MODELLING THE EVOLUTION OF OFFICE SPACE SUPPLY

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ABSTRACT

The spatial and temporal distribution of built-space supply plays an important role in shaping urban form and thus the general travel pattern in an urban area. Within an integrated framework, we are interested in modelling the decisions of a builder in terms of when, where, what type, and how much built-space to build. We present a discrete-continuous model formulation for the built-space supply decisions that are based on expected profit maximization. The framework is applied to estimate a model for supply of new office space in the Greater Toronto Area (GTA) for the duration of 1986 to 2006. To our knowledge, this work is the first that models the where, when, how much, and what type of office space to supply in a single framework at a fairly disaggregate spatial zoning system. The results indicates a risk taker behaviour on the builders' part, while market conditions and supply of resources (labour, construction cost etc.) are also found to be important factors in decision making.

Keywords: Choice Bundle, Discrete-Continuous model, Profit Maximization, Office Space Supply

1. INTRODUCTION

Understanding the factors that affect office location decisions plays an extremely important role in our greater understanding of travel behaviour in urban areas. Since office location strongly influences the spatial distribution of morning and afternoon peak-period travel, where firms choose to locate their offices greatly influences short-term individual-level decisions such as mode of transportation, and long-term household-level decisions such as residential location. Conditions in the office-space market affect firms' location and relocation decisions, and hence influence the general travel patterns in an urban area. Moreover, modelling and understanding the office space market in general and office space supply in particular has high economic benefits. The large capital requirements and long development periods make office investment riskier than other types of built space (Tse and Webb, 2003). Using office space models for forecasting and understanding of the working of building industry could decrease these investment risks.

Many aggregate (country, municipality or CBD-suburb level) office-space supply models can be found in the real estate and integrated land-use and transportation modelling literature. There are very few examples, however, of serious modelling efforts at the more disaggregate submarket level. This is in spite of the fact that there is strong evidence to suggest that sub-markets within a metropolitan area are temporally asynchronous from each other in terms of growth and are characterised by a high level of agglomeration by industry type. The availability of certain types of office floor-space has an effect on firm location and relocation decisions. Literature on modelling the quality of the new office-space supply is also relatively scarce.

Another important dimension in the modelling of office space market is that the location, quality, and quantity are interconnected decisions. At anytime, a location may have excess stock of one type, but is under stocked in other type. Similarly, some locations are suitable for only a few specific types of built-space while not suited for others. For instance, the downtown Toronto has a high concentration of Type A and B office space, but rarely Type C space[1]. The quantity that could be built at certain location is also influenced by the neighbourhood characteristics (zoning by-laws, technological constraints). In the real estate literature, mostly the quantity is modelled at a very high level of aggregation. Operation integrated urban systems modelling frameworks, models these decisions at a lower level of aggregation (census tracts, small grids), but do not treat them as related decisions within a single framework. Instead the individual dimensions are modelled separately, and then some kind of simulation or rule based allocation is used to simulate the built-space evolution. In ILUTE for instance, Miller *et al.* (2010) used a separate model for the location choice probabilities for each type of dwelling and another model for the quantities to produce in the study area. These two models are then used in a Monte Carlo simulation to allocate the new stock to individual locations.

[1] The BOMA classification of office space type is explained in the Data Description section

The building industry generally, and in the case of the GTA in particular, is an oligopoly with very few firms and these firms move in and out of the market very frequently with the boom and bust cycles of the built-space market (Buzzelli and Harris, 2003). It is important to bring in the builders' behaviour within the decision modelling framework. Their attitude towards risk taking and the expectation of profit might vary among individual builders. A modelling framework that can incorporate these issues is currently missing in the literature.

Farooq *et al.* (2010) reported a high spatial variation in the rent of office-space. It was also reported that there were not only inter-cluster variation, but also intra-cluster variations. Under the profit maximizing assumption for the builder of space, this variation in rent will influence the decision to select the best location for the new office space.

In this paper, we propose a novel approach that explicitly ties the location, quality, and quantity decisions for new built-space supply together into a single dynamic framework based on expected profit maximization. As an application, we then estimate a model for new office space development at a fairly disaggregate spatial resolution. This paper treats the problem of building new built space as a situation in which a builder as the decision maker is faced with the decision of selection of a choice bundle and the associated quantities, while optimizing his expected profits. By doing so, we were able to incorporate not only the relation between various decision dimensions, but also captured the behaviour of the decision maker (builder) and influence of changing sub-market conditions and regional economy.

There are various large scale demand models available in the choice modelling literature that model the choice of a discrete bundle of goods (e.g., types of activities in which to engage) and an associated continuous quantity (e.g., how much time to allocate to each activity) (Bhat, 2005, 2008; Habib *et al.*, 2008; Habib and Miller, 2009, Kim *et al.* 2002). These models predominantly use random utility modelling framework that assumes that the consumer is a utility maximizer. In terms of mathematical model formulation, the assumption of profit maximization by the producer (specifically builders in our case) is analogous to the utility maximization assumption in the large scale demand models. Moreover, profit from manufacturing a product is a more quantifiable concept than utility. Thus we can pose the problem of expected profit maximization in the same way as RUM does for the utility maximization of consumers faced with choice bundle selection and the associated quantities. This lets us use the same construct of optimization conditions (Kuhn-Tucker conditions) that is frequently used in the large scale demand models.

The rest of the paper is organized as follows: we first present a brief review of the state of research in modelling built space evolution, in general, and office space supply in particular, that is available in the real estate and integrated urban systems modelling literature. We then introduce the model specification and estimation procedure. This is followed by data description and presentation of estimated model for the office space supply in the Greater Toronto Area. In the last section, we discuss the conclusions and future research direction.

2. LITERATURE REVIEW

In the real estate literature on office-space markets, we see that most of the focus is on the analysis of valuation, absorption rates, and vacancy rates of the office space at the country or metropolitan area level. Another dimension that is frequently investigated in fair detail is the construction cycles involved in the office market. The studies that have addressed the supply side have mostly used a long-run equilibrium model in which supply is the result of a stock adjustment process to the time-lagged office sector employment growth and difference in the effective rent level from the natural level. There is, however, rarely any work available on the location choice decisions for new space at submarket or lower aggregation levels within a metropolitan area. In the integrated urban systems modelling literature, the decisions are modelled at higher aggregation, but the models there are lacking in terms of completeness and sound economic foundations.

