Modelling a tradable emission permit system for urban motorists

Julie Bulteau[1]

Abstract: This article deals with the feasibility of a tradable emission permit system (TEPs) for urban motorists. The objective is to develop a new microeconomic theoretical model to reduce urban pollution. We suppose that the city's regulating authority sets up a tradable emission permit system based on the number of kilometres covered by private cars. By the use of a Constant Elasticity of Substitution (CES) function, we determine the equilibrium under an environmental constraint and analyse the effects of a TEPs on social welfare. The aim is to find the optimal quantity of permits leading to the desired environmental objective. The analytical and numerical results of the model show the instrument's feasibility and efficiency. An important variable in the model must be taken into account: the knowledge of environmental damage. This variable will clearly influence the tool's success.

1. Introduction

The use of a tradable emission permit system (TEPs) as an economic tool to promote sustainable mobility is increasingly mentioned in research on urban transport policies (see CNT, 2001). Nevertheless, there is little evidence of this instrument being applied to motorists. The lack of an economic tool application may be due to undeveloped and incomplete or nonexistent theoretical foundations. This creates distrust of the instrument by policy makers and leads to it being shelved in city-scale experiments.

There are three main theoretical justifications of this economic instrument applied to motorists: Daganzo (1995), Goddard (1997) and Raux and Marlot (2005). However, it remains to be developed. Therefore, the development of a theoretical modelling tool applied to motorists is central to this article. We analyse the establishment of a TEPs by internalising the negative externalities of the car. The objective is to develop a new microeconomic

[1] Julie Bulteau, doctor of Economic Sciences, Laboratoire d'Economie et de Management de Nantes Atlantique (LEMNA) - Institut d'Economie et de Management de Nantes-IAE -Chemin de la Censive du tertre BP 52231 44322 Nantes Cedex 3

theoretical model to reduce urban pollution. In the first part, we suppose the city's regulating authority sets up a tradable emission permit system based on the number of kilometres covered by private cars.

In the second part, using a Constant Elasticity of Substitution (CES) function, we determine the equilibrium under an environmental constraint and analyse the effects of TEPs on social welfare. The aim is to find the optimal quantity of permits leading to the desired environmental objective. In the third part, we present the analytical and numerical results of the model.

2. The model

We consider N transport consumer indexes $i = 1, ..., N$. The representative consumer i may travel either by car, noted V_i and producing emissions (negative externalities), or by public transport, TC_i supposed non-polluting. We choose a consumer utility function following: $U_i(V_i, TC_i)$ and consider that V_i and TC_i are expressed in kilometres travelled. We suppose that U_i is increasing and quasi-concave in V_i and TC_i, despite a marked preference for car use. Both modes are perfect substitutes, which generates two corner solutions: $U_i(V_i, 0) > 0$ and $U_i(0, TC_i) > 0$.

We denote: R_{Ti} the "transportation" income that the agent i devotes to his travel, p_v the vehicle costs per kilometre travelled representing maintenance costs, fuel and insurance, and finally p_{TC} the price of public transport per kilometre travelled.

2. 1. Equilibrium with environmental control

We suppose the city's regulating authority sets up a tradable emission permit system to reduce urban pollution based on the number of kilometres covered by private cars. The aim is to obtain the first rank social optimum. We analyse the consequences of the implementation of this tool.

2.1.1. The link between emissions and car use

Environmental externalities, generated by cars, can be reduced if individuals replace their car travel by public transport travel. However, we suppose individuals prefer to use their car involving a coefficient $a_i > \frac{1}{2}$.

Emissions e_i increase with the number of kilometres travelled by car $e_i = V_i$ per individual i. We suppose each kilometre travelled emits one unit of pollution: $e_i = V_i$. We normalise these emissions by taking into account a standard of the "Grenelle de l'environnment bonus, malus" with regard to purchasing a car. The rule of this norm is to pay an environmental bonus to the first registration for any purchase of a new passenger car emitting fewer than 130 grams of CO_2 per kilometre. We use this standard in our model. We assume that one kilometre travelled is equivalent to an emission of 130 gCO_2 so the relationship is as follows: $e_i = V_i = 130\ gCO_2$. Emissions cause a degradation of the environment expressed by a convex and increasing function of environmental damage *D (e).* The objective of the regulatory authority is to maximise the welfare of society, i.e. to maximise consumer utility while taking account of the environmental damage caused. The regulatory authority implements a TEPs applied to motorists. Emissions are represented as follows: $e = \sum_{i=1}^{N} e_i = \sum_{i=1}^{N} V_i$, while the cap set on emissions by the authority is expressed by $\bar{e} = \bar{V} = \sum_{i=1}^{N} \overline{V_i}$. Each permit entitles the holder to emit one unit of pollution, i.e. each license allows the holder to travel one kilometre by car. The total number of permits sets the total allowable emissions.

2.1.2. Utility maximisation with a TEPs

The regulatory authority implements the TEPs. The permit allocation is free and individuals receive a number of licenses: $\bar{V_i}$. The individuals, in maximising their utility, consider the permit allocation as given, and the price of it noted: p_e. Thus, the individual must take into account the number of allowed kilometres and the permit price if they wish to travel more kilometres by car. Again, when the agent i maximises his utility function: $U_i(V_i, TC_i) = [a_i V_i^{\rho} + (1-a_i) TC_i^{\rho}]^{\frac{1}{\rho}}$ under the new budget constraint incorporating the permit price, the program is the following:

$$\left\{ \begin{array}{ll} Max U_i(V_i, TC_i) = \left[a_i V_i^{\rho} + (1-a_i) TC_i^{\rho} \right]^{\frac{1}{\rho}} & \\ s.c.\ p_V V_i + p_e \left(V_i - \bar{V_i} \right) + p_{TC} TC_i \leq R_{T_i} & (\lambda) \\ s.c. V_i \geq 0 & (\mu_V) \\ s.c. TC_i \geq 0 & (\mu_{TC}) \end{array} \right\} \quad \textbf{(4.21)}$$

The associated Lagrangian is:

$$L=\left[a_iV_i^{\rho}+(1-a_i)TC_i^{\rho}\right]^{\frac{1}{\rho}}-\lambda\left(p_vV_i+p_e(V_i-\overline{V}_i)+p_{TC}TC_i-R_{T_i}\right)+\mu_VV_i+\mu_{TC}TC_i \quad \textbf{(4.22)}$$

If $V_i>\overline{V}_i$, then agent *i* will buy permits at the end of the period.

If $V_i<\overline{V}_i$, the initial allocation of permits exceeds the number of licenses held by the end of the period, so the agent may sell them (or keep them for future use).

We note that the permit fee contributes a large amount to the price of car use. The ratio of marginal utilities equals the price ratio of modes in which the price of the car includes the cost of emission permits.

2.1.2.1. Different equilibria between modes of transport

Three cases emerge depending on the different uses of modes of transport; the situation where both modes are used (car and TC), the situation where the individual uses only the car and finally, the third situation where only TC is used.

- **CASE N°1**: Both modes are used.

If both modes(V_i and TC_i) are used, involving $\mu_V=0$ and $\mu_{TC}=0$, and $V_i>0$ and $TC_i>0$ then the program has an interior solution. With two first-order conditions presented in Appendix 1, we derive the marginal rate of substitution following:

$$\frac{a_i}{(1-a_i)}\left(\frac{V_i}{TC_i}\right)^{\rho-1}=\frac{p_V+p_e}{p_{TC}} \quad \textbf{(4.23)}$$

Equation (4.23) indicates that the ratio of marginal utilities equals the ratio of prices.

However, the price of car use increases the price of permits (compared to the baseline situation).

