TRAVEL FUNCTIONS ON ARCS FOR DYNAMIC TRAFFIC ASIGNMENT MODELS IN URBAN NETWORKS

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INTRODUCTION

This paper presents an analysis of several travel time functions on arcs, which are suitable for Dynamic Traffic Assignment (DTA) problems on urban networks. Arcs could have different physical and operative characteristics in urban networks, and then different travel time functions. We identify the appropriated characteristics of such functions, for being incorporated within Dynamic Traffic Assignment (DTA) methods, for traffic estimation on a short period.

In some algorithms for DTA problems, a dynamic network is loaded (with a known O-D matrix) for each time interval by means of relaxations such as a static User Equilibrium (UE) Assignment (Shin et al., 2004; Janson, 1991; Jayakrishnan et al., 1995; Ran and Boyce, 1996; Chen and Hsueh, 1998).

Some research on DTA propose to use travel time functions which are dependent on vehicle flows and other dynamic variables (Friesz et al.,1993; Xu et al. 1999; Zhu and Marcotte, 2000).

In literature, travel time functions for flow entering in an arc at a given instant, have been considered following two ways: whole-link travel time models (WTTF) and the Lighthill, Whitham and Richards' hydrodynamic model (LWR) (Lighthill and Whitham, 1955; Richards, 1956). These models don't takes into account delays due to traffic lights and produce results which are not so realistic for urban networks.

This paper reviews some travel time functions on arcs, which are flow dependent and have appropriate characteristics for DTA problems. These functions could be used for dynamic urban networks with two types of roads, controlled access arcs and arcs with traffic lights. We present the performance of such travel time functions in the Frank-Wolfe algorithm (Frank and Wolfe, 1956), on a case study. Static assignment results are analyzed and the suitable characteristic for the dynamic assignment are identified.

The paper first presents the introduction, a review of literature on functions for assignment models and a description of the main travel time functions. Then, the case study is described, the travel functions are analyzed and the results are presented. Finally, conclusion and references are included.

REVIEW OF LITERATURE ON FUNCTIONS FOR ASSIGNMENT MODELS

A review of literature, on DTA models and travel functions used by dynamic equilibrium models, was carried out. There are several proposals for modelling time dependent networks by means of an analytic approach (Boyce et al., 1995; Chen, 1998; Chen et al., 2003; Ban et al., 2005, 2008, Carey 1986, 1987, 1992, 2001, 2009, et al.,). Some of such proposals use optimization mathematical programming. However, although travel time functions are critical components of DTA models, there is limited research on it. Usually, authors use dynamic theoretical functions with their analytical proposals, whose results could not be realistic.

The functions for describing traffic's dynamic in a network (TDF), and travel time functions on arcs (TTF) which are used in DTA models, frequently are decisive to define a solution algorithm, to guarantee a good algorithm' performance and to obtain coherent and realistic results. Hence, it is interesting to study the properties of different functions, in order to identify those that can be used in DTA models and for describing traffic's dynamic.

Although the DTA formulation and solution has been researched during three decades, there is a gap in the following points: the analytic treatment of flow dynamics on the network; the functional relation among the dynamic variables; the flow performance on each arc belonging to a path which is chosen by travellers; and the flow propagation on the network.

The desirable properties of TDF, for the static UE traffic assignment are quite studied; however for DTA case, it is necessary to study the functional forms of the dynamic variables, their properties and their relations with the flow dependent TTF.

The traffic's dynamic function (TDF) is an analytical expression which relates dynamic variables of the DTA model. Several DTA formulations consider three dynamic variables which propagate flow on the network. These variables are presented in Figure 1 and are describes as follows:

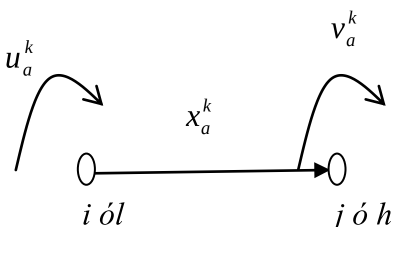

Figure 1. Arc (i, j) or (l,h) and propagation flows

$u_a^k(t)$ is the flow entering in arc *a* at time *t*;

$x_a^k(t)$ is the flow on arc *a* at time *t*;

$v_a^k(t)$ is the flow exiting from arc *a* at time *t*; and

k is the analysis time interval.

The flow on arc *a* at time *t* depends on the relation between $u_a^k(t)$ and $v_a^k(t)$.

Ran and Boyce (1996) define an ideal state for the UE ADT problem, where dynamic paths are obtained for each time instant, resulting from loading a network with a known trips pattern, as follows: "Travel time of a dynamic network has got equilibrium (UE), when for each O-D pair for each time instant, the travel time of travellers leaving at the same time is equal and the minimum."

A general form of a DTA model search to optimize a system, whose cost is shown in equation 1, where:

$$\Omega(k) = \sum_t \sum_{o-d} \sum_a u*_a(k)\tau_a(x*_a(k)) \quad \forall a \in A, \forall t \in T, \forall od \qquad (1)$$

a is an arc belonging to a set of arcs *A*;

o-d represents the number of trips between each origin-destination (O-D) pair;

t is a time instant into the analysis period T;

$\tau_a(x*_a(k))$ is the travel time on arc a, which is dependent on the flow on arc a at instant k in T. This travel time must guarantee equal minimum time for each O-D trip.

A modification of the DTA model proposed by Janson (1991) formulated as a mathematical program based on the formulation by Beckmann et al., (1956), where time is included, is shown in equations 2a to 2d and 3.

$$\underset{\mathbf{x},\mathbf{f}}{Minimize} \qquad z_J(\mathbf{x}) = \int_t \sum_a \int_{w=0}^{x_a(t)} c_a(w)dwdt \qquad (2a)$$

Subject to

$$\sum_{p \in R_{od}} f_p(t) = q^{od}(t) \qquad \forall od \quad \forall t \qquad (2b)$$

$$f_p(t) \geq 0 \qquad \forall od,\ \forall p \in R_{od} \qquad \forall t \qquad (2c)$$

$$x_a(t) = \int_s \sum_{od} \sum_{p \in R_{od}} f_p(s)\delta_{a,t}^p(s)ds \qquad \forall a \qquad \forall t \qquad (2d)$$

$$\delta_{a,t}^p(s) = \begin{cases} 1 & \text{If flow on path } p\text{, which started at time } s \text{ is on arc } a \text{ at time } t \\ 0 & \text{Otherwise} \end{cases} \qquad (3)$$

where,

$c_a(w)$ is the travel time on arc a, as a separable and increasing with time t function;

$f_p(t)$ is the instantaneous flow entering to path *p* at time t;

$q^{od}(t)$ is the travel demand between each O-D pair at time t; and
$\mathbf{x}$ is a vector of flow on arcs, which is time dependent.

This formulation has been the base for several DTA models, such as those proposed by Romph (1994), Jayakrishnan et al. (1995) and Ran and Boyce (1996), as is cited by Han y Heydecker (2006). Ran and Boyce (1996), Peeta and Ziliaskopoulos (2001) and Ban et al. (2008) are some papers that deal with the state of the art on DTA problems.