Rosen (1984) was the first to introduce a statistical modelling framework for the office space market. It was reported that the existing methodology to model the office space market was only rate-based and deemed it 'inadequate' for investment and development decisions. The model forecasted vacancy rate, rent, and quantity of new floor space using separate linear regression equations. The model was estimated for San Francisco based on data from 1961 to 1983. Supply was assumed to be a function of vacancy rate, rent, interest rates, construction costs, and tax laws affecting commercial real estate. The model lacked any spatial representation and there was no differentiation in the quality of the office space. The model also lacked dynamics in that there was no investigation of lagged effects.

Howarth and Malizia (1998) proposed strategies to improve our understanding of the office space market. It was recommended that more rigorous economic methods should be employed; the spatial variation should be incorporated in the model; and the long-term forecasting should incorporate the lag effects to capture the dynamics and the business and building cycles. It was also recommended to use changes in credit availability, trends in employment types, and that the regulatory environment should be taken into consideration. Product differentiation in terms of existing office space should be considered in decisions of differentiated new office space. The differences in the submarket dynamics should be incorporated in the location choice decision of new space. Risk and uncertainty faced by the builders should also be taken into account. This study provides a significant contribution in terms of exploration of the important explanatory measures of the office market dynamics. It didn't, however, indicate the probable statistical and econometric modelling frameworks that could be used to model the office space market.

The Real Estate Econometric Forecast Model (REEFM), proposed by Viezer (1999) pooled the office space market data for fifty-one metropolitan areas in United States over the years 1985–1996. The REEFM framework consisted of six stochastic equations for forecasting occupancy, real rents, capitalization rates, market value per square foot, net change in stock, and real construction costs. Individual models were estimated using linear regression. The dynamics was captured using lagged variables. It was reported that inflation rates and interest rates were insignificant in explaining the variation. Due to the highly aggregate nature of the REEFM model and very few explanatory variables, it was limited in use for behavioural and spatially detailed analysis of the office market.

A comprehensive Walrassian Equilibrium based dynamic adjustment model of office market can be found in Hendershott *et al.* (1999). The relationship between supply and demand was used to link new construction, absorption, vacancies, and rent to employment growth and interest rates. Using structural equations, the model was estimated for the City of London for the twenty year duration of 1977 to 1996. As reported in previous studies, the difference between actual and natural vacancy rates affected the rents. The construction responded to lagged changes in rent. Absorption rate was negatively related to rent while positively related to growth of financial services employment growth. While the proposed model was economically consistent and a complete model of office space market, it lacked submarket dynamics and spatial variation in the supply and rent levels.

Poterba's two-equation asset-market approach to modelling the housing market was adopted by Nanthakumaran *et al.* (2000) to model the office space market of the United Kingdom. This long-run equilibrium model treated supply as the “automatic stabiliser” to the changing rent level, resulting from changing demand. The parameter estimated for the capital value variable in the flow-supply equation was interpreted as price elasticity of supply. Other studies that used some variation of two-equations long-run equilibrium models for office space market include: Lentz and Tse (1977), McDonald (2000), Tse and Webb (2003), and Ho (2005).

Fürst (2006) used a three-stage simultaneous equation model to analyse the office space market of Manhattan, New York. This study modelled the absorption rate, rent, and new supply of office space. Fuerst also investigated the existence of submarkets and their working as asynchronous autonomous economic units. This work is unique in the sense that it is the only study found in the real estate literature where the office market is analysed at a more disaggregate level than the generally-used metropolitan area level. The study divided Manhattan into 15 submarkets.

In UrbanSim, developed parcels or gridcells are used as the basic unit of built space (Waddell *et al.*, 2003, 2008). The built space development is modelled as a discrete choice decision in which the land owner of a site (parcel/gridcell) decides on changing the state of site. This decision is modelled as a multinomial logit model. Land owners are faced with twenty four choices that are a combination of built space type (residential, mixed use, commercial, industrial, government, vacant, developable, and undevelopable) and associated range of development intensity, in term of number of units or floor space (Waddell and Ulfarsson, 2003, 2004). In the simulation, these probabilities are used in combination with random draws to update the yearly built space stock. The simplicity of this approach makes it easier to operationalize in the urban simulation context, but at the same time, makes it very limited in terms of its ability to capture the underlying behaviour and structure of built space evolution. In most of the cases, the development choices are rarely independent of one another. Haider and

Miller (2004) reported the phenomena of spatial inertia in which the existence of one type attracted the intensification of the same type of development at a location. The associated quantity for each type of development is also overly simplified by limited categorization in term of choices. The quantity of development is a continuous dimension which is influenced by the market conditions, built capacity and various other factors. Moreover, the assumption of homogenous decision maker is not behaviourally consistent. The landowner in downtown will have a different perspective of choices then a landowner in the suburbs.

MUSSA (Martínez, 1996), like UrbanSim, assumes a homogenous builder, but its supply subsystem has a better economic foundation. The builder is faced with the decision of selecting the combination of built-space type and location to maximize his profit (Martínez and Hurtubia, 2006). The probability of selection for each type and location combination is modelled using a multinomial logit model. MUSSA defines a linear profit function based on expected revenue and production cost function (Martínez and Henríquez, 2007). Using a strong equilibrium assumption, the share of each type of built-space is determined from the total difference in demand and available stock and the probabilities of selection. This model is only implemented for residential built-space. By imposing strong equilibrium and IIA assumptions, the solution for market clearing in the simulation becomes more tractable, but is not very representative of the actual built space markets.

PECAS simulates the evolution of spatial economics using an aggregate zoning system under a strong equilibrium structure (Hunt and Abraham, 2003). The land use development for each year is determined in a multidimensional input-output table. These totals are then disaggregated using a set of logit models for each type of built space. Like MUSSA, PECAS also first determines the aggregate change in the built-space from a strong equilibrium assumption, but unlike MUSSA, here the representation of different types of built-space is more complete and there is some representation (at aggregate level) of the interplay between the share of each type of built-space.

