AN ANALYSIS OF REFORMING BY OFF-PEAK FEE DISCOUNT FOR ETC TO REDUCE HIGHWAY CONGESTION

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ABSTRACT

This study analyses discounts on highway fee for the off-peak commuting with ETC as the application of the traditional bottleneck congestion model in order to consider self-select fee system and post payment related to historical usage. The ETC off-peak commuting discounts improves efficiency of road system due to utilization of off-peak capacity of highway so as to shift traffic demand from open road to highway, by contrast, ETC peak commuting discount which has been in place in Japanese highway. Furthermore, ETC off-peak commuting discount plus that is depend on historical highway usage is an effective measure to give an incentive for usage of ETC device, restraining aggravation of road congestion.

Keywords: Electric Toll Collection, Self-select fee system, Bottleneck congestion.

1. INTRODUCTION

Japanese highway fee system has been added characteristic features which are (1) self-select fee system, (2) non-stop toll collection, (3) post-payment by introduction of ETC: Electric Toll Collection. These features enable us to charge highway fee in variety payment structure to reduce congestion. First, (1) self-select fee system including ETC and non-ETC achieves Parato improvement between users by proper price discrimination. Second, (2) non-stop toll collection saves time and energy consumptions in toll gate congestion. Third, (3) post-payment related to usage history allow us charging highway fee which restrain demand intensive in peak period in repeated commuting.

In the almost road pricing policy for highway congestion, additional charge in peak period is their subject. However, many public transportation fees are discounted in off-peak period to level time varying demand. In fact additional charge in peak period and reduced charge in off-peak period have similar affection on congested transportation system.

Suzuki (2008a) investigated effects Japanese highway fee discount for ETC users in peak period on highway bottleneck congestion. This study applied traditional bottleneck congestion model which was conceived by Vickrey(1969) and extended Arnott de Palma and

Linsey (1990). As the result, the discount obviously deteriorate highway congestion in peak period in many cases however it may be workable if the toll gate performance would be improved by high usage rate of ETC booths. Furthermore, mixed use of ETC and non-ETC users restrain highway congestion from increase which is generated by fee discount for ETC users.

Considering the results of previous study, off-peak discounts for ETC are investigated to systematize in highway fee policies in this study. The analyses relate these discounts to (1) self-select fee system and (3) post-payment.

2. A BOTTLENECK CONGETION MODEL ASSOCIATED WITH SELF-SELECT ETC DISCOUNT FEE SYSTEM

The Japanese highway fee system changed to self-select in a sense that highway user can choose between normal fee system and discount fee system even if the application of discount fee are limited in specified possession of ETC device, time period , distance and area. This change seems to achieve Pareto improvement between users because user can choose current normal fee system if the ETC discount fee system is worse than current one for user as Train (1994) said. However, the assumption of consumers' independences is not satisfied because highway users' choices are influenced by other users' choices through the congestion.

The Japanese ETC commuting discount system targets highway commuters in rush hour to spread ETC usage rapidly. Sort of inconsequential discounts fee in congested time period such as Japanese one has not been attracted attention in congestion charging studies for the past. Therefore we need to know adequate ways of discounts highway fee in congested period if the appropriate ways exist. Suzuki (2008a) applied bottleneck congestion model to current Japanese highway fee system as self-select fee system to study effect introduction of ETC commuting discount on highway congestion. According to the results, the conditions which make ETC commuting discount system to keep functional to ensure the incentive to use ETC commuting discount without increase of highway congestion, and furthermore aggravating situation of bottleneck congestion and its mechanism were cleared up by comparative statics.

Suzuki (2008b) specified three bottle neck point and relations between them to consider improvement of performance of toll gate by increase of non-stop fee collection at bottle neck points. As the results, The ETC commuting discount fee system reduce congestion if the utilization rate of ETC booth can increase. On the other hand, it is also shown possibility of wasted case that the high discount rate occurring excessive concentration of discount period generates bottle neck congestion outside of toll gate such as main track on the highway even if the toll gate performance is improved.

ETC system can be imposed corresponding to several highway use attribute which are section, distance, time, vehicle type. This function helps to develop more efficient fee system. This study focuses on ETC off-peak commuting discount to reduce users' travel costs by intensive use of off-peak capacity and to give an incentive for ETC to highway users. For the sake of simplification the bottleneck point aren't specified.

At point of purchase of ETC device which postulate ETC commuting discount, users may choose highway fee system without considering variation in travel time due to change of travel demand, but with considering the costs of a ETC device and highway fees. For the above reason this study analyze the relationship between user's travel choices and bottle neck congestion in their morning commuting in short term regarding the long term choice of purchase of ETC device as a given condition.

The following assumptions are introduced for the simplification. The residential area and work place are connected by a highway route: A, and an open road route: B. Highway fees are collected at toll gate which is end of the highway.

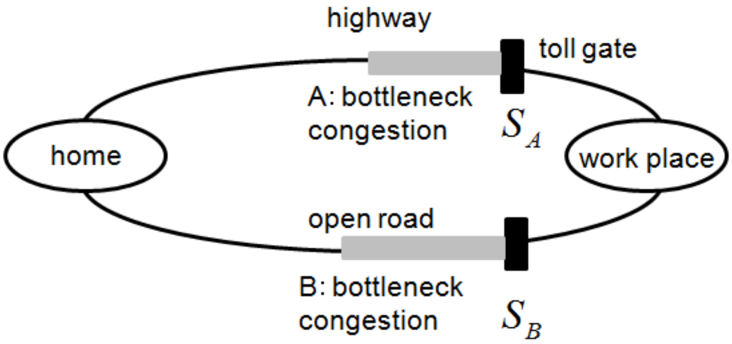

Figure 1 – Commuting network

The total number of commuters is constant: N in morning period.

$$N = N_A + N_B \tag{1}$$

The commuters choose their departure time and route under the following (i)-(iii) highway fee system.

(i) Normal fee system: τ for each commuting,

(ii)ETC off-peak commuting discount: δ discount from τ for each commuting in off-peak period with ETC,

(iii)ETC of-peak commuting discount plus: δ discount from τ for one commuting in peak period with ETC if P-1 times off-peak commuting out of P times commuting,

in addition, (ii),(iii) postulates purchase of ETC device.