At equilibrium, the kilometres travelled by car and public transport by agent *i* are the following:

$$V_i^{**}=\left(\frac{a_i}{p_V+p_e}\right)^{\sigma}\left(\frac{R_{T_i}+p_e\overline{V}_i}{a_i^{\sigma}\left(p_V+p_e\right)^{1-\sigma}+\left(1-a_i\right)^{\sigma}p_{TC}{}^{1-\sigma}}\right) \quad \textbf{(4.24a)}$$

$$TC_i^{**}=\left(\frac{1-a_i}{p_{TC}}\right)^{\sigma}\left(\frac{R_{T_i}+p_e\overline{V}_i}{a_i^{\sigma}\left(p_V+p_e\right)^{1-\sigma}+\left(1-a_i\right)^{\sigma}p_{TC}{}^{1-\sigma}}\right) \quad \textbf{(4.24b)}$$

$$U_i^{**}(V_i^{**},TC_i^{**},\overline{V}_i) = \left(R_{T_i} + p_e\overline{V}_i\right)\left(a_i^{\sigma}\left(p_V + p_e\right)^{1-\sigma} + \left(1-a_i\right)^{\sigma} p_{TC}^{\ 1-\sigma}\right)^{\frac{1}{\sigma-1}} \quad \textbf{(4.24c)}$$

where (**) corresponds to the situation with environmental regulations. For all N individuals, we have the following relations: $V^{**} = \sum_{i=1}^{N} V_i^{**}$, $TC^{**} = \sum_{i=1}^{N} TC_i^{**}$, $\overline{V} = \sum_{i=1}^{N} \overline{V}_i \geq V^{**}$

So, equilibrium with environmental constraints for N individuals is:

$$V^{**} = \left(\frac{1}{p_V + p_e}\right)^{\sigma} \sum_{i=1}^{N} \left(\frac{a_i^{\sigma}(R_{T_i} + p_e\overline{V}_i)}{a_i^{\sigma}\left(p_V + p_e\right)^{1-\sigma} + \left(1-a_i\right)^{\sigma} p_{TC}^{\ 1-\sigma}}\right) \quad \textbf{(4.25a)}$$

$$TC^{**} = \left(\frac{1}{p_{TC}}\right)^{\sigma} \sum_{i=1}^{N} (1-a_i)^{\sigma} \left(\frac{R_{T_i} + p_e\overline{V}_i}{a_i^{\sigma}\left(p_V + p_e\right)^{1-\sigma} + \left(1-a_i\right)^{\sigma} p_{TC}^{\ 1-\sigma}}\right) \quad \textbf{(4.25b)}$$

We now wish to determine the impact of the emission permit price (p_e). Firstly, the price of emission permits has two contradictory effects. On the one hand, it generates an increase in the cost of car use: ($p_V + p_e$). The rising cost of the car is comparable to that generated by a tax per kilometre travelled (p_eV_i). However, on the other hand, the price of permits increases the individual's transportation income ($R_{T_i} + p_e\overline{V}_i$). This increase in income may be treated as a transportation subsidy($p_e\overline{V}_i$) paid to individuals for their travel.

We study the effects of the emission permit price (p_e) on the number of kilometres travelled by car (V^{**}) and public transport (TC^{**}). In Appendix 2, we show the following relationship: $\frac{\partial V^{**}}{\partial p_e} < 0$ under the condition $R_{T_i} > p_V\overline{V}_i$. This means that when the permit price increases, the number of kilometres travelled by car falls if the "transportation" purchasing power for the automobile $\left(\frac{R_{T_i}}{p_V}\right)$ is higher than the allocation of permits $\left(\overline{V}_i\right)$.

Conversely, the relationship between the number of kilometres travelled by TC and the permit price is positive: $\frac{\partial TC^{**}}{\partial p_e} > 0$, so an increase in the permit price on the market increases the number of kilometres travelled by TC. Therefore, the permit price has an expected incentive effect on different modes of transport.

The implementation of the TEP system to reduce emissions works. Indeed, it should be noted that higher permit prices not only increase the use of TC, but also lead to a decline in car use resulting in lower emissions. We can already say that if there is a very high increase in the permit price, then the individual will use only TC and we will obtain Case n°3, where the car is not used. However, we also note that the quantity of permits allocated $\overline{V}_i$ to each individual provides an increase in "transportation" income, since we assumed a free allocation of permits to ensure mobility for all. Thus, the transportation income of individual *i* in the first case increased the amount $p_e\overline{V}_i$. Now we analyse the case where the individual only uses his car to commute to work.

- **CASE N°2**: Public transport is not used: first corner solution.

If only one mode of transportation is used, then a corner solution is determined. In this Case n°2, we assume that public transport is not used. This situation is reflected by the following system:

$$\begin{cases} TC_i = 0\,, \quad \mu_{TC} > 0 \\ V_i > 0\,, \quad \mu_V = 0 \end{cases}.$$

We obtain the marginal rate of substitution:

$\dfrac{a_i}{(1-a_i)}\left(\dfrac{V_i}{TC_i}\right)^{\rho-1} > \dfrac{p_V + p_e}{p_{TC}}$ (**4.26**), (calculations are in Appendix 1).

Equation (4.26) shows that the public transport price is relatively too high, causing the non-use of TC.

At equilibrium, the budget constraint is saturated, and the kilometres travelled by car and public transport are:

$V_i^{**'} = \dfrac{R_{T_i} + p_e\overline{V}_i}{p_V + p_e}$ (**4.27a**) and $TC_i^{**'} = 0$ (**4.27b**)

This corner solution $U_i^{**'}(V_i^{**'},0) > 0$ is possible for only a linear form of utility function, i.e. $\rho \to 1$. We will determine the existence conditions of this solution in the section (§2.1.2.2.) in the paragraph devoted to the linear form.

Thus, for N individuals, where public transport is not used, the equilibrium is determined by:

$V^{**'} = \frac{1}{p_V + p_e}\sum_{i=1}^{N}(R_{T_i} + p_e\bar{V}_i)$ (**4.28a**) and $TC^{**'} = 0$ (**4.28b**)

We study the effects of the emission permit price only on the use of a car $V_i^{**'}$, since TC is not used. We obtain the following equation: $\frac{\partial V^{**'}}{\partial p_e} < 0$ if $R_{T_i} > p_V\bar{V}_i$ (see calculations in Appendix 2).

This relationship indicates that an increase in permit price leads to a decrease in the number of kilometres travelled by car if the transport purchasing power of the individual for the car exceeds the allocation of permits $\left(\frac{R_{T_i}}{p_V} > \bar{V}_i\right)$. In other words, the number of kilometres travelled by car in the initial situation (with $TC_i^* = 0$) is higher than the allocation of permits($\bar{V}_i$). Thus, the implementation of the TEPs for motorists is working well as a constraint in relation to the initial situation. Moreover, as in Case n°1, we observe the same contradictory effects of the emission permit price because, on one hand, it increases the cost of the car ($p_V + p_e$) but, on the other hand, it increases the individual's transportation income ($R_{T_i} + p_e\bar{V}_i$). There is a direct consequence of a change in the permit price on the individual's utility level insofar as taking public transport is null. Thus, the individual's utility level will decrease if the permit price increases. However, if the permit price rises markedly, then we return to Case n°1 where there is a modal split between the car and public transport. If the rise in the permit price is even greater, we can obtain Case n°3 where only TC is used.

The use of both modes is significantly related to changes in the emission permit price. These developments provide information about the choice of modes of individual *i*.

- **CASE N°3**: The car is not used: second corner solution.

This situation involves a second corner solution of our program because the car is not used, referring to the following system

$$\begin{cases} TC_i > 0, & \mu_{TC} = 0 \\ V_i = 0, & \mu_V > 0 \end{cases}$$

Combining the two first-order conditions, (see Appendix 1), we obtain the marginal rate of substitution: $\frac{a_i}{(1-a_i)}\left(\frac{V_i}{TC_i}\right)^{\rho-1} < \frac{p_V + p_e}{p_{TC}}$ (**4.29**), meaning that the total cost of the car (price of the car and license price) is relatively too high. This may explain why individuals use only TC to travel.

At equilibrium, the kilometres travelled by car and public transport are:

$V_i^{***''} = 0$ (**4.30a**), and $TC_i^{***''} = \frac{R_{T_i} + p_e \overline{V}_i}{p_{TC}}$ (**4.30b**)

As in the previous case, the corner solution as $U_i^{***''}(0, TC_i^{***''}) > 0$ exists only if the utility function is the linear form function, i.e. $\rho \rightarrow 1$, when the modes are perfectly substitutable.

We will see the existence conditions of this solution in section (§2.1.2.2.).

