SustainCity: State of the art review of landuse transport modelling

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Abstract

In the context of the SustainCity project, three European cities (Brussels, Paris and Zurich) will be modelled using the land use microsimulation plaform UrbanSim. This platform relies on various models interacting with each other, to predict long term urban development. The aim of this paper is to present the different econometric tools used to estimate these models.

The models analyzed here are: Real estate prices, Household location choice, Employment location choice and Real estate development. The associated econometric models are mainly linear regression, binary logit and multinomial (or nested) logit.

Employment location choice models receive a special attention in Work Package 2.3 (firmographics). We attach here a particular attention on describing and comparing Real estate price models to Household location choice models (which are described in detail in Work Package 2.2). We start from hedonic approach, especially the seminal work of Rosen (1974), and compare it with discrete choice models, discussing each method's advantages and disadvantages. At the end, some examples of applications of the hedonic models to real estate markets are presented.

Keywords

SustainCity, Econometrics

1 Introduction

The word "hedonic" comes from Greek "hedone" or enjoyment. **Hedonic approach** in economics is used for evaluating the *economic value* of environmental goods such as landscape quality, noise-, air- and water pollution, etc. This approach is based on the assumption that goods can be considered as aggregates of different attributes.[1] Some attributes do not have an individual price, because they cannot be sold separately. On real estate markets, for example, it is not possible to purchase separately one room or preferred location, panoramic qualities or quality of air and surrounding landscape. Hence, the hedonic approach allows us to assess natural values based on market values, using implicit prices of single characteristics of a good on the basis of market values of the whole good. The use of this approach is of particular interest in the field of environmental valuation, as it can be assumed that the values attributed to natural resources are attributes of commodities which are sold on the market. Obviously, the more closely the market good is related to the use of a natural resource, the more suitable is this approach for the evaluation of the natural resource. Real estate properties are very interesting in this context, as their values are strongly influenced by location characteristics.

The hedonic approach assumes that the economic value of each attribute influences the total value of the commodity and can thus be revealed as a difference in price. Its correct application requires that the following conditions are satisfied:

- Good quality data are available with large number of market exchanges.
- The market under consideration is transparent.
- There are no expectations with regards to changes in environmental qualities.

The last condition translates into the fact that it is not possible to estimate the total economic value of the environmental good, but only the value connected to present or just close future uses.

Application of the hedonic approach in economics comes through **hedonic regression**, which is a revealed preference method of estimating demand or value of a specific good by decomposing it into its constituent characteristics, and obtaining estimates of the contributory value

[1] This idea was first introduced by Lancaster (1972).

of each characteristic. This requires that the composite good being valued can be reduced to its constituent parts and that the market values those constituent parts.

Hedonic models are usually estimated using regression analysis, although more generalized models, such as sales adjustment grids, are special cases of hedonic models. An attribute vector, which may be a dummy or continuous variable, is assigned to each characteristic or group of characteristics. Hedonic models can accommodate non-linearity, variable interaction, or other complex valuation situations.

Hedonic models are commonly used in real estate appraisal, real estate economics and Consumer Price Index (CPI) calculations. In CPI calculations hedonic regression is used to control the effect of changes in product quality. Price changes that are due to substitution effects are subject to hedonic quality adjustments. In real estate economics, hedonic approach is used to adjust for the problems associated with researching a good that is as heterogeneous as buildings. Because buildings are so different, it is difficult to estimate the demand for buildings generically. Instead, it is assumed that a house can be decomposed into characteristics such as number of bedrooms, size of plot, or distance to the city center. A hedonic regression equation treats these attributes (or bundles of attributes) separately, and estimates prices (in the case of an additive model) or elasticity (in the case of a log model) for each of them. This information can be used to construct a price index that can be used to compare the price of housing in different cities, or to do time series analysis. As with CPI calculations, hedonic pricing can be used to correct for quality changes in constructing a housing price index. It can also be used to assess the value of a property, in the absence of specific market transaction data. Demand for various housing characteristics and housing demand in general can be analyzed with these models as well. Finally, assumption testing can be derived from these methods.

2 Hedonic price models

2.1 Estimation of a hedonic model

In general, hedonic theory does not correspond to a specific functional form.[2] Functional forms for the hedonic price function that have been proposed and used in the literature include linear and quadratic functions as well as log-log, semi-log, inverse semi-log, exponential, and Box-Cox trans-formation (Freeman, 1993). The semi-log specification has some advantages, including the easy interpretation of coefficients as percentage changes in the price given a one-unit change in the characteristic. Moreover, it helps to minimize the problem of heteroscedasticity and it mitigates the impact of nonlinear relationships between market price and the explanatory variables (Malpezzi, 2003). Therefore, hedonic pricing equations are typically estimated using either linear or semi-logarithmic models (Sirmans et al., 2005). A second challenge is the adequacy of parametric specification. Some authors, e.g. Anglin and Gencay (1996), Martins-Filho and Bin (2005), indicate that this problem arises from the inability of economic theory to provide guidance on how characteristics of similar products relate functionally to their market price. Consequently, there have been attempts to use semi- or non-parametric methods[3], which allow for the possibility of nonlinearity in the hedonic price functions and flexible modelling of the influence of continuous covariates on the dependent variable.

On an operative level the hedonic approach proceeds in two phases. A value function of the private good is estimated first, then a demand function for its attributes.

2.2 Canonic model and its limits

The theory at the basis of hedonic methods have been formulated by Rosen (1974) and successively improved with regards to the valuation of environmental goods by Freeman (1979). Rosen's model simulates a competitive market and foresees the simultaneous estimation of demand and supply function. In the particular case of real estate markets, with a very rigid

[2] Sirmans, Macpherson and Zietz (2005) explain that studies have wrestled with the problem of the correct functional form and no consensus has been found of which is most appropriate.

[3] See, for example, Clapp (2003, 2004), Fahrländer (2006), Pace (1993, 1998), and Parmeter, Henderson and Kumbhakar (2007).

supply, the model can be simplified and traced back to the neoclassic scheme of the theory of consumer's demand (Diamond and Smith, 1985).