The commuters are also divided in ETC users: N^E and non-ETC users: N^N by fee types.

$$N = N^E + N^N \tag{2}$$

Furthermore, users who are applied ETC off-peak discount plus is discriminated as N^{EE} from N^E as necessary.

The bottle neck congestions arise in one place for each route. S_j represents the capacity of bottle neck j and $Q_j(t)$ represents the queue which a commuter who alights at t faces at the bottle neck j. Omitting uncongested part of travel, the travel time of a commuter who departs in t denotes

$$T_j(t) = \frac{Q_j(t)}{S_j}. \tag{3}$$

The departure rate of commuters who alight at t and choose route j and fee type k (k=E: ETC, =N: non-ETC) is given by $r_j^k(t)$. The variation of queue is formulated as follows

$$\frac{dQ_j(t)}{dt} = \begin{cases} 0, & Q_j(t) = 0 \ and \ \sum_{k=N,E} r_j^k(t) \le S_j \\ \sum_{k=N,E} r_j^k(t) - S_j \ , & \sum_{k=N,E} r_j^k(t) \ge S_j \end{cases} . \tag{4}$$

The travel costs of commuters consist of the following costs: the travel time cost, the schedule cost and the highway fee.

$$C_j^k(t) = \alpha T_j(t) + SD_j(t) + \left(1 - \delta_j^k(t)\right)\tau , \tag{5}$$

where α is shadow values of travel time, $SD_j(t)$ shows schedule cost, $\delta_j^k(t)$ means discount rate which changes corresponding to time period , commuting route and fee type.

All commuters wish to arrive at work at $t*$. Let $t_{n,j}$ be the departure time for which a commuter arrives at work on time using route j, then $t_{n,j} + T_j(t_{n,j}) = t*$. If a commuter departs earlier than $t_{n,j}$, he is early by $t* - t - T_j(t)$, while if he departs later than $t_{n,j}$, he is late by $t + T_j(t) - t*$. The travel costs of commuters are expressed as

$$C_j^k(t) = \begin{cases} \alpha T_j^k(t) + \beta\left(t* - t - T_j^k(t)\right) + \left(1 - \delta_j^k(t)\right)\tau & \left(t \le t_{n,j}\right) \\ \alpha T_j^k(t) + \gamma\left(t + T_j^k(t) - t*\right) + \left(1 - \delta_j^k(t)\right)\tau & \left(t \ge t_{n,j}\right) \end{cases} , \tag{6}$$

where β , γ are shadow value of time early and time late. The order of these values is assumed $\beta < \alpha < \gamma$ related to α as given in Small (1982). In addition, $\left(1 - \delta_j^k(t)\right)\tau$ shows highway fee which imposed for departure time: t, route choice: j and fee type: k. Then, commuters choose departure time, route, and fee type to minimize their travel costs.

3. ANALYSES OF REFORMING HIGHWAY FEE SYSTEM BY ETC OFF PEAK COMMUTING DISCOUNTS

3.1 The bottleneck congestions without ETC off-peak commuting discounts

An equilibrium without any discounts that is the case of $\delta_A^k(t)=0$, $\delta_B^k(t)=1$ in the equation (6) is solved as a benchmark for following highway fee discount policies. Here, only normal fee is imposed for the highway user therefore ETC users and non-ETC users are not discriminated. The case is named for (I). The equilibrium is defined since commuters are not incentivized to change their departure times and route. Let the queue continue from $t_{0,j}$ to $t_{e,j}$. The morning congested period equals the total number of commuters divided by the bottleneck capacity for each route j.

$$t_{e,j} - t_{0,j} = \frac{N_j}{S_j} \tag{7}$$

The first and last commuters do not face a queue. The travel cost for the first and last commuters are simplified from equation (5) by reducing $T_j^k(t)$ as shown below,

$$C_j\left(t_{0,j}\right) = \beta\left(t^* - t_{0,j}\right) + \left(1 - \delta_j^k(t)\right)\tau , \tag{8}$$

$$C_j\left(t_{e,j}\right) = \gamma\left(t_{e,j} - t^*\right) + \left(1 - \delta_j^k(t)\right)\tau . \tag{9}$$

The travel costs of all commuters should equal each other in this equilibrium then the departure time and travel cost are derived as

$$t_{0,j} = t^* - \frac{\gamma}{\beta+\gamma}\frac{N_j}{S_j}, \quad t_{n,j} = t^* - \frac{\beta\gamma}{\alpha(\beta+\gamma)}\frac{N_j}{S_j}, \quad t_{e,j} = t^* + \frac{\beta}{\beta+\gamma}\frac{N_j}{S_j}, \tag{10}$$

$$C_j = \frac{\beta\gamma}{\beta+\gamma}\frac{N_j}{S_j} + \left(1 - \delta_j^k(t)\right)\tau . \tag{11}$$

The travel time and the departure rate are calculated as follows.

$$T_j(t) = \begin{cases} \dfrac{\beta}{\alpha-\beta}\left(t - t_{0,j}\right) & for\ t \in \left[t_{0,j}, t_{n,j}\right] \\ \dfrac{\gamma}{\alpha+\gamma}\left(t_{e,j} - t\right) & for\ t \in \left[t_{n,j}, t_{e,j}\right] \end{cases} , \tag{12}$$

$$r_j^k(t)=\begin{cases}\dfrac{\alpha}{\alpha-\beta}S_j & for\ t\in[t_{0,j},t_{n,j}]\\ \dfrac{\alpha}{\alpha+\gamma}S_j & for\ t\in[t_{n,j},t_{e,j}]\end{cases} \quad . \tag{13}$$

The numbers of highway commuters and open road are calculated as following equations.

$$N_A=\frac{S_A}{S_A+S_b}\left(N-S_B\frac{\beta+\gamma}{\beta\gamma}\tau\right) \quad ,N_B=N-N_A, \tag{14}$$

The figure 2 depicts the transitions of travel cost components in benchmark equilibrium and implied equilibrium conditions are satisfied because the travel time costs $\alpha T_j(t)$, schedule cost $SD_j(t)$, highway fee $\tau(t)$ are different corresponding to arrival time but travel costs are constant between commuters. Vickrey (1969) insisted that social optimal is achieved by charging the time varying fee that corresponds to the travel time cost $\alpha T_j(t)$.