For all *N* individuals, the equilibrium is determined by:

$V^{***''} = 0$ (**4.31a**) and $TC^{***''} = \frac{1}{p_{TC}} \sum_{i=1}^{N} R_{T_i} + pe\overline{V}_i$ (**4.31b**)

We note that the emission permit price increases the individual's transportation income ($R_{T_i} + pe\overline{V}_i$) on the use of TC.

We analyse, as in previous cases, the influence of the emission permit price on the use of public transport in the equilibrium. We obtain the following equation:

$\frac{\partial TC^{***''}}{\partial p_e} > 0$ (see calculations in Appendix 2), implying an increase in the permit price will always increase the use of public transport.

We observe that a decrease in the permit price would result in Case n°1 where the two modes are used. However, a large decrease in the permit price would result in Case n°2 where only the car is used. Therefore, the changes in the emission permit price are a fundamental element in our model.

To obtain a complete analysis of the situation with the market system of TEP, various forms of utility function, depending on the value of elasticity, must be taken into account.

2.1.2.2. The substitution of transport modes

The CES form of utility function allows several situations to be taken into account depending on the value of elasticity. Through an analysis of situations, we determine the different types of mode.

- **Cobb-Douglas Function: Modes of transport imperfectly substitutable**

When the parameter* ρ *tends to zero*:* ρ →*0*

When ρ tends to zero, the coefficient of substitution elasticity is equal to unity. Thus, the marginal rate of substitution determined by equation (**4.23**) becomes the following:

$$\frac{a_i TC_i}{(1-a_i)V_i} = \frac{p_V + p_e}{p_{TC}} \quad (\mathbf{4.32})$$

This new marginal rate of substitution represents preferences from a utility function of Cobb-Douglas form: $U_i(V_i, TC_i) = V_i^{a_i} TC_i^{1-a_i}$, involving the possible substitutability between the two modes, as well as their strict positivity.

At equilibrium, the kilometres travelled by car and public transport, for the agent *i*, are the following:

$$V_i^{**} = \frac{a_i\left(R_{T_i} + p_e \bar{V}_i\right)}{p_V + p_e} \,(\mathbf{4.33a}) \text{ and } TC_i^{**} = \frac{(1-a_i)\left(R_{T_i} + p_e \bar{V}_i\right)}{p_{TC}} \,(\mathbf{4.33b})$$

The equilibrium, with an environmental constraint for *N* individuals, is:

$$V^{**} = \frac{1}{p_V + p_e}\sum_{i=1}^{N} a_i\left(R_{T_i} + p_e \bar{V}_i\right) (\mathbf{4.34a}) \text{ and } TC^{**} = \frac{1}{p_{TC}}\sum_{i=1}^{N}(1-a_i)\left(R_{T_i} + p_e \bar{V}_i\right) \; (\mathbf{4.34b})$$

Once again, we want to determine the effects of the emission permit price on the number of kilometres travelled by car (V^{**}) and public transport(TC^{**}).

According to Appendix 2, this leads to the following negative relationship: $\frac{\partial V^{**}}{\partial p_e} < 0$ if $p_V \bar{V}_i < R_{T_i}$. This relation demonstrates that if the permit price increases, then the number of kilometres travelled by car will decrease, provided that the purchasing power of "transportation" for the car is more than the allocation of permits.

Moreover, we obtain a positive relation to public transport: $\frac{\partial TC^{**}}{\partial p_e} > 0$, meaning, on the contrary, that an increase in the permit price on the market causes an increase in the kilometres travelled by public transport.

As in previous cases, we have the same opposite impacts caused by the emission permit price: it increases the cost of the car($p_V + p_e$) and, at the same time, increases the transport budget of the individual($R_{T_i} + p_e\overline{V}$).

We note a significant difference between the general case of the CES function and the particular case of the Cobb-Douglas form. The Cobb-Douglas function implies the strict positivity of the two modes; indeed the individual must use not only his car but also public transport as the modes are presumed substitutable. This condition for strictly positive use of both modes is restrictive. The use of a single transport mode to commute to work is often observed.

- **Linear Form: Modes of transport perfectly substitutable**

***When the parameter ρ tends to unity**: $\rho \to 1$*

If ρ tends to one, the TMS determined by equation (4.23) becomes the following:

$\frac{a_i}{(1-a_i)} = \frac{p_V + p_e}{p_{TC}}$ (**4.35**). This new relation involves a linear form utility function where the modes are regarded as perfectly substitutable.

One solution of the program is determined when the two modes are used under the condition: $a_i = \frac{p_V + p_e}{p_{TC} + p_V + p_e}$. Assuming the budget constraint is saturated, the indifference curve can be confused, at equilibrium, with the latter. We then get the equilibrium:

$$p_V V_i^{**} + p_e(V_i^{**} - \overline{V}_i) + p_{TC} TC_i^{**} = R_{T_i} \quad \textbf{(4.36)}$$

However, two corner solutions emerge. If $\frac{a_i}{(1-a_i)} > \frac{p_V + p_e}{p_{TC}}$ implying the following condition $a_i > \frac{p_V + p_e}{p_{TC} + p_V + p_e}$, then public transport is not used. The equilibrium is the following: $V_i^{**} = \frac{R_{T_i} + p_e\overline{V}_i}{p_V + p_e}$.

Instead, if $\frac{a_i}{(1-a_i)} < \frac{p_V + p_e}{p_{TC}}$, generating the following condition: $a_i < \frac{p_V + p_e}{p_{TC} + p_V + p_e}$ then the individual uses only TC. The equilibrium is the following: $TC_i^{**} = \frac{R_{T_i} + p_e\overline{V}_i}{p_{TC}}$. For N individuals, we obtain three situations:

If both transport modes are used:

$$p_V \sum_{i=1}^{N} V_i^{**} + p_e \sum_{i=1}^{N} (V_i^{**} - \bar{V}_i) + p_{TC} \sum_{i=1}^{N} TC_i^{**} = \sum_{i=1}^{N} R_{T_i} \quad \textbf{(4.37)}$$

If only the car is used:

$$V^{**} = \frac{1}{p_V + p_e} \sum_{i=1}^{N} \left(R_{T_i} + p_e \bar{V}_i \right) \ \textbf{(4.38a)} \text{ and } TC^{**} = 0 \ \textbf{(4.38b)}$$

If only public transport is used:

$$V^{**} = 0 \ \textbf{(4.39a)} \text{ and } TC^{**} = \frac{1}{p_{TC}} \sum_{i=1}^{N} \left(R_{T_i} + p_e \bar{V}_i \right) \textbf{(4.39b)}$$

Once again, we note two contradictory effects of permit price (p_e). On the one hand, it increases the cost of the car ($p_V + p_e$) as in the establishment of a tax per kilometre ($p_e V_i$), and secondly, it raises the individual's transport budget in the same manner as a subsidy ($p_e \bar{V}_i$).

The situation represented by (4.38) indicates that the number of kilometres travelled by car at equilibrium decreases with the emission permit price (see Appendix 2) under the condition that the purchasing power of "transportation" for the car is higher than the initial allocation of permits($\bar{V}_i < \frac{R_{T_i}}{p_V}$). The total transport budget is spent on car use.

The second corner solution (4.39) shows that the number of kilometres travelled by public transport, at equilibrium, increases with the emission permit price (see Appendix 2). The transport budget is totally devoted to the use of TC. These three situations better reflect the reality observed in the sense that the individual can combine both modes of transport or travel only by car or TC. However, we emphasise that the linear perfect substitutability between the two modes is a rare phenomenon.

- **Leontief Form: Modes of transport are complementary**

When the parameter ρ tends to infinity: $\rho \to \infty$

When the elasticity tends to infinity, the TMS determined by equation (4.23) tends to zero.

A utility function of the Leontief form is then obtained, determined by: $U_i(V_i, TC_i) = Min\left[a_i V_i^{**}; (1 - a_i) TC_i^{**} \right]$. At equilibrium, the number of kilometres travelled by car and TC are: $a_i V_i^{**} = (1 - a_i) TC_i^{**}$ (**4.40**).

The Leontief form postulates that cars and public transport are seen as complementary goods. For example: the individual is initially forced to use his vehicle to access a TC station before completing his journey by TC.

