EFFECTS OF SUBSIDIES DISTRIBUTION AND MARKET STRUCTURE CHANGE IN THE DUOPOLISTIC TRANSIT MARKET

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ABSTRACT:

By reviewing the existing organizational and operation arrangements of public transit system in China, this paper firstly explored a pricing competition model between profit maximizing operators and consumer surplus maximizing operators on a single route. Under this mixed duopoly market, effects of two widely used polices (fare subsidy and nationalization) on equilibrium fare changes are investigated through diagrammatic and numerical comparisons. Several conclusions from numerical calculations provide useful suggestions on setting policies in this duopolistic market. First, the private duopoly arrangement is more preferable than any mixed duopoly arrangement, as it produces more consumer and social surplus induced by lower equilibrium fares. Secondly, the application of any subsidy scheme can contribute to lowering equilibrium fares, improving consumer surplus and boosting public transit demand, which is in line with the standard results in the extant literature. Thirdly, given a certain amount of subsidies per trip, subsidizing on the low quality operator (bus) can achieve much better efficiency than any other subsidy schemes. Finally, it is noteworthy that in all of the scenarios reported, subsidization rules and the structure of transit market will definitely affect how optimal fares will be set.

Key words: Mixed duopoly, Subsidies redistribution, Bertrand pricing Game, nationalized degree

1. INTRODUCTION

Due to traffic growth contrasting with limited road capacity and worsening environment, currently, many municipalities in China have constructed or are constructing rail transit lines. It is clear that theses new rail transit lines would not only face substantial competition from automobiles. Service provided by bus companies would also be a major source of competition. In the face of intense competition, when any operator make decisions on fares, they not only need to consider the costs and profits of their own, the fare responses of travellers, but need to take account of reactions of their main rivals as well, as it will influence travellers' opportunity costs, which in turn will have an perceptible influence on ridership redistribution.

In the last decade, following the deregulation in transport sector, the organizational and operation arrangements of public transit in China have been dramatically changed. In the urban bus market, it exhibits a transition from stat-owned monopolistic form to regulated private competitive regime. However, because of the huge initial investments and subsequent operating costs, all rail transit lines in China are usually operated by public-owned operators. Therefore, with the entry of a new public transit mode (rail transit), the common operation arrangements in most Chinese cities present such a structure that one semi-public rail transit operator competes with one or several private bus companies[1], which substantially suggests the presence of mixed duopoly market. Under this mixed duopoly market, the activities chosen by semi-public and private operators may differ due to the different objective functions. Specifically, if the rail transit operator chooses to partially maximize consumer surplus (CS) rather than purely maximize its own profit, how does this affect market equilibrium status and could it really contribute to transferring some or entire benefits to consumers?

In addition, due to the existence of increasing scale economies (in production and consumption) and negative externalities (i.e. congestion on city roads), in a national context, it might be impossible for bus to cover its full costs without subsidies from authorities. On the other hand, as quasi-public goods, most rail transit services are financially supported by local governments. As a consequence of structure change in the transit market, initial financial supporting policy and fare setting rules should be re-assessed to conform to the new market environment. Then, one question with respect to the effectiveness of subsidy distribution is raised: with a certain amount of fare subsidy, what kind of subsidy distribution between bus and rail transit is efficient in terms of reducing fare levels, boosting total transit demand and improving social welfare?

With the purpose of exploring how equilibrium configurations change with different transport policy measures in the mixed duopoly market, strategic models are developed in this paper for a small realistic example based loosely on Nanjin traffic data. With detailed numerical calculations and sensitivity analysis, the key findings of this paper are summarized as follows: first, the private duopoly arrangement is more preferable than any mixed duopoly arrangements, as it produces more consumer and social surplus induced by lower equilibrium fares. Second, the application of

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[1] In this paper, for simplicity, we identify two modes, bus and rail, as the main modes of public transit. We thus leave another mode- taxi, which only takes very small portion of transit market, out of our consideration.

any subsidy scheme can contribute to lowing equilibrium fares, improving consumer surplus and boosting public transit ridership, which is in line with the standard results of extant literatures. Third, given a certain amount of subsidy per trip, subsidizing on the low quality operator (bus) can achieve much better efficiency than other subsidy schemes. Finally, it is noteworthy that subsidy scheme and the structure of transit market will definitely affect how efficient fares will be set.

The rest of the paper is organized as follows: in section 2, I briefly review some literatures in order to shed light on the contributions made by this paper. As a starting point, Section 3 provides more detailed analysis on Bertrand pricing game where one rail transit operator, which partially maximizes consumer surplus, competes with one private bus operator who only maximizes its profits. In section 4, to evaluate the effects of structural and subsidy distribution changes on equilibrium status, two detailed case analysis are applied to Nanjing traffic data. Lastly, conclusions and the directions for further research are presented in Section 5.

2. LITERATURE REVIEW

This analysis of demonstrating impacts of fare subsidy distribution and market structure on equilibrium status changes are inspired by two strands of literature: the application of game theory approach on transport modelling and the study of providing efficient subsidies to public transit.

Regarding modelling competition in public transit market, game theory is firstly applied as Cournort style in pure duopoly market. Viewed as a touchstone work, Victon (1981) first started to consider a Nash-Cournot competition in a private duopoly market with service quality and fare as controlled variables. By utilizing data from Bay Area and estimation results from McFadden' transit demand research, he concluded that if fares can be freely changed, neither modes need operate at a loss and the rail transit can cover its operating costs from fare-box revenue even if its rival offers money losing service. As a result, he pronounced that subsidy is not necessarily needed if the link has potential large traffic demand.

To shed more light on modelling competitive interactions in transit market, C.S. Fisk (1984) specified the difference between Stackelberg game and non-cooperative game and suggested that the price competition between bus and rail transit fits into Bertrand pricing game. After observing the change in British bus market , Oldfield and Emmerson(1986) modelled a Cournot-Nash competition between two bus operators along a single-high-density route to explain that transit price setting should follow the change of public transit organization. With the purpose of extending previous analysis to oligopolistic competition, Huw C.W.L etc (1993a, 1993b) adopted a multinomial Logit model to examine Cournot equilibrium fares and frequencies by incorporating a constant elastic demand form.

Besides these pioneer studies on Cournot competition, as a catalyst for many subsequent papers, Braid (1986) adopted a strategic modelling approach to model pricing and quantity decisions for two congested and symmetric facilities. Along these lines, A.de Palma and L. Leruth (1989) expanded Braid's one stage game into two-stage pricing-frequency games, and analyzed two

scenarios: where consumers are homogeneous and where they are differentiated in their wiliness to pay to avoid congestion.

However, to the best my knowledge, previous works on modelling transit competition have principally focused on pure private oligopoly or duopoly market, where two or more competitors only concentrate on maximizing their own profits. Actually, in reality, some public transit operators are totally or partially state-owned, which indicate the application of mixed oligopoly model. Recently, Sanchez and Colonques (2006) explored frequency and pricing competition in a mixed duopoly context where one of the operators is public owned, and whose objective is welfare maximization. Although this research is inspired by Sanchez and Colonques, by introducing the degree of nationalization (σ), this paper specifies that the public operator partially not totally maximize consumer welfare[2].

During the last two decades, much concern has been dedicated to transit subsidy issues. Extensive literatures have examined the social desirability of providing subsidies from different perspectives by using different methodologies. Mohring (1972) made a path–breaking contribution to firstly propound "Mohring effect", which addressed that an additional passenger could benefit all passengers on board by inducing higher service frequencies, which motives financial subsidies to this positive external effect. Inspired by Morhing's work, Vickrey(1980) continued to analyze basic justifications for subsidy policy and suggested three major considerations that should be entered into subsidy decision-making process.

By taking the cost of public funds and inefficient road pricing into consideration, Odd I Larson (1995) proposed a model from both supply and demand sides to discuss efficient subsidies for Oslo Public Transit Company (OPTC). Ian Savage and August Schupp (1997) presented a model to calculate the marginal effects of subsidy on reducing fare levels and improving service levels of public transit in Chicago. By differentiating the impacts in peak and off-peak periods for bus and rail service, they concluded that it is more advantageous to use subsidies to reduce fares than improve service levels and give bus the priority to be subsidized. More recently, Ian and Small (2009) empirically modelled welfare effects of fare adjustments and optimal service pricing by taking some externalities effects into consideration. A brief review of extant literatures on transit subsidy reveals that most studies only focus on the effectiveness of subsidy in a monopoly market; empirical work devoted to studying efficient allocation of subsidy across public transit modes in a duopoly contest market is scare.