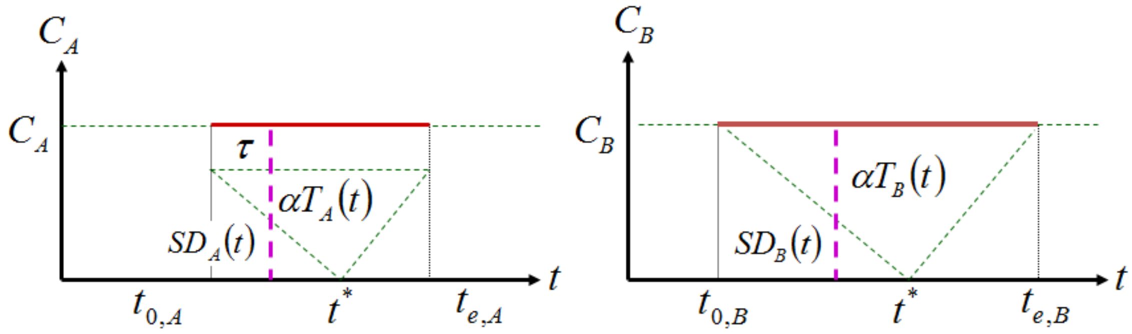

Figure 2 – Transition of travel cost components in case (I)

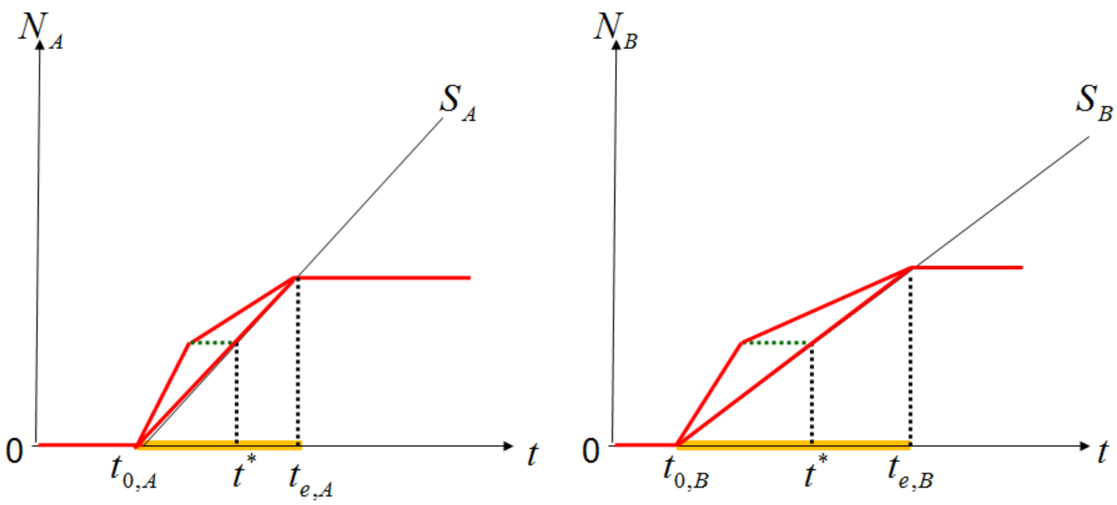

Figure 3 – Transitions of cumulative number of departures and arrivals in case (I)

3.2 The bottleneck congestions with ETC off-peak commuting discount

In this subsection, the combined highway fee system (i) and (ii) are analyzed. The case is called as (II). The commuters are divided in ETC users and non-ETC users according to fee type. As mentioned previous section, the commuter who arrival at t* faces the longest queue. Then let it be off-peak period that time period does not include desired arrival time t*, and denote $\left[t_{0,A}, t_A^+\right], \left[t_A^-, t_{e,A}\right]$. The travel costs are formulated related to fee type, route choice and departure time as equation (15).

$$C_j^k(t) = \begin{cases} \alpha T(t) + \beta\left(t^* - t - T_j(t)\right) + \left(1 - \delta_j^k(t)\right)\tau & \text{for } k \in (N, E), t \in \left[t_{0,j}, t_{n,j}\right] \\ \alpha T(t) + \gamma\left(t + T_j(t) - t^*\right) + \left(1 - \delta_j^k(t)\right)\tau & \text{for } k \in (N, E), t \in \left[t_{n,j}, t_{e,j}\right] \end{cases}, \qquad (15)$$

$$\text{where} \quad \delta_j^k(t) \begin{cases} = 0 & \text{for } j = A, k = N, t \in \left[t_{0,A}, t_{e,A}\right] \\ = 0 & \text{for } j = A, k = E, t \in \left[t_A^+, t_A^-\right] \\ = \delta & \text{for } j = A, k = E, t \in \left[t_{0,A}, t_A^+\right], \left[t_A^-, t_{e,A}\right] \\ = 1 & \text{for } j = B, k = N, E, t \in \left[t_{0,j}, t_{e,j}\right] \end{cases}.$$

3.2.1 Case of $\left(t_A^+ - t_{0,A} + t_{e,A} - t_A^-\right)S_A > N^E$:(IIa)

First of all, the maximum number of commuters which can be handled at the toll gate in discount period is defined as bottleneck capacity of discount period: $\left(t_A^+ - t_{0,A} + t_{e,A} - t_A^-\right)S_A$. Let us begin with the case in which the number of ETC users is less than the bottleneck capacity of discount time period under fee system (i) + (ii). This case: (IIa) corresponds to a relatively low discount rate, expensive ETC device, large capacity of bottleneck, long discount period, early spread period of ETC device and so on. The equilibrium holds under the conditions that no commuter has an incentive to change his/her departure time and route in order to reduce his/her travel cost.

