A FLIGHT SCHEDULE AND FLEET ASSIGNMENT MODEL

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ABSTRACT

The main objective of this paper is the proposition of a model to optimize the network to be served by an airline, solving, in an integrated way, the flight schedule definition and the fleet assignment problems. The model includes specific operational constraints like takeoff and landing slots and limited flexibility on airport options due to a traffic mainly composed by passengers. The model was successfully applied to a Brazilian regional airline case, which resulted in a complete schedule and provided information to support decisions on new possible flights or other aircraft type utilization.

Keywords: flight scheduling, fleet assignment, linear programming, tactical planning.

1. INTRODUCTION

Along with the increasing passenger demand over the years, there was an increase in the frequency of flights, and a slight drop in the number of passengers per flight (SWAN, 2002). This fact in addition to the increasing competition between airlines points toward models which allow the definition of a new air transport mesh according to the evolution of demand, and which aims at a reduction of operating costs (KLABJAN, 2004). However, such problems are relatively complex, leading to a model composed of a large number of variables and constraints (HANE *et al*. 1995; KLABJAN, 2004), typically leading to a division of the problem in stages, with simpler, more specific models to each stage (RABETANETY *et al*. 2006).

In Brazil, like in other countries, the problem has specific attributes, such as operational restrictions at the main airports (CGNA, 2009, MD, 2008; CAETANO and GUALDA, 2008). Due to the fact that the majority of the airport traffic consists of passengers (ANUÁRIO, 2007 *apud* Oliveira, 2009), the adoption of alternative airports to free resources on the saturated ones becomes impractical. In this context, this paper presents a model that deals with two of the several stages of the operational planning of an airline - the Schedule Generation Problem and the Fleet Assignment Problem, by applying it to the case of an airline operating in a regional market.

Initially, a brief review of the concepts involved in solving these problems will be described. This review will be followed by the presentation of specific aspects of the problem addressed, the proposed model and the results of a robustness analysis of this model, and its application on instances based on the network of a Brazilian airline.

2. AIRLINES OPERATIONAL PLANNING

The operational planning of airlines can be divided into three interrelated problems: the definition of which flights will be offered, which aircraft will be used on each flight, and which crew will perform these flights. These steps are shown in Figure 1.

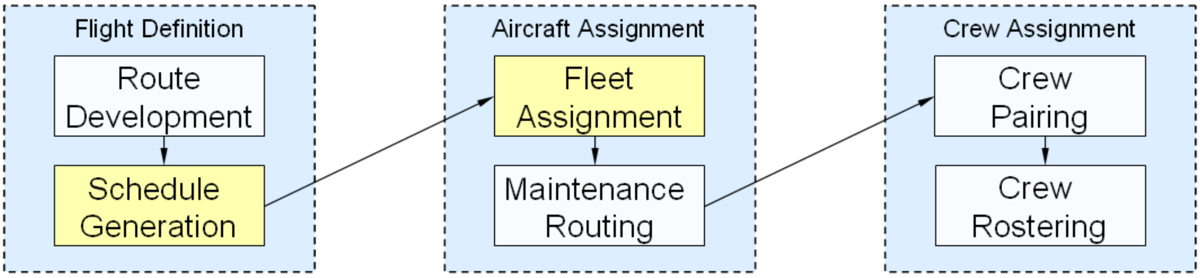

Figure 1 – Airline Operational Planning Stages

The flight definition is the process where the flight schedule is defined. In the first step of this process, the Route Development, demands between city pairs are identified and thus potential flights between airport pairs are defined. It is important to address the airports restrictions on this step, since some airports do not operate in a 24/7 basis. The second step of this process, the Schedule Generation, selects which flights will actually be on the flight schedule. This step shall cope with specific operating restrictions of each airport, such as fixed schedules available for a given airline, called slots (ANAC, 2008; KLABJAN, 2004; RABETANETY *et al*., 2006).

The aircraft assignment is the process that defines which aircraft will perform each scheduled flight. In the first step of this process, the Fleet Assignment, the best type of aircraft is assigned to each flight, maximizing profitability. The second step of this process, the Maintenance Routing, shall cope with operational restrictions of each aircraft, such as maintenance periods (ABARA, 1989; BARNHART *et al*. 2003; KLABJAN, 2004; RABETANETY *et al*., 2006).

The crew assignment is the process that defines which crew members will be scaled to each flight. In the first step, the Crew Pairing, the flights are grouped into sequences called journeys, which must respect labor laws and technical criteria. In the second step, the Crew Rostering, the crew members are assigned to each journey (BARNHART *et al*., 2003; GUALDA and GOMES, 2008; KLABJAN, 2004; RABETANETY *et al*., 2006).

The increasing complexity of aircraft, crew and passenger management, and the intensification of the competition between airlines led to the need to create models that include a growing number of routes and restrictions. Such large-scale models bring huge computational challenges because many of them are considered NP-hard (HANE et al., 1995). This behavior may be even worse when models are used to solve simultaneously two or more stages of the problem, seeking a global optimization (KLABJAN, 2004, SHERALI *et al*. 2006; RABETANETY *et al*., 2006).