To sum up, this study contributes to transport economic research from two aspects. First, while previous studies assume that the payoff functions in a classical game are identical, this paper considers the mixed duopolistic case in which different players pursue different objectives and

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2 In the context of public economics, economists model the objectives of public firms in two ways: one way is to maximize total social welfare and the second approach is to maximize partially social welfare. As the extension of the first one, the second approach is more general. Additionally, to study the effect of nationalization on equilibrium status, the second approach is more flexible and efficient.

thus have distinct payoff functions. Second, following White's work, I examine impacts of different subsidy distributions on equilibrium price and social welfare changes in urban transport sector.

3. THE DESCRIPTION OF THEORETICAL MODELS

To deal with a common situation where one semi-public rail transit operator competes with one private bus company along an isolated route, this section provides more details on theoretical modelling. First, the structure of a Bertrand-Nash pricing game and a constant elastic demand model are laid out. Second, under this mixed duopoly, private and semipublicly operators simultaneously maximize different objective functions with respect to fare as strategic variable.

3.1 The framework of Betrand-Nash pricing game in mixed duopoly

To address Bertrand Nash-equilibrium in price, this study mainly focuses on a mixed duopoly market, where bus and rail transit compete as rivals and afford services along an isolated corridor. Put it to different, one semi-public operator (rail transit), who concerns totally or partially users' benefits, is competing with one private operator (bus company) who only considers its own profits during a given time period[3]. Since each player is trying to "optimize its objective function without prior knowledge of other players' functions"[4], these two players make their choices simultaneously and receive payoffs depending on the fares they chosen. It also assumes that both the bus and the rail transit operators set their fares independently and without any collusion.

For manageability of the model, I assume that public transport services afforded by rail transit and bus operator are perfect substitutes and travellers are homogeneous in their willingness to pay. In terms of fare structure, in China, the fare charged by bus is only dependent on types of vehicle (air-conditioned or plain) regardless of distance travelled. While, fares of rail transit are zone-based, which increase the basic rate once the trip length beyond the average one. Thus, the assumption of flat fare is less problematic for rail transit if the assumed trip length is less than the average level. Thus, for simplicity of exposition, the assumption that both operators do not engage in price discrimination, but rather provide services using a simple flat fare structure, is reasonable. Hereby, this Bertrand-Nash game G_n has the following features:

1) Plays: This paper considers a simple two-player game in which one semi-public rail transit operator competes with one private bus company in an isolated route during a given time period.

2) Strategies: This game only considers fare as operators' strategy variable holding service frequency and vehicle size as constant and given exogenously.

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3 To be in line with the designed capacity constraints, we will only focus on transit demand for inbound commuting trips in one morning rush hour.

4 C. S. Fisk (1984) , Game theory and transportation systems Modeling, *Transportation research part B*, Vol.18 No. 4/5, pp 301-313

3) Payoff: The payoff for each operator corresponds to its objective function, denoted by OF_j. In this mixed duopoly context, one rail transit operator acts as semi-public owned that aims to mix profit maximization and consumer surplus maximization. Whereas, one bus Company acts as pure private firm who only concerns its own profit maximization. Table 1 depicts the framework of this Bertrand game under mixed duopoly market environment.

Table 1 the framework of Bertrand pricing game under mixed duopoly

Elements	Details
Players	Two players: rail transit operator (semi-public) and bus operator (private)
Strategies	Price decisions independently
Payoff	Rail transit aims to mix profit maximization and consumer surplus maximization.
(Objective function)	Bus companies only concern its own profit maximization.

3.2 The structure of Elastic Demand Function

To address the cross substitution between public transport modes and private transport modes, this subsection begin with constructing the travel demand function for operator i[5], which conventionally takes following form:

(1) $$Q_i = Q^* \{\xi, cc\} M_i \qquad i = r, b$$

Q_i is the number of passengers selecting mode i.

$Q(\xi, cc)$ represents the aggregate traffic demand of public transit in a fixed journey length L, which is elastic with respect to changes of composite costs(cc) .

M_i denotes the choice probabilities (or market share) of one type of transport services that afforded by operator i.

ξ is the absolute value of public transit elasticity (ξ >0), which is the average public transit demand change with respect to weighted transit fares.

Therefore, the resultant travelling demand for operator i is composed of two parts: the traffic demand of the whole public transit modes and the market share that operator i takes. Since the combination of elastic public transit demand with Logit market share function in a single model is not an easy task, I will discuss them in turn.

In this paper, the total demand for public transit service takes a simple negative exponential demand function with an elastic parameter ξ . The Explicit form of demand can be expressed as:

(2) $$Q^* \{\xi, cc\} = \bar{Q}^* \exp\left[-\xi(cc - \overline{cc})\right] \qquad i = r, b$$

Where, the composite cost (cc) presents all travelers costs of using public transit modes (bus and rail transit).

The presumably closest link to this paper in terms of the constant elastic demand function can be found in Williams et al(1993) .The bar over a variable denotes its value in a reference state before

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[5] As a mater of convention, we will use the subscript *r* to denote rail transit and the subscript *b* for bus.

price adjustments take place. So we define $\overline{Q}^*$ being the initial public transit demand in a benchmark situation before price competition takes place.

To reflect the elastic public transit demand changes with respect to travelers' composite costs, I specify the formulation of composite costs as:

$$cc = \frac{1}{-\theta}\log\left\{\sum_i \exp(-\theta C_i)\right\} \qquad i = r,b$$

(3)

Where, C_i denotes the generalized costs of traveling by mode i. θ is a positive dispersion parameter to describe fare effects on generalized costs, which we will discuss later.

With the same technical formulation, the composite cost for initial state is:

(4)
$$\overline{cc} = \frac{1}{-\theta}\log\left\{\sum_i \exp(-\theta \overline{C}_i)\right\} \qquad i = r,b$$

$\overline{C}_i$ is the generalized costs of traveling by mode i in initial state.

This study deals with an isolated corridor connected two traffic zones, where travelers have choices between two transport modes: bus and rail Transit. Furthermore, instead of utility maximization, individual travelers' mode choices are assumed to be based on minimizing generalized costs per journey, which equals the sum of the fare charged and other costs (i.e., riding time costs and waiting time costs). In this manner, the generalized costs, C_i , for a representative traveller to choose one type of service runned by operator i, can be specified as:

(5)
$$C_i = P_i + a_1\rho_v T_i + a_2\rho_w W(f_i) \qquad i = r,b$$

Where:

P_i is the fare charged by the operator i.

T_i is average riding time that operator i serving[6].

ρ_v and ρ_w are value of time (VOT) for riding and waiting, respectively.

$W(f_i)$ is the average expected waiting time at station for mode i (hours).

a_1 and a_2 are parameters that can be obtained form some empirical studies.

To focus on the principal aspects, this paper confine itself to be the case of one homogeneous passenger group, which indicate that all passengers' are uniformly distributed along the route and identical for value of time. Furthermore, for both bus and rail transit services, since the route, the location and number of stops are decided in the planning stage, accessing time costs are not relevant here and may not be included in the generalized costs function.

