shown in Figure 9. This means that when there is a close following situation in the shoulder lane, the median lane vehicles preferred to stay (longitudinally) away from this vehicle pair. We interpret this as an evidence for strong relationship (interaction) between the two neighbouring lanes running in the same direction. The existing microscopic models

currently lack incorporating such interactions. This feature was less clear in Figure 10 implying that more median lane (offside lane) vehicles adjust their position with respect to the situation in the shoulder lane (inside lane) rather than the shoulder lane vehicles adjusting their position with respect to the situation in the median lane.



Figure 9 – A55 Site 1 (23,564 vehicle pairs)



Figure 10 – A55 Site 1 (20,481 vehicle pairs)

It should be remembered that the data have not been categorised according to various flow levels yet (i.e. low, moderate, heavy). After removing the light flow portion of the data, much clearer results are expected to be found in terms of the above percentage values. This will be the next step of our ongoing analyses.





Figure 11 – A55 Site 2 (20,878 vehicle pairs)



Figure 12 – A55 Site 2 (21,653 vehicle pairs)

Finally, to be able to mimic an imaginary situation where two lanes are totally (physically) separated, we picked the inside lane of Site 1 and the outside lane of Site two (as if there is a physical barrier between the two unidirectional lanes), and compared the findings on the same diagram as shown in Figure 13.



Figure 13 – The shoulder lane of Site 1 and the median lane of Site 2 for comparison.

The most striking finding is the percentage levels, which remained below 15%, whereas the values reached 25-30% in Figure 7 and Figure 8. This difference highlights the fact that traffic (time headways in particular) in two unidirectional lanes shows interactions between them and should not be treated in isolation in a lane by lane fashion, as is the case at the moment in modelling arena.

CONCLUDING REMARKS

Existing car following theories may not be applicable in many situations due to some of the assumptions they posses. For instance, each vehicle is expected to be influenced directly by the one in front, which only happens in real traffic flow where lane discipline is extremely ideal and the lanes are separated. In reality, however, analyses become more complex and further research is worth undertaking. If there was no interaction between the vehicles travelling in different lanes, the headways between the consecutive two neighbouring vehicles would have been random. The only factor would have been the flow level. Namely, with increasing flow, more vehicles would have been expected to keep shorter headways, resulting in some increase in the distribution with decreasing time headways. However, below a certain value of time headways, fewer numbers of drivers were willing to keep short headways as opposed to the above expectation. This means that considerable amount of drivers preferred to lag behind the vehicle in the adjacent lane, rather than driving side by side. The next step of the research is to formulate this with the support of wavelet analogy for better interpretation of the results and this is subject to our ongoing work. We have recently applied this technology on an enhanced interpretation of traffic microsimulation of a sample network. But to analyse vehicular headways by means of wavelet transformation will be the first of its kind in the field of traffic engineering.

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