When a meaningful part of the traffic of a continental sized country is composed of passengers, including tourists, the demand will most certainly change along the year, thus modifying the profitability of some legs. In this context, the regional market may be a bigger challenge for ailines, since the network is composed of shorter legs, allowing the competition of *low cost-low fare* companies (OLIVEIRA A., 2009) or even the competition of cheaper transport modes, like high-speed trains or buses, that impose a pressure on ticket prices.

3. THE PROBLEM

Although the steps described above are all interdependent, the schedule generation and the fleet assignment problems are two good candidates for joint treatment, due to its strategic nature. The impact that the use of an aircraft of a particular type may have on the company's route network is one kind of response that such a model should provide. On another situation, taking into account airports with operational restrictions on takeoff and landing, the model can select the most interesting flight to fill one particular slot.

Depending on the the type of passenger of a specific leg, the demand can also change along the week. When the traffic is mainly composed by business travellers, some legs may be more demanded on Mondays in one direction and on Fridays on the opposite direction. In order to cope with this characteristic, the potential flight network must be built for an entire week.

3.1. Proposed Features for the Model

The model should help to answer strategic and tactical questions of an air transporter, including those related to the selection of flights to be performed and the aircraft type to be used on each of them, adjusting the number of offered seats as best as possible to the demand for seats. The goal is to avoid flights with empty seats and, at the same time, avoid the loss of passengers due to lack of seats, in the same proportion. The penalty for such cases should be proportional to the length of the flight.

The model must ensure the continuity of the movement of aircraft of each type, which must begin and end the trip in the same airport, and, when appropriate and economically efficient, flights should be created to reposition the aircraft. These flights must meet all requirements, such as minimal maintenance time between flights, which is considered constant for the purposes of this paper.

The selection of flights shall comply with the time slots when they are established. In the case of airports in which the operation is not restricted, there should be no restriction on the number of flights that depart or land at a given moment, since this is a strategic model, and there will be room for operational adjustments at these airports. However, only one type of aircraft should be assigned to each flight.

4. MATHEMATICAL MODELING

Traditional fleet assignment models are associated with two distinct approaches. One of them is based on networks of interconnections, where each node represents a possible connection between legs, and the model decides which connections should be done and which should not (ABARA, 1989; SHERALI *et al*., 2006). Another representational approach is through a space-time network, where the arrival or departure airports are represented by nodes, and the arcs represent the legs between the airports nodes to which they belong, or the time that aircraft are on the ground between one leg and another (BERGE and HOPERSTEAD, 1993 *apud* SHERALI *et al*., 2006; HANE *et al*. 1995).

These classical models assume that the flights schedule are set and that every flight should be covered and, traditionally, they do not include operational restrictions at airports. To overcome these limitations, it is necessary to define a more comprehensive model. The proposed model is based on the concept of space-time modelling (BERGE and HOPERSTEAD, 1993 *apud* SHERALI *et al*. 2006; HANE *et al*. 1995), although it has been extended to meet the requirements described in Section 3.

$$[Min] \sum_{f \in F} \sum_{(i,j) \in Lv} (D_{ij} - C^f)^2 T_{ij} x_{ij}^f + \sum_{(i,j) \in Lvd} D_{ij}{}^2 T_{ij} y_{ij} \quad (1)$$

Subject to:

$$\sum_{f \in F} x_{ij}^f + y_{ij} = 1 \qquad \forall (i,j) \in Lvd \quad (2)$$

$$\sum_{o|(o,k) \in L} x_{ok}^f - \sum_{d|(k,d) \in L} x_{kd}^f = 0 \qquad \forall k \in N, \forall f \in F \quad (3)$$

$$\sum_{(i,j) \in Lt} x_{ij}^f \le A_f \qquad \forall f \in F \quad (4)$$

$$\sum_{f \in F} \sum_{d|(i,d) \in Lv} x_{id}^f \le 1 \qquad \forall i \in Nrd \quad (5)$$

$$\sum_{f \in F} \sum_{o|(o,j) \in Lv} x_{oj}^f \le 1 \qquad \forall j \in Nra \quad (6)$$

Binary variables:

$$x_{ij}^f \text{ for } \forall (i,j) \in (Lpd \cup Lvd) \quad (7)$$

$$y_{ij} \text{ for } \forall (i,j) \in Lvd \quad (8)$$

Integer Variables:

$$x_{ij}^f \ge 0 \text{ for } \forall (i,j) \in L \backslash (Lpd \cup Lvd) \quad (9)$$

Where: F: set of all types of aircraft, indexed by f.

N: set of all nodes in the system, indexed by i, j, o, d or k, representing an airport at a specific time.

L: set of arcs that represent the movement of aircraft, maintenance, waiting in the ground or wrap, indexed by (i, j), being the source node i and j the destination node of the movement.

Lv: set of arcs that represent movements of flight.

Lvd: set of arcs representing flights with demand.

D_{ij}: passenger demand from node i to node j.

C_f: number of seats of aircraft of type f

T_{ij}: flight time from node i to node j.

x^f_{ij}: indicates the number of aircraft of type f through the arc (i, j).

y_{ij}: 1 indicates that the flight from i to j is not done, 0 otherwise.

L_t: set of arcs whose origin time is equal to or less than t and destination time is after t. The time t is set to a valid time according to the problem.