Rosen (1974) assumes that goods are valued for their utility-bearing attributes or characteristics. He defines **hedonic prices** as the implicit prices of attributes revealed to economic agents from observed prices of differentiated products and the specific amounts of characteristics associated with them. When objectively measured goods' attributes are mapped to observed equilibrium market prices in a competitive economy, the marginal implicit worth of output characteristics can be derived from a hedonic price function that traces the behaviour of consumers and producers of differentiated products.

Rosen's (1974) hedonic model is composed of two parts. Firstly, a hedonic price function is estimated which allows calculation of the marginal implicit prices for each product characteristic. In the second stage, these marginal prices and consumers' socioeconomic characteristics are used to estimate the parameters of the equation representing consumers' behaviour.

Application of the Rosen's approach in practice is, however, quite limited given that his model assumes the following strong assumptions:

- Market offers a continuous range of choices, i.e. all combinations between private good and environmental conditions.
- Purchasers are able to behave according to the principle of decreasing marginal utility with respect to the environmental characteristics.
- All purchasers have the same opportunity of access to the market. In other words, they have the same cost of information, transaction and transfer, the same income available and the same mobility.
- Characteristics are perfectly observed and prices are transparent.
- Prices adapt immediately to changes in the demand for environmental goods.

These hypotheses are quite restrictive, as they describe a market which is perfectly transparent on the side of the offer, and homogeneous and perfectly competitive on the demand side.

The Rosen's (1974) approach was also criticized by Brown and Rosen (1982), Epple (1987) and Bartik (1987), since it attempts to obtain higher order approximations to the utility function by imposing homogeneity across individuals, and runs into an *identification problem* in the process. If Ekeland, Heckman and Nesheim (2002) provide a solution to this identification problem, however, their approach only allows for a single dimensional characteristic which must be observed. Bajari and Benkard (2004) solve the identification problem by allowing each individual to have different utility parameters but relying on parametric restrictions on the utility function. Moreover, they generalize the Rosen's approach by allowing for imperfect competition, a discrete product space with discrete characteristics, and one product characteristic that is not observed by the econometrician. These generalizations make it much easier to apply hedonic demand models in empirical work in a wider set of applications.

Identification of preferences is well understood for the case when the data contains many observations for each consumer under widely varying pricing regimes; see Mas-Colell (1977). However, real data sets (e.g. from real estate markets) are seldom this rich, frequently containing only a single observation per consumer. If the choice set is continuous, then the household level preference parameters must satisfy a set of first order conditions that require the marginal rate of substitution between a continuous product characteristic and the composite commodity to equal the implicit price of that product characteristic. If the functional form of the utility function is known and the parameter vector is of equal or lesser dimension than the characteristics vector, then these first order conditions can be used to recover household level random coefficients. By aggregating household level random coefficients, the population distribution of random coefficients can be obtained nonparametrically.

However, it is more common in empirical applications for the market to contain a small number of products. In this case, an individual consumer's random coefficients typically are not identified from the revealed preference conditions even if the parametric form of utility is known. Instead, the revealed preference conditions imply that each individual's taste coefficients lie in a set. This set tends to be smaller when there are more products in the market, eventually converging to a singleton. Bajari and Benkard (2004) show how to use these sets for each individual to construct bounds on the population distribution of random coefficients. Their procedure is shown to converge to the population distribution of taste coefficients as the number of products becomes large.

3 Comparison with discrete choice models

One reason why the hedonic approach has not been yet widely used in empirical literature is because of its strict assumption that all product characteristics are perfectly observed. In practice, typically only a very small number of product characteristics are observed. As a result, it is common for the perfect observability assumption to lead to revealed preference violations. For example, it is common for data sets to contain two products with positive demand in the same period, where one of the products is "better" in every dimension of characteristics space, and also has a lower price. In such cases, there is no set of parameters under which the hedonic model can rationalize the observed demands. This problem has led to the wide use of discrete choice econometric models (such as logit, probit, etc.) that allow for further product differentiation through a random error term.[4] In practice, these two approaches can lead to different results and since it is very difficult to know which approach is more correct, the hedonic models are seen as a valid alternative to the standard discrete choice models.[5]

One advantage of the hedonic model is that it facilitates flexible nonparametric estimation of the individual preference parameters. It also provides an alternative way of estimating the demand curve for prices that are not observed in the data. The primary disadvantages of the hedonic model seem to be its data requirement, since the estimation of a hedonic model is likely to require significantly more data than other alternatives, and the fact that the assumptions of perfect information and independence may be too strict in practice.

An alternative approach used in the housing literature is the multinomial logit model (McFadden, 1978). This is a discrete choice approach that attempts to identify the importance of various housing characteristics on the housing purchase decision. This model does account for the inherent hierarchy in the data (Chattopadhyay, 2000; Nechyba and Strauss, 1998; and Quigley, 1985). Unlike the hedonic model, however, this approach does not attempt to explain housing prices, but instead examines the variables that determine whether a household buys a dwelling or not.

In empirical studies of differentiated product markets, it is common practice to estimate random utility models (RUM) of consumer demand. In a RUM, the consumer's utility is a func-

[4] Manski (1977) suggests that there are four sources of uncertainty that justify the use of the random error term in the model: unobserved product characteristics, unobserved consumer heterogeneity, measurement error and functional misspecification.

[5] See Ackerberg and Rysman (2002), Bajari and Benkard (2003) and Berry and Pakes (2001) for further discussion of these issues.

tion of product characteristics, a composite commodity and a stochastic, household specific, product level taste shock. In many markets, such as autos, computers or housing, there are thousands, if not tens of thousands of unique products available to consumers. As a result, each household's utility function contains a very large number of these random taste shocks. Since households choose the good with the maximum utility, it is important to pay attention to how demand models are affected by large draws of the taste shocks.

In general, RUMs provide a quite flexible framework for modeling consumer preferences. However, in practice, in order to facilitate estimation of RUMs, practitioners typically make restrictive functional form assumptions about the distribution of the taste shocks. For example, the logit model is attractive from a computational point of view because the likelihood function can be evaluated in closed form. The multinomial probit model can be studied in a computationally efficient manner using simulation.