In macroscopic models for dynamic networks, there are two ways for estimating travel time on arc a at instant t: whole-link travel time models (WTTF) and the Lighthill, Whitham, and Richards hydrodynamic model (LWR) (Lighthill and Whitham, 1955; Richards, 1956). WTTF models analytically represent the flow propagation on the arc at time t, by means of the relations of the following dynamic variables: $\tau_a^k(t)$, $x_a^k(t)$, $u_a^k(t)$ or $v_a^k(t)$. WTTF model is based on kinematic wave theory, where traffic is likened to a hydraulic fluid. García et al., (2006) describe this model as "a traffic representation, by means of differential equations where the traffic behaviour in a space-time point is only affected by the system's state in a vicinity of such point. This model is formulated in terms of density, flow and speed on arcs."

Ran and Boyce (1996) consider WTTF as the sum of two flow components, one for free-flow, which depends on $u_a^k(t)$ and $x_a^k(t)$, and the other for delay related to queues which depends on $v_a^k(t)$ and $x_a^k(t)$; travel time function is considered as dependent on density, speed and number of vehicles on an arc. Later, Ran et al., (1997) conclude that the TDF which are used in many researches on DTA, have lack of bases on traffic flow theory.

That is one of the reasons why we present a framework for the formulation of DTA models, with base on traffic engineering. Hence, available delay models are studied, identifying useful functions and incorporating such function to TDF, in order to analyze their use in DTA models.

Carey and McCartney (2002) analyze WTTF (whole-link travel time) models, which are widely used in mathematical programming models for the DTA problem. They analyze the WTTF behaviour, assuming that flow-entering on arc a is known; however, this analysis do not considers a network. Some analytic properties for flow dependent travel time functions (on an arc), with constant entering and exiting flows, are identified; such functions are linear, quadratic and n-polynomial. The realism of functions relations is not questioned.

In literature, several papers on DTA use flow dependent travel time functions, $\tau_a(x_a(k))$, i.e., $\tau(t) = f(x(t))$, where $\tau(t)$ is travel time on arc a for vehicles entering at time t, and $x(t)$ is the flow (number of vehicles) on arc a at time t (Carey and Ge, 2003). Friesz et al. (1993) use a linear travel time function for a DTA model, $\tau(t) = a + bx(t)$, where *a* and *b* are constants, $x(t) = \int_0^t (u(s) - v(s))ds$ is the number of vehicles on arc a at time t, and $u(t)$ and $v(t)$ are entering and exiting flows for arc a at time t, respectively. The quadratic function is as follows $\tau = a + bx + cx^2$ (Carey y Ge 2001); specifically, Chen et al. (2003) use the function $c_a(t) = 1 + 0.01(u_a(t))^2 + 0.001(x_a(t))^2$, $\forall a, t$, where $u_a(t)$ is the entering flow on arc a during interval t, and $x_a(t)$ is the flow on arc a at the beginning of interval t.

Ran and Boyce (1996; 2002) present some non linear functions for travel time on arcs: $\tau_a(t) = \tau_a[x_a(t), u_a(t), v_a(t)]$ $\forall a$, i.e. $\tau_a(k) = g_{1a}(k) + g_{2a}(k)$, where: $g_{1a}[x_a(t), u_a(t)]$ is the instantaneous travel time of flow flowing on arc a; $g_{2a}[x_a(t), v_a(t)]$ is the instantaneous delay time on queue; and $\tau_a(t)$ is the current travel time of travellers on arc a at time t. For example,
$g_{1a}(k) = \beta_{1a} + \beta_{2a}[u_a(k)]^2 + \beta_{3a}[x_a(k)]^2$ and $g_{2a}(k) = \beta_{4a} + \beta_{5a}[v_a(k)]^2 + \beta_{6a}[x_a(k)]^2$.

Ran and Boyce propose two type functions, which can be used for representing controlled access arcs and arcs with traffic lights.

1. For controlled access arcs, Ran y Boyce (1996) propose the following:

   Under prevalent constant traffic conditions, instantaneous travel time $\tau_a(t)$ is equal to the travel time of travellers on arc a, i.e.

   $$\tau_a(k) = D_{a1}(k) + D_{a2}(k) \ \ \forall a, \ D_{a1} = 3600\frac{l_a}{w_{a0}} \ \text{ and } \ D_{a2} = \alpha[e_a(k)]^m$$

   where:

   D_{a1}: intersection free-flow time (seconds)
   D_{a2}: travel time, or delay at a queue on arc a plus time for crossing the corresponding intersection (seconds)
   w_{a0} : free-flow speed on arc a (km/h),
   l_a : length of arc a (km),
   m : integer calibration parameter,
   α : real calibration parameter,
   $e_a(k) = \frac{[(u_a(k) - v_a(k))(\Delta k/2)] + x_a(k)}{l_a}$: flow entering in arc a at the beginning of interval k,
   $u_a(k)$: flow entering in arc a at interval k
   $v_a(k)$: flow exiting from arc a at interval k
   $x_a(k)$: flow on arc a at interval k

2. For arcs with traffic lights, Ran y Boyce (1996) propose the following:

   Under prevalent constant traffic conditions, the travel time stochastic function on arc a during time interval k, where $\Delta k \geq 5$ minutes, is the following:

$\tau_a(t) = D_{a1}(t) + D_{a2}(t)$, with $D_{a2}(t) = \alpha_a d_{a1}(k) + \beta_a d_{a2}(k)$ i.e., $\tau_a(t) = D_{ai}(t) + \alpha_a d_{a1}(k) + \beta_a d_{a2}(k)$,

where:

$a1$ corresponds to the uncongested part of arc a, and $a2$ to the congested part,

$D_{a1}(t)$ is the travel time or crossing time in the uncongested part of arc a (seconds),

$D_{a2}(t)$ is the travel time or delay in a queue, in the congested part of arc a (seconds) plus the time for crossing the corresponding intersection (seconds).

$$D_{a1}(t) = 1800 \frac{2l_a e_{am} - [x_{a2}(k) + x_{a2}(k+1)]}{w_{a1}(k) e_{am}}$$

where:

l_a: length of arc a (km),

e_{am} : maximum density (vehicles/km),

$x_{a2}(k)/e_{am}$: queue length in the congested part of the arc, $a2$, at the beginning of interval k,

$x_{a2}(k+1)/e_{am}$: queue length in the congested part of the arc, $a2$, at the end of interval k,

$[x_{a2}(k) + x_{a2}(k+1)]/2e_{am}$: queue length,

$w_{a1}(k)$: free-flow speed in the uncongested part of arc a (km/h)

To evaluate the travel time in the congested part of arc a, the exit capacity of such arc is considered (saturation flow). This involves the analysis of deterministic and stochastic components. The average delay for vehicles on arc a, which arrive at intersection during interval k, is the following sum (Ran and Boyce, 1996):

$D_{a2}(k) = \alpha_a d_{a1}(k) + \beta_a d_{a2}(k)$, where $d_{a1}(k)$ is the uniform delay which is a consequence of traffic control.