In urban transport, waiting time cost constitutes an appreciable proportion of travel time costs. For relative short headways (less than 5 minutes), the average waiting time can be estimated from

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[6] Considering the frequency and vehicle size are assumed to be exogenously given, we simplify riding time by disregarding the effect of passengers boarding /alighting time and the possibility of on-road congestion.

one-half the headway between successive buses or rail trains. Therefore, the function of expected waiting time can be written as:

(6)
$$W(f_i) = \frac{1}{2f_i} \qquad i = r,b$$

f_i stands for the service frequency afforded by operator i, which is the number of vehicles passing a station in one direction during one hour . In order to keep equations brief and simple for further resolving, we introduce another conception, headway, denoted as h_i. Obviously, headway is the inverse measure of service frequency:

(7)
$$h = \frac{1}{f}$$

By incorporating this well-known "wait equals half the headway" rule into the above generalized cost function, the generalize costs can be re-interpreted as:

(8)
$$C_i = P_i + a_1\rho_v T_i + a_2\rho_w \frac{h_i}{2} \qquad i = r,b$$

Turning to the specification of market share, the mode-choice probability of bus or rail transit is given by the well-known binary Logit form. As a result, the market share of operator i take the forms as[7]:

(9)
$$M_i = \frac{\exp(-\theta C_i)}{\sum_i \exp(-\theta C_i)} \qquad i = r,b$$

Where, M_i denotes the choice probabilities of selecting operator i on the basis of its generalized costs. Then, the resulting demand function for operator i may now be expressed in the following form (Detailed Mathematical derivations are found in the Appendix 1):

(10)
$$Q_i = \bar{Q}^* \left[\frac{\sum_i \exp(-\theta C_i)}{\sum_i \exp(-\theta \bar{C}_i)} \right]^{\frac{\xi}{\theta}} \bullet \frac{\exp(-\theta C_i)}{\sum_i \exp(-\theta C_i)} \qquad i = r,b$$

To keep the explosion brief and simple, we let $\delta = \left[\sum_i \exp(-\theta \bar{C}_i) \right]^{\xi/\theta}$, which is constant based on

the reference state circumstance before any adjustment in fares. Then, it is convenient to recast the demand function as:

(11)
$$Q_i = \bar{Q}^* \delta \frac{\exp(-\theta C_i)}{\left[\sum_i \exp(-\theta C_i) \right]^{1-\frac{\xi}{\theta}}} \qquad i = r,b$$

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[7] A more advanced approach developed most rigorously by McFadden (1973) is called "random utility" approach, which used detailed information about the individual travellers to estimate mode-choices probability. We will leave it to further study.

3.3 Strategic pricing competition in a mixed oligopoly market

As demand function is well defined in equation (11), specified models are formulated for this one-short game where operators make strategic pricing decisions to maximize different objective functions(OF).To step further into the effects of strategic price decisions, a duopolistic arrangement is described, where there is a competition between two transport modes (bus and rail transit). In the real word, although multiple private bus companies are prevailing in most cities, few overlapping operating situation leads us to view them as one virtual operator. Thus, I can theoretically assume that only one bus operator competes with one rail transit operator.

For this given set of market conditions, the bus operator is always characterized as pure private company, which only concerns maximizing its own profit, which can be expressed like:

(12) $$OF_b = \prod_b = P_b Q_b + S_b Q_b - OC_b$$

Where $\prod_b$ is the profit function of bus operator.
S_b stands for the subsidies of each trip that bus operator receive.
OC_b denotes the operating costs of services provision from bus operator.

From semi-public operator (rail transit) point of view, it aims to maximize total or partial consumer surplus. As a consequence, its objective function is

(13) $$OF_r = \sigma CS + (1-\sigma) \prod_r$$

Where $\prod_r$ stands for the profit function of rail transit and CS denotes the consumer surplus.

The parameter σ , which lines between 0 and 1, can be regarded as the importance level attributed to the consumer surplus objective, in contrast with the profit objective. To put it another way, it gives a measurement of nationalization degree.

σ =0 signifies that rail transit company solely concerns about its profits, and under this condition, the market can be viewed as pure private oligopoly. σ =1 means that rail transit operator only aims to maximize consumer surplus and disregard its own profits. For this reason, the higher σ , the higher the rail transit concerns consumer surplus instead of its profits.

By a trick borrowed from Odd I Larsen (1997), the functional form of consumer surplus is the log of the denominator of the mode-choice probability model. To aggregate individual surplus, the log-sum exponentials form are multiplied by the number of travellers (Q^*). It follows that

(14) $$CS = \frac{Q^*}{-\theta} \times \ln\left[\sum_i \exp(-\theta C_i) \right] \qquad i = r,b$$

Thus, the final version of rail transit's objective function can be modeled as:

(15) $$OF_r = \sigma \left\{ \frac{Q^*}{-\theta} \times \ln\left[\sum_i \exp(-\theta C_i) \right] \right\} + (1-\sigma) \left[P_r Q_r + S_r Q_r - OC_r \right] \qquad i = r,b$$

In this mixed duopoly market, operators compete with each other in prices, meaning that one operator adjusts its pricing strategy according to the other operator's price. To seek a Bertrand Nash Equilibrium, each operator will attempt to maximize its objective functions with respect to price subject to capacity constraint.

(16)
$$Max\ OF_i$$
$$s.t.\ \ Q_i \le Q_{im} \qquad i = r, b$$

For the case of bus operation, to derive its price reaction function, the objective function (equation (16)) is maximized with respect to price. By setting the result equal to zero and rearranging terms, we unambiguously obtain one of the central results of this paper—the price reaction of bus. (In order to not to break the flow of the main argument, the detailed mathematic derivations are showed in Appendix 2)

(17)
$$\frac{1}{\theta}\ln\left[\theta(P_b + S_b) - 1\right] - \frac{1}{\theta}\ln\left[1 - \xi(P_b + S_b)\right] + P_b - P_r + \alpha_1\rho_v(T_b - T_r) + \frac{\rho_w\alpha_2}{2}(h_b - h_r) = 0$$

Based on the equation (17), we can easily derive the price reaction function $\phi_b^R(P_r)$ (where the R superscript denotes reaction function), which is the implicit function of P_r[8].

A similar procedure is used to show that the first order condition of OF_r with respect to P_r (A detailed description of computations is provided in Appendix.3):

(18)
$$\frac{1}{\theta}\ln\left[\theta(1-\sigma)(P_r + S_r) - 1\right] - \frac{1}{\theta}\ln\left[1 - \xi(1-\sigma)(P_r + S_r)\right] + P_r - P_b + \alpha_1\rho_v(T_r - T_b) + \frac{\rho_w\alpha_2}{2}(h_r - h_b) = 0$$

By analogy, equation (18) gives an implicit representation of the price function of rail transit $\phi_r^R(P_b)$, which is the implicit function of P_b.

The Bertrand Nash equilibrium prices can be pursed further only if we jointly solve above two price reaction functions. The solution yields Bertrand Nash Equilibrium, which can be defined as Nash equilibrium prices, denoted as P_b^{NE} and P_r^{NE} respectively[9].

(19)
$$\begin{cases} P_r^{NE} = P_r^{NE}(P_b) \\ P_b^{NE} = P_b^{NE}(P_r) \end{cases}$$

Figure 1 shows an illustrative Bertrand Nash equilibrium calculated for a set of particular parameters. The price reaction curve of bus has upward sloping with very steeper curvature, which indicates that $\varphi_b'^R(P_r) > 0, \varphi_b''^R(P_r) > 0$ and $\lim \phi_b^R(P_r) = P_0$ .Similarly, $\phi_r^R(P_b)$ presents the same characteristic. These two curves can only intersect once and do not cross again since neither can meet any line of unit slope more than once. Thus, the intersection point (A) is a "stable" Bertrand-Nash equilibrium with asymmetric price results. Obviously, the eqilibrium rail transit fare is higher than the bus equilibrium fare in this mixed duopoly. Viewing above results could help us understand that prices are strategic complements, which mean if one operator chooses a higher fare, its rival has an incentive to raise fare too.

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[8] The second order conditions are satisfied and do not impose any additional requirements.
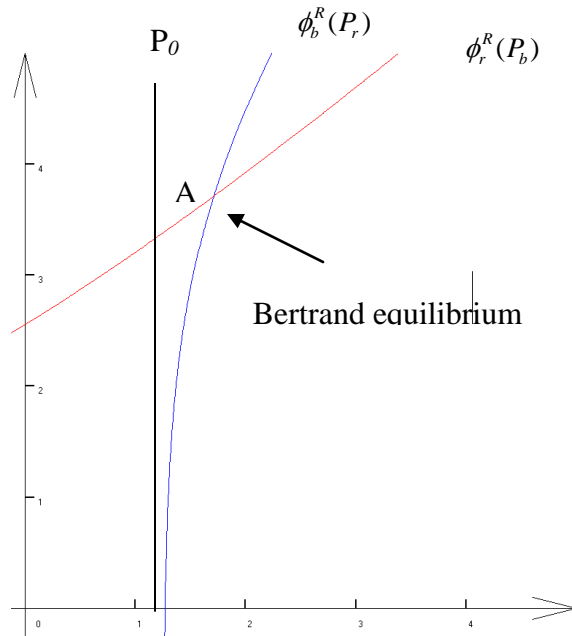[9] Where *NE* superscripts denote their Nash equilibrium values

Figure 1 Price Reaction Functions to Duopoly Game

4. A NUMERICAL CASE ANALYSES

In welfare economics, social Surplus[10] (or social welfare) usually can be viewed as one primary measure to evaluate the effectiveness of a proposed policy. As a result, the purpose of following numerical analysis is to investigate the sensitivity of Nash equilibrium fares, market shares, patronage, social welfare and its constituent parts to variations in two key parameters (nationalized degree σ and subsidy levels, S) .To establish orders of magnitude for key parameters, we use Nanjing traffic data to gauge results.