While commonly used RUM models greatly simplify the computational burden of estimation, some recent papers have found that such simplifications can also lead to implausible results. For example, Petrin (2002) studies the problem of measuring the welfare benefits from the introduction of the minivan. In his logit estimates, he finds that the average compensating variation for minivan purchasers is $7,414, for a vehicle that sold for just $8,722, with 10% of consumers willing to pay over $20,000 for the option to purchase a minivan. Even his random coefficients logit estimates have several percent of consumers willing to pay over $20,000 for this option. Similarly, in Ackerberg and Rysman's (2001) study of the demand for phone books, the optimal number of products goes from greater than 10 to just 5 when a model with a logit error term is used versus a model that uses a more general error structure. In both cases, the authors found that their results were being generated from large realizations of the logit error term in the demand system.

Bajari and Benkard (2003) investigate the economic implications of common assumptions on the error term in RUMs. Many RUMs have the following properties: 1) the model includes an additive error term whose conditional support is unbounded, 2) the deterministic part of the utility function satisfies standard continuity and monotonicity conditions, and 3) the hazard rate of the error distribution is bounded above. They show that these assumptions can lead to several potentially unappealing features of the demand model.

First, demand for every product is positive for all sets of prices. This implies that some consumers are unwilling to substitute away from their preferred product at any price. For many products, this property is a priori unreasonable.

Second, when the number of products becomes large, the share of the outside good must go to zero, meaning that every consumer purchases some variety of the good. In many markets this property is also a priori unreasonable. Similarly, if the share of the outside good can always be bounded away from zero, then the deterministic part of utility from the inside goods for all of the consumers who purchase the outside good must tend to negative infinity as the number of products becomes large, suggesting that the parameters of the model are not stable to changes in the number of goods.

Third, in Generalized Extreme Value (GEV) based models, even if the number of products becomes infinite, no product has a perfect substitute. That is, each individual would almost surely suffer a utility loss bounded away from zero if he was forced to consume a product other than her preferred alternative. This property shows that the product space can never become crowded as implied by standard models of horizontal and vertical product differentiation with a continuum of goods.

Fourth, in GEV based models, as the number of products becomes large, a Bertrand-Nash equilibrium does not tend toward the perfectly competitive outcome where all firms price at marginal cost. This shows that RUMs always build in “excess” market power for the firms. As a result, some RUMs may generate misleading implications if they are used, for example, in merger analysis. Note that the last two properties do not hold for probit models, suggesting that when there are a large number of products, the probit model may be preferable to the logit model in some applications.

Fifth, as the number of products becomes large, the ratio of the error term to the deterministic part of utility at the chosen product becomes one. That is, all of the consumer's utility is derived from the random error term. This suggests that RUMs may tend to understate the consumers' welfare from the observed product characteristics.

Sixth, as the number of products becomes large, the compensating variation for removing all the inside goods tends to infinity for each individual. This last result is consistent with the empirical findings of Petrin (2002) and Ackerberg and Rysman (2001).

Other issues, raised by Ackerberg and Rysman (2001), Berry and Pakes (2000), Caplin and Nalebuff (1991), and Petrin (2002), are that the logit model tends to overstate the benefits from product variety. Also, Anderson, de Palma and Thisse (1992) establish that Bertrand competition does not converge to perfect competition in the logit model.

Thus, many commonly used RUMs have some unappealing economic properties as the number of products becomes large. One way to avoid this set of assumptions is to allow the distribution of the error term to change with the choice set. Such an approach is outlined in detail in Ackerberg and Rysman (2001) who allow the mean or variance of the error term to fall as more products are added to the market. Another alternative is to use a *pure hedonic* demand model which eliminates the *iid* error term in the utility function. This is the approach of Berry and Pakes (2000) and Bajari and Benkard (2003).

Hedonic models of demand for differentiated products have been used extensively in the past. Examples include models of horizontal product differentiation such as Hotelling (1929), Gorman (1980) and Lancaster (1966), models of vertical product differentiation, such as Shaked and Sutton (1987) and Bresnahan (1987), as well as Rosen's (1974) model.

The hedonic model does not impose many of the undesirable assumptions of standard discrete choice models. First, in the hedonic model, it is not always the case that demand is positive at any price (Bajari and Benkard, 2003).

Furthermore, for individuals with low preference for characteristics of the inside good relative to their preference for the composite commodity, only a very low price will induce them to purchase the good. If a consumer's willingness to pay for characteristics of the good is below the marginal cost of production, then it may be that no rational price is low enough to induce purchase. Thus, the share of the outside good does not necessarily tend to zero as more products enter the market.

If the distance between the characteristics of two products is small and preferences are Lipschitz continuous, then in the pure hedonic model these products will be close substitutes. As a result, as the number of products becomes infinite, all products will have perfect substitutes and markups in Bertrand price competition will tend to zero.

Finally, the pure hedonic model does not imply that a continuum of products provides consumers with infinite utility relative to income or price. Thus, the compensating variation for removing all inside goods remains bounded even in that case.

However, the hedonic model also has limitations. In the hedonic model, not all products are required to be strong gross substitutes. The number of cross price elasticities for a particular product that are strictly greater than zero is a function of the number of product characteristics. This is potentially unappealing in markets where there are only a handful of observed characteristics.

In addition, the empirical results in Bajari and Benkard (2003) suggest that if many products are included in the choice set, then the perfect information assumption in the pure hedonic model can lead to demand curves that are too elastic. To summarize their results, one reason for including the random error term in the utility function is that it could represent imperfect information (e.g. due to a cost of acquiring information about products). Leaving out this imperfect information may imply too high a degree of substitutability across products.

To conclude, standard discrete choice models have some undesirable economic properties when viewed as structural models of demand. These properties are primarily driven by functional form assumptions about the random error term introduced into the model for estimation purposes. They hold not only in the logit model, but in any RUM with the three properties recalled page 6: the conditional support of the error term is unbounded; the deterministic part of the utility function satisfies standard continuity and monotonicity conditions; the hazard rate of the error term is bounded above. Due to the computational complexity of estimating RUMs, these properties are maintained in most applications. However, there may also be alternative RUM models that eliminate these properties, which is an area for further research.