A delay for just one green period, considering uniform arrival, was proposed by Webster (1958) (Ran and Boyce, 1996): $d_{a1}(k) = \dfrac{0.5c[1 - g(k)/c(k)]^2}{1 - \rho_a(k)\, g(k)/c(k)}$, where,

$c(k)$: a cycle during interval k (seconds)

$g(k)$: the green time in a cycle during interval k (seconds)

$\rho_a(k) = \dfrac{u_{a2}(k)}{\mu_a(k)}$: the saturation degree in the exit of arc a during interval k,

$u_{a2}(k)$: the arrival rate at the intersection,

$\mu_a(k)$: the saturation flow or discharge rate (vehicles/h),

$d_{a2}(k)$: the overflow delay, including the effects of random arrivals, as well as the overflow delays which are suffered by vehicles arriving during the specific flow period.

Linear and some non-linear WTTF of Ran and Boyce (1996) have been widely used by several authors: Astarita (1995, 1996), Wu et al. (1995, 1996), Xu et al. (1999), Carey and McCartney (2002), Zhu and Marcotte (2000). Nevertheless, such functions are criticized because their lack of realism (Carey, Ge and McCartney, 2003). Carey et al., (2003, 2004, 2005) present an extensive review of literature on WTTF, and analyze FIFO and other desirable properties.

Carey et al. (2003) propose a travel time model as the estimated mean flow, $w(t)$, of the entering rate in arc a at time t, $u_a^k(t)$, and the exiting rate after going though arc a, $v_a^k(t+\tau(t))$, i.e., $\tau(t)=h(w(t))$; then $w(t)=\beta u(t)+(1-\beta)v(t+\tau(t))$ with $0\le\beta<1$. They show that their model has FIFO and other desirable properties.

Carey and Ge (2003) compare several WTTF forms, combined with the estimated travel time on arc a, as a estimation of the mean flow rate $w(t)$, i.e., $\tau(t)=h(w(t))$, with the Lighthill, Whitham, Richards (LWR) model. The analysis divides the arc into segments. The convergence of a discretised travel time model is review by Carey and Ge (2005).

Some authors analyze characteristics and properties of the most used functions, in terms of their performance and adaptability to be incorporated to DTA models (Carey and Ge, 2005). They also review FIFO, causation and consistency performance with the static model.

Carey and Ge (2007) present a discretised version of the WTTF model, proposed by Carey et al., (2003), and analyze FIFO properties for only one arc, assuming that $u_a^k(t)$ is constant.

Nie and Zhang (2005) affirm that, if FIFO rule is not respected for the linear, linear in tracts and non linear WTTF, which are broadly used in literature, then the obtained exit flows are negative. For these authors, none of such functions respects the FIFO rule, and some classes of linear in tracts and non-linear functions could respect FIFO rule with a constraint on the entering flow. They stand out the difficulty of finding delay functions as $\tau(t)=f(x(t))$, which can be realistic and respect FIFO rule.

Jayakrishnan et al., (1995) introduce a travel time function on arc a, as a modified version of the Greenshields model, in terms of density and speed. Shin et al., (2004) verify the behaviour of this function, in a DTA model, and use the Greenshield model (1934) in a macroscopic analysis of roads without traffic lights.

García et al., (2005) propose a WTTF continuous model, based on the one proposed by Ran and Boyce (1996) adding an arc's capacity constraint. They carry out a numeric example, for a single arc outside of a network, considering a constant entrance flow. The result shows coincidence with the LWR model.

The WTDFs which are thoroughly used in dynamic loading problems has strong limitations, since they should have desirable mathematical characteristics, as being non-decreasing, continuous and differentiable, besides of other desirable properties such as to respect FIFO rule (Tortoiseshell and Ge, 2007). Zhu and Marcotte (2000), Tortoiseshell and McCartney

(2002) and, Huang and Lam (2002) satisfy this requirement by means of travel time lineal functions on arcs, which is inexact given that the travel time on an arc has a non-linear behaviour in a congested network.

Hence, this paper starts from the Ran et al. (1997) framework; then some delay functions in arcs with traffic lights are upgraded, such functions are introduced in DTA problems, and combinations of functions are used with the Frank-Wolfe algorithm, as a sub-routine of a process for the dynamic flow estimation in urban networks with arteries and arcs with traffic lights. Although it is accepted that the flow propagation in a dynamic network depends on the dynamic variables, such as entering flow, exiting flow and flow on the arc, in a time instant, we just consider a travel time function dependent on the number of vehicles in the arc in a time instant. A relation among the dynamic variables is proposed in a bi-level DTA model, whose inferior problem is a temporary user equilibrium traffic assignment in order to fix momentarily, travel time in arcs. The DTA model is studied later in this paper. It is necessary that any analytic approach proposal for the DTA problem coincide with the network behaviour when time is constant, under certain conditions.

A review of some travel time functions in arcs is presented below; these function which consider delay, are useful to estimate travel times in arcs with traffic lights and controlled access arcs. Additionally, a case study is presented, where a user equilibrium traffic assignment Wardrop (1956) is used.

TRAVEL TIME FUNCTIONS

The travel functions to be analyzed are flow dependent for two types of roads, those with controlled access and those controlled by traffic lights. Functions BRP, Akcelik and Webster are among the analyzed travel functions.

In an urban network could have arterial corridors (arcs with traffic lights) and controlled access roads. The former have coordinated programs, generally of two phases (eliminating turn to left), and can be macroscopically analyzed by means of functions which at least include a fix delay and an overflow delay.

BRP Function

BRP function was proposed by the U.S. Bureau of Public Roads (1964). It has been widely used in traffic assignment problems in urban networks. This function is continuous, increasing, differentiable, and have a parabolic form. It has two terms, the free-flow travel time and another term which is a factor of the former and increase with the flow-capacity rate (x/Q). This function does not include delays for traffic lights. The BRP function is the following: $t(x_a^n) = t_a^0\left(1+\alpha_a\left(\dfrac{x_a^n}{Q_a}\right)^{\beta_a}\right)$,

where

x_a^n : flow on arc a ,

$t_a^0 = \frac{L_a}{v_f}$: cost of a free-flow trip ($f_a(0)$),

L_a : length of arc a ,

v_f : free-flow speed or maximum speed on arc a ,

Q_a : capacity of arc a ,

α_a y β_a : arc' parameters

The BPR function is simple and has a good performance in network loading processes. Then, they are widely used in planning transportation models.

Travel Time Function with Webster Function Delay

The analytic traffic models incorporate three delay types (Roess et al., 2004):

- Uniform delay: in an intersection with traffic light, this delay considers uniform arrival, stable flow and no individual cyclical characteristics.
- Random delay: it is an additional delay to the uniform one, where it is assumed that the flow is randomly distributed; it usually exists in isolated intersections.
- Overflow delay: it is an additional delay which exists when the capacity of an individual phase or a series of phases is smaller than the demand or the flow arrival rate.