The current operational version of ILUTE (Miller *et al.*, 2010), like the above mentioned urban simulations, first determines the total built-space and then allocates it to individual locations. But it doesn’t impose any equilibrium assumption on the market to determine the aggregate totals. Instead, the aggregate supply for each type of built-space is determined using a dynamic econometric model that represents the builders’ behaviour in different market conditions and the natural built cycles of the building industry (Farooq et al., 2008). Separate logit models for each type of built-space are estimated (Haider and Miller, 2004) that determine the probability of selection of a location by a type of built-space. For each simulation year, the totals are computed and probabilities are updated. A Monte Carlo simulation is then used to assign location to the individual unit of built-space (Farooq *et al.*, 2008). Currently, only the new housing supply model is operational. ILUTE’s land use evolution is economically more consistent as it incorporates various behavioural and market dimensions, including: risk taking attitude of the builders, spatial inertia, and lagged market conditions driving the new supply.

Here we only have discussed the more recent, operational, and widely used urban systems model. A more detailed review of urban space evolution within various integrated urban systems modelling frameworks is discussed in Wegner (1995), Timmermans (2003), Hunt (2005), and Miller (2006).

In general it could be concluded that there a lack of a single large-scale built-space supply modelling framework that is spatially disaggregate, econometrically consistent, captures decision makers' behaviour and the associated heterogeneity, and is able to capture the interplay between various dimensions of decision making (where, when, how much, what type). Moreover, there is also a need of modelling framework that moves away from the conventional equilibrium based aggregate approach and towards a dynamic disequilibrium microsimulation approach which is more representative of the actual built space markets.

3. MODEL STRUCTURE

The decision makers here are a set of building construction firms that are active in the urban area at certain time "*t*". They are faced with the decision of choosing the quantity of different types of built-space to be built, and the location where to build them. It is assumed that builders take these decisions so as to maximize their expected profit. Profit is determined by the difference of expected revenue and cost.

3.1. Theoretical Framework

The expected profit (Π) of a building construction firm, from *N* differentiated products that it can decide to build at certain decision point "*t*" could be represented by:

$$\Pi = \sum_{i=1}^{N} \frac{\gamma_i}{\alpha_i} \left\{ (f^r(X_i^r) - f^c(X_i^c))^{\vartheta} \left(\left(\frac{q_i}{\gamma_i} + 1 \right)^{\alpha_i} - 1 \right) \right\} + f^z(z) \quad (1)$$

Where:

$f^r(X_i^r)$ represents the expected unit revenue from selling product *i*

X_i^r is a vector of variables related to product attributes, location features, and built-space market conditions that influence the revenue

$f^c(X_i^c)$ represents the expected unit cost in building product *i*

X_i^c is a vector of variables related to product attributes, location, state of regional economy, and conditions in various associated markets (labour, material *etc.*) that influence the cost

q_i is the quantity of product *i* that is decided to be built

The formulation here treats the share of profit from individual type of floor-space *i* in the same manner as Bhat (2005) and Kim *et al.* (2002) treated the share of individual choices in their utility function for large scale demand systems. The translation parameter, γ_i makes sure that there a possibility of zero production of any given type of floor-space. The parameter, α_i represents the scale parameter and ϑ here represents the risk behaviour of the builder and the structure of space market in the region. In the simplest case ϑ could be a constant parameter, but in a more elaborate case it will be parameterized based on combination of the builder's and the market's characteristics.

3.2. Concept of Hicksian Product

As the builder has other options of investments besides the set of built-space types that we are interested to model, we introduce the concept of Hicksian/composite product in our general formulation, $f^z(z)$. Profit from the Hicksian product in (1) represents the expected loss that is avoided at a given interest rate at the decision time, by not building the floor-space that could have been built under the technological/zoning constraint. For this, we use a separate profit generation function similar to the one used by von Haefen and Phaneuf (2004), and Habib and Miller (2009) for the composite activity.

If we assume that the revenue and cost functions are linear in parameters and the modeller's inability to perfectly observe builder's expected profit is represented by the error term ε_i, then (1) can be rewritten as:

$$\Pi = \sum_{i=1}^{N} \frac{\gamma_i}{\alpha_i} \left\{ (\beta_i^r X_i^r - \beta_i^c X_i^c)^{\vartheta} e^{\vartheta \varepsilon_i} \left(\left(\frac{q_i}{\gamma_i} + 1 \right)^{\alpha_i} - 1 \right) \right\} + \frac{1}{1-e^{\rho}} z^{(1-e^{\rho})} \qquad (2)$$

The form of the profit function for composite product here guarantees a positive profit from a nonzero composite product (Habib and Miller, 2009). Note that there is no error term associated with the profit from the Hicksian product. The rational here is that the error terms from rest of the products in (2) are the differences from the Hicksian part.

3.3. Estimation Problem

Using (2) our optimization problem can be defined as:

Maximize

$$\Pi = \sum_{i=1}^{N} \frac{\gamma_i}{\alpha_i} \left\{ (\beta_i^r X_i^r - \beta_i^c X_i^c)^{\vartheta} e^{\vartheta \varepsilon_i} \left(\left(\frac{q_i}{\gamma_i} + 1 \right)^{\alpha_i} - 1 \right) \right\} + \frac{1}{1-e^{\rho}} z^{(1-e^{\rho})} \qquad (3a)$$

Subject to

$$\sum_{i=1}^{N} q_i + z = K_T \qquad (3b)$$

$q_i \geq 0$ *i* = 1, 2,…, N (3c)

$z > 0$ (3d)

K_T is the maximum possible space that could be built in the time interval under zoning and technological constraints

The Lagrangian function for the problem in (3) becomes:

$$\mathcal{L} = \left[\sum_{i=1}^{N} \frac{\gamma_i}{\alpha_i} \left\{ (\beta_i^r X_i^r - \beta_i^c X_i^c)^{\vartheta} e^{\vartheta \varepsilon_i} \left(\left(\frac{q_i}{\gamma_i} + 1 \right)^{\alpha_i} - 1 \right) \right\} + \frac{1}{1-e^{\rho}} z^{(1-e^{\rho})} \right]$$

$$-\lambda \left[\sum_{i=1}^{N} q_i + z - K_T \right] \quad (4)$$

λ=Lagrangian multiplier

The Khun-Tucker (KT) first order conditions for optimal allocations here will be:

$$\frac{\partial \Pi}{\partial q_i} - \lambda \leq 0 \qquad \text{for } i = 1,2,\ldots n \quad (5a)$$

&

$$\frac{\partial \Pi}{\partial z} - \lambda \geq 0 \quad (5b)$$

(5a) ensures that for the selected levels of the product bundle, any further increase in the quantity of product *i* will have no further positive effect on the total profit. (5b) ensures that the quantity of the composite product (not investing) is at the level where it has no negative effect on the profit.