According to the results obtained, we find that the emission permit price is crucial because it affects the efficiency of TEPs. However, the authority can influence it indirectly through the choice of the number of licences distributed. We bring together the main results in the following tables to obtain an overview:

Situation with environmental regulation: implementation of aTEP system			
	Interior solution	***1st corner solution: only a car is used***	***2nd corner solution: only TC is used***
*V***	$V^{**}=\left(\frac{1}{p_V+p_e}\right)^{\sigma}\sum_{i=1}^{N}\left(\frac{a_i^{\sigma}(R_{T_i}+p_e\bar{V}_i)}{a_i^{\sigma}\left(p_V+p_e\right)^{1-\sigma}+\left(1-a_i\right)^{\sigma}p_{TC}^{\,1-\sigma}}\right)$	$V^{**'}=\frac{1}{p_V+p_e}\sum_{i=1}^{N}R_{T_i}+p_e\bar{V}_i$	$V^{**''}=0$
*TC***	$TC^{**}=\left(\frac{1}{p_{TC}}\right)^{\sigma}\sum_{i=1}^{N}(1-a_i)^{\sigma}\left(\frac{R_{T_i}+p_e\bar{V}_i}{a_i^{\sigma}\left(p_V+p_e\right)^{1-\sigma}+\left(1-a_i\right)^{\sigma}p_{TC}^{\,1-\sigma}}\right)$	$TC^{**'}=0$	$TC^{**''}=\frac{1}{p_{TC}}\sum_{i=1}^{N}R_{T_i}+pe\bar{V}_i$

Situation with environmental regulation: implementation of a TEP system			
	Cobb-Douglas function : $\rho\to 0$ $\left(V^{**}>0 \text{ et } TC^{**}>0\right)$	***Linear form function:*** $\rho\to 1$ $\left(V^{**}\geq 0 \text{ et } TC^{**}\geq 0\right)$	***Leontief function :*** $\rho\to\infty$
*V***	$V^{**}=\frac{1}{p_V+p_e}\sum_{i=1}^{N}a_i\left(R_{T_i}+p_e\bar{V}_i\right)$	$p_V\sum_{i=1}^{N}V_i^{**}+p_e\sum_{i=1}^{N}(V_i^{**}-\bar{V}_i)+p_{TC}\sum_{i=1}^{N}TC_i^{**}=\sum_{i=1}^{N}R_{T_i}$ if $a_i=\frac{p_V+p_e}{p_{TC}+p_V+p_e}$ or $V^{**}=\frac{1}{p_V+p_e}\sum_{i=1}^{N}\left(R_{T_i}+p_e\bar{V}_i\right)$ and $TC^{**}=0$, if $a_i>\frac{p_V+p_e}{p_{TC}+p_V+p_e}$	Equilibrium solutions minimise this function : $U(V_i^{**},TC_i^{**})=Min\left[\sum_{i=1}^{N}a_iV_i^{**};\sum_{i=1}^{N}(1-a_i)TC_i^{**}\right]$ as: $\sum_{i=1}^{N}a_iV_i^{**}=\sum_{i=1}^{N}(1-a_i)TC_i^{**}$
*TC***	$TC^{**}=\frac{1}{p_{TC}}\sum_{i=1}^{N}(1-a_i)\left(R_{T_i}+p_e\bar{V}_i\right)$	or $TC^{**}=\frac{1}{p_{TC}}\sum_{i=1}^{N}\left(R_{T_i}+p_e\bar{V}_i\right)$ and $V^{**}=0$, if $a_i<\frac{p_V+p_e}{p_{TC}+p_V+p_e}$	

2.2. Equilibrium in the market for emission permits

After determining the behaviour of individuals regarding the use of transport modes following the introduction of a TEP system, it seems essential to analyse the market characteristics of TEP.

The market is balanced when the supply of permits is equal to the demand. Depending on various parameters, we want to know if people will behave as a seller or a buyer of permits on the market. The parameter a_i represents the proportion of transportation income spent on car use. It is assumed that individuals have a preference for the car involving $a_i > \frac{1}{2}$. Thus, other things being equal, the higher a_i, an individual has, the more he would use his car and, therefore, he should buy emission permits. The individual's transportation income (R_{T_i}) also influences the behaviour of buying and selling emission permits. In fact, people prefer to use their car, which implies an increase in income leads to increased car use. Other things being equal, the higher the income the individual has, the more emission permits he should buy. Thus, the a_i parameter and the transportation income (R_{T_i}) influence the behaviour of the exchange of emission permits on the market.

In order for this market to be balanced and consistent with the environmental standard set by the regulatory authority, the price of emission permits is fixed so that permit demand, kilometres travelled by car demand, which represent the total emissions, and the constraint of pollution, $e = \overline{e} = \sum_{i=1}^{N} \overline{V}_i$, cancel. Equilibrium is thus obtained when:

$$\sum_{i=1}^{N} V_i^{**} = \sum_{i=1}^{N} \overline{V}_i \Leftrightarrow \sum_{i=1}^{N} \left(V_i^{**} - \overline{V}_i \right) = 0 \quad \textbf{(4.41)}$$

This relation (**4.41**) represents a necessary condition to achieve the desired environmental standard fixed by the regulatory authority. The second prerequisite for the achievement of equilibrium in the market was determined before the optimal condition by equation (4.23), i.e.

$$\frac{Um_{V_i}}{Um_{TC_i}} = \frac{p_V + p_e}{p_{TC}}$$

Each individual fulfils the optimality condition of the utility maximisation program. This condition implies the following relation for N individuals:

$$\frac{Um_{V_1}}{Um_{TC_1}} = \frac{Um_{V_2}}{Um_{TC_2}} = \ldots = \frac{Um_{V_N}}{Um_{TC_N}} = \frac{p_V + p_e}{p_{TC}} \quad \textbf{(4.42)}$$

This relationship indicates that the market equilibrium is obtained when the rates of marginal substitution of each individual are not only equal, but equal to the ratio of prices, which generates a Pareto optimum.

According to the conditions (**4.41**) and (**4.42**), the emission permit price can be determined by the number of permits allocated.

Therefore, we believe, initially, the general case of the CES function with the interior solution where both modes are used. By combining equations (**4.41**) and (**4.25a**), we obtain the following relation:

$$p_e \approx \left(\frac{\sum_{i=1}^{N} \frac{a_i^{\sigma}\left(R_i + p_e \overline{V}_i\right)}{a_i^{\sigma}\left(p_V + p_e\right)^{1-\sigma} + (1-a_i)^{\sigma} p_{TC}{}^{1-\sigma}}}{\sum_{i=1}^{N} \overline{V}_i} \right)^{\frac{1}{\sigma}} - p_V \quad \textbf{(4.43)}$$

Relation (4.43) does not provide an explicit solution of the emission permit price.

An iterative solution is needed to determine this. Furthermore, we analyse the case of the Cobb-Douglas function to examine the relation between the emission permit price and the quantity of permits allocated.

We deduce from equations (**4.34a**) and (**4.41**) the emission permit price following:

$$p_e = \frac{\sum_{i=1}^{N} a_i R_{T_i} - p_V \sum_{i=1}^{N} \overline{V}_i}{\sum_{i=1}^{N} \overline{V}_i \left(1 - a_i\right)} \quad \textbf{(4.44)}$$

The study of equation (**4.44**) allows us to determine the influence of the number of permits allocated on the permit price. We thus obtain the following equation: $\frac{\partial p_e}{\partial \overline{V}_i} < 0$

(the calculations are presented in Appendix 3). The emission permit price is a decreasing function of the number of kilometres allocated. Therefore, if the city decides to pursue a stricter environmental policy, it can reduce the number of permits, which will increase their price (p_e) and lower car use.

The formation of equilibrium in the market with the Cobb-Douglas function has enabled us to analyse the impact of the emission permit price, *via* their allocation, on the effectiveness of an environmental policy. The price of permits is one of the key variables of the system's success, but the regulator can only act on it through the number of allowances allocated. It is also important and necessary to examine the influence of this system's implementation on social welfare in order to determine the optimal policy.

2.3. Impact on welfare

The establishment of an economic tool involves modifications of well-being. Therefore, the consequences of creating a system of TEP applied to motorists must be identified and analysed.