The delay due to the influence of the platoon flow is introduced as an adjustment to the uniform delay. In many current models, the random delay and the overflow delay are combined in a single function, which is just named overflow delay.

The Webster Delay Function (1958) includes delays for traffic lights. This function is composed of the free-flow travel time and the uniform Webster delay (in s/vehicles) which becomes constant when x/Q is equal to one. At this moment, the travel time also includes an overflow delay, i.e., $t_a(x) = t_a^0 + d_a^1 + d_a^2$,

where:

$t_a^0 = \frac{L_a}{v_f}$: free-flow travel time ($f_a(0)$)

$$d_a^1 = \left(\frac{C}{2}\right) * \left\{ \frac{\left(1 - g/C\right)^2}{1 - \left[\min\left(1, (X) * g/C\right)\right]} \right\} \text{ (s/vehicle)}$$

$d_a^2 = \frac{T}{2}\left[1 - X\right]$: (s/vehicle)

d_a^1 : uniform delay in arc a ,

C_a : cycle of traffic light (seconds),

Q_a : capacity of arc a (vehicles/h),

$X = (x/Q)$: saturation degree,
x: flow arrival rate (vehicles/h),
d_a^2: average overflow delay,
T: analysis time.

Akcelik Function

The travel time function with delay produced by congestion, called Akcelik Function, is the following: $t = t_a^0 + d_a^1 + \frac{1}{4} T_f \left\{ (X-1) + \sqrt{(X-1)^2 + \frac{8A}{CT_f} X} \right\}$.

As in the previously mentioned TTFs, $t_a^0 = \frac{L_a}{v_f}$ is the free-flow travel time ($f_a(0)$) and d_a^1 is the Webster uniform delay in arc a. The difference is that, the overflow delay is a time function which depends on the time of the Akcelik congestion function (Akcelik, 1991), i.e., $t = \frac{1}{4} T_f \left\{ (X-1) + \sqrt{(X-1)^2 + \frac{8A}{QT_f} X} \right\}$ (s/km),

where:
$X = (x/Q)$: saturation degree,
x : flow arrival rate (veh/h),
T_f: analysis time, which is defined by the analyst (h),
Q_a: capacity of arc a (veh/h),
A: parameter of arc a (an attribute in the network).

Delay is larger as time T_f is increased. The time dependent function includes overflow periods, defined by means of saturation degree (volume/capacity rate, $X = (x/Q)$) larger than one.

Other Functions

The best function for modeling travel time when the delay at a intersection has a large influence in the total travel time, is the Akcelik congestion function (Akcelik, 1991). It is the following:

$$t = t_0 \left(1 + \frac{AX}{Qt_0(1-X)} \right), \text{ (s/km)},$$

where:

A: coefficient delay under the current conditions, defined as $A = I\kappa$,

I : factor which represents intensity of delay elements in the arc (as density of intersections per kilometer), and
k : delay parameter which contains the randomness level (or rate) of arrivals and services in points on the arcs where traffic is interrupted.

This function is limited to $x/Q < 1$.

The travel time function with HCM delay (HCM 2000) is the following:

$$t_a(x) = t_a^0 + d_a^1 PF + d_a^2 + d_a^3,$$

where,
$t_a^0 = \frac{L_a}{v_f}$ is the free-flow travel cost ($f_a(0)$) and d_a^1 is the uniform Webster delay of arc a ,
$d_a^2 = 900T\left[(X-1) + \sqrt{(X-1)^2 + \left(\frac{8klX}{QT}\right)}\right]$,
PF : adjustment factor due progression,
$X = \left(x/Q \right)$: saturation degree
x : flow arrival rate (veh/h),
T : analysis time (h),
Q : capacity of arc a (veh/h),
k : increasing delay factor for acted controls (0.5 for programmed or fix time controls),
I: adjustment factor for a filter in the entering up-stream flow (coordinated traffic lights) (1.00 for the análisis of a single intersection),
d_a^3: delay due a pre-existing queue (s/veh).
The delay due a pre-existing queue requires a complex model; their detail level would correspond to a microscopic analysis.

Functions Comparison

The behaviour of five travel time functions was compared, for arcs with traffic lights.

Figure 2 shows the behaviour of some travel time functions on arcs with traffic lights; in the horizontal axis, the volume-capacity rate ($X = (x/Q)$) for different flow (x) is represented, while that in the axe Y, the rate $t(x)/t(0)$ is represented. Each function includes free-flow travel time (t_a^0) for a 60 km/h, uniform Webster delay (d_a^1), and overflow delay (d_a^2), i.e. $t_a(x) = t_a^0 + d_a^1 + d_a^2$.

The example presented in Figure 2, considers the following characteristics: the arc has 8 lanes, S= 951 veh/h, C= 120 s, g=60 and Q=7,611 veh/h.

For the BPR function, α=0.55 and β= 2 were used (adjustment for comparison); for the Akcelik's time dependent function, T_f=2 min and A=0.75 were used.

The HCM function considered k=0.5 (arcs with fix phases), *I*=0.922 (for upstream X=0.4), and T=2 min, where the "*I*" factor has a small influence in the function.

In the example, t(0) changes in each function and includes fix and overflow delays, except for the BPR and the Akcelik without delay functions Akcelik (1991).