A_f: Indicates the number of aircraft of type f available.

Nrd: node-set of airports with restrictions on takeoff.

Nra: node-set of airports with landing restrictions.

Lpd: set of arcs that represent the minimal maintenance of aircraft after landing flights with demand.

The objective function (expression 1) seeks to minimize the sum of quadratic differences between the demand and the number of seats on each flight made by any aircraft, plus the non executed flights lost demand. All these values are weighted by the length of the flight.

The constraint represented in the expression 2 ensures that no more than one aircraft will perform a demanded flight. If no aircraft is assigned to a demanded flight, the variable y_{ij} is used to indicate the total loss of demand because that flight is uncovered. The presence of the variable y_{ij} is important, so a penalty for not performed demanded flights can be applied, leading to the selection of flights that minimize the number of passenger lost and empty seats.

The constraint represented in the expression 3 ensures the balance on each node, for each fleet. Once the network is cyclic, this restriction is the same for all nodes, including the origin and final destination of each aircraft. The constraint represented in the expression 4 ensures that the number of available aircraft on each fleet is observed by the aircraft count through all the arcs that include the time t.

The constraint represented in the expression 5 ensures that only one flight departs from a takeoff slot on operational restricted airports. The restriction presented on expression 6 has a similar purpose, ensuring that only one flight arrive at a landing slot on operational restricted airports.

The variables representing demanded flight arcs, post-flight maintenance, and the variables that indicate whether a flight was uncovered are binary, and are specified on expressions 7 and 8. All other arc variables are integers greater than or equal to zero, as stated in the expression 9.

An interesting feature of this model shows up when the model processes several fleets with a single aircraft in each of them. In that case, the computed solution already presents the routes of each specific aircraft. However, although this solution represents the allocation of flight of an aircraft, it cannot be considered the final solution of the aircraft assignment problem, because it did not take all the rules of scheduled maintenance of an aircraft into account.

5. SOLUTION ENVIRONMENT

The input data should be provided through text files, which must contain all necessary information. These files are explained below.

- **Potential Flights File**: this file contains a table of potential flights and all flights whose demand is relevant. The demand for each flight must be specified.
- **Flight Time File**: this file presents the length of each possible flight between city pairs. If a flight does not exist in this table, the model will consider it as impossible.
- **Aircraft File**: this file describes available aircraft and their characteristics, including minimum maintenance time after flight, aircraft capacity etc.
- **Restricted Airports File**: this file lists all airports where operations are restricted. It is of great concern that flight departures and arrivals at these airports are included on Potential Flight File, even if their demands are zero. The reason for this procedure is to define the time of available slots.

The transformation of these data in a complete network is done by a software written in Java, designed to generate the mathematical model. The potential flight network is expanded with flights to reposition aircraft (empty flights, without demand). The arcs created for aircraft repositioning should also comply with each airport operational restrictions. After the creation of repositioning arcs, soil arcs will be created, representing aircraft waiting times at airports. Finally, an arc is created from last to first node of each airport, so that the final positioning of each aircraft type can be the same as the initial one.

With the network ready, the software generates the mathematical model, thus creating a file in LP format, which can be solved with most computer packages in the market. For this study, the package ILOG CPLEX 11.0 was selected, running on a computer with an Intel Core2Duo 2.4GHz processor and 2GB of RAM.

6. ROBUSTNESS ANALYSIS

The model robustness analysis was based on a series of tests consisting of critical situations for which the solutions were known beforehand.

The results of all tests described below were in line with the expectations, with fast and accurate results, indicating that the model is robust to deal with the problems of the air carriers which fit into the characteristics described in Section 3.

6.1. Best Choice of Aircraft Type for a Set of Flights

The first tests aimed at verifying the correct choice of aircraft for different flights. Thus, two paths involving 3 cities were defined, one with an 100 passenger demand per leg and another with an 116 passenger demand per leg as shown in Table I.

Table I – Routes with continuous demand (indicating the time of departure matches)

Route Number	Demand	Complete Proposed Route
1	100	A(1:40) – B(7:40) – A(13:40) – C(23:00) – A(32:20)
25	116	A(1:40) – C(11:00) – B(18:40) – C(26:20) – A(35:40)

Table II indicates the duration of all possible flights.

Table II – Possible flights (both directions)

Airport 1	Airport 2	Length (min)
A	B	300
B	C	400
A	C	500

In the first test it was defined the availability of two fleets, one 100 passenger aircraft and one 116 passenger aircraft, each fleet composed of 3 aircraft. The required maintenance time after flight was set to 45 minutes.