To examine the system's impact on social welfare, we assume that the costs of setting up the instrument and those caused by the pollution control are null. We believe that social welfare consists of the utility of individuals and environmental damage related to car use. Thus, we define the damage function *D (e)* as $D(e) = D\left(\sum_{i=1}^{N} \gamma V_i\right)$ with γ taking a value between 0 and 1.

D (e) is an increasing function of the number of kilometres travelled by car.

An increase in V_i causes an increase in environmental damage in proportion γ. The function of the well-being of society reads:

$$W\left(V_i, TC_i, \bar{V}_i\right) = \sum_{i=1}^{N} U_i\left(V_i, TC_i, \bar{V}_i\right) - D(\gamma \sum_{i=1}^{N} V_i) \quad \textbf{(4.45)}$$

We substitute in the utility function of agent *i*, p_e by his equilibrium value found previously (**4.43**). We integrate in the damage function the equilibrium relation (**4.41**) which tells us that the number of allowances should be equal to the number of kilometres travelled by car. In this way, we are able to express the function of social welfare only in terms of $\bar{V}_i$:

$$W\left(\bar{V}\right) = \sum_{i=1}^{N} U_i\left(\bar{V}_i\right) - D\left(\gamma \sum_{i=1}^{N} \bar{V}_i\right) \quad \textbf{(4.46)}$$

Thus, the regulator maximises the function of social welfare (**4.46**) to determine the optimal quota:

$$\frac{\partial W}{\partial \bar{V}_i} = \frac{\partial \sum_{i=1}^{N} U_i}{\partial \bar{V}_i} - \frac{\partial \sum_{i=1}^{N} D_i}{\partial \bar{V}_i} = 0 \quad \textbf{(4.47)}$$

We obtain the following equation $Um_i = Dm_i$ where $Dm_i = \gamma$, which implies $Um_i = \gamma$ (**4.48**). This relation indicates that the optimal quantity of permits on the market is obtained when the marginal utility of a kilometre travelled by car is equal to the marginal damage γ. In other words, when there is an additional unit of permits on the market, it increases consumer

utility at the same time as damaging the environment. These results are consistent with the standard results of environmental economics. This work has enabled us to highlight the impacts caused by the creation of a TEP system, not only on the individual's travel behaviour but also on welfare. The role of the emission permit price has proved crucial for ensuring the effectiveness of the economic instrument. However, it should be noted that the determination of prices permitted by the contract is dependent on the initial allocation of permits conducted by the regulatory authority. It is therefore essential to determine the optimal number of quotas to ensure the effectiveness and proper functioning of the market system of TEP.
To support the results of our modelling, we present numerical simulations in the next section.

3. Applications for two individuals with identical incomes

We focus primarily on interpretations of the results of numerical applications; we carry out sensitivity tests on the number of permits distributed, the emission permit price, and the elasticity of substitution.

3.1. Numerical applications and interpretations

We consider only two individuals who have different preferences a_i. We assume that agent *1* has preferences $a_1 = \frac{5}{6}$ and agent *2* $a_2 = \frac{2}{3}$. Thus, agent *1* has a more pronounced preference for car use compared to agent *2*. We also believe that the operating system runs for a year. However, like Raux (2007b), we suggest that the distribution of permits is conducted each week. Furthermore, we assume that the weekly sum dedicated to transportation by individuals is about € 20 and is identical for both agents. Regarding the cost per kilometre travelled by car and public transport, we rely on a study of FNAUT[2] (2007). This study indicates that a kilometre travelled by car in urban areas costs € 0.30 ($p_V = 0.30$) while one travelled by public transport costs € 0.10 ($p_{TC} = 0.10$). We use the parameter $\rho = 0.6$.This choice creates the lack of a corner solution and implies an elasticity of substitution of value $\sigma = \frac{5}{2}$ This coefficient is a very important variable in our model; we

.

[2]FNAUT: Fédération nationale des associations d'usagers des transports

carry out sensitivity testing thereafter. Finally, we set the marginal damage: $\gamma = € \ 0.25$. This value is indicative. Nevertheless, we consider it not only takes into account the environmental damage, but includes other types of damage caused by car use (e.g. accidents, noise pollution, etc.).The results of this first numerical application are presented in the following table:

TABLE N°4.3: Numerical application n°1: situation without environmental regulation

Parameter values: $p_V = 0.30$; $p_{TC} = 0.10$; $R_{T_1} = R_{T_2} = 20$; $a_1 = \frac{5}{6}$; $a_2 = \frac{2}{3}$; $\sigma = \frac{5}{2}$		
At equilibrium	***Agent 1***	***Agent 2***
V_i^*	61	35
TC_i^*	17	96
Total number of kilometres travelled by car	$V^* = 96$	
Total number of kilometres travelled by public transport	$TC^* = 113$	
Social welfare	$W^* = 80.83$	

Thus, in the situation without environmental regulation, in equilibrium with the parameters chosen, individuals 1 and 2 travel, respectively, 61 and 35 km by car and 17 and 96 km by TC[3]. In addition, the total number of kilometres travelled by car is 96 and 113 by TC, involving a total of 209 kilometres. The social welfare is then valued at € 80.83.

The relation between CO_2 emissions and the number of kilometres travelled has been defined by $e_i = V_i = 130 \, gCO_2$. We can deduce the total emissions due to the use of the car e_{T1} = 12 480 gCO_2.

Now, we suppose that the regulatory authority of the city decides to reduce these emissions by 20% so the desired amount will then be e_{T2} = 9 984 gCO_2, equivalent to 76.8 km which we round up to 77 km. In order to meet the desired objective, the regulatory authority decides to introduce the instrument of tradable emission permits. We assume that the permits are distributed in proportion to past emissions. Thus, individual *1* would receive a total of 49 permits($\overline{V}_1 = 49$), and individual 2 a total of 28 licenses ($\overline{V}_2 = 28$).

We determine the permit price using an iterative solution. The principle is to build a solution by successive approximation (calculations performed with the software Mathematica); the

3 Note that the kilometres travelled by both modes are rounded to whole numbers

price is found to be: p_e = € 0.1079. Thus, we are able to define the new equilibrium reached with the introduction of the TEP system, whose results are presented in the following table:

TABLE N°4.4: Numerical application n°2: situation with environmental regulation

Parameter values: $p_V = 0.30;\ p_{TC} = 0.10;\ R_{T_1} = R_{T_2} = 20;\ a_1 = \frac{5}{6}; a_2 = \frac{2}{3}; \sigma = \frac{5}{2}; p_e = 0.1010;$ $\bar{V}_1 = 49; \bar{V}_2 = 28$		
At equilibrium	***Agent 1***	***Agent 2***
V_i^{**}	54	23
TC_i^{**}	32	136
Total number of kilometres travelled by car	$V^{**} = 77$	
Total number of kilometres travelled by public transport	$TC^{**} = 168$	
Social welfare	W^{**}= 82.95	

After this second simulation, we note that the individual *1* exceeds his quota of kilometres travelled by car. Indeed, he travelled 54 km, while his quota allowed him to travel only 49. To respect the system put in place, he must buy additional permits at a cost of € 0.5395. In contrast, individual *2* has a surplus of permits; he can sell them on the market and thereby recover the sum of € 0.5395. We note that the individual *1*, who has a greater preference for the car, is a buyer of permits. This underpins our anticipation regarding the influence of the parameter a_i on the behaviour of permit exchange.