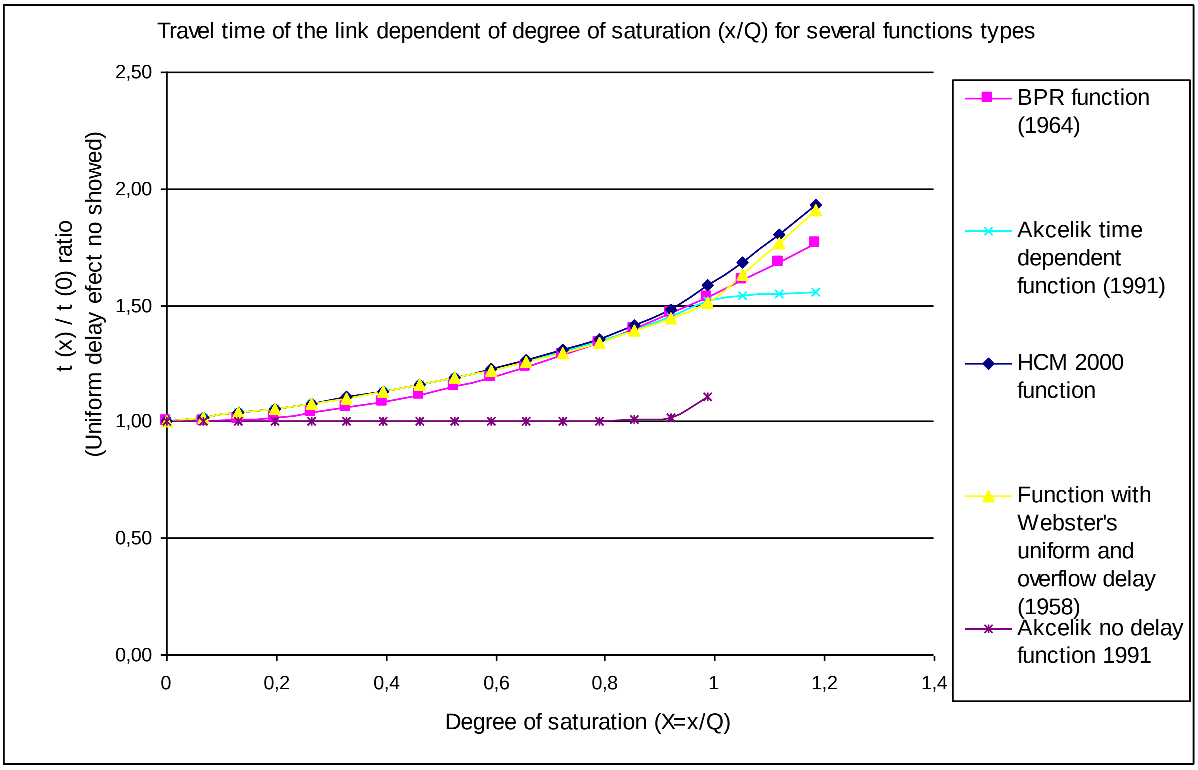

Figure 2. Behaviour of five travel time functions, for arcs with traffic lights

Figure 3 shows the behaviour of the same functions of Figure 2, but in a different representation; the difference is that the travel time in the arc, which varies with the number of vehicles x, is now divided by the flow-free travel time $t(0)$ without considering delays, i.e., it is the free-flow travel time constant. This representation let us to identify the differences among, the functions with fixed and overflow delays in the arcs with traffic lights, and the functions without delays due to traffic controls. Obviously, the BPR function ignores the real delay effects due to traffic lights.

Figure 3 contains x/Q is in the horizontal axis, for different flow (x), and $t(x)/t(0)$ in the vertical axis, and considers the same parameters for the functions that Figure 2.

An interesting finding from the functions comparison is that, the Webster delay function has a similar behavior that the HCM function. Both functions are criticized because they overestimate the travel time when the flow surpasses the capacity.

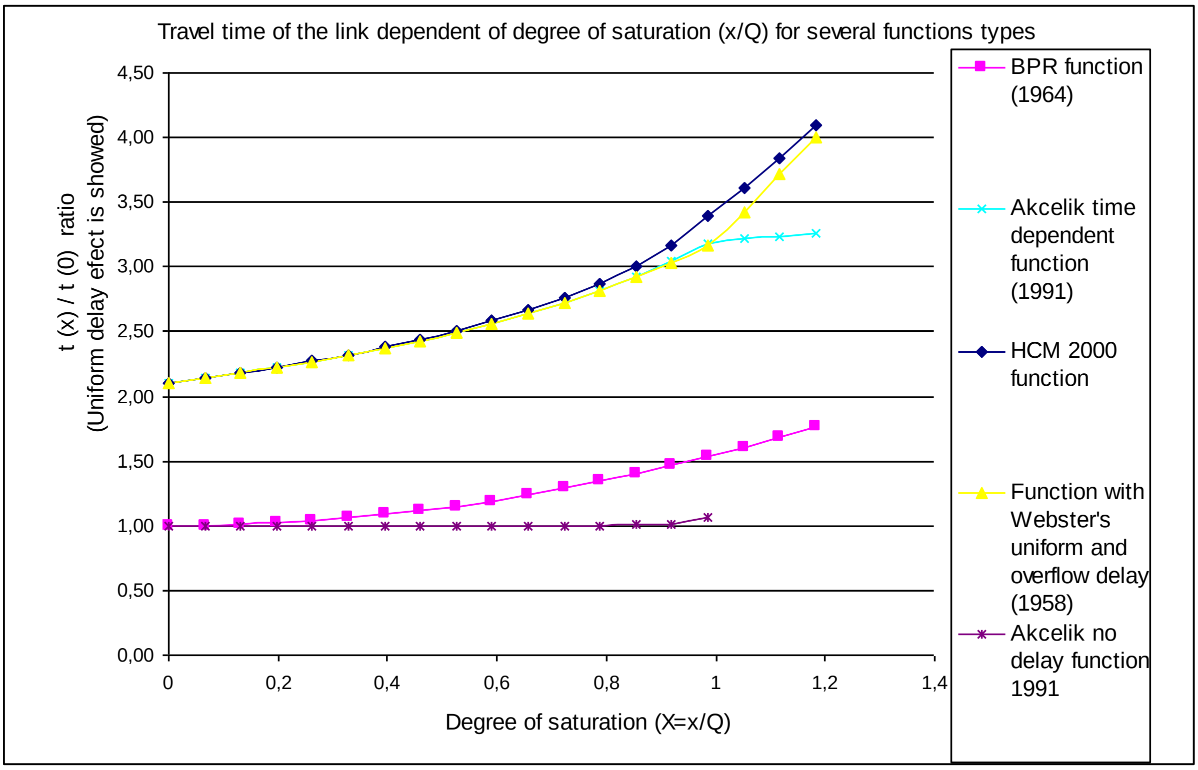

Figure 3. Behaviour of five travel time functions, where x/Q is in the horizontal axis and $t(x)/t(0)$ in the vertical axis

CASE STUDY (NETWORK)

In order to compare the travel time functions in a real situation, a real network was used, which is formed of two paths, called Revolución y Segundo-Piso-Periférico, between one O-D pair. This is a sub-network of the Mexico-City network. The Revolución path is composed of arcs with traffic lights, and the Segundo-Piso-Periférico path is a corridor with controlled access arcs. The length of each path is approximately 7.3km.

Therefore, there are two kinds of arcs and, they need two different travel functions on arcs.

Figure 4 shows a map of the network, and Table 1 shows the characteristics and parameters of the travel time functions in the arcs.

The case study features are the following:

a) The network has 32 arcs, 32 nodes, one origin, one destination and two paths.

b) The attributes of arcs are length, number o lanes, lane's width, flow direction, traffic count volume and free-flow speed.

c) The attributes of intersections are traffic light phases, number of lanes for each movement, traffic count volume per each movement, control type, cycle time, effective green time, saturation flow, arrival rate, saturation degree and fix delay.

d) A known O-D matrix, which can vary with the time.

e) The traffic count volumes, which vary with the time.

The described network and information are used in the next section to analyze the dynamic behavior of several travel time functions, using the same network loading process (user equilibrium).