$$t_{0,j} = t^* - \frac{\gamma}{\beta+\gamma}\frac{N_j}{S_j}, \quad t_{n,j} = t^* - \frac{\beta\gamma}{\alpha(\beta+\gamma)}\frac{N_j}{S_j}, \quad t_{e,j} = t^* + \frac{\beta}{\beta+\gamma}\frac{N_j}{S_j} \qquad (16)$$

$$t_{n,A}^+ = \frac{\alpha-\beta}{\alpha}t_A^+ + \frac{\beta}{\alpha}t_{0,A}, \quad t_{n,A}^- = \frac{\alpha+\gamma}{\alpha}t_A^- - \frac{\gamma}{\alpha}t_{e,A} \qquad (17)$$

$$T_j(t) = \begin{cases} \frac{\beta}{\alpha-\beta}\left(t - t_{0,j}\right) & \text{for } t \in \left[t_{0,j}, t_{n,j}\right] \\ \frac{\gamma}{\alpha+\gamma}\left(t_{e,j} - t\right) & \text{for } t \in \left[t_{n,j}, t_{e,j}\right] \end{cases} \qquad (18)$$

$$C_A^N = C_B^N \qquad (19)$$

$$C_A^N = \frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A} + \tau \qquad \text{for } t \in \left[t_{0,A}, t_{e,A}\right] \qquad (20)$$

$$C_A^E = \frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A} + \left(1 - \delta_A^E(t)\right)\tau \qquad \text{for } t \in \left[t_{0,A}, t_{e,A}\right] \qquad (21)$$

$$C_B^N, C_B^E = \frac{\beta\gamma}{\beta+\gamma}\frac{N_B}{S_B} \qquad for\ t \in \left[t_{0,B}, t_{e,B}\right] \tag{22}$$

In this equilibrium illustrated in figure 4 and 5, only the travel cost of ETC users which are drawn in faint colour decreases within the off-peak period. The difference between the travel costs of ETC and non-ETC users arise from the following mechanism. ETC users will try to pass the toll gate within the off-peak period, while non-ETC users are not strongly interested in the off-peak period because they cannot enjoy the ETC off-peak commuting discount. From the given condition, i.e. $\left(t_A^+ - t_{0,A} + t_{e,A} - t_A^-\right)S_A > N^E$, all ETC users can use the highway within the off-peak period, including some non-ETC users. ETC users change their departure time within the off-peak period and non-ETC users change their departure time within both off-peak and peak period, furthermore they also change their route choices between the highway and the open road, in order to reduce their travel costs. As a result, all ETC users pass the toll gate within the discount period. The travel times for ETC users and non-ETC users are exactly the same on highway route corresponding to each departure time in the equilibrium; therefore, a commuter cannot distinguish ETC users from other commuters except at the toll gate. Even if the highway fee is discounted, the congestions will not change in both highway and open road as shown in figure 5. On the other hand, as for travel costs, only ETC users enjoy lower travel costs due to off-peak discount. The incentive for ETC exists therefore the number of ETC users will increase in the long term.

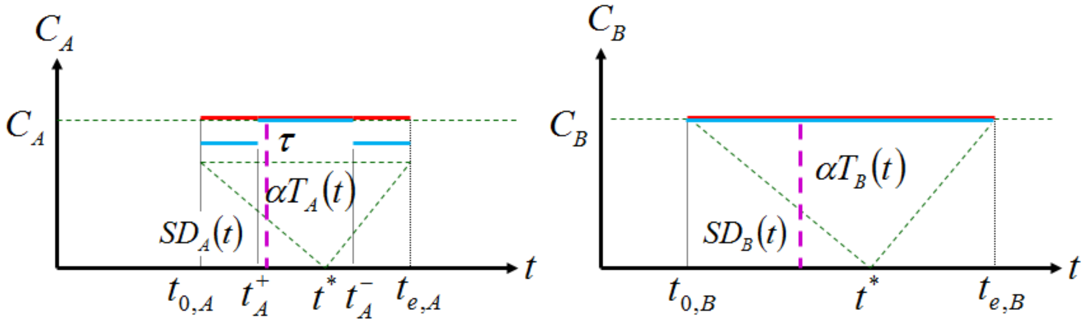

Figure 4 – Transitions of travel cost components in case (IIa)

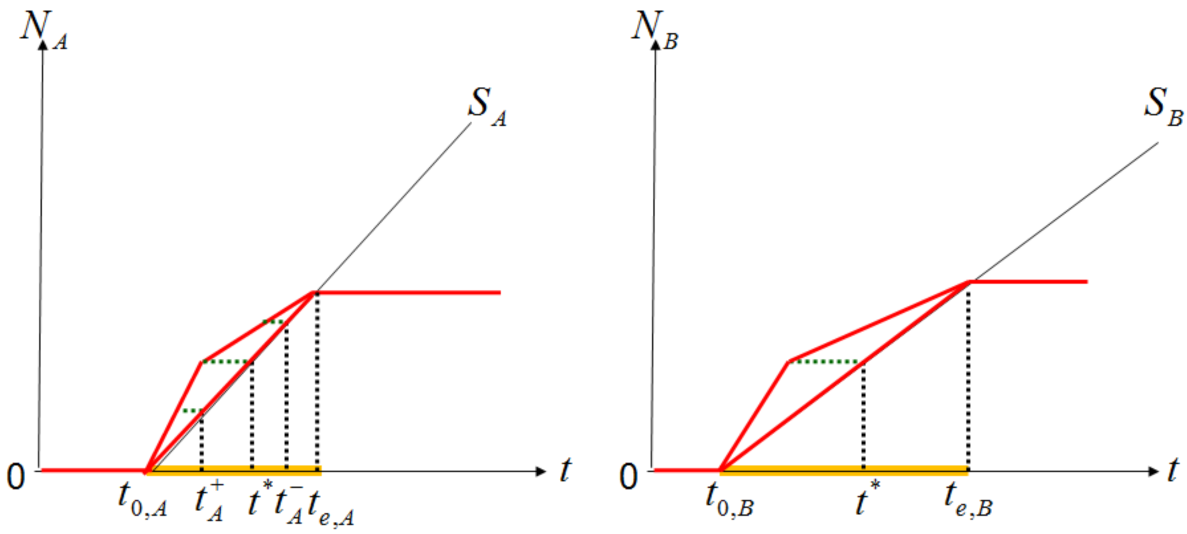

Figure 5 – Transitions of cumulative number of departures and arrivals in case (IIa)