Form (5a) and (5b)

$$\frac{\partial \Pi}{\partial q_i} \leq \frac{\partial \Pi}{\partial z} \qquad \text{for } i = 1,2,\ldots n$$

$$(\beta_i^r X_i^r - \beta_i^c X_i^c)^{\vartheta} e^{\vartheta \varepsilon_i} \left(\frac{q_i}{\gamma_i} + 1 \right)^{(\alpha_i - 1)} \leq z^{-e^{\rho}} \quad (6)$$

We can show that $\frac{\partial^2 \Pi}{\partial q_i \, \partial \varepsilon_i}$ is a *i*×*i* non-singular matrix and $\frac{\partial^2 \Pi}{\partial z \, \partial \varepsilon_i}$ is a zero valued vector. Thus using explicit function theorem (Habib and Miller, 2009), we can express the error term as:

$$\varepsilon_i \leq g_i(\beta_i^r, X_i^r, \beta_i^c, X_i^c, q_i, \vartheta, \gamma_i, \alpha_i, \rho)$$

$$\varepsilon_i \leq \frac{1}{\vartheta} \left[(1 - \alpha_i) log \left(\frac{q_i}{\gamma_i} + 1 \right) - \vartheta log(\beta_i^r X_i^r - \beta_i^c X_i^c) - e^{\rho} log \left(K_T - \sum_{j=1}^{n} q_j \right) \right], \forall i \quad (7)$$

3.4. Econometric Model Structure

If the joint probability density function, $f(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$ of the error terms is known then the probability associated with the quantities of a certain bundle of product that is selected by the building firm for construction is given by:

$$P(Q) = \int_{-\infty}^{g_{m+1}} \ldots \int_{-\infty}^{g_n} f(g_1, g_2, \ldots g_m, \varepsilon_{m+1}, \ldots, \varepsilon_n) |J| d\varepsilon_{m+1} \ldots d\varepsilon_n \quad (8)$$

Where

$Q = [q_1^*, q_2^*, \dots, q_m^*, 0,0 \dots 0]$ is the vector of quantities of each type selected by the builder to build.

$|J|$ is the determinant of *m*×*m* Jacobian matrix, whose individual elements are defined by: $\partial\varepsilon_i/\partial q_i$ (Kim *et al.,* 2002; Bhat, 2005; and Habib and Miller, 2009).

$$|J| = \prod_{i=1}^{N} \frac{1}{\vartheta}\left[\frac{(1-\alpha_i)}{(q_i+\gamma_i)} + \frac{e^{\rho}}{(K_T+q_i)}\right]$$

Most of the discrete-continuous large scale demand models including (Bhat, 2005, 2008; Habib, 2008, 2009; Pinjari and Bhat, 2009), have assumed the error terms to be IID with Type I extreme value distribution. This assumption simplifies (8) and gives a closed form solution for the calculation of choice probabilities. The estimation of model parameters also becomes computationally manageable in cases where the size of choice-set is large.

However, we think that this assumption is not valid in the case of new built-space. Most of the time, the types of the space that are built by the builder are highly correlated to each other. Builders are localized in terms of their operations (Buzzelli and Harris, 2003). Moreover, builders and their associated contractors/sub-contractors are specialized in building certain types of space only. The builder that builds detached dwellings is more likely to build semi-detached and attached dwellings then building high rise apartment building. The location case is similar: A zone (business node) that primarily has Type-A office space will unlikely to get built an inferior, Type-C office space. A more appropriate assumption, therefore is that the error terms are jointly normally distributed with a mean of 0 and covariance matrix of Ω. Hence:

$$P(Q) = \frac{1}{(2\pi)^{n/2}|\Omega|^{1/2}} \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \int_{-\infty}^{g_{m+1}} \dots \int_{-\infty}^{g_n} exp\left(-\frac{1}{2}\mathrm{E}'\Omega^{-1}\mathrm{E}\right)|J| d\varepsilon_1 \dots d\varepsilon_n \qquad (10)$$

Where $\mathrm{E} = [\varepsilon_1, \dots, \varepsilon_m, \varepsilon_{m+1} \dots \varepsilon_n]$

Equation (10) involves computing an (n–m) dimensional integral of the function that will have a high computational cost associated for large choice sets. In the case of built-space however, the builder is faced with very few choices (e.g. 3 in case of office space and 4 to 5 in the case of housing). Thus the evaluation of (10) remains computationally viable.

The resulting likelihood function from (10) for all the builders thus becomes:

$$L(Q|\beta_i^r, \beta_i^c, \vartheta, \gamma_i, \alpha_i, \rho, \Omega) =$$

$$\prod_{b=1}^{B} \frac{1}{(2\pi)^{n/2}|\Omega|^{1/2}} \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \int_{-\infty}^{g_{k+1}} \dots \int_{-\infty}^{g_n} exp\left(-\frac{1}{2}\mathrm{E}'\Omega^{-1}\mathrm{E}\right)|J| d\varepsilon_1 \dots d\varepsilon_n \qquad (11)$$

3.5. Parameter Estimation

The likelihood maximization based parameter estimation process involves two basic steps, the generation/evaluation of the candidate parameter values, and the evaluation of the likelihood function. In the logit-based conventional discrete choice models, the likelihood function has a closed form, so the evaluation of likelihood, gradient, and hessian of the function is trivial. Gradient based search methods like Newton-Raphson (NS), Broyden–Fletcher–Goldfarb–

Shanno (BFGS), Berndt–Hall–Hall–Hausman (BHHH), David–Fletcher–Powell, Polak–Ribiere conjugate gradient, and simulated annealing (Ben-Akiva and Lerman, 1985; Washington *et al.*, 2003; and Train, 2009) are used to estimate the parameters and their statistical properties.