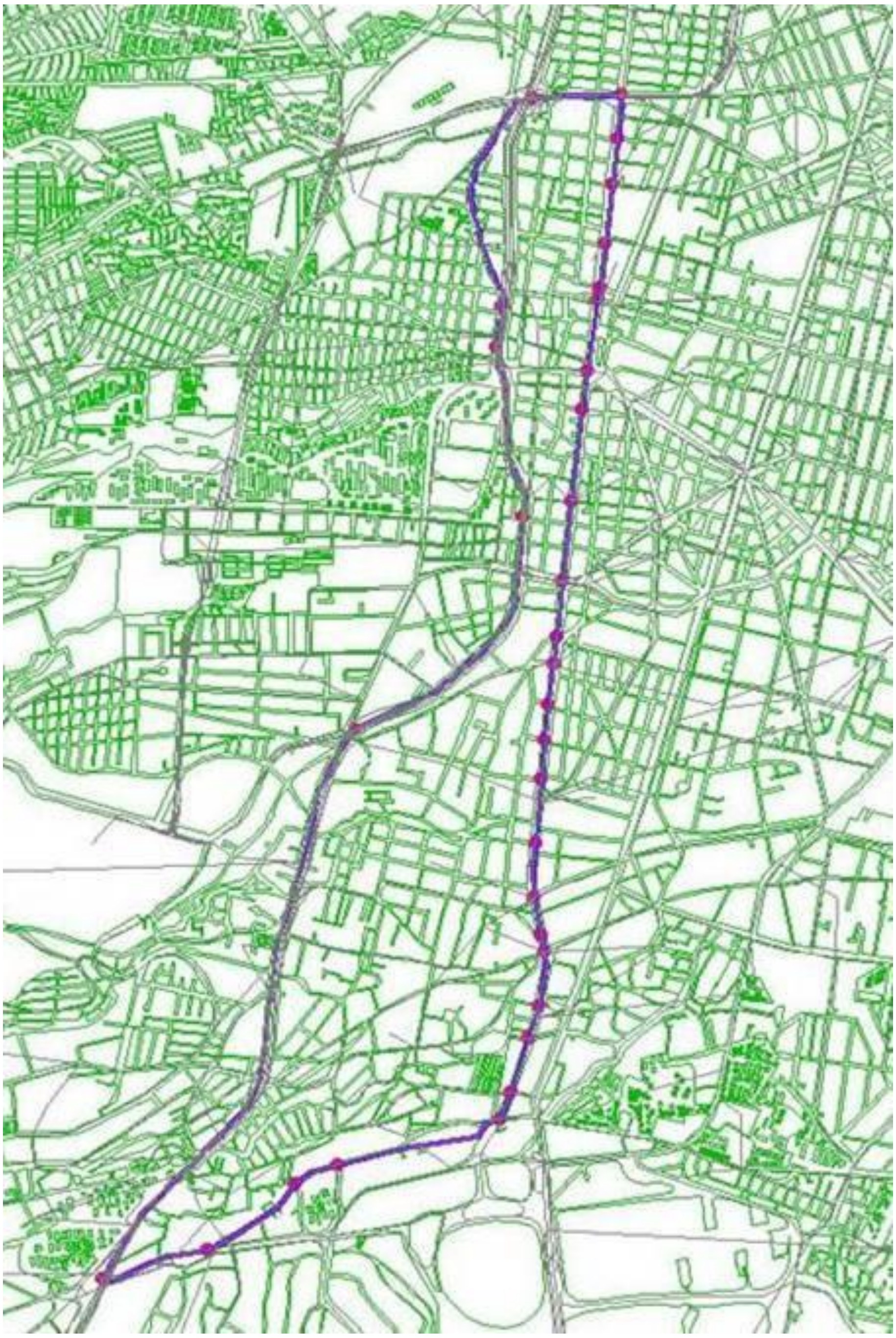

Figura 4. Modelo de red con arcos cuyas funciones son diferentes. Elaboración en TransCAD.

Table 1. Characteristics and parameters of the travel time functions in the arcs of a Mexico City sub-network with two paths.

		Nª Arco	Effective green time (s)	Signal cycle length	Saturation flow rate (veh/h/c)	Capacity of lane (veh/h/c)	Number of lanes T	Total capacity (veh/h/c)	Max speed (km/h)	t (0) (s / m of link)	link length SIG (m)	t (0) link (s/km)	Alfa (α)	Beta (β)	A
Revolución	Andrea de Castagn	29	60,31	120,44	1900	951	8	7611	60	0,06	227,67	0,0037945000	0,550	2,000	0,75
	Rembrandt	57	64,84	120	1900	1027	8	8213	60	0,06	226,84	0,0037806667	0,550	2,000	0,75
	Antonio Van Dick	31	64,84	120	1900	1027	8	8213	60	0,06	306,46	0,0051076667	0,550	2,300	0,75
	Mixcoac (2 sem)	58 y 60	69,77	120	1900	1105	5	5523	60	0,06	280,98	0,0046830000	0,600	2,400	0,75
2	Molinos	59	55,12	120	1900	873	6	5236	60	0,06	361,57	0,0060261667	0,400	2,500	0,75
	Castañeda	62	64	120	1900	1013	4	4053	60	0,06	199,77	0,0033295000	0,600	2,350	0,75
	Merced Gómez	61	79,52	120	1900	1259	4	5036	60	0,06	463,25	0,0077208333	0,420	2,600	0,75
3	Barranca del Muert	40	54,66	120	1900	865	4	3462	60	0,06	402,26	0,0067043333	0,420	2,000	0,75
	Macedonio Alcalá	41	70,21	120	1900	1112	4	4447	60	0,06	283,11	0,0047185000	0,590	2,300	0,75
	Corregidora	63	84,97	120	1900	1345	4	5381	60	0,06	131,75	0,0021958333	0,900	3,000	0,75
	Sofia	42	77,46	120	1900	1226	4	4906	60	0,06	212,05	0,0035341667	0,800	2,300	0,75
	R. Boker	64	62,06	120	1900	983	4	3930	60	0,06	180,14	0,0030023333	0,750	2,000	0,75
	Tlacopac	65	76,57	120	1900	1212	4	4849	60	0,06	196,04	0,0032673333	0,800	2,400	0,75
	Maria Luisa	66	76,19	120	1900	1206	4	4825	60	0,06	323,82	0,0053970000	0,600	2,300	0,75
	Desierto de los Leo	67	104,65	120	1900	1657	4	6628	60	0,06	269,95	0,0044991667	0,400	4,800	0,75
	Altavista	45	63,97	120	1900	1013	4	4051	60	0,06	194,89	0,0032481667	0,700	2,150	0,75
4	Madero	78	61,06	120	1900	967	4	3867	60	0,06	84,14	0,0014023333	0,950	2,000	0,75
	Dr Gálvez	46	72,29	120	1900	1145	4	4578	60	0,06	281,4	0,0046900000	0,600	2,300	0,75
	Rey Cuauctémoc	68	74,7	120	1900	1183	4	4731	60	0,06	168,87	0,0028145000	0,850	2,350	0,75
	Altamirano	48	59,69	120	1900	945	4	3780	60	0,06	283,9	0,0047316667	0,600	1,900	0,75
	Eje 10 Sur	49	52,21	120	1900	827	4	3307	60	0,06	142,43	0,0023738333	0,700	1,800	0,75
	Río MagdalenaxIgl	50	87	120	1900	1378	5	6888	60	0,06	832,11	0,0138685000	0,250	3,300	0,75
	Río MagdalenaxFra	69	87	120	1900	1378	5	6888	60	0,06	235,44	0,0039240000	0,700	3,000	0,75
	S Jerónimo	70	87	120	1900	1378	5	6888	60	0,06	544,65	0,0090775000	0,350	3,000	0,75
5	G. San Jerónimo	71	44,73	120	1900	708	5	3541	60	0,06	533,87	0,0088978333	0,320	2,000	0,75
	Sum Revolución										7367,36	0,1227893333			
	Sum flow														
Segundo Piso P	San Antonio Segur	1			2200	2200	3	4400	60	0,06	436,6	0,0072766667	0,040	3,000	0,1
Periférico		74			2200	2200	2	4400	60	0,06	1188,13	0,0198021667	0,035	3,400	0,1
		73			2200	2200	3	4400	60	0,06	201,27	0,0033545000	0,040	3,000	0,1
		75			2200	2200	3	4400	60	0,06	878,91	0,0146485000	0,040	3,000	0,1
		76			2200	2200	3	4400	60	0,06	1464,02	0,0244003333	0,040	3,000	0,1
		77			2200	2200	3	4400	60	0,06	3104,63	0,0517438333	0,040	3,000	0,1
	Sum Segundo Piso Periférico										7273,56	0,1212260000			