Table III – Fleets assigned to each leg (including maintenance and stand by)

Fleet (Airplanes)	Event	From (Day – Time)	To (Day – Time)
100 pax (1)	Flight	A (0 - 01:40)	B (0 – 06:40)
100 pax (1)	Maintenance	B (0 – 06:40)	B (0 – 07:25)
100 pax (1)	Stand By	B (0 – 07:25)	B (0 – 07:40)
100 pax (1)	Flight	B (0 – 07:40)	A (0 – 12:40)
100 pax (1)	Maintenance	A (0 – 12:40)	A (0 – 13:25)
100 pax (1)	Stand By	A (0 – 13:25)	A (0 – 13:40)
100 pax (1)	Flight	A (0 – 13:40)	C (0 – 22:00)
100 pax (1)	Maintenance	C (0 – 22:00)	C (0 – 22:45)
100 pax (1)	Stand By	C (0 – 22:45)	C (0 – 23:00)
100 pax (1)	Flight	C (0 – 23:00)	A (1 – 07:20)
100 pax (1)	Maintenance	A (1 – 07:20)	A (1 – 08:05)

Fleet (Airplanes)	Event	From (Day – Time)	To (Day – Time)
116 pax (1)	Flight	A (0 – 01:40)	C (0 – 10:00)
116 pax (1)	Maintenance	C (0 – 10:00)	C (0 – 10:45)
116 pax (1)	Stand By	C (0 – 10:45)	C (0 – 11:00)
116 pax (1)	Flight	C (0 – 11:00)	B (0 – 17:40)
116 pax (1)	Maintenance	B (0 – 17:40)	B (0 – 18:25)
116 pax (1)	Stand By	B (0 – 18:25)	B (0 – 18:40)
116 pax (1)	Flight	B (0 – 18:40)	C (1 – 01:20)
116 pax (1)	Maintenance	C (1 – 01:20)	C (1 – 02:05)
116 pax (1)	Stand By	C (1 – 02:05)	C (1 – 02:20)
116 pax (1)	Flight	C (1 – 02:20)	A (1 – 10:40)
116 pax (1)	Maintenance	A (1 – 10:40)	C (1 – 11:25)

The generation of the model took about 2 seconds, resulting in model with 540 constraints and 948 variables. Its resolution by Cplex took no more than 0.04s, and the result is shown in Table III, indicating the expected allocation of one 100 passenger aircraft for the route 1 and one Embraer 195 to the route 2. The other aircraft were on standby and are not presented in the table.

It should be noticed that no aircraft will be able to accomplish their flights on consecutive days at the proposed route, since the completion of the final flight takes place at a later time than it would be necessary to achieve the daily cycle. To generate solutions that meet the daily cycles requirement, it is necessary to generate solutions for a problem that presents flights for several consecutive days. One application of this type will be seen in section 7.

Building a model for the previous problem, but allowing a single 100 passenger aircraft, will result in a model with 274 constraints and 478 variables. The allocation solved by Cplex in about 0.01s is presented in Table IV.

Table IV – 100 passenger aircraft assignment (flights only)

Fleet (Airplanes)	Event	From (Day – Time)	To (Day – Time)
100 pax (1)	Flight	A (0 – 01:40)	C (0 – 10:00)
116 pax (1)	Flight	C (0 – 11:00)	B (0 – 17:40)
116 pax (1)	Flight	B (0 – 18:40)	C (1 – 01:20)
116 pax (1)	Flight	C (1 – 02:20)	A (1 – 10:40)
- (0)	Flight	A (0 - 01:40)	B (0 – 06:40)
- (0)	Flight	B (0 – 07:40)	A (0 – 12:40)
- (0)	Flight	A (0 – 13:40)	C (0 – 22:00)
- (0)	Flight	C (0 – 23:00)	A (1 – 07:20)

In this case, the 100 passenger aircraft was allocated to the route 2, with an 116 passenger demand. This behavior is predictable and appropriate to this type of model: once it is not possible to meet all the demand, the aircraft will be allocated to the flights where the greatest demand occurs, even though this demand exceeds the capacity of the aircraft.

When solving the same problem with only one 116 passenger aircraft, the results are identical to those in Table 4, but the 116 passenger aircraft will be replacing the 100 passenger aircraft on route 2.

6.2. Creating Suitable Repositioning Flights

In tests presented on section 6.1 there was not a reason to create any flight, beyond those present in the table of potential flights, so the model could fulfill efficiently the desired flights. However, this is not always a valid assumption: the most efficient solution may involve some extra flights to reposition aircraft.

The tests proposed here are aimed at checking the appropriate generation of such repositioning flights. Thus, three flights will be created in the table of potential flights, such as those presented in Table V. The possible flights will be the same previously listed in Table 2, and a single 100 passenger aircraft will be used, with a minimum of maintenance after 45 minutes of flight.

Table V – Potential flights

From	To	Departure	Demand
A	B	01:40	100
A	C	01:40	10
C	A	15:00	100

The flights schedule was defined to create a conflict on the selection between the flight from A to B or the flight from A to C, both starting at 01:40. The most straightforward choice, avoiding the creation of an aircraft repositioning flight is the flight from A to C, which prepares the aircraft for the next departure also.

However, this “A to C” is a long flight (500 minutes) with a very low demand, causing a great loss of passengers of a more demanded flight from A to B. In this way, the best alternative is performing flight from A to B, performing an empty flight from B to C, thus losing only a small demand of the flight from A to C.

The generation of the model takes less than 1 second, resulting in a model with 99 constraints and 167 variables. Its resolution by Cplex took no more than 0.03s, and it is shown in Table VI, which presents the selection of the flight from A to B, the creation of a flight from B to C, and the selection of the flight from C to A. The events in italics denote repositioning flights.