4.1 The description of Nanjing Traffic Data

In this sub-section, the traffic data leading to selection of specific parameter values used is mainly from *Nanjing urban transport planning (2006)*. The statistics shows the average trip length is 5.8 kilometres. In this numerical calculation, the link length is assumed to be 6 Km, which is slightly longer than the average trip length that obtained from travel survey.

The rest of traffic data sources from *the annual report of Nanjing urban transportation in 2008*. Measured in CNY (¥), most buses charge a flat fare with 1 ¥ for plain buses and 2 ¥ for air-conditioned ones. Consequently, a rough average of 1.5 is settled for bus fare in the following calculations. Furthermore, like many rail transit systems in the world, the fares of rail transit are distance–based, ranging from 3 ¥ for journeys under 6 km to 4 ¥ for journeys over 21 km. Since the assumed route length is 6 km, I can approximately assume that average fare rate is 3 ¥.

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[10] In this case, the social welfare consists of the sum of consumer surplus and operator profit.

Regarding service characteristics, the average speed of bus is 24 kilometers per hour (ignoring congestion on the surface road) and the rail transit's average speed is 35 kilometers per hour. Consequently, the average travel time by taking bus is 15 min and 9 min for rail transit to run along this assumed 6-Km link. Besides travel speed, frequency is another important service characteristic. Some related data show that the headway of bus falls in the range of 7-9 minutes (7 min for the peak time and 9 min for off-peak period), for rail transit in the range of 5-7 minutes. Mode shares for bus and train are 18.56% and 81.44% respectively during the morning rush hour. Table 2 lists all parameters used in the analysis and values assumed for calculation.

For bus operating costs, the information stems from *Nanjing Bus Group Annual Report (2008)*, which illustrates that the variable operating costs of running an extra bus kilometre is 4.4 ¥ and fixed operating costs per coach per hour as 81.3 ¥. In terms of rail transit, values for fixed operating costs (2530 ¥) and variable costs (50 ¥ per kilometre) are provided by Nanjing Metro Company. The Table 2 below also compiles operating costs for these two modes.

Additionally, for this 6-Km corridor, the designed capacity of rail transit in one rush hour is 22320 passengers, while the maximum passenger volume that 6 bus lines can totally take is 6000. Table 2 simultaneously summarizes the physical capacity constraints to be used in sequel.

Table 2 Traffic Data on this 6-kilmeter link

		Rail Transit	Bus
Operating Data	Speed(Km/hr)	35	24
	Travel time(hr)	0.17	0.25
	Fares (¥)	3	1.5
	Frequency(veh/hr)	10	7.5
	Headway((hr/veh)	0.1	0.1333
	Modal share	20.77%	79.23%
	Transit Lines	1	6
Operating costs (¥)	Fixed costs (¥/veh.hr)	2530	81.3
	Variable costs (¥/veh.km)	50	4.4
	Operating vehicles	12	10
Capacity	Vehicle capacity(pers/veh)	1860	100
	Transit lines	1	611
	Vehicles for each line	12	10
	Line capacity (pers/hr)	22320	6000

Since little evidence is available to fix the value of time for the city of Nanjing, plausible values can be transferred from the studies of Jiang Yin and Richard F. DiBona (2009) for the case of Tianjing City. Although, they reported different VOT across different travel modes, purposes and time of day, in this case, we only use a single value of riding time, 9.25 ¥ per hour, regardless of travel

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[11] According to ongoing arrangement, along this corridor from Xuewumen to Zhangfuyuan, 6 bus lines (Bus 1, Bus 28, Bus 38, Bus 35, Bus 26, Bus 100) are parallel operated by Nanjing Bus group and competed with Metro Company.

modes and purposes . Assuming that the mean value of travel time only varies with average personal income, we can conclude that the value of riding time for Nanjing City is 10.1 CNY /hour.

On this route segment, the number of travelers served by bus and rail transit is 22,100 commuters during one morning rush hour for one direction. Moreover, on the basis of on board surveys, Hong Sun and Zuo ren Yan (2006) estimated the sensitive of traffic demand to fares, which indicates, for one percent reduction in fares, there is a consequent 0.17-0.41 increase in patronage. Here, we adopt the value of 0.2 as the fare elasticity for public transit.

Table 3 summarizes parameters used for following numerical application. The numerical values of α_1, α_2 and θ were estimated from public trip assignment model, which have been done for Nanjing area by using Transtar 2005 (*Public Transport Version*). In this numerical test, we assume these parameters are constant when moving from the base to the other cases.

Table 3 Parameters Lists for calculations

Value	α_1	α_2	θ	ρ_v	ρ_w	ξ
Parameter	3	3.3	1	10.1 ¥ /hr	20.1 ¥ /hr	0.2

To make the following cases more illustrative and comparable, Table 4 outlets the current operational status for bus and rail transit. By setting current position as benchmark case, in the subsequent discussions, we can compare how fares, market shares, social welfare and its constituent parts change with two key parameters.

Table 4 Current operational status

	Benchmark Case
P_r (Rail transit fare)	3
P_b (Bus fare)	1.5
M_r (Market share of rail transit)	81.44%
M_b (Market share of bus)	18.56%
Q (Total transit demand)	22100
Q_r (Travel demand of rail transit)	17998
Q_b (Travel demand for bus)	4102
R_r(Fare-box revenue for rail transit)	53995
R_b (Fare-box revenue for bus)	6153
OC_r(operating costs of rail transit)	33960
OC_b(operating costs of bus)	6462
SS (Social Surplus)	232151
PS (Producer Surplus)	19726
CS (Consumer Surplus)	212425

With 22,100 commuters per morning peak hour, this new rail transit line catches a significantly larger market share (81.44 %) than bus system does (18.56 %). Although there does exist the shortfall between costs of providing bus services and fare-box revenues, rail transit can cover its

operating costs from fare box[12]. This result is consistent with Viton' the study (1981), which indicates that, in some links with high-potential demand, some transit modes can be profitable without subsidy.

4.2 Case 1: the effect of nationalized degree (σ) on equilibrium states

In Case 1, to clarify the impact of nationalized degree (σ) on competitive fares, modal shares and the elements of social surplus, we begin with analyzing the move from the standard private duopoly[13] (SD, $\sigma = 0$) to the mixed duopoly (MD, $\sigma = 0.8$) case through solving numerically for different values of σ. Applying this procedure yields the results, which gathered in Table 5.

Table 5 the sensitivity analysis of nationalized (σ) on equilibrium status

	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$	$\sigma = 0.5$	$\sigma = 0.6$	$\sigma = 0.7$	$\sigma = 0.8$
P_r	2.78	2.94	3.13	3.34	3.61	3.95	4.4	5.09	6.37
P_b	1.19	1.22	1.25	1.3	1.36	1.45	1.59	1.84	2.41
M_r	80.03%	77.87%	75%	71.88%	67.45%	61.73%	54.2%	43.26%	27.26%
M_b	19.97%	22.13%	25%	28.12%	32.55%	38.27%	45.8%	56.74%	72.74%
Q	23175	22568	21891	21170	20313	19316	18120	16512	14020
Q_r	18547	17574	16418	15217	13701	11923	9821	7143	3822
Q_b	4628	4994	5473	5953	6612	7393	8299	9369	10198
PS	16645	17339	17447	18142	18031	17394	15985	13175	8501
R_r	51560	51668	51388	50825	49461	47096	43212	36358	24346
R_b	5507	6093	6841	7739	8992	7393	13195	17239	24577
CS	217259	214564	211463	208044	203814	198670	192163	182778	166665
SS	233904	231903	228910	226186	221845	216064	208148	195953	175165

The first two rows of Table 3 show that, for different magnitudes of nationalized degree (σ), the pair of equilibrium fares are lowest in the standard duopoly situation ($\sigma = 0$) with only 1.19 ¥ for bus and 2.78 ¥ for rail transit. The numerical results illustrate that the bus fare and the rail transit fare tend to coincide for the lower nationalized degree. But, the divergences become more and more pronounced when σ increases gradually.