TRAVEL FUNCTIONS ANALYSIS

Two network loading processes were carried out for the previously described network; the assignment used the convex combinations o Frank-Wolf algorithm (Frank-Wolfe, 1956). Each process involved two travel time functions, which were suitable for each type of arc.

The cost function and the algorithm for the estimation of flow on each arc, have interesting characteristics. It is important to guarantee that the travel time functions ($t_a(x)$) are positive, increasing and differentiable, that the total cost function ($z(x_a)$) is increasing, differentiable and convex.

Also, it is important to guarantee that existence and unique solution requirements are accomplished by the optimization sub-problem (it is a cost minimization problem). Then, an analysis for determining the limits of the parameters (as the analysis time parameter, which depends on the flow/capacity rate) was carried out.

The network loading processed and the travel time function are described below.

Process 1: Network loading with a mixed travel time function

The mixed travel time function is composed of, the BPR function for the controlled access arcs, and the free-flow travel time plus the Webster uniform delay plus the Webster overflow delay, for the arcs with traffic lights.

For a controlled access arc, the travel time on the arc is always positive, and the function is increasing. The contribution of this arc to the system' cost ($z(x_a)$) is positive and contribute to make that it be increasing. It is as follows:

$z(x_a)=\sum_a\left[t_a^0 x_a^n+\frac{t_a^0\alpha\left(x_a^n\right)^{(\beta+1)}}{(\beta+1)Q_a^{\beta}}\right]+z(x_a^n)$, where $z(x_a^n)$ is the cost of other type of arcs.

The contribution of a controlled access arc in the minimization subproblem is the following:

Minimize $_{0\le\lambda\le 1}$ $z\left[x_a^n+\lambda(y_a^n-x_a^n)\right]=\sum_a\int_0^{x_a^n+\lambda_n(y_a^n-x_a^n)}t_a(\omega)d\omega$

$\sum_a(y_a^n-x_a^n)t_a^0\left\{1+\frac{\alpha}{Q_a^{\beta}}\left[x_a^n+\lambda(y_a^n-x_a^n)\right]^{\beta}\right\}+\frac{dz\left[x_a^n+\lambda(y_a^n-x_a^n)\right]}{d\lambda}=0$, where the last term corresponds to other type of arcs.

The terms of the minimization sub-problem, for the searching of λ, are dominated by the difference $(y_a^n-x_a^n)$. Hence, the algorithm requires that this factor is positive. For the BPR function, this factor is always positive.

For an arc with traffic lights, the travel time on the arc is composed of: the free-flow travel time plus the uniform Webster delay (which becomes a constant when x/Q=1) and plus the overflow delay, i.e., $t_a(x)=t_a^0+d_a^1+d_a^2$, where:

$$d_a^1=\left(\frac{C_a}{7200}\right)*\left\{\frac{\left(1-{g_a}/{C_a}\right)^2}{1-\left[\min\left(1,(X_a)*{g_a}/{C_a}\right)\right]}\right\}$$

$d_a^2=\frac{T}{2}\left[X_a-1\right]$ (it is used when $X_a=\left({x_a^n}/{Q_a}\right)\ge 1$).

T is the analysis time, which is defined by the analyst previously to the network loading process.

This function has a discontinuity in $X_a=1$, but it is increasing and differentiable respect to x_a, before and alter that $X_a=1$. All of its terms are positive and contribute to the increasing of the function.

The arc contribution to the system' cost, $z(x_a)$, is the following:

For $0<\left({x_a^n}/{Q_a}\right)<1$, $z(x_a)=\sum_a\left\{t_a^0x_a^n-\left[\left(\frac{C_a}{7200}\right)\left(\frac{Q_aC_a}{g_a}\right)\left[1-\left(\frac{g_a}{C_a}\right)\right]^2\ln\left|1-\left(\frac{x_a^n g_a}{Q_aC_a}\right)\right|\right]\right\}+z(x_a^n)$,

where $z(x_a^n)$ is the cost of other type of arcs.

The logarithm of the second term of the sum is negative, then it is a positive contribution to $z(x_a)$. Hence, $z(x_a)$ is an increasing function.

For $\left(x_a^n \big/ Q_a \right) \geq 1$, $\quad z(x_a) = \sum_a \left\{ t_a^0 x_a^n + \left[\left(\frac{C_a}{7200} \right) \left[1 - \left(\frac{g_a}{C_a} \right) \right] x_a^n + \left[\frac{T}{2} x_a^n \left(\frac{x_a^n}{2Q_a} - 1 \right) \right] \right] \right\} + z(x_a^n)$,

where $z(x_a^n)$ is the cost of other type of arcs.

If $1 < \left(\frac{x_a}{Q_a} \right) < 2$, the contribution of the arc to the system's cost is negative. Hence, in order to guarantee that $z(x_a)$ be increasing, T has to be constrained as follows: $T < 4\left[t_a^0 + \left(\frac{C_a}{2} \right)\left(1 - \left(\frac{g_a}{C_a} \right) \right) \right]$ (for arcs with traffic lights).

The contribution of a controlled access arc in the minimization sub-problem is the following:

Minimize $_{0 \leq \lambda \leq 1}$ $z\left[x_a^n + \lambda(y_a^n - x_a^n) \right] = \sum_a \int_0^{x_a^n + \lambda_n (y_a^n - x_a^n)} t_a(\omega) d\omega$

A λ is searched such that it minimizes $z\left[x_a^n + \lambda(y_a^n - x_a^n) \right]$, considering $dz\left[x_a^n + \lambda(y_a^n - x_a^n) \right] \big/ d\lambda$.