3.2.2 Case of $\left(t_A^+ - t_{0,A} + t_{e,A} - t_A^-\right)S_A < N^E$ *:(IIb)*

Herein, the number of ETC users is more than the capacity of the discount period. This case:(IIb) may be caused by a relatively high discount rate, cheap ETC device, small capacity of bottleneck, short discount period, late spread period of ETC device and so on. The equilibrium also holds under the condition that no commuter has an incentive to change his/her departure time and route to reduce his/her travel cost

$$t_{0,j} = t^* - \frac{\gamma}{\beta+\gamma}\frac{N_j}{S_j}, \quad t_{n,j} = t^* - \frac{\beta\gamma}{\alpha(\beta+\gamma)}\frac{N_j}{S_j}, \quad t_{e,j} = t^* + \frac{\beta}{\beta+\gamma}\frac{N_j}{S_j} \tag{23}$$

$$t_{n,A}^+ = \frac{\alpha-\beta}{\alpha}t_A^+ + \frac{\beta}{\alpha}t_{0,A}, \quad t_{n,A}^- = \frac{\alpha+\gamma}{\alpha}t_A^- - \frac{\gamma}{\alpha}t_{e,A} \tag{24}$$

$$T_A(t) = \begin{cases} \dfrac{\beta}{\alpha-\beta}\left(t - t_{0,A}\right) + \dfrac{\delta\tau}{\alpha-\beta} & \text{for } t \in \left[t_{0,A}, t_{n,A}^+\right] \\ \dfrac{\beta}{\alpha-\beta}\left(t - t_{0,A}\right) & \text{for } t \in \left[t_{n,A}^+, t_{n,A}\right] \\ \dfrac{\gamma}{\alpha+\gamma}\left(t_{e,A} - t\right) & \text{for } t \in \left[t_{n,A}, t_{n,A}^-\right] \\ \dfrac{\gamma}{\alpha+\gamma}\left(t_e - t\right) + \dfrac{\delta\tau}{\alpha+\gamma} & \text{for } t \in \left[t_n^-, t_e\right] \end{cases} \tag{25}$$

$$T_B(t) = \begin{cases} \dfrac{\beta}{\alpha-\beta}\left(t - t_{0,B}\right) & \text{for } t \in \left[t_{0,B}, t_{n,B}\right] \\ \dfrac{\gamma}{\alpha+\gamma}\left(t_{e,B} - t\right) & \text{for } t \in \left[t_{n,B}, t_{e,B}\right] \end{cases} \tag{26}$$

$$C_A^N = C_B^N \tag{27}$$

$$C_A^N = \frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A} + \left(1 + \delta_A^E(t)\right)\tau \qquad \text{for } t \in \left[t_{0,A}, t_{e,A}\right] \tag{28}$$

$$C_A^E = \frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A} + \tau \qquad \text{for } t \in \left[t_{0,A}, t_{e,A}\right] \tag{29}$$

$$C_B^E, C_B^N = \frac{\beta\gamma}{\beta+\gamma}\frac{N_B}{S_B} \qquad \text{for } t \in \left[t_{0,B}, t_{e,B}\right] \tag{30}$$

As shown in the equilibrium in figure 6, travel costs are a little decreased except only non-ETC users within off-peak period. The difference between travel costs of ETC and non-ETC users cause by the following mechanism. ETC users try to pass the toll gate within off-peak period; however, a section of ETC users cannot do so because the number of ETC users is more than the capacity of bottleneck in the discount period. Then, ETC users extend the queue within the off-peak period until their travel costs equal to the travel costs in the peak period on the highway, or the open road. On the other hand, not-ETC users do not need to pass the gate within the off-peak period. Therefore, non-ETC users pass the toll gate within peak period or use open road. In the equilibrium, all commuters who pass the toll gate within the off-peak period are ETC users, and commuters who do so within peak period or use open road are both ETC users and non-ETC users. Hence, Parate improvement is achieved within commuters because equilibrium travel costs are decreased for all users compared with case (I). It is caused by demand shift from open road to highway use despite repressing increase of congestion within peak period shown as figure 6. The incentive for ETC use is disappeared therefore the number of ETC users may not increase in the long term.

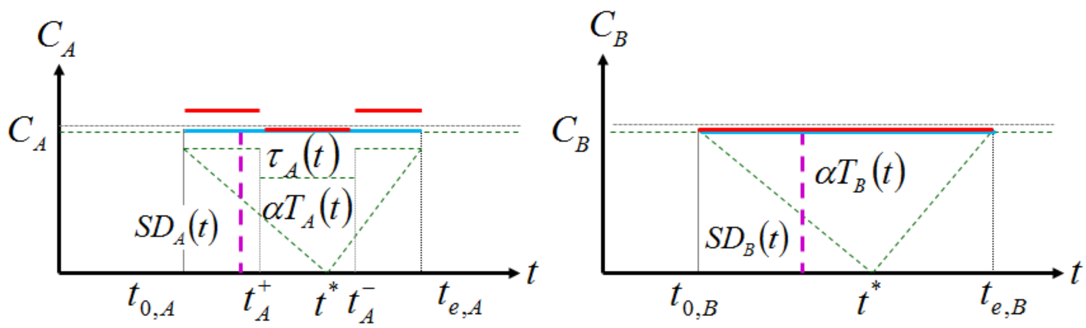

Figure 6 – Transitions of travel cost components in case (IIb)