In case of parameter estimation for probit models, the evaluation of the likelihood function becomes non-trivial, because of the involvement of the multi-dimensional integral. In case of the classic probit model the multidimensional integral involved in the likelihood function is approximated using one of several methods: numerical integration, tabulation, numerical approximation, and Monte Carlo simulation (Sheffi *et al.* 1982). MC Simulation is most widely used in which the likelihood function is evaluated using various simulation techniques like Accept–Reject (AR), smoothed AR, and Geweke–Hajivassiliou–Keane (GHK). The resulting approximate log-likelihood function is called as Simulated Log-Likelihood (SLL). The gradient of the function required for optimization problem can be approximated by dividing the change in SLL by the change in the parameter values (Train, 2009). Bolduc (1999) suggested a simulation based procedure for the analytical solution of the gradient in the GHK-simulator. Another approach for the probit model parameter estimation is the Bayesian based Markov Chain Monte Carlo (MCMC) simulation technique that avoids the direct evaluation of the likelihood function. Instead it derives the posterior distributions from the prior belief and the data. The moments and other statistical properties are derived by sampling from the posterior distribution using simulation techniques like Metropolis-Hasting (M-H), Adaptive M-H, and Gibb's sampler (Kim *et al.,* 1999; Kim *et al.*, 2002; and Train, 2009). Kim *et al.* (1999) used Markov chain Gibb's sampler to draw directly from posterior distribution and performed finite sample likelihood inference.

Bhat (2001) used a quasi-random Monte Carlo simulation technique to estimate parameters for a mixed logit model. A Halton sequence for each dimension of the integral in the likelihood function was drawn by paring k-sequences. The sequence ensured that the whole region under the integral is uniformly covered. The cyclic nature of the Halton sequence results in correlation issues. To avoid this problem, a scrambling technique was used, but this adds an exponential overhead with each dimension, so as to produce a "good" permutation (Hess and Polak, 2003). It is however not very clear what maximization criteria were used and how the approximate gradient/scores and hessian were calculated. It is also not very clear how the local maxima were avoided in the estimation process.

Train (2009) outlined a Bayesian based MCMC method for the parameter estimation in mixed logit models. Bayesian methods relax the constraint of maximizing the simulated-likelihood function which could be complicated in cases where there might be various local maxima and thus might result in identification problems. In Bayesian methods, the prior distribution plays an important role and is assumed to be near the values that globally maximizes the likelihood function. Bayesian methods are also superior from standard simulated-likelihood maximization methods in terms of consistency and efficiency.

In the case of the large-scale demand model estimation, Monte Carlo simulation, quasi-Monte Carlo simulation, and Markov Chain based Monte Carlo simulation methods are commonly used (Bhat, 2001, Kim *et al.,* 2002, Habib, 2009). Bhat (2005) and Habib and Miller (2009) used the quasi-random Monte Carlo simulation procedure outlined in Bhat (2001) for the likelihood function that had extreme valued error terms and normally distributed parameters.

Kim *et al.* (2002), von Haefen and Phaneuf (2004), and Habib (2009), used Markov Chain Monte Carlo (MCMC) based on Metropolis-Hasting method to estimate their parameters from the likelihood function involving the normal distribution. The likelihood functions in the cases of von Haefen and Phaneuf (2004), and Habib (2009) had extreme valued error terms and normally distributed correlated parameters. Kim *et al.* on the other hand had a normally distributed correlated error terms as well. Kim *et al.* (2002) used GHK simulator to evaluate the multidimensional integral involved within the log likelihood function. The statistical properties of the estimated parameters were computed using Gibb's sampling.

The likelihood function in equation (11) also involves correlated error terms that are normally distributed. In the estimation of parameters from this function, we also decided to use Bayesian MCMC with Gibb's sampling approach. For the evaluation of the multidimensional normal probability function involved in equation (11), we used a technique based on randomized lattice rules that seeks to fill the hyper integration space evenly using a deterministic process. In principle, these lattice rules construct regular patterns, such that the projections of the integration points onto each axis produce an equidistant subdivision of the axis (Genz, & Bretz 2002, 2009). Robust integration error bounds are obtained by introducing additional shifts of the entire set of integration nodes in random directions. Since this additional randomization step is only performed to introduce a robust Monte Carlo error bound, 10 simulation runs are usually sufficient. We preferred this method from the more widely used Halton sequence based simulation procedure, because it has been numerically proven to outperform Halton or Sobel sequences in terms of efficiency and doesn't suffer from the correlation issues (Lai, 2009).

3.6. Estimation Procedure

The procedure that we used to estimate parameters in equation (11) is as follows:

Let the parameters in the likelihood function are represented as $\zeta_b = (\beta_b^r, \beta_b^c, \gamma, \rho)$

1. Initialize $\zeta_b, \alpha, \bar{\zeta}_b, \Omega_\zeta$

2. Generate $\{\zeta_b, b = 1, \dots, B\}$ from

$$\psi(\zeta_b|\{q_{bt}, t = 1, \dots, T\}, \alpha, \bar{\zeta}_b, \Omega_\zeta\} \propto det\left|\Omega_\zeta\right|^{\frac{1}{2}} exp[-\frac{1}{2}(\zeta_b - \bar{\zeta}_b)'\Omega_\zeta^{-1}(\zeta_b - \bar{\zeta}_b)]\prod_t^T L_{bt}$$

Where

ψ is a $N \times 1$ vector representing all the alternatives

t represents the decision occasion

Generate a random number $\tau_\psi -\succ N(0,0.0025)$, then the candidate value of ζ_b for iteration *k* will be:

$$\zeta_b^k = \zeta_b^{(k-1)} + \tau_\psi$$

Accept this new value with the probability:

$$min\left[\frac{exp[-\frac{1}{2}(\zeta_b^{(k)} - \bar{\zeta}^{(k)})'\Omega_\zeta(\zeta_b^{(k)} - \bar{\zeta}^{(k)})]\prod_t^T L_{bt}^k}{exp[-\frac{1}{2}(\zeta_b^{(k-1)} - \bar{\zeta}^{(k-1)})'\Omega_\zeta(\zeta_b^{(k-1)} - \bar{\zeta}^{(k-1)})]\prod_t^T L_{bt}^{k-1}}, 1\right]$$