The key point of the argument on how equilibrium fares are determined by σ can be illustrated diagrammatically through Figure 2. In figure 2, several sets of price reaction curves are plotted in the in the space of $\{P_b, P_r\}$. Concerning the effects of nationalization, the move to a Mixed Duopoly (MD) implies an outward shift of rail transit's price reaction curve. Thus, in Figure 2, we can observe that the point where two reaction curves intersect moves up, which consequently leads to higher equilibrium prices for both parties.

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[12] As for the huge construction costs, rail transit is expected to rely on general obligation bonds secured by the property tax. Thus, it is only expected to pay for all rolling stock and operating expenses out of the fare box.

[13] In this paper, Standard Duopoly (SD) means both rail transit and bus companies, only maximize profits. In contrast, the Mixed Duopoly case addresses one of the operators (rail transit) will choose partially maximize consumer surplus and its profit function.

Based on previous discussions, it is possible to make a rough idea here that, the more rail transit operator concerning consumer surplus, the higher equilibrium prices for both two modes will be, which in turn leads to lower consumer surplus. Consequently, in contrast with initial objective to increase consumer surplus, a higher degree of nationalization system involves higher fares and lower consumer surplus comparing with standard private duopoly.
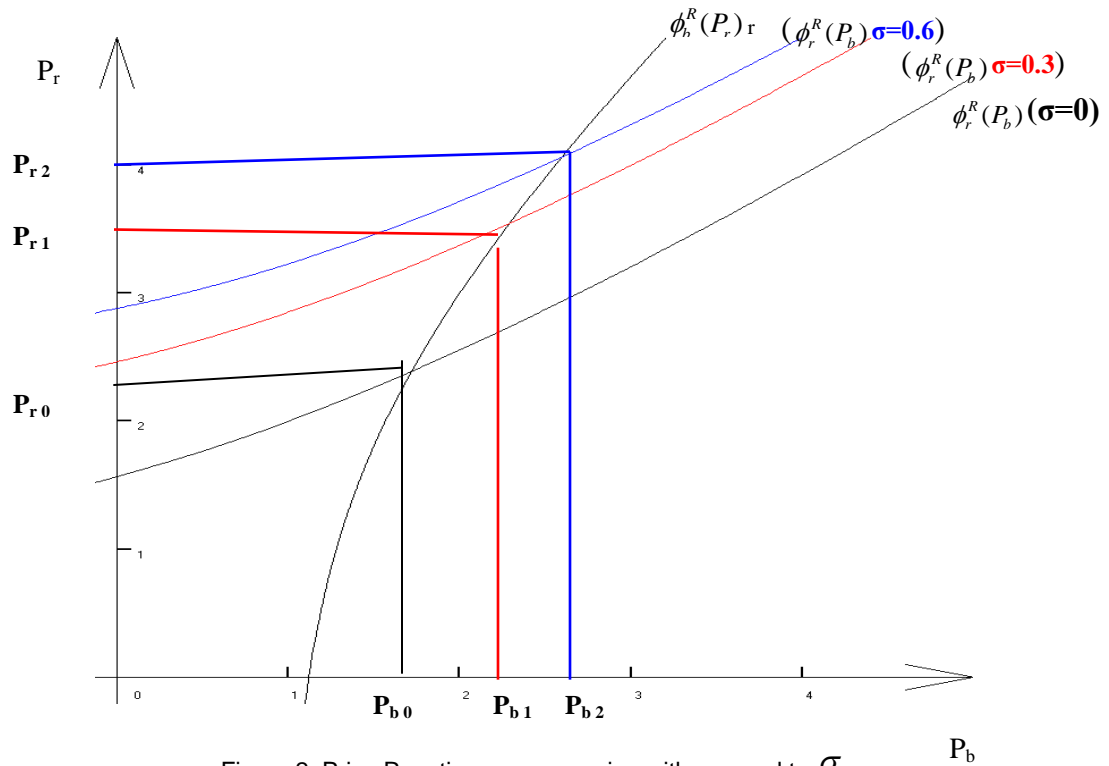

Figure 2 Price Reaction curves moving with respond to σ

With reference to the Figure 3, it could be observed very clearly that the total demand of public transit (Q^*) and rail transit (Q_r) are reduced gradually with an increase in the strength of nationalization (σ). However, in contrast, since the bus attracts more and more travellers from its rival due to the relative lower fares, its market share increases quickly. The explanation for these curious results can be explained as follows. Generally speaking, the overall impact of nationalized degree on patronage is the combination of two terms: the first is the decreasing effect on total public transit demand, and the second is decreasing or increasing impact on mode's market share. Regarding rail transit, with the more extend to which it cares about consumer surplus, decreased total transit demand and the reduction of its market share result in the substantial decline in its aggregate demand. Whereas, for bus, since the increasing effects on mode share dominates the decreasing effects on total transit demand, the equilibrium patronage goes up with the increasing steps of nationalized degree (σ). Based on rough calculations, a clear result is that: driven by more concerned with consumer surplus, the rail transit operator loses its competitive advantage in public transit market. On the other hand, although there is a drop in total traffic demand, market share and travel demand of bus significantly rise.
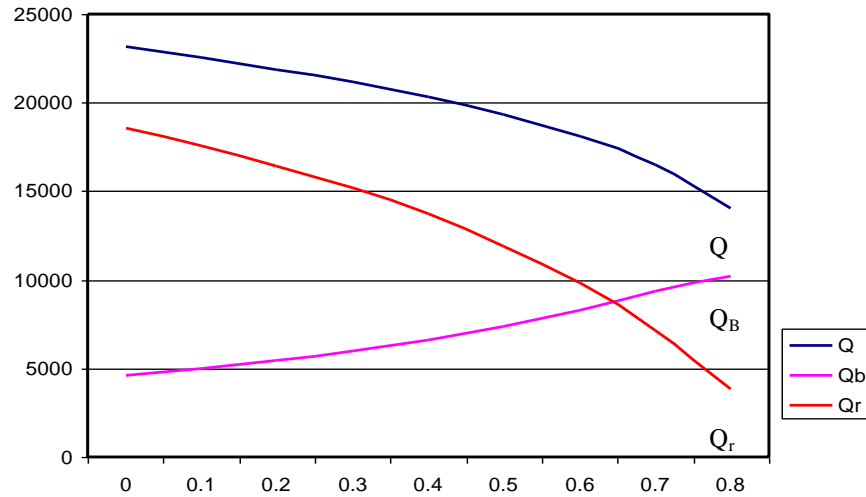
Figure 3 the change of traffic demand with nationalized degree (σ)

As revealed by Figure 4, with a declining number of passengers taking rail transit, the rail transit operator surfers greatly from fewer trips, causing its fare-box revenue to tail off rapidly. In contrast, the ability of bus operator to convert consumer surplus into its profits can be raised, which leads to the growth of bus company's profits. Thus, an obvious reflection is that the bus operators have a competitive edge in generating fare box revenue.
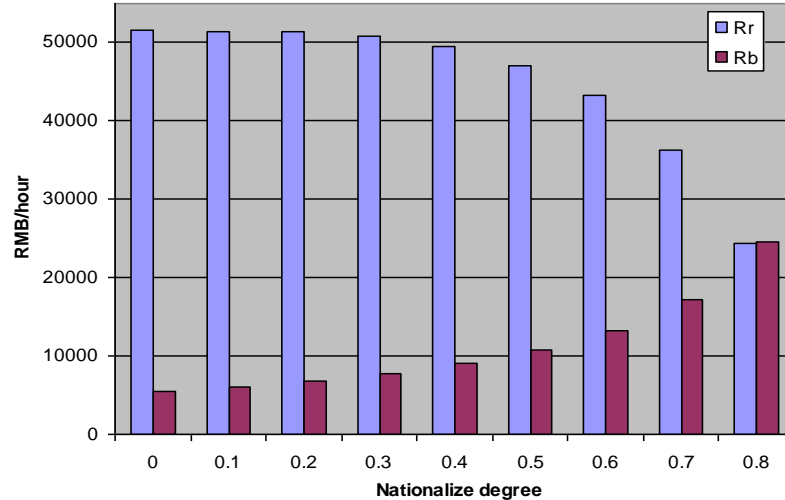