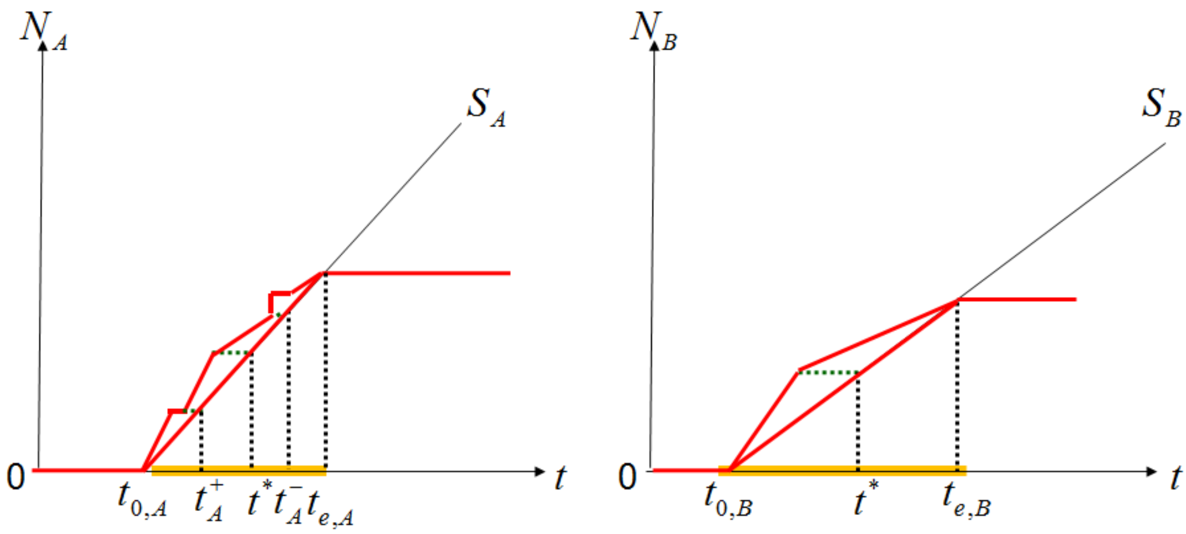

Figure 7 – Transitions of cumulative number of departures and arrivals in case (IIb)

3.3 The bottleneck congestions with ETC off-peak commuting discount plus

In this subsection, the ETC off-peak commuting discount i.e. (iii) is combined to (i) and (ii) as the extra incentive to promote off-peak highway commuting. The commuters are divided in ETC users: N^E and non-ETC users: N^N, in addition the ETC users who travel off-peak period frequently are discriminated from the ETC users as N^{EE}.

$$C_j^k(t)=\begin{cases}\alpha T(t)+\beta(t^*-t-T_j(t))+(1-\delta_j^k(t))\tau & for\ k\in(N,E), t\in[t_{0,j},t_{n,j}]\\ \alpha T(t)+\gamma(t+T_j(t)-t^*)+(1-\delta_j^k(t))\tau & for\ k\in(N,E), t\in[t_{n,j},t_{e,j}]\end{cases}, \quad (31)$$

where $\delta_j^k(t)\begin{cases} =0 & for\ j=A, k=N, t\in[t_{0,A}, t_{e,A}] \\ =0 & for\ j=A, k=E, t\in[t_A^+, t_A^-] \\ =\delta & for\ j=A, k=EE, t\in[t_A^+, t_A^-] \\ =\delta & for\ j=A, k=E,EE, t\in[t_{0,A}, t_A^+],[t_A^-, t_{e,A}] \\ =1 & for\ j=B, k=N,E,EE, t\in[t_{0,j}, t_{e,j}] \end{cases}$.

3.3.1 Case of $(t_A^+ - t_{0,A} + t_{e,A} - t_A^-)S_A > N^E - N^{EE}$:(IIIa)

Herein, the number of ETC users who commute within the off-peak period not frequently $N^E - N^{EE}$ is less than bottleneck capacity of discount period: $(t_A^+ - t_{0,A} + t_{e,A} - t_A^-)S_A$. The case is labeled as (IIIa). The equilibrium is solved in the same way as previous section.

$$t_{0,j} = t^* - \frac{\gamma}{\beta+\gamma}\frac{N_j}{S_j}, \quad t_{n,j} = t^* - \frac{\beta\gamma}{\alpha(\beta+\gamma)}\frac{N_j}{S_j}, \quad t_{e,j} = t^* + \frac{\beta}{\beta+\gamma}\frac{N_j}{S_j} \tag{32}$$

$$t_{n,A}^+ = \frac{\alpha-\beta}{\alpha}t_A^+ + \frac{\beta}{\alpha}t_{0,A}, \quad t_{n,A}^- = \frac{\alpha+\gamma}{\alpha}t_A^- - \frac{\gamma}{\alpha}t_{e,A} \tag{33}$$

$$T_j(t) = \begin{cases} \frac{\beta}{\alpha-\beta}(t - t_{0,j}) & for\ t\in[t_{0,j}, t_{n,j}] \\ \frac{\gamma}{\alpha+\gamma}(t_{e,j} - t) & for\ t\in[t_{n,j}, t_{e,j}] \end{cases} \tag{34}$$

$$C_A^N = C_B^N \tag{35}$$

$$C_A^N = \frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A} + \tau \qquad for\ t\in[t_{0,A}, t_{e,A}] \tag{36}$$

$$C_A^E = \frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A} + (1 - \delta_A^E(t))\tau \qquad for\ t\in[t_{0,A}, t_{e,A}] \tag{37}$$

$$C_A^{EE} = \frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A} + (1 - \delta_A^{EE}(t))\tau \qquad for\ t\in[t_{0,A}, t_{e,A}] \tag{38}$$

$$C_B^E, C_B^N, C_B^{EE} = \frac{\beta\gamma}{\beta+\gamma}\frac{N_B}{S_B} \qquad for\ t\in[t_{0,B}, t_{e,B}] \tag{39}$$

In the equilibrium illustrated in figure 8, the travel cost of ETC users: N^E decrease within the off-peak period and travel cost of frequently off-peak commuting ETC users who commute off-peak period frequently: N^{EE} decrease outside of off-peak period too. The differences of travel costs of users are accrued in below process. The ETC users who travel off-peak not frequently cannot get discount within peak-period then they try to commute within off-peak period. From the given condition, i.e. $(t_A^+ - t_{0,A} + t_{e,A} - t_A^-)S_A > N^E - N^{EE}$, all of them can commute within off-peak period, including some non-ETC users and ETC users who commute off-peak period frequently. ETC users who commute off-peak not frequently change their departure time within the off-peak period, ETC users who commute off-peak period frequently change their departure time within both off-peak and peak period, and non-

ETC users change their departure time and route within both off-peak and peak, in addition highway and open road in order to reduce their travel costs. The equilibrium is valid when no commuters can find alternative departure time and route to reduce his/her travel costs. As a result, travel times for each fee type user are exactly the same corresponding to each departure time and route choice in this equilibrium. Even if the users' departure time and route choice are discriminated by self-selecting, the congestions are not change in both highway and open road from (I) and (IIa). On the other hand, as for travel costs, ETC users which include frequently commuting off-peak period enjoy lower travel costs due to off-peak discount as represented in figure 8. Furthermore, ETC users who commute within off-peak period frequently get benefit from ETC off-peak commuting discount plus within peak period. The incentive for ETC and commuting off-peak period exist therefore the number of ETC users will increase in the long term.