Table VI – Best flights selected by the model for an 100 passenger aircraft

Fleet (Airplanes)	Event	From (Day – Time)	To (Day – Time)
100 pax (1)	Flight	A (0 – 01:40)	B (0 – 06:40)
100 pax (1)	*Flight*	*B (0 – 07:25)*	*C (0 – 14:05)*
100 pax (1)	Flight	C (0 – 15:00)	A (0 – 23:20)
- (0)	Flight	A (0 – 01:40)	C (0 – 10:00)

As it may be observed in this solution, a new landing node was created at the airport C. If the airport C had operational restrictions, that is, operating with the system of slots, this solution probably would not be possible.

Based on the table of potential flights, it is possible to observe that the arrival of a flight from A to C occurs at 10:00, setting the time for the landing slot. Since the arrival of the repositioning flight occurs at 14:05 at airport C, this solution would not be acceptable. Thus, the optimal solution in the case of a restricted airport C, would be the flight from A to C, and then the flight from C to A.

When considering the operational restrictions of airport C, the creation of the model takes less than 1 second, resulting in a model with 34 constraints and 44 variables. Its resolution by Cplex took no more than 0.04 s, and the results are shown in Table VII, indicating the selected flights.

Table VII – Best flights selected for an 100 passenger aircraft, with operational restrictions at airport C

Fleet (Airplanes)	Event	From (Day – Time)	To (Day – Time)
100 pax (1)	Flight	A (0 – 01:40)	C (0 – 10:00)
100 pax (1)	Flight	C (0 – 15:00)	A (0 – 23:20)
- (0)	Flight	A (0 – 01:40)	B (0 – 06:40)

6.3. Selecting Better Alternatives in Operation Restricted Airports

In some cases it may be necessary to create repositioning flights using the slots on operationally restricted airports. The tests proposed here are intended to verify if the model correctly address this kind of situation, that is, the generation of repositioning flights based on restricted airports.

The first tests must verify the generation of arcs arriving at landing slots. Three flights are defined in the table of potential flights, presented in Table VIII, and the possible flights are shown in Table IX. A single 100 passenger aircraft will be used, with a minimum maintenance time after flight of 45 minutes. The operation is restricted at airport C.

Table VIII – Potential flights

From	To	Departure	Demand
A	B	01:40	100
A	C	01:40	10
C	A	15:15	100

Table IX – Possible flights (both directions)

Airport 1	Airport 2	Length (min)
A	B	300
B	C	400
A	C	760

It is possible to verify on these tables that the landing slot in C is currently 1:40 + 760 minutes, that is, 14:20. Based on this organization of flights, it should be possible to complete the sequence of flights ABCA, even with restricted operation in C.

For this case, the creation of the model takes less than 1 second, resulting in a model with 34 constraints and 44 variables. Its resolution by Cplex took no more than 0.03 s, and it is shown in Table X, indicating the selected flights.

Table X – Best flights selected for an 100 passenger aircraft, with operational restrictions at airport C

Fleet (Airplanes)	Event	From (Day – Time)	To (Day – Time)
100 pax (1)	Flight	A (0 – 01:40)	B (0 – 06:40)
100 pax (1)	*Flight*	*B (0 – 07:40)*	*C (0 – 14:20)*
100 pax (1)	Flight	C (0 – 15:15)	A (1 – 03:55)
- (0)	Flight	A (0 – 01:40)	C (0 – 14:20)

Another check is required to verify the creation of arcs originating from the take-off slots. Thus, three potential flights will be used, indicated on Table XI, with the possible flights, the same as previously indicated in Table II. A single 100 passenger aircraft will be used, with a minimum maintenance time after flight of 45 minutes. The operation is restricted at airport B.

Table XI – Potential flights

From	To	Departure	Demand
A	B	01:40	100
B	A	08:20	10
C	A	15:50	100

Because the takeoff slot in B is at 8:20, it should be possible to complete the sequence of flights ABCA, even with restricted operation in B. In this case, the creation of the model takes less than 1 second, resulting in a model with 41 constraints and 54 variables. Its resolution by Cplex took no more than 0.03 s, and the results are shown in Table XII, presenting the selected flights.

Table XII – Best flights selected for an 100 passenger aircraft, with operational restrictions at airport B

Fleet (Airplanes)	Event	From (Day – Time)	To (Day – Time)
100 pax (1)	Flight	A (0 – 01:40)	B (0 – 06:40)
100 pax (1)	*Flight*	*B (0 – 08:20)*	*C (0 – 15:00)*
100 pax (1)	Flight	C (0 – 15:50)	A (1 – 00:10)
- (0)	Flight	C (0 – 08:20)	A (0 – 13:20)

6.4. Conflict between the Number of Landing and Takeoff Slots

It is possible that the table of potential flights may define an airport with more landing slots than takeoff slots. It's a hypothetical situation, but it may be interesting to evaluate the model behavior in this kind of situation.

Table XIII – Potential flights

From	To	Departure	Demand	From
A	B	01:40	06:40	100
A	C	01:40	13:20	50
A	B	01:45	06:45	100
B	C	07:40	13:20	100
B	C	07:45	13:20	100
C	A	15:15	02:55	50
C	A	15:20	03:00	50

To test it, a problem with 7 potential flights was defined, which are presented on Table XIII. The possible flights are shown in Table XIV and two aircraft will be available: a 74 passenger aircraft, and one 100 passenger aircraft. The minimum maintenance time after flight is 45 minutes. The operation is restricted at airport C.