3. Generate $\bar{\zeta}_b$ from

$$\psi\left(\bar{\zeta}_b \middle| \{\zeta_b, b = 1, \ldots, B\}, \Omega_\zeta\right\} = N\left(\frac{\sum_1^B \zeta_b}{B}, \frac{\Omega_\zeta}{B}\right)$$

4. Generate Ω_ζ from

$$\psi\left(\Omega_\zeta \middle| \{\zeta_b, b = 1, \ldots, B\}, \bar{\zeta}_b, \right\} \propto Inverted\ Wishart\left(d_0 + B.D_0 + \sum_1^B (\zeta_b - \bar{\zeta}_b)'(\zeta_b - \bar{\zeta}_b)\right)$$

Where d_0 is the prior degrees of freedom and D_0 is the sum of squares of Ω_ζ

5. Generate α from

$$\psi(\alpha|\{q_{bt}, b = 1 \ldots B\ and\ t = 1, \ldots, T\}, \{\zeta_b, b = 1 \ldots B\}, \bar{\alpha}_0, \Sigma_0\} \propto det|\Omega_0|^{\frac{1}{2}} \exp[-\frac{1}{2}(\alpha - \bar{\alpha}_0)'\Omega_0^{-1}(\alpha - \bar{\alpha}_0)] \prod_b^B \prod_t^T L_{bt}$$

$\bar{\alpha}_0$and Ω_0 are the prior parameters

Generate a random number $\tau_\alpha -\succ N(0,0.01)$, then the candidate value of $\alpha$for iteration *k* will be:

$$\alpha^k = \alpha^{(k-1)} + \tau_\alpha$$

Accept this new value with the probability:

$$min\left[\frac{exp[-\frac{1}{2}(\alpha^k - \bar{\alpha}_0)'\Omega_0^{-1}(\alpha^k - \bar{\alpha}_0)]\prod_b^B \prod_t^T L_{bt}^k}{exp[-\frac{1}{2}(\alpha^{(k-1)} - \bar{\alpha}_0)'\Omega_0^{-1}(\alpha^{(k-1)} - \bar{\alpha}_0)]\prod_b^B \prod_t^T L_{bt}^{(k-1)}}, 1\right]$$

6. Generate ϑ from

$$\psi(\vartheta|\{q_{bt}, b = 1 \ldots B\ and\ t = 1, \ldots, T\}, \{\zeta_b, b = 1 \ldots B\}, \bar{\vartheta}_0, \Sigma_0\} \propto det|\Omega_{0'}|^{\frac{1}{2}} exp[-\frac{1}{2}(\vartheta - \bar{\vartheta}_0)'\Omega_0^{-1}(\vartheta - \bar{\vartheta}_0)] \prod_b^B \prod_t^T L_{bt}$$

$\bar{\vartheta}_0$and $\Omega_{0'}$ are the prior parameters

Generate a random number $\tau_\vartheta -> N(0,0.01)$, then the candidate value of ϑ for iteration *k* will be:

$$\vartheta^k = \vartheta^{(k-1)} + \tau_\vartheta$$

Accept this new value with the probability:

$$min\left[\frac{exp[-\frac{1}{2}(\vartheta^k - \bar{\vartheta}_0)'{\Omega_0}^{-1}(\vartheta^k - \bar{\vartheta}_0)]\prod_b^B \prod_t^T L_{bt}^k}{exp[-\frac{1}{2}(\vartheta^{(k-1)} - \bar{\vartheta}_0)'{\Omega_0}^{-1}(\vartheta^{(k-1)} - \bar{\vartheta}_0)]\prod_b^B \prod_t^T L_{bt}^{(k-1)}}, 1\right]$$

7. Iterate back to step 1

The simulation has to be run for a sufficient numbers of iterations before drawing inferences. It is suggested that around 25,000 iterations should be enough for the burn-in (Kim *et al.,* 2002; von Haefen and Phaneuf, 2004; Train, 2009; and Habib, 2009). Gibb's sampling is then done to construct the distributional summary statistics for $\zeta_b, \bar{\zeta}_b, \Omega_\zeta, \alpha, \vartheta$. Gibb's sampling induces a serial correlation in the parameters. To avoid this correlation, it is also suggested that every 10th iteration is used in the simulation after warm up (von Haefen and Phaneuf, 2004; and Train, 2009).

3.7. Identification Problem

Parameter estimation from the data based on the underlying model structure is fundamentally an optimization problem that may have a non-unique solution set. The identification problem is the problem of determining what conclusions drawn from the data about a model parameters are feasible (Manski, 1995; Train, 2009). Walker *et al.* (2007) defined the identification problem as the problem of determining the set of restrictions to impose in order to obtain a unique vector of consistent parameter estimates.

MCMC with Gibb's sampling does a better job in regards to the identification problem, as compared to the quasi-MC methods because of its ability to first of all base the search on a prior distribution. We can thus control the direction of search based on our prior beliefs about the solution. Secondly, the Metropolis-Hasting based search process itself is more controlled and directed. Lastly, the statistical properties of the solution are draws from the posterior distribution of the parameters that are not just based on the likelihood values from the data, but also on the prior distribution and the search process.

4. DATA DESCRIPTION

The modelling framework outlined in the previous section was applied to estimate a model for new office space supply. We used the office space supply data for the Greater Toronto Area from 1986 to 2005 to estimate the model. The dataset includes all the buildings with an office-space of 20,000 sq. ft. or more. Office buildings are distributed across the study area in various identifiable clusters. Based on their geographic concentration in various regions, the study area was divided into 36 unique business nodes (Figure1) in the survey that generated the dataset.

FIGURE 1: Study area and approximate location of the business nodes.

Table 1 lists the business nodes used in this study, their share of office space in year 2005, and the average gross prices for three different types of office space that is available in the GTA market. We see that a high concentration of office space is in the downtown Toronto followed by the regional centres of Mississauga, North York, and airport area.