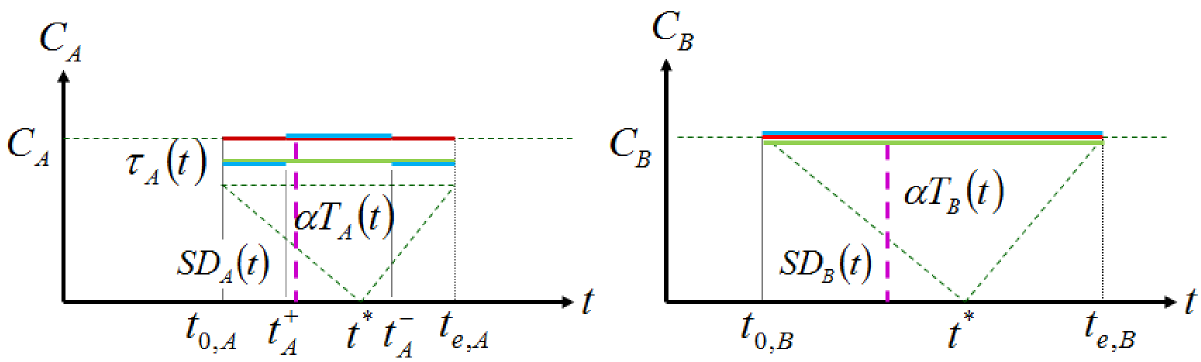

Figure 8 – Transitions of travel cost components in case (IIIa)

3.3.2 Case of $\left(t_A^+ - t_{0,A} + t_{e,A} - t_A^-\right)S_A < N^E - N^{EE}$ *:(IIIb)*

Next, the number of ETC users who travel off-peak period not frequently $N^E - N^{EE}$ is more than bottleneck capacity of discount time period: $\left(t_A^+ - t_{0,A} + t_{e,A} - t_A^-\right)S_A$. The case is described as (IIIb). The equilibrium is solved in the same manner as previous section.

$$t_{0,j} = t^* - \frac{\gamma}{\beta+\gamma}\frac{N_j}{S_j}, \quad t_{n,j} = t^* - \frac{\beta\gamma}{\alpha(\beta+\gamma)}\frac{N_j}{S_j}, \quad t_{e,j} = t^* + \frac{\beta}{\beta+\gamma}\frac{N_j}{S_j} \tag{40}$$

$$t_{n,A}^+ = \frac{\alpha-\beta}{\alpha}t_A^+ + \frac{\beta}{\alpha}t_{0,A}, \quad t_{n,A}^- = \frac{\alpha+\gamma}{\alpha}t_A^- - \frac{\gamma}{\alpha}t_{e,A} \tag{41}$$

$$T_A(t) = \begin{cases} \frac{\beta}{\alpha-\beta}\left(t-t_{0,A}\right) + \frac{(1+1/P)\delta\tau}{\alpha-\beta} & \text{for } t \in \left[t_{0,A}, t_{n,A}^+\right] \\ \frac{\beta}{\alpha-\beta}\left(t-t_{0,A}\right) & \text{for } t \in \left[t_{n,A}^+, t_{n,A}\right] \\ \frac{\gamma}{\alpha+\gamma}\left(t_{e,A}-t\right) & \text{for } t \in \left[t_{n,A}, t_{n,A}^-\right] \\ \frac{\gamma}{\alpha+\gamma}\left(t_e-t\right) + \frac{(1+1/P)\delta\tau}{\alpha+\gamma} & \text{for } t \in \left[t_n^-, t_e\right] \end{cases} \tag{42}$$

$$T_B(t)=\begin{cases}\dfrac{\beta}{\alpha-\beta}\left(t-t_{0,B}\right) & for\ t\in\left[t_{0,B},t_{n,B}\right]\\ \dfrac{\gamma}{\alpha+\gamma}\left(t_{e,B}-t\right) & for\ t\in\left[t_{n,B},t_{e,B}\right]\end{cases} \tag{43}$$

$$C_A^N=C_B^N \tag{44}$$

$$C_A^N=\frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A}+\left(1+\left(1+1/P\right)\delta_A^E(t)\right)\tau \qquad for\ t\in\left[t_{0,A},t_{e,A}\right] \tag{45}$$

$$C_A^E=\frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A}+\tau \qquad for\ t\in\left[t_{0,A},t_{e,A}\right] \tag{46}$$

$$C_A^{EE}=\frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A}+\left(1-\left(1+1/P\right)\delta_A^{EE}(t)\right)\tau \qquad for\ t\in\left[t_{n,A}^+,t_{n,A}^-\right] \tag{47}$$

$$C_A^{EE}=\frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A}+\tau \qquad for\ t\in\left[t_{0,A},t_{n,A}^+\right]\cup\left[t_{n,A}^-,t_{e,A}\right] \tag{48}$$

$$C_B^E,C_B^N,C_B^{EE}=\frac{\beta\gamma}{\beta+\gamma}\frac{N_B}{S_B} \qquad for\ t\in\left[t_{0,B},t_{e,B}\right] \tag{49}$$

In this equilibrium, the travel costs of N^N users increase $\delta\tau$ compared to N^E within the off-peak period and travel costs of N^{EE} decrease $\delta\tau$ compared to others within the peak period shown as figure 9.