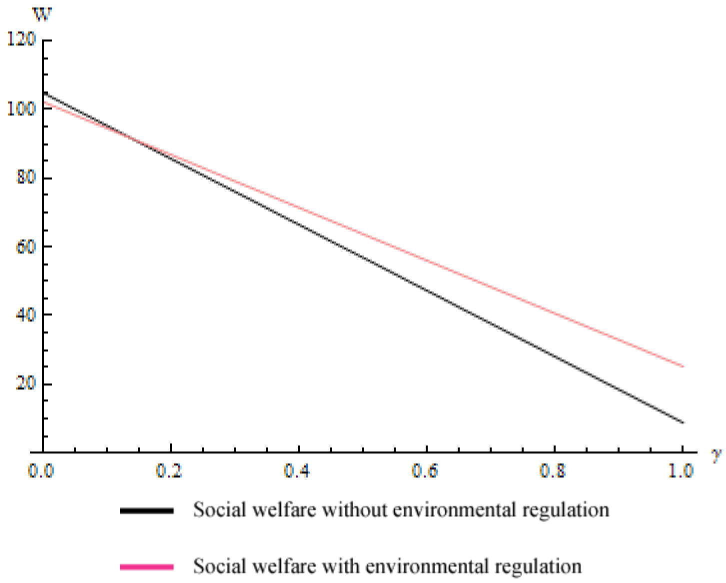
Nevertheless, we also observe that the total number of kilometres travelled has increased. It has grown from an initial situation of 209 km to 245 km when the regulatory authority has established an environmental constraint. As we noted in the modelling, the individual's income is increased by the allocation of permits (a subsidy). Thus, a higher budget is devoted to the number of kilometres travelled (because it cannot be used for the consumption of other goods in the model): there is an income effect. This observation leads us to wonder about the possible addition of an extension to the model to moderate the total number of kilometres travelled. However, we can say that the TEP system is effective because social welfare has increased slightly, from € 80.83 to 82. 95, and the set objective for pollution control is met. We synthesise the results of the situation with and without environmental regulation in the following table:

TABLEN°4.5: Pollution and social welfare for two individuals

	Initial situation	**Situation with TEP**
Pollution: emissions in gCO_2	12 480	9 984
Social welfare in €	80.83	82.95

This table shows that, during initiation of the TEP system, reducing emissions is respected and well-being is increased slightly. The graph below illustrates social welfare in the two situations depending on the marginal damage.

GRAPH N°4.1: Social welfare and marginal damage for two individuals

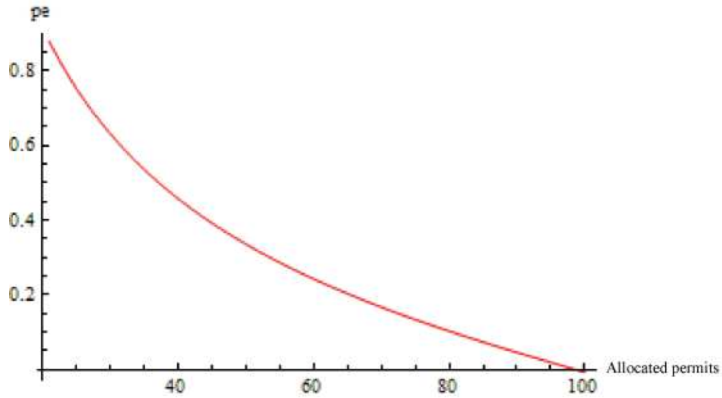


Graph 4.1 shows that well-being with environmental regulation is weaker than in the initial situation, that is to say “laissez-faire”, where the marginal damage is less than € 0.1383. From this value, the welfare obtained with the TEP system is always higher than the baseline. The welfare is maximised. Moreover, we observe that the higher the marginal damage, the greater the gap between the two curves of well-being. To supplement the numerical analysis and support our theoretical results, we perform sensitivity tests related to allowances distributed to the permit price and the elasticity of substitution.

3.2. Sensitivity tests on the number of emission permits

We determined, by the theoretical model, the importance of the impact of the number of permits distributed on the pricing of emission permits generating efficiency or otherwise of the TEP system. Therefore, with the following graph, we illustrate the development of the permit price based on the quantity of allocated permits.

GRAPH N°4.2: Permit price (p_e) and allocated permits (*V*)

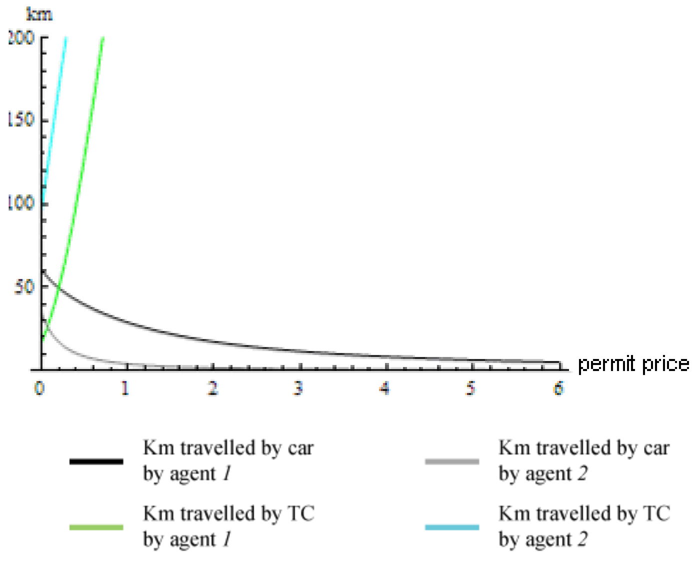


On this graph, we observe that the permit price is a decreasing function of the quantity of allocated permits. Therefore, a small number of permits available in the market entails a high price. Beyond a certain number of permits granted, the permit price becomes zero. In our simulation, this number is 97. The environmental policy is thus ineffective if too many licenses are available on the market.

3.3. Sensitivity tests for the emission permit price

As the emission permit price is a key variable in the market system of TEP, it is necessary to study the impact of this parameter on the number of kilometres travelled. The following graph shows the evolution of kilometres travelled by car and by TC, for two individuals, depending on the permit price.

GRAPH N°4.3: Kilometres travelled by car and by TC and permit price

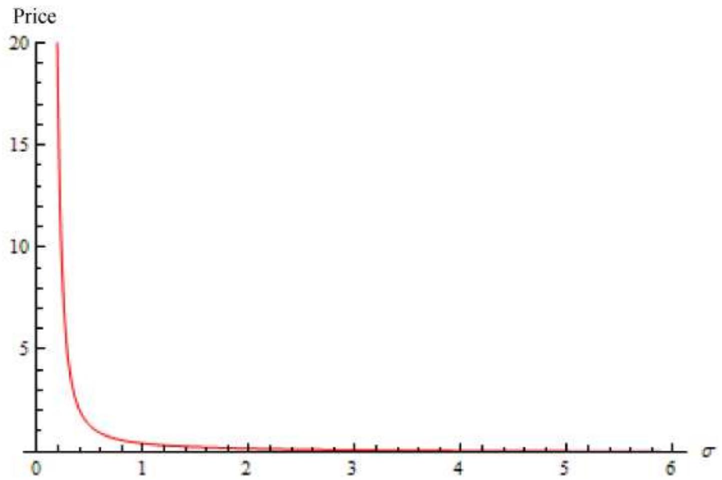


On this graph, we observe that when the emission permit price is situated within the interval [0,0.2], then individual *1* travels a number of kilometres by car greater than or equal to that travelled by TC. Beyond a permit price of € 0.2, however, he travels more kilometres by TC that by car. Note that individual *2* uses TC more than the car, whatever the emission permit price. The reason is that his preference for the car is lower than that of individual *1*. Note also that the increase in the permit price slows car use for both individuals. The price increase comes from a decrease in the quantity of emission permits.

3.4. Sensitivity tests on the coefficient of substitution elasticity

The diversity of forms obtained by the CES utility function includes different values taken by the coefficient of substitution elasticity σ. For this reason, we perform sensitivity tests on this parameter to illustrate our theoretical results.

GRAPH N°4.4: Emission permit price (p_e) and substitution of elasticity (σ)



We observe in Figure n°4.4, the emission permit equals € 0.3942 when the coefficient of substitution elasticity is equal to unity ($\sigma = 1$). This represents a Cobb-Douglas utility function when the two modes should be strictly used. We note that when the elasticity tends to zero, the permit price tends to infinity. The reason is that this case reflects a Leontief utility function where the modes are complementary. This implies that, whatever the permit price, people will still use the two modes. Moreover, we note that the permit price is zero when the coefficient of substitution elasticity is about 5.85. This represents a linear form of utility function that allows perfect substitutability between modes. The emission permit price is really sensitive to the coefficient of substitution elasticity. Finally, the applications of numerical and sensitivity testing, and the theoretical results of the model are illustrated.