4 Applications in real estate market

Analyzing the demand for housing is a challenging empirical problem. A home is a bundle of four types of characteristics. The first characteristic type comprises the *physical attributes of the home*, such as the number of rooms, the lot size, and whether or not the unit is attached. The second includes the *attributes of the community*, such as geographic and demographic characteristics of the surrounding neighbourhood (proportion of green areas, foreigners, etc.). A third characteristic is *accessibility* to an urban park, a central business district or one's place of work. The last type of characteristics relates to *environmental quality*, such as quietness, air pollution or proximity to recreational areas. Even if there is a missing market for environmental quality (such as open spaces), by unbundling the housing product it is possible to assess the (implicit) value that individuals are revealing by their (explicit) choice in the housing market.

There is considerable heterogeneity in preferences for housing attributes. These preferences depend on observed demographic characteristics of the household, such as the number of children and marital status. Also, there is considerable heterogeneity in preferences even after accounting for all household demographics. Recently, there have been major advances in estimating the demand for local public goods that explicitly incorporate heterogeneity into the estimation procedure. Epple and Sieg (1999) presented an equilibrium model in which households who differ with respect to income and local public good preferences sort across Boston communities. Sieg, Smith, Banzhaf, and Walsh (2002) extended this framework to allow for a heterogeneous housing stock and showed how to use hedonic methods to construct price indices. Smith, Sieg, Banzhaf, and Walsh (2004) used a locational equilibrium model to estimate the environmental benefits of Clean Air Act regulation. Bayer, McMillan, and Rueben (2002) also estimated a rich model of the demand for housing attributes, and given their demand estimates, they simulated an equilibrium model of the housing market.

There are many other empirical works applying the hedonic approach to real estate markets.[6] Here we will discuss only two recent ones: Bajari and Kahn (2005), and Löchl and Axhausen (2009).

[6] For some surveys, see e.g. Palmquist (2005) and Sheppard (1999).

4.1 Housing demand and racial segregation – Bajari and Kahn (2005)

Bajari and Kahn (2005) use hedonic approach to estimate housing demand and to explain racial segregation in big American cities. They present a three-stage, nonparametric estimation procedure to recover willingness to pay for housing attributes. In the first stage they estimate a nonparametric hedonic house price function using local polynomial methods. In the second stage they recover each consumer's taste parameters for product characteristics using first-order conditions for utility maximization. Finally, in the third stage they estimate the distribution of household tastes as a function of household demographics. Their empirical model of housing demand is innovative in the way that it explicitly incorporates heterogeneity into the estimation procedure by allowing preferences to be a flexible function of observed demographics and idiosyncratic taste shocks for housing characteristics. Furthermore, the model accommodates both discrete and continuous characteristics, and accounts for the simultaneity of the choice of where to live and where to work. The authors use approach that incorporates some of the attractive features of the Rosen's (1974) hedonic two-step model while at the same time captures some of the insights from recent work on differentiated product demand estimation in industrial organization (Berry et al. 1995; Petrin 2002). Similar to the hedonic literature, they estimate a hedonic pricing function to recover the marginal prices of housing attributes by deriving preferences not only for observed but also unobserved product characteristics. Recent advances in industrial organization have emphasized the importance of accounting for product characteristics observed by the consumer but not by the economist, since ignoring such attribute biases estimated price elasticities toward zero.[7]

Finally, Bajari and Kahn apply their methods to explore why the average white household lives in the suburbs while the average black household lives in the city center. They estimate their housing demand model for three major American cities: Atlanta, Chicago and Dallas, to explore the merits of potential explanations that include differences in income, differences in place of work, and differences in willingness to pay for access to peers. Their estimates yield new insights about household demand for structure, peers, and commuting. The proposed methods are computationally straightforward and can be estimated using standard statistical packages.

[7] See, for example, Berry 1994; Berry et al. 1995; Petrin 2002 or Ackerberg and Rysman 2002.

4.1.1 Model

Let's denote by $i = 1, \dots, I$ individuals and by $j = 1, \dots, J$ housing units. Each housing unit is described by a vector $\mathbf{x}_j$ of observed characteristics and by a scalar ξ_j of unobserved characteristics. The price p_j of a housing unit j is a function $\mathbf{p}$, which maps the characteristics of housing units into prices as follows:

$$p_j = \mathrm{p}(\mathrm{x}_j, \xi_j) .$$

Household utility is a function of housing characteristics ($\mathbf{x}_\mathrm{j}$, ξ_j) and consumption of a composite commodity c_{ij}, which price is normalized to 1. The utility that consumer i receives from product j is then

$$u_{ij} = u_i(\mathrm{x}_j, \xi_j, c_{ij}) .$$

Households are rational utility maximisers who choose their preferred bundle given their income y_i. Product $j^*(i)$ is utility maximizing for household i if

$$j^*(i) = \arg\max_j u_i(\mathrm{x}_j, \xi_j, c_{ij}) ,$$

with the budget constraint $c_{ij} = y_i - \mathrm{p}(\mathrm{x}_j, \xi_j)$.

Suppose that product characteristic $x_{j,k}$ is a continuous variable and that product j^* is utility maximizing for household i. Then the following first-order condition must hold:

$$\frac{\partial u_i(\mathrm{x}_{j^*}, \xi_{j^*}, c_{ij^*})}{\partial x_{j,k}} + \frac{\partial u_i(\mathrm{x}_{j^*}, \xi_{j^*}, c_{ij^*})}{\partial c_{ij}} \frac{\partial c_{ij^*}}{\partial x_{j,k}} = 0 ,$$

which gives

$$\frac{\partial u_i(\mathrm{x}_{j^*}, \xi_{j^*}, c_{ij^*}) / \partial x_{j,k}}{\partial u_i(\mathrm{x}_{j^*}, \xi_{j^*}, c_{ij^*}) / \partial c_{ij}} = -\frac{\partial c_{ij^*}}{\partial x_{j,k}} = \frac{\partial \mathrm{p}(\mathrm{x}_{j^*}, \xi_{j^*})}{\partial x_{j,k}} . \qquad (1)$$

This is the familiar condition that the marginal rate of substitution between a continuous characteristic and the composite commodity is equal to the partial derivative of the hedonic.