Table XIV – Possible flights (both directions)

Airport 1	Airport 2	Length (min)
A	B	300
B	C	400
A	C	700

The first solution will ignore any restrictions on airports A, B and C. In this case, the generation of the model took about 2 seconds, resulting in a model of 585 constraints and 1013 variables. The optimal solution, obtained in less than 0.04 s, is presented in Table XV.

Table XV – Best flights selected by the model for each aircraft, with no operational restrictions at any airport.

Fleet (Airplanes)	Event	From (Day – Time)	To (Day – Time)
74 pax (1)	Flight	A (0 – 01:45)	B (0 – 06:45)
74 pax (1)	Flight	B (0 – 7:40)	C (0 – 13:20)
74 pax (1)	Flight	C (0 – 15:20)	A (1 – 03:00)
100 pax (1)	Flight	A (0 – 01:40)	B (0 – 06:40)
100 pax (1)	Flight	B (0 – 07:45)	C (0 – 13:20)
100 pax (1)	Flight	C (0 – 15:15)	A (1 – 02:55)
- (0)	Flight	A (0 – 01:40)	C (0 – 13:20)

The result shows that the two aircraft were assigned to the best path, ABCA. All the flights with the biggest demand present on the potential flights table were selected to be performed.

When considering the operational restrictions at the airport C, the time for landing and takeoff will be limited to 13:20 and 15:15 for landing, and 15:20 for takeoff. With this configuration, it is no longer possible to choose the same flights as before, and therefore the 100 passenger aircraft should be allocated primarily on possible flights with an 100 passenger demand, and those not previously selected should be assigned to the 74 passenger aircraft.

Table XVI – Best flights selected by the model for each aircraft, with operational restrictions at airport C

Fleet (Airplanes)	Event	From (Day – Time)	To (Day – Time)
74 pax (1)	Flight	A (0 – 01:45)	B (0 – 06:45)
74 pax (1)	*Flight*	*B (1 – 09:25)*	*A (1 – 14:25)*
100 pax (1)	Flight	A (0 – 01:40)	B (0 – 06:40)
100 pax (1)	Flight	B (0 – 07:45)	C (0 – 13:20)
100 pax (1)	Flight	C (0 – 15:20)	A (1 – 03:00)
- (0)	Flight	A (0 – 01:40)	C (0 – 13:20)
- (0)	Flight	B (0 – 7:40)	C (0 – 13:20)
- (0)	Flight	C (0 – 15:15)	A (1 – 02:55)

The generation of this model, with operational restrictions at the airport C, took about 2 seconds, resulting in a model of 142 constraints and 199 variables. The solution took no more than 0.03 s to be obtained and the results are shown in Table XVI, representing exactly the predicted solution.

7. APPLICATION

After the robustness test, the model was applied to instances based on the regional air transportation network of a Brazilian company, considering the data made available by the company on the Internet. The problem comprises five cities and 104 weekly flights, composed of 20 daily flights from Monday to Friday, and only two repositioning flights on Saturdays and Sundays.

In order to accomplish the schedule, the company uses three ATR-42/300 model aircraft, with capacity for 50 passengers. Since the company works with small aircraft, the maintenance time required between flights is only 15 minutes. The instances generated for analysis were divided into 3 categories, as specified below.

- **Category A** is a coverage problem, using the original mesh of the company. The demand for each flight is equal to the capacity of the aircraft (homogeneous fleet).
- **Category B** is composed of problems which combine the schedule generation and the fleet assignment, using the original mesh of the company, but considering average demands per flight, based on data from 2007 (ANAC, 2007).
- **Category C** is also composed of problems which combine the schedule generation and the fleet assignment, using the original mesh of the company, and average demands for each flight are also used. However, in this case it was considered that the company had received two takeoff slots and two landing slots at Congonhas airport, at 9:00, at 17:30, at 10:00, and at 18:30, respectively.

Each category was represented by problems with different compositions of aircraft fleets, and were solved for a 7 days period (a week), with results quite varied with respect to flights that must be performed by the company.

Table 17 shows the characteristics of models and solution times for categories A and B, and the model generation time was always less than 15s. In these cases, the table of potential flights includes all 104 flights of the company.

Table XVII – Characteristics of the model and solution times for categories A and B

Category	**A**	**B**		
Available Fleets	**3x AT43**	**3x AT43**	**2x AT43 + E140**	**2x AT43 + E120**
Assigned Fleets	3x AT43	2x AT43	2x AT43 + E140	2x AT43 + E120
Variables	35456	35456	35456	35456
Constraints	19526	19526	19526	19526
Objective Function	0	4,0mi	3,8mi	3,2mi
Solution Time (s)	0,43	0,43	0,42	0,41

Table 18 presents the results of the model processing in each of these settings. In category A, all flights were covered with the three company's aircraft, as expected.