ETC users who are not applied ETC off-peak commuting discount plus try to pass the toll gate within off-peak period; however a part of them cannot do so because N^E-N^{EE} is more than the capacity of bottleneck in discount period. Then, the queue is extended within the off-peak period. As for ETC users who are applied ETC off-peak commuting discount plus would like to pass the toll gate with peak period because their travel costs within peak period lower than off-peak period relatively. Even if the queue is prolonged until other ETC users' travel costs within off-peak period equal to their travel costs within peak period, the ETC users who are applied ETC off-peak discount plus can still enjoy their benefits within peak-period every Pth commuting. Therefore the queue is extended until the gaps of travel times are equivalent to $(1+1/P)\delta\tau$ between off-peak and peak period. The travel costs between ETC users are indifferent in the expected value. Hence, all ETC users who go through the toll gate within the off-peak period are potential N^{EE} who will be applied ETC off-peak commuting discount plus within peak period every Pth commuting, then $N^{EE}=\left(t_A^+-t_{0,A}+t_{e,A}-t_A^-\right)S_A/(P-1)$.

Meanwhile, non-ETC users have no incentive to attend the extended queue within off-peak period. Therefore, N^N go through the toll gate within peak period if they use highway route. Therefore, The three different groups; i.e. N^{EE} , N^E , N^N commute mixing within peak period on highway route. If setting short peak period or small value of P which cause $\left(t_A^--t_A^+\right)S_A<N^{EE}$ may disturb onset of mitigation of bottleneck congestions. The open road may be used by both N^N and N^E . As the result, the commuting time period on the highway route is expanded and shift travel demand from open road to highway route depicted in figure 10. Then travel costs of both open road and highway are decreased and efficiency of road

system is improved even if bottleneck congestion on the highway route is aggravated within off-peak period.

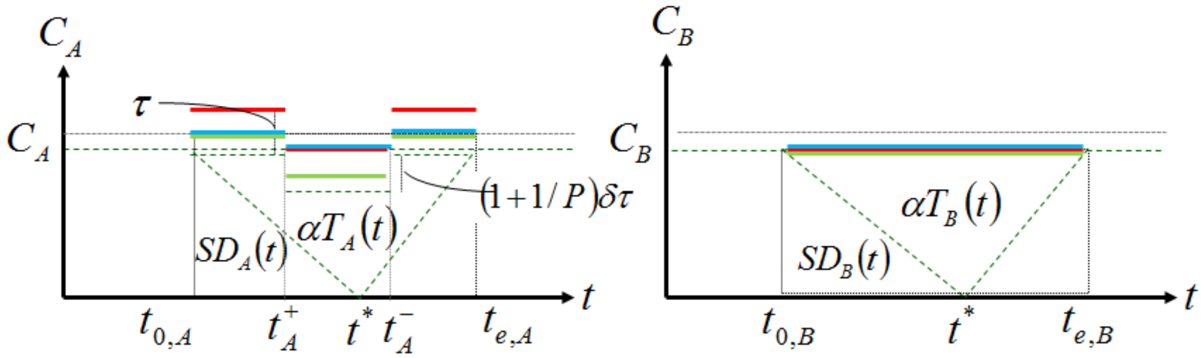

Figure 9 – Transitions of travel cost components in case (IIIb)

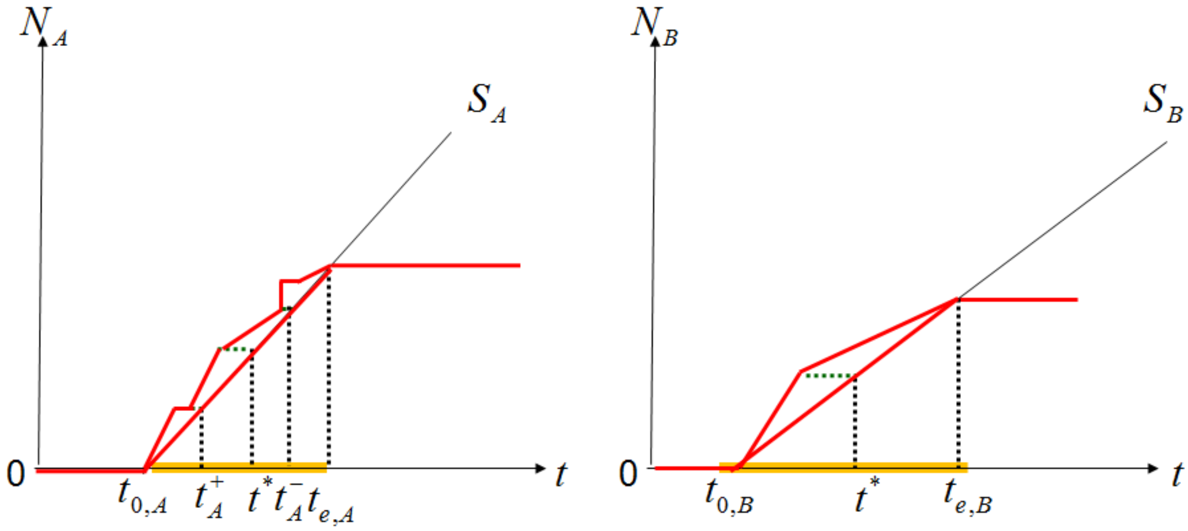

Figure 10 – Transitions of cumulative number of departures and arrivals in case (IIb)

4. CONCLUSIONS

This study analyses discounts on highway fee for the off-peak commuting with ETC as the application of the traditional bottleneck congestion model in order to consider self-select fee system and post payment related to historical usage.

The ETC off-peak commuting discount improves efficiency of road system due to utilization of off-peak capacity of highway so as to shift traffic demand from open road to highway, by contrast, ETC peak commuting discount which was in place in Japanese highway. However, if the number of ETC users who are applied the discounts less than the bottleneck capacity in discount period then the introduction of discount dose not reduce bottleneck congestion even if they can save their highway fees. Furthermore, the scale of effect of mitigation of road congestion depends on level of lowering travel costs which caused by demand shift from open road to highway.

It is confirmed that the additional discount; i.e. ETC off-peak commuting discount plus is an effective measure to give an incentive for usage of ETC device, restraining aggravation of road congestion. That is, if the policy maker gives incentive for usage of ETC device more according to ETC off-peak commuting discount, the discount period must be expanded then increase of road congestion within peak period is inevitable. Meanwhile, ETC off-peak commuting discount plus allows giving incentive for ETC more without aggravation of road congestion. Considering near-ubiquity of ETC, this discount system must be effective in Japanese highway fee policy.

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