For identification reasons, the authors use the following specification for consumer preferences:

$$u_{ij} = \beta_{i,1}\log(room_j) + \beta_{i,2}\log(age_j) + \beta_{i,3}\log(\xi_j) + \beta_{i,4}own_j + \beta_{i,5}single_j + \beta_{i,6}\log(mblack_j) + \beta_{i,7}\log(mba_j) + \beta_{i,8}city_j + c_{ij}, \tag{2}$$

where

$$\beta_{i,k} = f_k(d_i) + \eta_{i,k} \text{ with } E(\eta_i \mid d_i) = 0. \tag{3}$$

Thus the utility function is a linear or log-linear function of all house attributes: physical ones, those related to community and unobserved characteristic. The variables above have the following interpretation:

- $room_j$ is the number of rooms in housing unit j.
- age_j is the age in years of housing unit j.
- ξ_j is the value of the characteristic seen to the consumer, but not by the economist, associated with housing unit j.
- own_j is a dichotomous variable equal to 1 if the housing unit j is owner occupied and 0 otherwise.
- $single_j$ is a dichotomous variable equal to 1 if the housing unit j is detached and 0 otherwise.
- $mblack_j$ is the proportion of black heads of household where the housing unit j is located.
- mba_j is the proportion of college-educated heads of household where housing unit j is located.
- $city_j$ is a dichotomous variable equal to 1 if the housing unit j is located in the centre city.

The taste parameters $\beta_{i,k}$ are household-specific and are modelled as a function f_k of demographic characteristics d_i and an orthogonal residual $\eta_{i,k}$. The demographic characteristics d_i correspond to the age of the head of household, annual household income, household size, and dummy variables for whether the head of household is male, married, black, a college graduate, and a centre city worker.

With the functional form assumption (2), the first order condition (1) gives

$$\beta_{i,k} = x_{j^*,k} \frac{\partial \mathbf{p}(\mathbf{x}_{j^*}, \xi_{j^*})}{\partial x_{j,k}}. \quad (4)$$

Thus, each household-specific taste parameter $\beta_{i,k}$ can be recovered from the observed characteristic $x_{j^*,k}$ chosen by the household i, and the first partial derivative of the hedonic price function with respect to that characteristic, which can be flexibly estimated using nonparametric methods.

For product characteristics that take on dichotomous values (0 or 1), there is no first-order condition for utility maximization. Instead, utility maximization implies a simple threshold decision making rule. For instance, suppose that household i chooses product j^*. Define $\hat{\mathbf{x}}_i$ as a vector of observed characteristics with $single_i = 1$ and all other elements set equal to their corresponding values in $\mathbf{x}_{j^*}$. Define $\bar{\mathbf{x}}_i$ similarly, except for $single_i = 0$. The implicit price faced by household i for single detached housing in the market is then $\frac{\Delta \mathbf{p}}{\Delta single} = \mathbf{p}(\hat{\mathbf{x}}_i, \xi_j) - \mathbf{p}(\bar{\mathbf{x}}_i, \xi_j)$. Utility maximization implies that

$$\begin{aligned} [single_i = 1] &\Rightarrow \left[\beta_{i,5} > \frac{\Delta \mathbf{p}}{\Delta single} \right], \\ [single_i = 0] &\Rightarrow \left[\beta_{i,5} < \frac{\Delta \mathbf{p}}{\Delta single} \right]. \end{aligned} \quad (5)$$

That is, if household i lives in single detached housing, then we can infer that i's preference parameter exceeds the implicit price for this characteristic. Analogous conditions can be derived for the other dichotomous characteristics.

4.1.2 Estimation

Bajari and Kahn estimate their model in three steps. In the first step, they estimate the housing hedonic price using flexible nonparametric methods. In the second step, they use the first-order condition (4) to infer household-level preference parameters for continuous product characteristics. In the third step, they estimate (3) by regressing household-level preference parameters on the demographic characteristics d_i. The authors also discuss how to estimate preferences for dichotomous characteristics by maximum likelihood.

First Step: Estimating the Hedonic Price Function

Bajari and Kahn suppose the following functional form for the hedonic price function:

$$p_j = \alpha_{0,j^*} + \alpha_{1,j^*}\log(room_j) + \alpha_{2,j^*}\log(age_j) + \alpha_{3,j^*}own_j + \alpha_{4,j^*}single_j + \alpha_{5,j^*}\log(mblack_j) \\ + \alpha_{6,j^*}\log(mba_j) + \alpha_{7,j^*}city_j + \xi_{j^*}, \tag{6}$$

where $\alpha_{0,j^*} - \alpha_{7,j^*}$ are hedonic coefficients corresponding to implicit prices for housing attributes, and ξ_{j^*} is the error term corresponding to a vertical product characteristic observed by the consumer, but not by the econometrist. The coefficients are estimated using weighted least squares method, i.e.

$$\alpha_{j^*} = \arg\min_{\alpha}(\mathbf{p} - \mathbf{X}\alpha')\mathbf{W}(\mathbf{p} - \mathbf{X}\alpha'),$$

where $\mathbf{p} = [p_j]$ is a vector of all prices, $\mathbf{X} = [x_j]$ is a matrix of regressors and $\mathbf{W} = \text{diag}\{K_h(x_j - x_{j^*}\}$ is a matrix of kernel weights, which are a function of the distance between the characteristics of product j^* and product j.[8] Thus the local linear regression assigns greater importance to observations with characteristics close to j^*. It is worth to mention that local linear methods have the same asymptotic variance and a lower asymptotic bias than the Nadaraya–Watson estimator. The unobserved product characteristic ξ_{j^*} is estimated as the residual to the hedonic regression (6). The standard hedonic assumption is that the unobserved product characteristics are independent of the observed product characteristics, which is maintained also in this paper.