4. CONCLUSION

The objective of this article was to develop a theoretical system for the TEP market for motorists in an urban area. In the first section, we laid the foundation for modelling. In a simple microeconomic framework with the tools of environmental economics, we have developed a theoretical model of a market TEPs applied to motorists. The model uses a CES utility form to analyse the different possible cases of transport mode use and the nature of their substitutability. Thus, we have highlighted the role of the emission permit price and the quantity of allocated permits for the success or failure of this instrument. The regulatory authority may act indirectly on the prices provided through the quantity of allowances distributed. Too many permits available in the market cause the price of permits to be low or zero, leading to an inefficient system. Therefore, we have produced the following result: the optimal amount of permits on the market is obtained when the marginal utility and the marginal damage of one kilometre travelled by car are equal. We also emphasise the important role played by the coefficient of substitution elasticity, insofar as it determines the nature of the modes (substitutability or complementarity). We have noticed several times the contradictory effects of the permit fee. On the one hand, it increases the cost of car use but, on the other hand, it raises the individual's transportation income. An increase in income spent on travel involves an increase in the total number of kilometres travelled. However, as the environmental standard is met, the economic tool has reached its goal of sustainable mobility.

In the third section of this article (§3), we performed numerical simulations to illustrate our theoretical results. For simplicity, these involved two individuals. The results of these simulations illustrate the effectiveness of the TEP tool insofar as social welfare is increased compared to the initial situation, beyond a certain value of marginal damage. In addition, we conducted sensitivity tests on the quantity of emission permits, the permit price and the coefficient of substitution elasticity to show their influence in the model. When the system of TEP is implemented, we note that increasing the total number of kilometres travelled is also proved numerically.

Finally, we showed that the instrument of tradable emission permits can be applied not only to motorists of a city but this tool also gives effective access to sustainable mobility.

The modelling started in this paper provides evidence of theoretical and empirical answers about the functioning of a market of TEP in urban areas.

REFERENCES:

CNT (Conseil national des transports) (2001), *L'effet de serre et les transports : les potentialités des permis d'émission négociables*, Rapport du groupe de travail présidé par BONNAFOUS A., Paris, 117 p.

DAGANZO C.F. (1995), « A Pareto optimum congestion reduction scheme », *Transportation Research Part B*, vol. 29, pp. 139-154.

GODDARD H.C. (1997), « Using tradeable permits to achieve sustainability in the world's large cities », *Environmental and Resource Economics*, vol. 10, pp. 63-99.

RAUX C. et MARLOT G. (2005), « A system of tradable CO_2 permits applied to fuel consumption by motorists », *Transport Policy*, vol. 12, pp. 255-265.

APPENDIX 1:

- *Calculation of first order conditions in Case n°1*

The program is the following:

$$L=\left[a_iV_i^{\rho}+(1-a_i)TC_i^{\rho}\right]^{\frac{1}{\rho}}-\lambda\left(p_vV_i+p_e(V_i-\overline{V}_i)+p_{TC}TC_i-R_{T_i}\right)$$

(**4.22**)

The first order conditions are:

$$CPO_1:\frac{\partial L}{\partial V_i}=0\Leftrightarrow a_iV_i^{\rho-1}\times\left(a_iV_i^{\rho}+\left(1-a_i\right)TC_i^{\rho}\right)^{\frac{1}{\rho}-1}-\left(p_V+p_e\right)\lambda=0$$

$$CPO_2:\frac{\partial L}{\partial TC_i}=0\Leftrightarrow\left(1-a_i\right)TC_i^{\rho-1}\times\left(a_iV_i^{\rho}+\left(1-a_i\right)TC_i^{\rho}\right)^{\frac{1}{\rho}-1}-p_{TC}\lambda=0$$

With these two conditions, we obtain:

$$\frac{a_iV_i^{\rho-1}}{\left(1-a_i\right)TC_i^{\rho-1}}=\frac{p_V+p_e}{p_{TC}}\text{ so }TMS=\frac{a_i}{\left(1-a_i\right)}\left(\frac{V_i}{TC_i}\right)^{\rho-1}=\frac{p_V+p_e}{p_{TC}}\quad\textbf{(4.23)}$$

- *Calculation of first order conditions in Case n°2, and determining existence conditions of the corner solution:*

Consider the following Lagrangian:

$$L=\left[a_iV_i^{\rho}+(1-a_i)TC_i^{\rho}\right]^{\frac{1}{\rho}}-\lambda\left(p_vV_i+p_{TC}TC_i-R_{T_i}\right)+\mu_{TC}TC_i\quad\textbf{(4.22)}$$

The first order conditions are the followings:

$$CPO_1 : \frac{\partial L}{\partial V_i} = 0 \Leftrightarrow a_i V_i^{\rho-1} \times \left(a_i V_i^{\rho} + (1-a_i) TC_i^{\rho}\right)^{\frac{1}{\rho}-1} - \left(p_V + p_e\right)\lambda = 0$$

$$\Rightarrow \frac{a_i V_i^{\rho-1} \times \left(a_i V_i^{\rho} + (1-a_i) TC_i^{\rho}\right)^{\frac{1}{\rho}-1}}{p_V + p_e} = \lambda$$

$$CPO_2 : \frac{\partial L}{\partial TC_i} = 0 \Leftrightarrow (1-a_i) TC_i^{\rho-1} \times \left(a_i V_i^{\rho} + (1-a_i) TC_i^{\rho}\right)^{\frac{1}{\rho}-1} - p_{TC}\lambda + \mu_{TC} = 0$$

avec $\mu_{TC} > 0$

We have: $\mu_{TC} = -(1-a_i) TC_i^{\rho-1} \times \left(a_i V_i^{\rho} + (1-a_i) TC_i^{\rho}\right)^{\frac{1}{\rho}-1} + p_{TC}\lambda > 0$; we replace λ by his expression and we obtain:

$$\frac{a_i}{(1-a_i)} \left(\frac{V_i}{TC_i}\right)^{\rho-1} > \frac{p_V + p_e}{p_{TC}} \quad \textbf{(4.26)}$$

- *<u>Calculation of first order conditions in Case n°3, and determining existence conditions of the corner solution:</u>*

We consider the following Lagrangian:

$$L = \left[a_i V_i^{\rho} + (1-a_i) TC_i^{\rho}\right]^{\frac{1}{\rho}} - \lambda\left(p_v V_i + p_e(V_i - \bar{V}_i) + p_{TC} TC_i - R_{T_i}\right) + \mu_V V_i \quad \textbf{(4.22)}$$

The first order conditions are :

$$CPO_1 : \frac{\partial L}{\partial V_i} = 0 \Leftrightarrow a_i V_i^{\rho-1} \times \left(a_i V_i^{\rho} + (1-a_i) TC_i^{\rho}\right)^{\frac{1}{\rho}-1} - \left(p_V + p_e\right)\lambda + \mu_V = 0$$

avec $\mu_V > 0$

$$CPO_2 : \frac{\partial L}{\partial TC_i} = 0 \Leftrightarrow (1-a_i) TC_i^{\rho-1} \times \left(a_i V_i^{\rho} + (1-a_i) TC_i^{\rho}\right)^{\frac{1}{\rho}-1} - p_{TC}\lambda = 0$$

$$\Rightarrow \lambda = \frac{(1-a_i) TC_i^{\rho-1} \times \left(a_i V_i^{\rho} + (1-a_i) TC^{\rho}\right)^{\frac{1}{\rho}-1}}{p_{TC}}$$

So, we have: $\mu_V = -(a_i) V_i^{\rho-1} \times \left(a_i V_i^{\rho} + (1-a_i) TC_i^{\rho}\right)^{\frac{1}{\rho}-1} + \left(p_V + p_e\right)\lambda > 0$; is replaced by λ its expression gives:

$$\frac{a_i}{(1-a_i)}\left(\frac{V_i}{TC_i}\right)^{\rho-1} < \frac{p_V + p_e}{p_{TC}} \quad \textbf{(4.29)}$$

APPENDIX 2:

- *Case n°1 : determining the effects of the emission permits price (pe) on equilibrium quantities*