Table XVIII – Results of the fleet assignment for categories A and B

Category		**A**		**B**					
Fleet		**3x AT43**		**2x AT43**		**2x AT43 + E140**		**2x AT43 + E120**	
Day	**A-B**	**A=>B**	**B=>A**	**A=>B**	**B=>A**	**A=>B**	**B=>A**	**A=>B**	**B=>A**
Sun	PLU-UBA	1	0	1	0	1	0	1	0
Sun	UBA-UDI	1	0	1	0	1	0	1	0
Mon	PLU-CGH	-	-	-	-	-	-	-	-
Mon	PLU-GVR	2	2	2	2	2	2	2	2
Mon	PLU-IPN	3	3	0	0	3	3	1	1
Mon	PLU-UDI	4	2	2	2	1	2	4	2
Mon	UDI-CGH	-	-	-	-	-	-	-	-
Mon	UDI-UBA	2	0	0	0	0	0	2	0
Mon	UBA-PLU	2	0	0	0	0	0	2	0
Mon	UBA-CGH	-	-	-	-	-	-	-	-
Tue-Thu	PLU-CGH	-	-	-	-	-	-	-	-
Tue-Thu	PLU-GVR	2	2	2	2	2	2	2	2
Tue-Thu	PLU-IPN	3	3	0	0	3	3	1	1
Tue-Thu	PLU-UDI	4	2	2	2	2	2	4	2
Tue-Thu	UDI-CGH	-	-	-	-	-	-	-	-
Tue-Thu	UDI-UBA	2	0	0	0	0	0	2	0
Tue-Thu	UBA-PLU	2	0	0	0	0	0	2	0
Tue-Thu	UBA-CGH	-	-	-	-	-	-	-	-
Fri	PLU-CGH	-	-	-	-	-	-	-	-
Fri	PLU-GVR	2	2	2	2	2	2	2	2
Fri	PLU-IPN	3	3	0	0	3	3	1	1
Fri	PLU-UDI	4	2	1	2	4	2	4	2
Fri	UDI-CGH	-	-	-	-	-	-	-	-
Fri	UDI-UBA	2	0	0	0	2	0	2	0
Fri	UBA-PLU	2	0	0	0	2	0	2	0
Fri	UBA-CGH	-	-	-	-	-	-	-	-
Sat	UDI-UBA	1	0	0	0	0	0	1	0
Sat	UBA-PLU	1	0	0	0	0	0	1	0

In this case, however, the problem was solved as a coverage problem, which does not mean that all these flights are indeed interesting for the balance between supply and demand for seats.

Furthermore, the results of category B show that in terms of supply / demand balance, it makes no sense to perform some flights, as can be seen in the "2x AT43" column.

The exchange of one of the three company's aircraft by a smaller aircraft, such as the Embraer 140 (44 seats) or Embraer 120 (30 passengers), would cause an almost complete coverage, while maintaining the balance between demand and supply.

Table 19 shows the characteristics of models and solution times for the category C. These model generation times were always inferior to 35s. In these cases, the table of potential flights consists of 164 flights, including new flights to Congonhas.

Table XIX – Characteristics of the model and solution times for category C

Configuration	**C**			
Available Fleets	**3x AT43**	**2x AT43 + E170**	**E120 + 2x AT43 + E170**	**2x E120 + 2x AT43 + E170**
Assigned Fleets	3x AT43	2x AT43 + E170	E120 + 2x AT43 + E170	2x E120 + 2x AT43 + E170
Variables	45053	45053	60016	74979
Constraints	24286	24286	32320	40354
Objective Function	16,1mi	14,5mi	14,4mi	14,4mi
Solution Time (s)	0,91	0,61	0,95	10,4

Table 20 presents the results of the model processing in each of these settings. The original fleet of the company is unable to cover all flights offered in this case. There is a shift of aircraft to meet the flights that involve Congonhas, replacing several flights that occurred in previous configurations.

The exchange of an AT43 aircraft by a larger one, as the Embraer 170 (70 passengers) will not modify the number of flights, but shows a better balance between supply and demand, which can be detected by the reduction of the value of the objective function.

The addition of an aircraft like the Embraer 120 to the previous configuration, on the other hand, improves coverage and increases the number of flights.

The addition of a fifth aircraft, another like the Embraer 120, provides full coverage - excluding only impossible flights to and from Congonhas, since the slot was occupied by the flight between Belo Horizonte (PLU) and Congonhas (CGH).