Second Step: Applying the First-Order Conditions

Using estimates of implicit prices $\hat{\alpha}_{j^*}$ obtained from the first step and the observed choice of $x_{j^*,k}$, the taste parameters $\beta_{i,k}$ for continuous characteristics can be estimated from (4) as follows:

$$\hat{\beta}_{i,k} = x_{j^*,k}\frac{\partial \hat{\mathbf{p}}(\mathbf{x}_{j^*}, \xi_{j^*})}{\partial x_{j,k}} = \hat{\alpha}_{k,j^*}\,. \tag{7}$$

Third Step: Modelling the Joint Distribution of Tastes and Demographics

After recovering household-level preference parameters the authors estimate (3) as a linear function of demographic characteristics. For continuous characteristics, they let

[8] The authors chose normal kernel function with a bandwidth of 3.

$$\hat{\beta}_{i,k} = \theta_{k,0} + \sum_s \theta_{k,s} d_{i,s} + \eta_{i,k} , \tag{8}$$

where residuals $\eta_{i,k}$ are parameter-free and can be interpreted as household-specific taste shocks.

The preference parameters for dichotomous characteristics are not identified by equation (5), from which we can only infer that the preferences for a particular household are above or below the threshold value equal to the implicit price of the discrete characteristic. Given this lack of identification, the authors use a parametric model for these taste coefficients. They use the same functional form as in (8), but assume that $\eta_{i,k}$ are normally distributed with mean 0 and standard deviation σ. Then by (5), the probability that household i chooses to live in a single detached housing is

$$\begin{aligned} P\left[single_i = 1\right] &= P\left[\beta_{i,5} > \frac{\Delta \mathrm{p}}{\Delta single}\right] = P\left[\theta_{5,0} + \sum_s \theta_{5,s} d_{i,s} + \eta_{i,5} > \frac{\Delta \mathrm{p}}{\Delta single}\right] \\ &= 1 - P\left[\eta_{i,5} < \frac{\Delta \mathrm{p}}{\Delta single} - \theta_{5,0} - \sum_s \theta_{5,s} d_{i,s}\right] = 1 - N\left(\frac{\Delta \mathrm{p}}{\Delta single} - h(d_i, \theta_5); \sigma\right), \end{aligned}$$

where N is the normal cdf and $h(d_i, \theta_5) = \theta_{5,0} + \sum_s \theta_{5,s} d_{i,s}$. The likelihood function for the population distribution of tastes for single detached housing can be written as

$$L(\theta, \sigma) = \prod_{i=1}^{I} N\left(\frac{\Delta \mathrm{p}}{\Delta single} - h(d_i, \theta_5); \sigma\right)^{1 - single_{j^*(i)}} \times \left[1 - N\left(\frac{\Delta \mathrm{p}}{\Delta single} - h(d_i, \theta_5); \sigma\right)\right]^{single_{j^*(i)}}$$

where $single_{j^*(i)}$ is an indicator variable that is equal to 1 if household i purchased single detached housing and 0 otherwise. The authors estimate this model for each dichotomous product characteristic independently by maximum likelihood.

Results

The authors estimate the willingness to pay for different housing attributes in order to explain racial segregation in American cities. In particular, their empirical application focused on answering why blacks choose to live in cities while whites suburbanize. Based on the structural estimates, the authors conclude that white suburbanization is driven by their greater demand for larger single detached housing units and their greater demand for living in high human capital communities. Black demand for suburban housing products would significantly increase if their incomes, educational attainment, and marriage rates were as high as those of whites. Black urbanization is also explained by their propensity to work in centre cities. All else equal, the disutility from commuting provides an incentive to choose urban residential communities. The hypothesis that white demand for the suburbs is driven by the desire to live among other whites is rejected.

4.2 Modelling hedonic residential rents – Löchl and Axhausen (2009)

Löchl and Axhausen (2009) apply hedonic approach to residential rents in Zurich. They use three models: ordinary least square (OLS), spatial autoregressive (SAR) and geographically weighted regression (GWR). The OLS model lacks the ability to consider spatial dependency and spatial heterogeneity, consequently leading to biased and inefficient estimations. Therefore the other two models are more appropriate. In fact, GWR performs best with regard to model fit, but the issue of correlated coefficients gives favour to SAR.

4.2.1 Spatial Autoregressive Model (SAR)

SAR is a popular approach to incorporate spatial effects in a regression model. It assumes that the response variable at each location is a function not only of the explanatory variable at that location, but of the response at neighbouring locations as well. A comprehensive introduction might be found in Anselin (1988). The models are based on maximum-likelihood estimations and commonly applied in the fields of regional science, sociology, political science and economics[9].

Depending on where the spatial autoregressive process is believed to occur, the following three SAR models are usually distinguished:

1. spatial autoregressive lag model (SARlag): $P = \rho WP + \beta X + \varepsilon$
2. spatial error model (SARerr): $P = \beta X + u, \quad u = \lambda Wu + \varepsilon$
3. spatial mixed model (SARmix): $P = \rho WP + \beta X + WX\gamma + \varepsilon$

where P denotes a vector of rents, ρ is a spatial autocorrelation parameter, W is a $N \times N$ spatial weight matrix (where N is the number of observations), β is a vector of regression coefficients, X is a matrix with observations on structural and spatial explanatory characteristics, λ is the spatial autoregressive coefficient and ε is assumed to be a vector of independent and

[9] For some applications of SAR method in real estate appraisal, see e.g. Kim, Phipps and Anselin (2003), Shin, Washington and Choi (2007), and Willhelmsson (2002).

identically distributed (*iid*) error terms. Typically, the definition of neighbours used in the weights matrix is based on a notion of distance decay or contiguity.

4.2.2 Geographically Weighted Regression (GWR)

The GWR method, developed by Brunsdon, Fotheringham and Charlton (1998), attempts to incorporate geographical information into a regression model using a series of distance related weights. Essentially, it consists of a series of locally linear regressions that utilize distance-weighted overlapping samples of the data. The method explicitly allows parameter estimates to vary over space which leads to independent spatial error terms. Rather than specifying a single global model to characterize the entire housing market, GWR estimates a separate model for each data point and weights observations by their distance to this point, thus allowing unique marginal price estimates at each location.