Our dataset classified office space into four standard types (A, B, C, and G), as defined by the Building Owners and Managers Association (BOMA). This is a subjective classification that uses a combination of factors including rent, building finishes, system standards and efficiency, building amenities, location/accessibility, and market perception (BOMA, 2009). Type A buildings have high quality standard finishes, state of the art systems, exceptional accessibility and a definite market presence. Downtown Toronto and regional centres are dominated by Type A office space. Type A space has higher than average rents for the area. Type B office space has fair to good facilities and infrastructure, while Type C buildings are only providing a functional space at a lower rent level compared to the area average (BOMA, 2009; AtlusInSite, 2009). Type G buildings are government owned buildings and were not used in the model developed in this paper.

Yearly average gross rent rate and vacancy rates were generated from the dataset to incorporate the effect of market conditions on the supply of new office space. Under the homogenous builder assumption, we created 720 observations of types of space built and the associated quantity, from the combination of 36 nodes and 20 years. It was assumed that the construction time of the project was 1 year. For the market indicator variables, a lag of 1 year was used.

Statistics Canada was used as the main source for the data related to hourly wage rates of construction worker and number of construction workers in the labour force for each year in the study duration. The construction cost estimates data for office space was collected from the yearly per square ft. estimates published by RSMean Inc. for the office space construction in Toronto region. These estimates are the major sources used by the builders to develop construction estimates for the construction projects.

Table 2 provides the summary statistics and description of the dependent and explanatory variables used in the estimated model.

TABLE 1 Business Nodes in the study area

No.	Node	% of Total Office Area in the Node	Avg. Number of Floors	Avg. Gross Rent A	Avg. Gross Rent B	Avg. Gross Rent C
1	Financial Core	20.07	18	43.6	32.8	26.9
2	Downtown North	9.13	11	32.9	26.3	21.8
3	Downtown West	6.62	7	28.5	20.2	22.6
4	Downtown East	1.92	5	28.5	20.2	22.6
5	Downtown South	1.45	10	37.2	29.8	18.3
6	Bloor & Yonge	5.39	10	30.7	24.7	24.1
7	St. Clair & Yonge	1.86	11	30.1	25.6	21.8
8	Eglinton & Yonge	2.95	8	26.8	23.22	22.5
9	North Yonge	5.46	10	30.7	26.1	21.7
10	Heartland	1.92	4	30.7	26.1	21.7
11	Yorkdale	1.57	4	19.2	17.0	18.3
12	Downsview	0.47	4	19.2	17.0	18.3
13	Dufferin and Finch	0.37	6	19.3	17.0	18.3
14	Vaughan	0.89	5	18.9	17.5	15.7
15	Bloor & Islington	0.94	9	28.6	22.3	18.5
16	King & Dufferin	1.76	3	28.6	22.3	18.5
17	Sheridan	0.68	2	25.1	21.0	17.8
18	Airport Dispersed	2.82	4	21.4	19.9	15.8
19	Highway 427 Corridor	1.53	6	22.5	17.4	15.3
20	Airport Corporate Centre	3.71	4	24.3	19.4	25.8
21	Etobicoke Dispersed	0.40	5	21.0	17.0	15.7
22	Mississauga Dispersed	0.50	4	21.0	17.0	15.7
23	Mississauga City Centre	2.08	9	28.7	26.8	17.4
24	Cooksville	0.45	6	22.0	18.4	15.5
25	Brampton	1.26	4	20.4	15.4	-
26	Meadowvale	2.47	3	25.5	20.5	18.5
27	Oakville	1.09	4	22.5	19.8	20.8
28	Burlington	2.00	3	22.5	17.4	15.3
29	Don Mills and Eglinton	2.58	6	22.6	19.0	16.4
30	Duncan Mill	1.14	6	18.5	18.1	14.3
31	Consumers Road	2.76	7	27.1	21.4	20.3
32	Scarborough	2.93	5	25.9	19.1	14.7
33	Markham & Pickering	0.48	3	26.0	19.1	14.7
34	Highway 404 & Highway 7	4.81	4	22.4	16.8	-
35	Highway 404 & Steeles	3.37	4	19.4	16.9	12.7
36	Richmond Hill	0.16	5	23.8	20.6	17.6
		100.00	6	25.5	20.8	18.7

TABLE 2 Summary Statistics

Variable	Description	Mean/Proportion	Std. Dev.	Min.	Max.
Dependent Variable					
Supply_A	Supply of Type A (1000 sq. ft.)	76.57	226.78	0	2601.88
Supply_B	Supply of Type B (1000 sq. ft.)	11.92	44.99	0	525.00
Supply_C	Supply of Type C (1000 sq. ft.)	2.14	12.58	0	150
Independent Variable					
Built_A	Already Built Type A in the Node (million sq. ft.)	1.833	3.815	0	24.667
Built_B	Already Built Type B in the Node (million sq. ft.)	1.307	1.448	0	5.987
Built_C	Already Built Type C in the Node (million sq. ft.)	0.368	0.438	0	1.640
Gr_Rtl_Rt_A	Average Rent of Type A in the Node (CAN $)	25.46	6.888	2.30	50.30
Gr_Rtl_Rt_B	Average Rent of Type B in the Node (CAN $)	20.82	5.525	1.77	40.89
Gr_Rtl_Rt_C	Average Rent of Type C in the Node (CAN $)	18.73	5.000	7.40	37.62
Vac_Rt_A	Vacancy Rate of Type A in the Node	0.139	0.077	0	0.442
Vac_Rt_B	Vacancy Rate of Type B in the Node	0.160	0.096	0	0.56
Vac_Rt_C	Vacancy Rate of Type C in the Node	0.158	0.175	0	1
Con_Wrks	Number of Construction Workers in the GTA (× 1000)	41.93	9.87	2.73	61.30
Wage_Rt	Hourly Wage Rate for Construction Workers	18.27	2.69	12.83	22.91
Con_Cost	Construction Cost per sq. ft.	85.15	13.75	70.5	117.55

Total observations: 720

5. MODEL ESTIMATION

Using the dataset described in the previous section, we estimated the model of office space supply for the Greater Toronto Area. The dependent variable here is the probability of selection of a vector (3×1) of quantities for each type of office space to be built in a business node "*n*" at certain year "*t*" from 1986 to 2005. The explanatory variables used in the model represent the market conditions and land use characteristics of the business node and the state of regional economy at the time of decision to build. Parameter estimates and associated statistics are reported in Table 3. Table 4 reports the correlations between different types of office space.