Figure 4 the change of operator profit with nationalized degree

As expected, higher equilibrium prices and lower traffic demand have negative impacts on consumer surplus, which indicates that the consumer surplus deride from traveling decreases as nationalized degree increases (See Figure 5). Since the decrease of consumer surplus can not be compensated by the increase of producer surplus, there is a drop in social surplus.
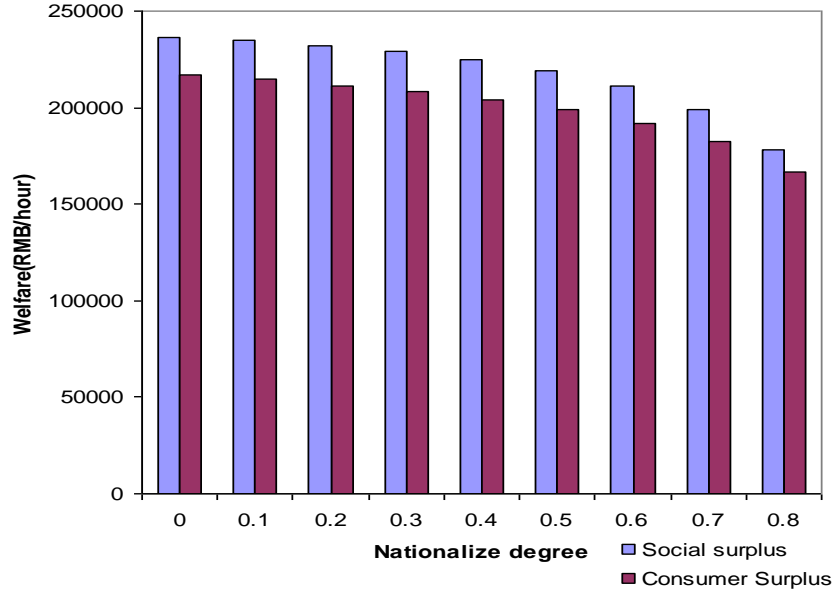
Figure 5 the change of welfare measurements with nationalized degree (σ)

The preceding discussions are summarized in Proposition 1:

Proposition 1: *With increasing nationalized degree, equilibrium prices go up. Since travellers pay more in a move from private to mixed duopoly, consumer surplus and social surplus necessarily fall. In brief, the private duopoly is more preferable than any mixed duopoly arrangements, as it produce more consumer and social surplus induced by lower equilibrium fares.*

The policy implication of above analysis is that the private duopoly is more beneficial than any mixed duopoly as it gives rise to two benefits: 1) lower equilibrium transit fares, and 2) higher social surplus, induced mainly by a higher consumer surplus and more rail transit revenue.

4.3 Case2: The effect of subsidy distribution on equilibrium status

In case 2, to investigate the effect of subsidies distribution between bus and rail transit, four subsidies scenarios are compared in details:

Scenario 1 (Point A: $S_b = 0, S_r = 0$): No subsidies for either rail transit or bus;

Scenario 2 (Point B: $S_b = s, S_r = 0$): Only subsidize s per trip to bus operator;

Scenario 3(Point C: $S_b = 0, S_r = s$): Only subsidize s per trip to rail transit;

Scenario 4 (Point D: $S_b = s/2, S_r = s/2$): Subsidize s/2 per trip for both rail transit and Bus;

Obviously, the approach taken here is to make the amount of subsidies (S_b and S_r) as variable parameters in these four scenarios and then to compare corresponding economic performances.

The following comparisons in Table 5 can give us some primary intuitions for what would happen when we set fiscal supporting polices for public transit.

Table 5 the resulting equilibrium fares, market share and social welfare

	Scenario 1 $S_b=0, S_r=0$	Scenario 2 $S_b=s, S_r=0$	Scenario 3 $S_b=0, S_r=s$	Scenario 4 $S_b=s/2, S_r=s/2$
P$_r$	2.78	2.56	2.65	2.53
P$_b$	1.19	0.74	1.18	0.94
M$_r$	80.03%	76.11%	81.88%	80.03%
M$_b$	19.97%	23.89%	18.12%	19.97%
Q	23175	24462	23676	24362
Q$_r$	18547	18618	19386	19497
Q$_b$	4628	5844	4290	4865
R_r	51560	47662	51373	49327
R_b	5507	4325	5062	4573
S	0	2922	9693	5080
S$_r$	0	0	9693	4874
S$_b$	0	2922	0	1216
SS	233904	234278	235434	235766
PS	16645	11565	16013	13478
CS	217259	222713	219421	222298
r	0	80.18	24.29	46.41

S$_r$---subsidies for rail transit   S$_b$---subsidies for bus   r-subsidies recover rates

Firstly, the simulations indicate that subsidizing any modes implies price reduction and social welfare improvement. Besides this obvious result, It might be interesting to note that with the same rate of subsidy (i.e., 0.5 ¥ per trip), the price difference between Scenario 1 and Scenario 2 is much more striking than the difference between Scenario 1 and Scenario 3. Thus, only subsidizing bus is more efficient than subsidizing only rail transit. To further explain the impact of subsidies distribution on equilibrium prices, we plot Figure 6.

In Figure 6, ϕ_b^{*R} denotes the price reaction curve when public transport authority decides to only subsidize bus operator. And, another curve ϕ_r^{*R} represents only rail transit operator receives subsidies. In this case, given theses price reaction curves ($\phi_b^{*R}, \phi_b^R, \phi_r^R$ and ϕ_r^{*R}), four different intersecting points (A, B, C and D) have defined four different Bertrand equilibrium statuses, which stand for four scenarios mentioned before. In terms of the bus equilibrium prices, we can observe that the fare distance between Scenario 1 and Scenario 2 ($P_A^{'} - P_B^{'}$) is much larger than the fare distance between Scenario 1 and Scenario 3 ($P_A^{'} - P_C^{'}$). The same result is holding for rail transit

fare ($|P_A - P_B| > |P_A - P_C|$). Through this graphic representation, a straight conclusion can be drawn: under duopoly market structure, if authorities want to reduce fare levels through subsidies schemes, only subsidizing bus can obtain much better results than only subsidizing rail transit.
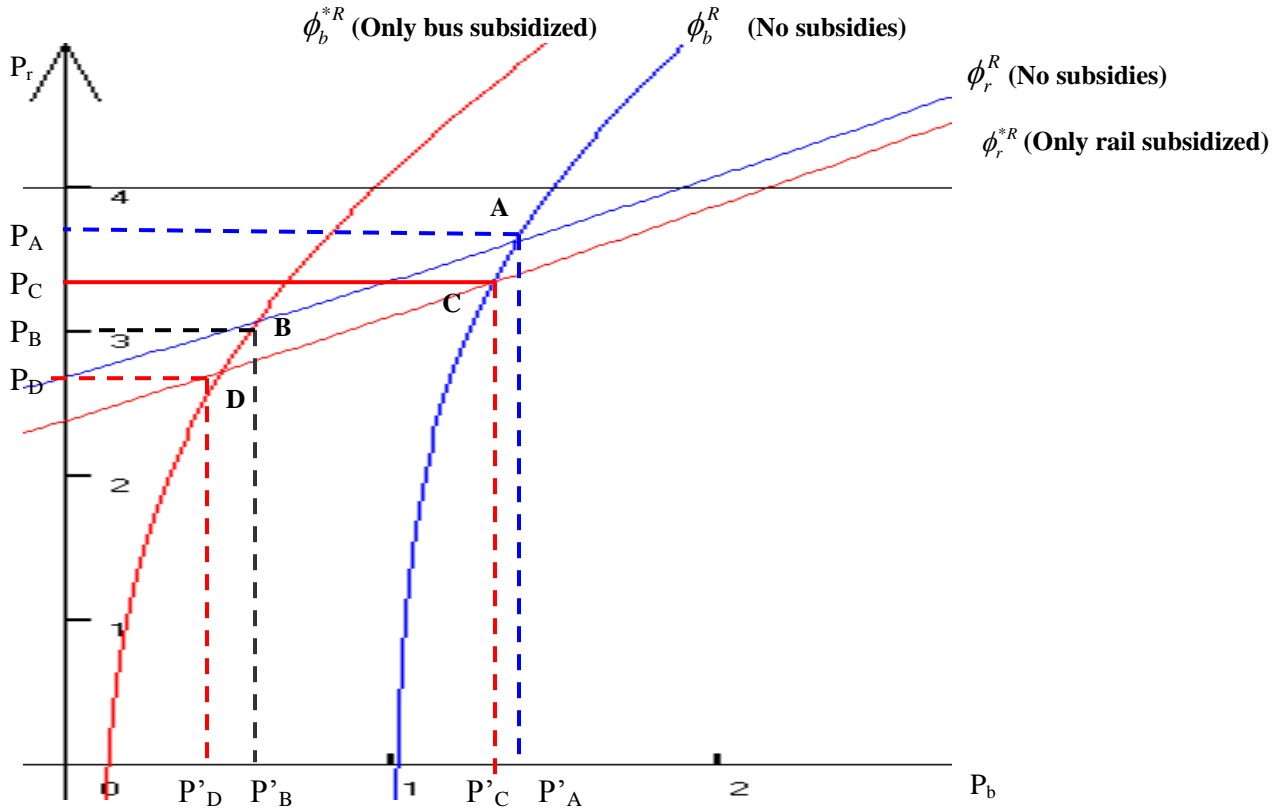