$V_i^{**} = \left(\frac{a_i}{p_V + p_e}\right)^{\sigma}\left(\frac{R_{T_i} + p_e\overline{V}_i}{a_i^{\sigma}\left(p_V + p_e\right)^{1-\sigma} + \left(1-a_i\right)^{\sigma} p_{TC}{}^{1-\sigma}}\right)$, we develop this function to find

$$V_i^{**} = \frac{a_i{}^{\sigma}R_{T_i} + a_i{}^{\sigma} p_e\overline{V}_i}{a_i{}^{\sigma}(p_V + p_e) + (1-a_i)^{\sigma} p_{TC}{}^{1-\sigma}(p_V + p_e)^{\sigma}}$$

This relations is $\left(\frac{u}{v}\right)$ form, then the derivative with respect to p_e is $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

with

$u(p_e) = a_i{}^{\sigma}R_{T_i} + a_i{}^{\sigma} p_e\overline{V}_i \quad \Rightarrow u'(p_e) = a_i{}^{\sigma}\overline{V}_i$

$v(p_e) = a_i{}^{\sigma} p_V + a_i{}^{\sigma} p_e + (1-a_i)^{\sigma} p_{TC}{}^{1-\sigma}(p_V + p_e)^{\sigma} \quad \Rightarrow v'(p_e) = a_i{}^{\sigma} + \sigma(1-a_i)^{\sigma} p_{TC}{}^{1-\sigma}(p_V + p_e)^{\sigma-1}$

So

$$\frac{\partial V_i^{**}}{\partial p_e} = \frac{a_i{}^{\sigma} p_{TC}{}^{\sigma}\left(a_i{}^{\sigma} p_{TC}{}^{\sigma}\left(p_e + p_V\right)\left(p_V\overline{V}_i - R_{T_i}\right) - \left(1-a_i\right)^{\sigma} p_{TC}\left(p_e + p_V\right)^{\sigma}\left(\sigma R_{T_i} - p_e\overline{V}_i - p_V\overline{V}_i + p_e\sigma\right)\right)}{\left(p_e + p_V\right)\left(a_i{}^{\sigma} p_{TC}{}^{\sigma}\left(p_e + p_V\right) + \left(1-a_i\right)^{\sigma} p_{TC}\left(p_e + p_V\right)^{\sigma}\right)^2}$$

If $R_{T_i} > p_V\overline{V}_i$, so we have $\frac{\partial V_i^{**}}{\partial p_e} < 0$, causing $\frac{\partial V^{**}}{\partial p_e} < 0$

$TC_i^{**} = \left(\frac{1-a_i}{p_{TC}}\right)^{\sigma}\left(\frac{R_{T_i} + p_e\overline{V}_i}{a_i^{\sigma}\left(p_V + p_e\right)^{1-\sigma} + \left(1-a_i\right)^{\sigma} p_{TC}{}^{1-\sigma}}\right)$, we develop this function to find

$$TC_i^{**} = \left(\frac{(1-a_i)^{\sigma} R_{T_i} + (1-a_i)^{\sigma} p_e\overline{V}_i}{p_{TC}{}^{\sigma} a_i{}^{\sigma}\left(p_V + p_e\right)^{1-\sigma} + \left(1-a_i\right)^{\sigma} p_{TC}}\right)$$

So

$$\frac{\partial TC_i^{**}}{\partial p_e} = \frac{\left(1-a_i\right)^{\sigma}\left(p_e + p_V\right)^{\sigma}\left(\left(1-a_i\right)^{\sigma}\overline{V}_i p_{TC}\left(p_e + p_V\right)^{\sigma} + a_i{}^{\sigma} p_{TC}{}^{\sigma}\left(\sigma R_{T_i} - R_{T_i}\right) + \overline{V}_i\left(p_V + p_e\sigma\right)\right)}{\left(a_i{}^{\sigma} p_{TC}{}^{\sigma}\left(p_e + p_V\right) + \left(1-a_i\right)^{\sigma} p_{TC}\left(p_e + p_V\right)^{\sigma}\right)^2}$$

We have $\frac{\partial TC_i^{**}}{\partial p_e} > 0$, causing $\frac{\partial TC^{**}}{\partial p_e} > 0$.

- *Case n°2: determining the effects of emission permits price p_e on the equilibrium amount $V_i^{**'}$*

$V_i^{**'} = \frac{R_{T_i} + p_e \bar{V}_i}{p_V + p_e}$, then derived $V_i^{**'}$ from p_e is the following :

$$\frac{\partial V_i^{*'}}{\partial p_e} = \frac{p_V \bar{V}_i - R_{T_i}}{(p_V + p_e)^2} < 0 \text{ si } p_V \bar{V}_i < R_{T_i}$$

- *Case n°3: determining the effects of the emission permits price p_e on the equilibrium amount $TC_i^{**''}$*

$TC_i^{**''} = \frac{R_{T_i} + p_e \bar{V}_i}{p_{TC}}$, so the derivated of $TC_i^{**''}$ from p_e is the following:

$$\frac{\partial TC_i^{**''}}{\partial p_e} = \frac{\bar{V}_i}{p_{TC}} > 0 \text{ with } p_{TC} > 0$$

- *Case of Cobb-Douglas function: determining the effects of the emission permits price p_e on equilibrium quantities $\left(V_i^{**}\right)$ et $\left(TC_i^{**}\right)$*

$V_i^{**} = \frac{a_i\left(R_{T_i} + p_e \bar{V}_i\right)}{p_V + p_e}$, then the derivative of $\left(V_i^{**}\right)$ compared to the price p_e is the following:

$$\frac{\partial V_i^{**}}{\partial p_e} = \frac{a_i\left(p_V \bar{V}_i - R_{T_i}\right)}{\left(p_V + p_e\right)^2} < 0 \text{ if } p_V \bar{V}_i < R_{T_i}$$

$TC_i^{**} = \frac{(1-a_i)\left(R_{T_i} + p_e \bar{V}_i\right)}{p_{TC}}$, then the derivative of $\left(TC_i^{**}\right)$ compared to p_e is the following:

$$\frac{\partial TC_i^{**}}{\partial p_e} = \frac{(1-a_i)\bar{V}_i}{p_{TC}} > 0 \text{ if } p_{TC} > 0$$

- *The case of linear function: determining the effects of the emission permits p_e on equilibrium quantities $\left(V_i^{**}\right)$ et $\left(TC_i^{**}\right)$*

$V_i^{**} = \dfrac{R_{T_i} + p_e\bar{V}_i}{p_V + p_e}$, then the derivative of $\left(V_i^{**}\right)$ compared to the price p_e is the following:

$$\frac{\partial V_i^{**}}{\partial p_e} = \frac{\left(p_V\bar{V}_i - R_{T_i}\right)}{\left(p_V + p_e\right)^2} < 0 \text{ if } p_V\bar{V}_i < R_{T_i}$$

$TC_i^{**} = \dfrac{R_{T_i} + p_e\bar{V}_i}{p_{TC}}$, then the derivative of $\left(TC_i^{**}\right)$ compared to p_e is the following:

$$\frac{\partial TC_i^{**}}{\partial p_e} = \frac{\bar{V}_i}{p_{TC}} > 0 \text{ if } p_{TC} > 0$$

APPENDIX 3:

- *Determination of the effects of the emission permits quantity on the permit market price: special case where $\rho \to 0$:Cobb-Douglas*

Consider $p_e = \dfrac{\sum_{i=1}^{N} a_i R_{T_i} - p_V \sum_{i=1}^{N}\bar{V}_i}{\sum_{i=1}^{N}\bar{V}_i\left(1-a_i\right)}$ **(4.44)**

Taking the relation $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$ and differentiating with respect to $\bar{V}_i$, we get:

$$\frac{\partial p_e}{\partial \bar{V}_i} = \frac{-Np_V\sum_{i=1}^{N}(1-a_i)\bar{V}_i + \left(\sum_{i=1}^{N}(1-a_i)\right)\left(pv\sum_{i=1}^{N}\bar{V}_i - \sum_{i=1}^{N}a_iR_{T_i}\right)}{\left(\sum_{i=1}^{N}(1-a_i)\bar{V}_i\right)^2} < 0$$ knowing that

N>0 and $0<a_i<1$ and $\sum_{i=1}^{N} a_i R_{T_i} \geq p_v \sum_{i=1}^{N}\bar{V}_i$