The typical output from a GWR model is a set of parameters that can be mapped in the geographic space to represent non-stationarity or parameter drift. Similarly, local measures of standard errors and goodness-of-fit statistics are obtained. Therefore, the additional benefit of the GWR approach is that it offers the potential of increased understanding of the nature of varying relationships between variables across space. Du and Mulley (2006) describe GWR as an alternative to spatial autoregressive models which "is perhaps more intuitive". The GWR approach has been recently applied in the fields like climatology, ecology, education, marketing research, regional science, political science, transport research and housing field[10].

In GWR, a weighting function is applied in order to give greater influence to close data points whereas a spatial kernel is usually used. Both the choice of the shape of the kernel and the bandwidth of it are important issues and various options are available (Fotheringham et al., 2002). The bandwidth choice is not a parameter relating to the model itself, but is essentially part of the calibration strategy for a given sample.

In the case of rent estimations, a GWR model can be written as follows:

$$P_i = \beta_{i0} + \sum_k \beta_{ik} X_{ik} + \varepsilon_i$$

where P_i is the ith observation of the rent, β_{i0} is the constant estimated for local regression i, β_{ik} is the regression coefficient estimated for the ith regression of the kth structural or spatial

[10] See for example Bitter, Mulligan and Dall'erba (2007), Farber and Yeates (2006), Fotheringham, Brunsdon and Charlton (2002), Kestens, Thériault and Des Rosiers (2006), Páez, Long and Farber (2008) and Yu (2004).

explanatory variable, X_{ik} is the ith observation of variable k and ε_i is the ith value of a normally distributed error vector with mean equal to zero. This differs from OLS by utilizing distinct constants and regression parameters for each data point. The estimation algorithm iterates through N OLS, each one modified by a unique distance-decay weight matrix. The estimation takes the form:

$$\beta_i = \left(X^T W_i X\right)^{-1} X^T W_i P$$

where β_i is the vector of estimated coefficients for observation i, X is the $N \times K$ matrix of explanatory variables, W_i is a diagonal distance-decay weight matrix customised for i's location relative to the surrounding observations and P is the vector of observed rents.

4.2.3 Results

The authors estimated the aforementioned models by using the data of rent offers from various Swiss real estate online platforms. These data included the following characteristics: monthly net asking rent, floor area in square meters, year of building construction, whether building is a single family house, whether it has a lift, whether dwelling unit has a fireplace, balcony or garden terrace, etc. These data were extended by the available data from a geographic information system, like average travel time to Zürich central business district by car, accessibility to employment by car or public transport, distance to next railway station and to highway, daily average air noise, number of jobs in hotel and restaurant industry within 1 km, number of inhabitants and proportion of foreigners in hectare. The authors also generated supplementary geographical variables, like solar radiation and visibility variables, using a digital elevation model. Their aim was to choose a broad set of statistically significant variables which would provide expected signs of estimates and moderate the negative impact of multicollinearity. For this purpose they did exploratory data analysis through OLS stepwise regression.

The OLS model itself gave poor results, apparently because it does not take into account spatial dependency of the underlying variables. Therefore, several SAR models and the GWR approach were tested. Although, all goodness-of-fit measures indicated slightly better performance for the GWR model compared to the SAR models, its residuals were spatially autocorrelated. Additionally, the resulting GWR coefficients were correlated, which severely reduced confidence in the method. The SARerr model has been preferred against the SARmix model, because the latter had several insignificant variables, including public transport accessibility, which is certainly a crucial measure in land use and transport modelling. Additionally, the SARerr model showed good accuracy of the predicted values compared to observed values.

A practical conclusion of the Löchl and Axhausen's work is that the rent increases with floor surface, number of rooms, the presence of lift, fireplace, balcony or garden terrace, accessibility to employment and central business district, number of hotels and restaurants in proximity, solar exposition, and visibility to lake and surrounding terrain. The rent is also higher if the building is a single family house. On the other hand, the rent decreases with noise pollution, distance to next railway station or to highway, population density and proportion of foreigners.[11]

[11] Foreigners are defined as inhabitants with nationalities outside of North-Western Europe, North America and Australia.

5 Land-use models

The interaction of land-use and transport is well understood and actually can be used to explain a large part of urban development patterns over the past few decades (including urban sprawl, radial development of cities along transportation corridors and the resulting quicker-than-expected saturation of major capital infrastructure projects). The evolution of land use states is a complex problem that depends on many parameters and can be (and has been) formulated in many different ways, including econometric (Frazier and Kockelman, 2005) and ecological (Irwin, 2010). In this section, an overview of modelling of land-use models is provided, in order to provide some background for possible enhancements of UrbanSim in this direction. UrbanSim (Waddell, 2002) uses microscopic models to simulate the effects of location, land use, and policy decisions by households, workers, developers and policymakers on the land use patterns and rents across a region. Depending on the level at which it operates, land use and development decisions may be made at a parcel, grid, or zone level.

In the context of reviewing land use evolution models, Frazier and Kockelman (2005) provide a review of spatial econometric models for panel data, which is summarized in the following few paragraphs. Parker et al. (2003) provide a review of land-use/cover change (LUCC) models, concluding that no single model can be conclusively considered superior to the others [see, e.g., Candau (2002); Clarke and Gaydos (1998); Parker, Berger and Manson (2001)].

Klosterman's (1999) "What if?" model of land use assigns land uses to a set of homogeneous zones in a bottom-up fashion, derived from socioeconomic, geographic, transportation and zoning information. Landis and Zhang's (1998) California Urban Futures 2 (CUF2) model employs multinomial models of land-use change per hectare (or other unit of observation) to predict future land use patterns.

Frazier and Kockelman (2005) conclude that while methods for dealing with temporal and spatial dependence are available, many of the models used in this context focus primarily at the effect of spatial autocorrelation and do not attempt to incorporate temporal correlations, which are particularly relevant for transportation-related models (and certainly for the UrbanSim model applications). Recognizing this void, Frazier and Kockelman formulate and estimate models for developed, residential and agricultural land cover, and use them to simulate population and land cover projections for 2020.