Figure 6: Price reaction curve shifts resulting from subsidy

Secondly, the interchange comparisons of alternative subsidy regimes suggest that if only the bus operates as a subsidized utility, the reduction of fares definitely enjoys competitive advantage, which sufficiently accelerate a ridership shift from rail transit to bus. By analogy, only subsidizing rail transit does contribute greatly to raising its modal share. Naturally, however, if both rail transit and bus receive the same amount of subsidies per trip (i.e Scenario 4, $S_b = S_r = 0.25$ ¥), the market share of both modes are unchanged.

Resulting from equilibrium fare reduction, any subsidy scheme can contribute to reducing composite costs that perceived by passengers, which in turn gives an overall increase in equilibrium traffic demand (Q^*, Q_b and Q_r). It is also noteworthy that, given a certain amount of subsidies per trip, the patronage increase in Scenario 2 is greater than other two scenarios.
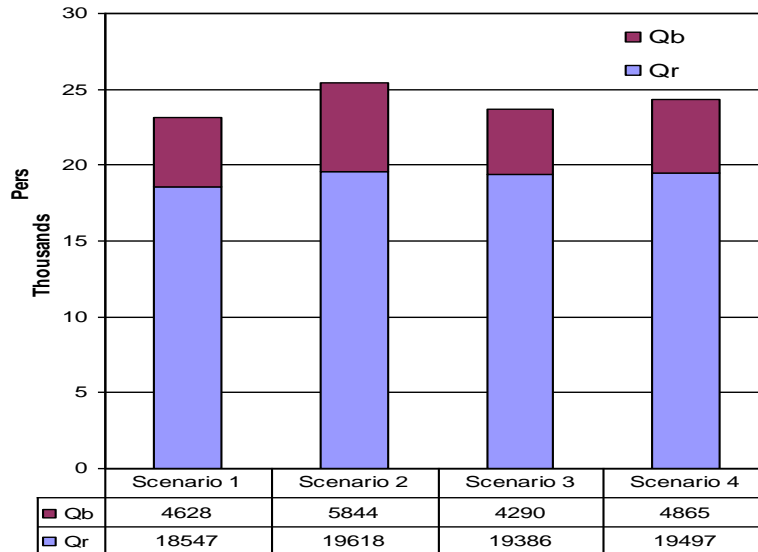
Figure 7 the comparisons of traffic demand

Another clear result that emerges from Table 5 is that as demonstrated by comparing with Scenario 1, producer surplus reduce 26% (Scenario 2), 3.3% (Scenario 3) and 16.38% (Scenario 4) respectively, which means only subsidizing bus implies the highest reduction in producer surplus. On the other hand, the change of consumer surplus in Scenario 2 is much larger than Scenario 3 and Scenario 4 would have been.

To exam the welfare impacts of different subsidies distribution, we take the ratio of social surplus in equilibrium relative to the total amount of subsidies (r) as the efficiency measurement and find only subsidizing bus operator can obtain the largest efficiency at the expense of smallest subsidies. Finally, to yield more transparent results, Table 6 summarizes main results of above discussions. By comparing with Scenario 1 (no subsidies), the ranking of other three scenarios suggests that subsidizing on the low-quality service (bus) has a relative high return in terms of reducing equilibrium fares, stimulating ridership and improving consumer surplus.

Table 6 Ranking Scenarios with respect to economic performances

	Scenario 2	Scenario 3	Scenario 4
The reduction effect in fares	1	3	2
The increase effect in total transit demand	1	3	2
Decrease in Produce Surplus	1	3	2
Increase in Consumer Surplus	1	3	2
r (the ratio of social surplus to subsidies)	1	3	2

The effects of different subsidy distribution schemes on equilibrium results are summarized in Proposition 2:

Proposition 2: *Under duopolistic competition arrangement, any subsidy proposals substantially contribute to reducing equilibrium fares, boosting traffic demand and increasing consumer surplus. Furthermore, from efficiency point of view, only subsidizing bus operator can achieve the best improvement of social welfare by using smallest subsidies.*

5. CONCLUSION

In this paper, the urban transit competition is portrayed as a 2-player, non-cooperative Bertrand game, in which bus and rail transit set fares as decision variables to maximize different objective functions. To evaluate the impacts of two ongoing changes (market structure and subsidy distribution), two numerical simulations based on Nanjing case are given and some useful findings are presented as follows:

(1) Viewing the structural change in the urban transit market, the first simulation directly relates to answer whether the mixed duopoly arrangement can help improve social benefits. A first conclusion to be draw from variation in nationalized degree (σ) concerns that the market change from pure private duopoly to mixed duopoly typically leads to the decrease of efficiency.

(2) Our second simulation assists in evaluating the effects of a change in the distribution of limited subsidies between two modes. To the end, a second conclusion can be draw from interchange comparisons of four subsidy scenarios, which states that, with limited subsidies, only subsidizing bus can achieve the best improvement of social welfare.

It is clear that the real-life case is far more complex than the setting constructed in this paper. In the future research, we can take several extensions into consideration. One is to extend this one stage price game into two-stage price–frequency game through accounting the impact of frequency on strategic decisions. Since there still exists cooperation between bus and rail transit, another further extension would be to examine the cooperative possibility between bus and rail transit.

Appendix 1 Derivation of elastic demand model

Re-writing the formulation for of composite costs, we get equation (A.1) and (A.2) in details:

(A.1)
$$cc = \frac{1}{-\theta} \ln \left[\sum_i \exp(-\theta C_i) \right]$$

(A.2)
$$\overline{cc} = \frac{1}{-\theta} \ln \left[\sum_i \exp(-\theta \overline{C}_i) \right]$$

Where the bar hat for variables indicates the reference state.
Substituting equations (A.1) and (A.2) into equation (2) yields

$$\exp\left[-\xi(cc - \overline{cc})\right] = \exp\left\{-\xi\left[\frac{1}{-\theta} \ln \sum_i \exp(-\theta C_i) - \frac{1}{-\theta} \ln \sum_i \exp(-\theta \overline{C}_i)\right]\right\}$$

$$= \left[\frac{\exp \ln \sum_i \exp(-\theta C_i)}{\exp \ln \sum_i \exp(-\theta \overline{C}_i)}\right]^{\frac{\xi}{\theta}} = \left[\frac{\sum_i \exp(-\theta C_i)}{\sum_i \exp(-\theta \overline{C}_i)}\right]^{\frac{\xi}{\theta}}$$

(A.3)

After some manipulation, this expression reduces to

$$Q_i = \overline{Q}^* \left[\frac{\sum_i \exp(-\theta C_i)}{\sum_i \exp(-\theta \overline{C}_i)}\right]^{\frac{\xi}{\theta}} \bullet \frac{\exp(-\theta C_i)}{\sum_i \exp(-\theta C_i)}$$