TABLE 3 Model Parameter Estimates

Parameters	Estimates	Std. Error	t-statistics
Explanatory Variable			
Const_A	29.71	0.182	163.29
Const_B	19.99	0.112	178.74
Const_C	-10.72	0.119	-89.75
Built_A	1.76	0.202	8.68
Built_B	0.68	0.057	11.94
Built_C	0.57	0.082	6.99
Gr_Rtl_Rt_A	0.69	0.219	3.13
Gr_Rtl_Rt_B	0.80	0.166	4.81
Gr_Rtl_Rt_C	1.11	0.108	10.31
Vac_Rt_A	-2.44	0.140	-17.44
Vac_Rt_B	-1.15	0.167	-6.91
Vac_Rt_C	-0.37	0.236	-1.58
Con_Wrks	0.84	0.072	11.73
Wage_Rt	-1.32	0.116	-11.37
Con_Cost	-0.64	0.135	-4.73
Model structure parameters			
Gamma_A	100.10	0.080	1246
Gamma_B	100.06	0.088	1135
Gamma_C	99.88	0.138	723.28
Rho	4.49	0.108	41.64
Alpha	0.66	0.173	1.107
Theta	1.59	0.226	15.44

The constant term for Type A space is the highest followed by Type B. For Type C the constant term is negative. This suggests that builders in general prefer to build higher quality space. The office employment sector in the GTA is dominated by financial, accounting, law, and technology firms that generate the demand for high quality office space. Higher constants for Type A and B seems to be the response of builders to this demand and higher profit margins.

TABLE 4 Correlation Matrix between the error terms

	Type of Office Space		
	A	B	C
A	1	0.25	-0.25
B		1	-0.10
C			1

Haider and Miller (2004) reported the phenomena of spatial inertia in the new housing supply of the GTA. We observe the same phenomena in the office space supply. The attractiveness which is captured by the amount of office space that is already available (Buit_*), is the highest in case of Type A, while it is lowest in the case of Type C.

The rent per sq. ft. of the type office space at the time of decision was used as the indicator of market and the growth of office based employment. In general there is a positive effect of the supply decisions with the higher rents and this effect is highest in the case of Type C buildings. This result is unexpected as one would expect that the higher quality space will be more sensitive to the increase in the rent. One reason for this might be the fact that in general there is a higher temporal variation found in the rent of Type C office space. While in case of Type A and B, the variation is both in terms of time and space.

We used average vacancy rate in the node at time of decision as another indicator of the demand for office space. The model reports negative sensitivity of the supply decisions to the increase in vacancy rates. This effect is highest in case of Type A space. A higher project cost is associated with Type A space and at the same time the revenue (indicated by rents) from it is the highest as well. This explains the higher sensitivity to vacant space in case of Type A supply decisions.

The number of construction workers in the labour force at the time of decision is used as the indicator of building inertia and state of the regional economy. A positive effect is found on the supply decisions due to the increase in number of construction workers.

With the increase in the wage rates the cost of the project increases and thus effects the new office space supply decision in the negative fashion. Similar behaviour is evident in the case of increase in the construction cost.

The greater than one theta variable that represents the behaviour of the builders towards risk shows that they are risk takers. Builders over-build in the boom of construction cycles anticipating future revenues. This fact is evident from the data. The translation parameters that make the corner solutions possible, are almost the same for all three types. The scale parameter seems to be in the acceptable range. The rho parameter that is associated with the parameterization for the Hicksian good is also in the right range.

A limitation that arises due to the use of Bayesian-based MCMC estimation process is the inability to generate model level goodness-of-fit statistics. The goodness-of-fit test for these types of method is an evolving research topic. An alternate approach to test the goodness of fit for this model could be to use simulation forecasting and compare the results with the observed data. Once new data for the years after 2005 are available, we plan on performing the simulation tests.

6. CONCLUSION AND FUTURE WORK

In this paper we presented a generic modelling framework for the supply of new built space and as an application estimated a model for the new office space supply in the Greater Toronto Area. The modelling framework is based on expected profit maximization attitude of the builders. This model is part of our ongoing efforts towards operationalization of the office space market within Integrated Land Use Transportation and Environment (ILUTE) modelling framework, currently under development at the University of Toronto. In the general market-clearing framework of ILUTE, the asking rent model captures the role of accessibility, neighbourhood characteristics, quality of space, and market conditions to determine the asking price at each simulation year. The models for demand for office space in the Greater Toronto Area have already been developed by Elgar *et al.* (2009). With the available demand and supply, these asking rents are then to be used in the market clearing module to match the space to the demander at a transaction rent that is endogenously determined. In next simulation year, the lagged transaction rents then influence the builders' decisions of where, how much, and what type of office space to supply.

The estimated model is dynamic in the sense that it captures the lagged effects of market conditions on the new supply. We observed the phenomena of spatial inertia in terms of location choices for different types of office space. The behaviour of the builders in terms of risk is explicitly incorporated and estimated in the model. The changes in the construction project's expected cost on the builder's decisions is also modelled. To our knowledge, this work is the first that models the where, when, how much, and what type of office space to supply in a single framework at a fairly disaggregate spatial zoning system.

In future, we intend to assess the goodness of fit for the model using simulation and comparison with the observed data. We also intend to bring in more detail in terms of builders' behaviour and heterogeneity in terms of decision making.

The Government of Ontario recently released a growth plan for Greater Golden Horseshoe Area that includes Greater Toronto Area. This plan forecasts an increase of 3.7 million in the population and 1.8 million in the employment of the region by 2031. The long-range forecast suggests high levels of growth in employment sectors that are housed in office-space. 25 nodes across the region are identified as high density employment centres and corridors linking them are given high priority for transit investment. The Province is also in the process of introducing a comprehensive transportation plan to support this growth. To quantitatively assess the implications of these policies and their effect on the urban growth pattern, demographics, and more specifically on the travel behaviour of the population, we intend to operationalize a comprehensive office market model within the Integrated Land-Use, Transportation, and Environment (ILUTE) modelling framework. This modelling effort is a major step forward in achieving this objective.

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