Irwin (2010) approaches the topic from an ecological perspective. Cadenasso et al. (2006) outline three critical dimensions of spatial complexity of ecosystem structure: heterogeneity (patch patterns), connectivity (patch functions) and contingency (patch history). While these approaches seem at first foreign to the land use - transport system interaction at large, and the development of UrbanSim in particular, it is possible that some ideas from this field could be transferable and potentially useful.

Econometric models of land use change derive from economic models of individual land use decisions in which landowners choose a land use in a given time period such that net expected returns over time are maximized. The theoretical framework for these models is well-established in urban economics (e.g., Arnott 1980; Arnott and Lewis 1979; Capozza and Helsley 1989, 1990; Capozza and Li 1994). While the models vary in their assumptions about space, expectations, durability of capital and uncertainty, they are forward-looking given that landowners make intertemporal land use decisions conditional on expectations over changes in land rents, e.g., due to population growth.

The remainder of this section is based on Irwin (2010), who identify two steps in the development of econometric models of spatially heterogeneous land use patterns. The first step is the specification of the econometric model based on hypotheses regarding the factors that influence expected land rents, which typically include multiple spatially heterogeneous landscape and location features and policy constraints. This model is then estimated using spatial microscopic panel data on land use over time at the scale of land ownership, e.g., land parcels, and additional spatially detailed data on the factors hypothesized to influence expected land rents. A variety of estimation models are possible, including binary or multinomial discrete choice models (Bockstael 1996; Nelson and Hellerstein 1997), duration models that account for time-varying variables (Irwin and Bockstael 2002; Towe et al. 2008) and option value models that account for the influence of uncertainty over future prices (Cunningham 2007; Towe et al. 2008). Following the model estimation, parameter estimates are used to simulate hypothetical changes in land use pattern, e.g., under baseline and alternative scenarios. This can be achieved using a GIS-based model of the actual landscape and / or different development directions. The use of hypothetical scenarios in an explicit and rigorous framework allows the investigation of the role of individual-level factors in generating regional land use patterns, including land use policies. The results can then be compared using spatial statistics or landscape metrics to draw conclusions regarding the predicted influence of these factors on the concentration, fragmentation or other spatial dimensions of land use. Such a process could allow the incorporation of an additional component into UrbanSim, which would explore the impact of different land-use related policies on urban development.

This two-step approach has been used to model urbanization and sprawl (e.g., Carrion-Flores and Irwin 2004; Irwin and Bockstael 2002); the effects of land policies on urbanization patterns (e.g., Irwin et al. 2003; Irwin and Bockstael 2004; Langpap et al. 2008; Lewis et al. forthcoming; Newburn and Berck 2006); and the conversion of forest and agricultural land (e.g., Lewis and Plantinga 2007). Because of their ability to account for multiple sources of spatial heterogeneity, ecological features can be readily incorporated. In addition, the land use simulations can be linked with environmental impact models in which land use is the driver of environmental change to permit a fuller examination of the predicted effects of policy and other variables on quality of life and sustainability. Linking the microscopic land-use trends with models of externalities can be helpful in quantifying these externalities and developing appropriate metrics and indicators for the succinct assessment of the evolution of urban development. This approach has been used, for example, to study the impacts of conservation payments on landowner decisions and biodiversity loss (Lewis et al. 2008), the effectiveness of targeting strategies on land conservation (Newburn et al. 2006) and the effect of land use policies on watersheds (Langpap et al. 2008).

6 Recommendations for UrbanSim

UrbanSim is a simulation model developed since the late 1990's to simulate the spatial and temporal evolution of household location, job location, and real estate supply and prices using microsimulation to allow complete disaggregation in agents, locations, and the representation of time. This model has been applied to numerous cities in the United States and Europe, but until now has been connected to traditional four-step travel models that provide static equilibrium traffic assignment, usually for only a small number of time periods during the day.

One of the key components of UrbanSim is the use of land or real estate price data. These are applied in the model system as an indicator of the relative market valuations for attributes of housing, non-residential space, and location (Waddell et al., 2003). However, finding suitable data sources of real estate transaction prices and rents might be a challenge while setting up an UrbanSim application when transaction and rent price data from data suppliers are unavailable to researchers. As a minimum prerequisite for modelling purposes, sufficient information about a decent amount of properties is needed, including price (transaction cost or rent) and some explanatory locational variables the model system should be sensitive to in the application. This typically includes at least some kind of regional accessibility, proximities and neighbourhood characteristics. The hedonic approach is a suitable method to model price values for every cell or parcel in the UrbanSim application based on such a subsample. Moreover, the analysis exposes the implicit prices of housing and location characteristics.

The previous empirical analysis highlighted the complex spatial structure of housing markets. The need to explicitly address spatial effects is obvious since a failure to do so may result in loss of explanatory power and erroneous estimates. The GWR method might not always be the best choice, although an additional benefit of GWR is to provide a means to visualize the spatial structure of housing markets. Bitter et al. (2007) have highlighted that the GWR method is also helpful in situations where locational information is difficult to obtain or when knowledge of local submarkets is unavailable. However, locally correlated GWR coefficient estimates are a remaining problem as presented in the study of Löchl and Axhausen (2009). Methodological improvements of the GWR approach have been suggested in the literature, but are subject to an ongoing debate among econometricians. Simultaneous autoregressive approaches proved to be a reasonable alternative in the analysis, which can be implemented in UrbanSim more easily because of its structure of a single set of resulting parameters.

Technically, UrbanSim is somewhat flexible concerning variable selection for the various needed models, as long as the variables are available and constantly updated. This system can be particularly useful for applications of hedonic modelling to not only real estate price data, but also to rental data. Given that local rent and buying prices are not perfectly correlated, combining rent models with transaction price models would additionally incorporate the rent/purchasing decision of households. With regard to model methodology, it became obvious once again that hedonic housing models considering spatial effects are more reliable and are therefore suggested for further exploration in future applications of UrbanSim.

While models with solely spatial explanatory variables can be used in UrbanSim currently, it is hoped for the future that the simulation system becomes sensitive to structural variables of the building stock since this gives the opportunity to better reflect the local situation and to make the simulation more realistic.

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