(A.4)

$$Q_i = \overline{Q}^* \left[\sum_i \exp(-\theta \overline{C}_i)\right]^{\frac{-\xi}{\theta}} \bullet \frac{\exp(-\theta C_i)}{\left[\sum_i \exp(-\theta C_i)\right]^{1-\frac{\xi}{\theta}}}$$

(A.5)

To derive the impact of price changes on travel demand, by differentiating Equation (11) with respect to price, we can obtain the following results:

(A.7)
$$\frac{\partial Q_i}{\partial P_j} = \overline{Q}^* \bullet \delta \bullet (\theta - \xi) \exp(-\theta C_i) \exp(-\theta C_j) \left[\sum_i \exp(-\theta C_i)\right]^{\frac{\xi}{\theta}-2}$$

Unfortunately, despite the simplicities of these models, the signs of the partial effects of price are hard to determine analytically. According to the characteristics of normal good, a high fare of bus will reduce its own demand and increase the demand for its competitor. Thus we can confirm that

$$\frac{\partial Q_i}{\partial P_i} < 0 \text{ and } \frac{\partial Q_i}{\partial P_j} > 0$$

Appendix 2 Derivation of price reaction function for bus

Again, the objective function of bus operator can be expressed like:

$$OF_b = \Pi_b = P_b Q_b + S_b Q_b - OC_b$$

Differentiating this equation with respect to P_b leads to

$$\frac{\partial \Pi_b}{\partial P_b} = Q_b + P_b \frac{\partial Q_b}{\partial P_b} + S_b \frac{\partial Q_b}{\partial P_b} = Q_b + (P_b + S_b) \frac{\partial Q_b}{\partial P_b} = 0$$

(A.8)

Summing Equations (A.6) and (10) into (A.8) and dividing $\bar{Q}^* \delta$ in both sides leads to the following equation:

$$\frac{\exp(-\theta C_b)}{\left[\sum_{i=b,r} \exp(-\theta C_i)\right]^{1-\frac{\xi}{\theta}}} + (P_b + S_b) \left\{ \frac{(-\theta)\exp(-\theta C_b)\left[\sum_{i=b,r} \exp(-\theta C_i)\right] - (\xi - \theta)\left[\exp(-\theta C_b)\right]^2}{\left[\sum_{i=b,r} \exp(-\theta C_i)\right]^{2-\frac{\xi}{\theta}}} \right\} = 0$$

(A.9)

Multiplying $\left[\sum_{i=b,r} \exp(-\theta C_i)\right]^{2-\frac{\xi}{\theta}} \bigg/ \exp(-\theta C_b)$ in both sides, equation (A.3) reduces to

$$\exp(-\theta C_b) + \exp(-\theta C_r) + (P_b + S_b)\left\{(-\theta)\left[\exp(-\theta C_b) + \exp(-\theta C_r)\right] + (\theta - \xi)\exp(-\theta C_b)\right\} = 0$$

(A. 10)

After some manipulation, this expression reduces to

(A. 11)
$$\left[1 - \theta(P_b + S_b)\right]\exp(-\theta C_r) + \left[1 - \xi(P_b + S_b)\right]\exp(-\theta C_b) = 0$$

$$\left[\theta(P_b + S_b) - 1\right]\exp(-\theta C_r) = \left[1 - \xi(P_b + S_b)\right]\exp(-\theta C_b)$$

$$\frac{\left[\theta(P_b + S_b) - 1\right]}{\left[1 - \xi(P_b + S_b)\right]} = \frac{\exp(-\theta C_b)}{\exp(-\theta C_r)}$$

(A.12)

Finally to simplify this equation, we take logarithm of two sides

$$C_r - C_b = \frac{1}{\theta}\ln\left[\theta(P_b + S_b) - 1\right] - \frac{1}{\theta}\ln\left[1 - \xi(P_b + S_b)\right]$$

(A.13)

It is easy to obtain the price reaction function for bus and this is one of the centre results of this paper. It is easy to verify the following inequality

(A.14) $$\frac{1}{\theta}\ln\left[\theta(P_b + S_b) - 1\right] - \frac{1}{\theta}\ln\left[1 - \xi(P_b + S_b)\right] + P_b - P_r + \alpha_1 \rho_v (T_b - T_r) + \frac{\alpha_3 \rho_w}{2}(h_b - h_r) = 0$$

Appendix 3: Derivation of price reaction function for rail transit

We rearrangement the objective function for rail transit, which is owned partially by stated.

$$OF_r = \sigma \frac{Q^*}{-\theta} \times \ln\left[\sum_i \exp(-\theta C_i)\right] + (1-\sigma)\left[(P_r + S_r)Q_r - OC_r\right]$$

(A.15)

For simplicity of expression, we write $\varphi = \sum_i \exp(-\theta C_i)$, Equation (A.15) can be rewritten as :

$$OF_r = \sigma \frac{\overline{Q}^*\delta}{-\theta} \times \varphi^{\frac{\xi}{\theta}} \ln\varphi + (1-\sigma)\left[(P_r + S_r)Q_r - OC_r\right]$$

(A.16)

To determine the reaction function for rail transit, the first order condition is

$$\frac{\partial OF_r}{\partial P_r} = \sigma \frac{\overline{Q}^*\delta}{-\theta} \times \left[\varphi^{\frac{\xi}{\theta}-1} \exp(-\theta C_r)(-\theta) + \frac{\xi}{\theta}\varphi^{\frac{\xi}{\theta}-1}\exp(-\theta C_r)(-\theta)\ln(\varphi)\right]$$

$$+ (1-\sigma)\overline{Q}^*\delta\left[\frac{\exp(-\theta C_r)}{(\varphi)^{1-\frac{\xi}{\theta}}} + (P_r + S_r)\left\{\frac{(-\theta)\exp(-\theta C_r)\varphi - (\xi-\theta)\left[\exp(-\theta C_r)\right]^2}{\varphi^{2-\frac{\xi}{\theta}}}\right\}\right]$$

(A.17)

Dividing Equation (A.17) by $\overline{Q}^*\delta\exp(-\theta C_r)\varphi^{2-\frac{\xi}{\theta}}$, we can get

$$\sigma\left[\varphi + \frac{\xi}{\theta}\varphi\ln(\varphi)\right] + (1-\sigma)\left[\varphi + (P_r + S_r)\left[(-\theta)\varphi - (\xi-\theta)\exp(-\theta C_r)\right]\right] = 0$$

(A.18)

Since, in this case, φ converges to zero with negative exponential values, we can get

$$\lim_{\varphi \to 0}\varphi\ln(\varphi) = 0$$

Then Equation (A. 18) yields these sequence calculations:

(A.19) $\quad \sigma\left[\varphi + 0\right] + (1-\sigma)\left[\varphi + (P_r + S_r)\left[(-\theta)\varphi - (\xi-\theta)\exp(-\theta C_r)\right]\right] = 0$

(A.20) $\quad \varphi + (1-\sigma)(P_r + S_r)\left[(-\theta)\varphi - (\xi-\theta)\exp(-\theta C_r)\right] = 0$

Substituting $\varphi = \sum_i \exp(-\theta C_i)$ into Eqn (A.20) and rearranging gives

(A.21) $\quad \left[1 - \xi(1-\sigma)(P_r + S_r)\right]\exp(-\theta C_r) = \left[\theta(1-\sigma)(P_r + S_r) - 1\right]\exp(-\theta C_b)$

(A.22) $\quad \dfrac{\left[\theta(P_r + S_s)(1-\sigma) - 1\right]}{\left[1 - \xi(1-\sigma)(P_r + S_r)\right]} = \dfrac{\exp(-\theta C_r)}{\exp(-\theta C_b)}$

By taking the logarithm on both sides of equation (A.22) and shifting some iterms, we can obtain the reaction function for rail transit:

(A.24) $\quad \dfrac{1}{\theta}\ln\left[\theta(P_r + S_r)(1-\sigma) - 1\right] - \dfrac{1}{\theta}\ln\left[1 - \xi(P_r + S_r)(1-\sigma)\right] + P_r - P_b + \alpha_1\gamma(T_r - T_b) + \dfrac{\alpha_3\gamma}{2}(h_r - h_b) = 0$

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