Table XX – Results of the fleet assignment for category C

Configuration		C							
Fleet		3x AT43		2x AT43 + E170		E120 + 2x AT43 + E170		2x E120 + 2x AT43 + E170	
Day	A-B	A=>B	B=>A	A=>B	B=>A	A=>B	B=>A	A=>B	B=>A
Sun	PLU-UBA	1	0	1	0	1	0	1	0
Sun	UBA-UDI	1	0	1	0	1	0	1	0
Mon	PLU-CGH	2	2	2	2	2	2	2	2
Mon	PLU-GVR	2	2	2	2	2	2	2	2
Mon	PLU-IPN	0	0	0	0	1	1	3	3
Mon	PLU-UDI	2	2	2	2	4	2	4	2
Mon	UDI-CGH	0	0	0	0	0	0	0	0
Mon	UDI-UBA	0	0	0	0	2	0	2	0
Mon	UBA-PLU	0	0	0	0	2	0	2	0
Mon	UBA-CGH	0	0	0	0	0	0	0	0
Tue-Thu	PLU-CGH	2	2	2	2	2	2	2	2
Tue-Thu	PLU-GVR	2	2	2	2	2	2	2	2
Tue-Thu	PLU-IPN	0	0	0	0	1	1	3	3
Tue-Thu	PLU-UDI	2	2	2	2	4	2	4	2
Tue-Thu	UDI-CGH	0	0	0	0	0	0	0	0
Tue-Thu	UDI-UBA	0	0	0	0	2	0	2	0
Tue-Thu	UBA-PLU	0	0	0	0	2	0	2	0
Tue-Thu	UBA-CGH	0	0	0	0	0	0	0	0
Fri	PLU-CGH	2	2	2	2	2	2	2	2
Fri	PLU-GVR	2	2	2	2	2	2	2	2
Fri	PLU-IPN	0	0	0	0	1	1	3	3
Fri	PLU-UDI	1	2	2	2	4	2	4	2
Fri	UDI-CGH	0	0	0	0	0	0	0	0
Fri	UDI-UBA	0	0	0	0	2	0	2	0
Fri	UBA-PLU	0	0	0	0	2	0	2	0
Fri	UBA-CGH	0	0	0	0	0	0	0	0
Sat	UDI-UBA	0	0	0	0	1	0	1	0
Sat	UBA-PLU	0	0	0	0	1	0	1	0

8. CONCLUSIONS

The present study sought to model, in an integrated manner, the problem of fleet assignment and the problem of schedule generation, taking into account real world characteristics and constraints.

A linear programming model associated with a space-time network design was proposed, tested, and successfully applied to instances related to a real case of a Brazilian regional airline.

For its implementation the model requires knowledge of the potential demand of each potential flight, and the availability of aircraft of each type. The tests and applications were compatible with reality, at least in the case of Brazilian airlines operating regionally.

The model is presently being extended to incorporate different weights for empty seats and lost demand, operating restrictions for certain types of aircraft at specific airports, and the redistribution of demand among different flights.

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REFERENCES

Abara, J. (1989) Applying Integer Linear Programming to the Fleet Assignment Problem. Interfaces , v.19, n.4, p.21-28.

ANAC (2007) Anuário Estatístico da Agência Nacional de Aviação Civil vol.I . Available at http://www.anac.gov.br/estatistica/estatisticas1.asp . Accessed on 10/07/2009.

ANAC (2008). ANAC cria regra de slots para aumentar concorrência em aeroportos saturados . Available at: http://www.anac.gov.br/imprensa/AnacCriaRegraDeSlots011008.asp . Accessed on 10/07/2009. Agência Nacional de Aviação Civil, Brasília, DF.

Barnhart, C.; AM Cohn; EL Johnson; D. Klabjan; GL Nemhauser e PH Vance. (2003) Airline Crew Scheduling in Handbook of Transportation Science. 2nd ed. Kluwer's International Series.

Caetano, DJ e NDF Gualda. (2008). Modelagem do Problema de Alocação de Frota de uma Empresa Aérea Brasileira. Annals of XXII Congresso de Pesquisa e Ensino em Transportes , ANPET, Fortaleza, v. 1, p. 2173-2176.

CGNA. (2009) Capacidades dos Aeroportos . Available at: http://www.cgna.gov.br/aeroportos/aeroportos.htm . Accessed on 10/07/2009. Centro de Gerenciamento de Navegação Aérea, Brasília, DF.

Dantzig, GB (1974) Linear programming and extensions . New Jersey: Princeton University Press.

Dantzig, GB e DR Fulkerson (1954) Minimizing the number of carriers to meet a fixed schedule, Naval Res. Log. Vol 1.

Gomes, WP e NDF Gualda. (2008). Otimização da Formação de Viagens no Processo de Alocação de Tripulantes e Aeronaves (Crew Pairing Problem). Annals of XXII Congresso de Pesquisa e Ensino em Transportes, ANPET, Fortaleza, v. 1, p. 1010-1020.

Hane, C.; C. Barnhart; E. Johnson; R. Marsten; G. Nemhauser e G. Sigismondi. (1994) A fleet assignment problem: Solving a large-scale integer program, Technical report , Georgia Institute of Technology , School of Industrial and System Engineering. Report Series 92-04.

Klabjan, D. (2004) Large-scale models in the airline industry, in G. Desaulniers, J. Desroriers, MM Solomon, editors, Column Generation, Kluwer Academic Publishers .

MD (2008) Congonhas voltará a ter escalas e conexões . Available at: http://www.defesa.gov.br/mostra_materia.php?ID_MATERIA=31859. Accessed on 10/07/2009. Ministério da Defesa, Brasília, DF.

Oliveira, A. (2009) Transporte Aéreo – Economia e Políticas Públicas. Pezco Editora. São Paulo.

Rabetanety, A.; J. Calmet e C. Schoen. (2006) Airline Schedule Planning Integrated Flight Schedule Design and Product Line Design. Tese. Universität Karlsruhe. Karlsruhe .

Sherali, HD; EK Bish; X. Zhu. (2006) Airline fleet assignment concepts, models, and algorithms, European Journal of Operational Research , n.172, p.1-30.

Swan, WM (2002) Airline route developments: a review of history. Journal of Air Transport Management . n.8. p.349-353.