If $0 < \left(\left[x_a^n + \lambda(y_a^n - x_a^n) \right] \big/ Q \right) < 1$,

$$\sum_a (y_a^n - x_a^n) \left\{ t_a^0 + \left[\left[\left(\frac{C_a}{7200} \right) \left[1 - \left(\frac{g_a}{C_a} \right) \right] \right]^2 \frac{1}{\left[1 - \left[\frac{g\left(x_a^n + \lambda(y_a^n - x_a^n) \right)}{Q_a C_a} \right] \right]} \right] \right\} + \frac{dz\left[x_a^n + \lambda(y_a^n - x_a^n) \right]}{d\lambda} = 0, \quad \text{where}$$

the last term corresponds to other type of arcs.

Given that $0 \leq \lambda \leq 1$, then all of the $(y_a^n - x_a^n)$ factors are positive and then the algorithm for searching λ would have a good behaviour.

If $\left(\left[x_a^n + \lambda(y_a^n - x_a^n) \right] \big/ Q \right) \geq 1$,

$$\sum_a (y_a^n - x_a^n) \left\{ t_a^0 + \left[\left(\frac{C}{7200} \right) \left[1 - \left(\frac{g}{C} \right) \right] \right] + \left[\frac{T}{2Q} \left(x_a^n + \lambda(y_a^n - x_a^n) \right) \right] - \frac{T}{2} \right\} + \frac{dz\left[x_a^n + \lambda(y_a^n - x_a^n) \right]}{d\lambda} = 0, \quad \text{where}$$

the last term corresponds to other type of arcs.

Given that $0 \le \lambda \le 1$, and $\left[\frac{\left(x_a^n + \lambda(y_a^n - x_a^n)\right)}{Q} - 1\right] > 0$, then the $(y_a^n - x_a^n)$ factors are positive.

Therefore, the network loading process requires that $T < 4\left[t_a^0 + \left(\frac{C_a}{2}\right)\left(1 - \left(\frac{g_a}{C_a}\right)\right)\right]$; this guarantees the requirements for using the Frank-Wolfe algorithm for solving the traffic assignment problem.

Process 2: Network loading with a mixed travel time function

The mixed travel time function is composed of, the free-flow travel time plus the Akcelik time dependent overflow delay, for the controlled access arcs, and the free-flow travel time plus the Webster uniform delay and plus the the Akcelik time dependent overflow delay, for the arcs with traffic lights.

For a controlled access arc, the travel time on the arc is $t_a(x) = t_a^0 + d_a^2$, where $d_a^2 = 0.25T\left\{\left(\frac{x_a^n}{Q_a} - 1\right) + \sqrt{\left[\left(\frac{x_a^n}{Q_a} - 1\right)^2 + \frac{8Ax_a^n}{Q_a{}^2 T}\right]}\right\}$.

The contribution of this arc to the system' cost ($z(x_a)$) is the following:

where $z(x_a^n)$ is the cost of other type of arcs.

If $r_a = \frac{TQ_a}{8}$, $s_a = \left[1 - \left(\frac{4A}{Q_a T}\right)\right]$ and $a_a{}^2 = \left[\frac{4A}{Q_a T}\left(2 - \frac{4A}{Q_a T}\right)\right]$, the function is increasing even when $0 < \left({x_a^n}/{Q_a}\right) < 1$.

The contribution of a controlled access arc in the minimization sub-problem is the following:

Minimize $_{0 \le \lambda \le 1}$ $z\left[x_a^n + \lambda(y_a^n - x_a^n)\right] = \sum_a \int_0^{x_a^n + \lambda_n(y_a^n - x_a^n)} t_a(\omega)\, d\omega$.

If $0 \le \lambda \le 1$,

$$\sum_a \left(y_a^n - x_a^n\right)\left\langle \begin{array}{l} t_a^0 + \dfrac{2r}{Q^2}\left\{\left[x_a^n + \lambda\left(y_a^n - x_a^n\right)\right] - Q\right\} \\ + 2\dfrac{r}{Q}\sqrt{\left\{\dfrac{\left[x_a^n + \lambda\left(y_a^n - x_a^n\right) - sQ\right]}{Q}\right\}^2 + a^2} \end{array}\right\rangle + \frac{dz\left[x_a^n + \lambda(y_a^n - x_a^n)\right]}{d\lambda} = 0,$$

where the last term corresponds to other type of arcs.

All of the $\left(y_a^n - x_a^n\right)$ factors are positive, even when $0 < \left(x_a^n + \lambda(y_a^n - x_a^n) \big/ Q_a \right) < 1$.

For an arc with traffic lights, the travel time on the arc is the following: $t_a(x) = t_a^0 + d_a^1 + d_a^2$, where

$$d_a^1 = \left(\frac{C_a}{7200}\right) * \left\{ \frac{\left(1 - g_a \big/ C_a\right)^2}{1 - \left[\min\left(1, (X_a) * g_a \big/ C_a\right)\right]} \right\}$$ (average overflow delay), and

$$d_a^2 = 0.25T \left\{ \left(\frac{x_a^n}{Q_a} - 1\right) + \sqrt{\left[\left(\frac{x_a^n}{Q_a} - 1\right)^2 + \frac{8Ax_a^n}{Q_a{}^2 T}\right]} \right\}.$$

The arc contribution to the system' cost, $z(x_a)$, is the following:

For $0 < \left(x_a^n \big/ Q_a \right) < 1$,

$$z(x) = \sum_a \left\{ \begin{array}{l} t_a^0 x_a^n - \left[\left(\frac{C_a}{7200}\right)\left(\frac{QC_a}{g_a}\right)\left[1 - \left(\frac{g_a}{C_a}\right)\right]^2 \ln\left|1 - \left(\frac{x_a^n g_a}{Q_a C_a}\right)\right| \right] \\ + r_a\left(\frac{x_a^n}{Q_a} - 1\right)^2 - r_a + r_a\left(\frac{x_a^n}{Q_a} - s_a\right)\sqrt{\left(\frac{x_a^n}{Q_a} - s_a\right)^2 + a_a{}^2} \\ + r_a a_a{}^2 \ln\left\{ \sqrt{\left(\frac{x_a^n}{Q_a} - s_a\right)^2 + a_a{}^2} + \left(\frac{x_a^n}{Q_a} - s_a\right) \right\} + r_a s_a - r_a a_a{}^2 \ln\{1 - s_a\} \end{array} \right\} + z(x_a^n),$$

where $z(x_a^n)$ is the cost of other type of arcs.

Let be $r_a = \frac{TQ_a}{8}$, $s_a = \left[1 - \left(\frac{4A}{Q_a T}\right)\right]$ and $a_a{}^2 = \left[\frac{4A}{Q_a T}\left(2 - \frac{4A}{Q_a T}\right)\right]$.

For $\left(x_a^n \big/ Q_a \right) \geq 1$,

$$\boxed{z(x) = \sum_a \left\{ \begin{array}{l} t_a^0 x_a^n + \left[\left(\frac{C_a}{7200}\right)\left[1 - \left(\frac{g_a}{C_a}\right)\right] x_a^n \right] + r\left(\frac{x_a^n}{Q_a} - 1\right)^2 - r_a + r_a\left(\frac{x_a^n}{Q_a} - s_a\right)\sqrt{\left(\frac{x_a^n}{Q_a} - s_a\right)^2 + a_a{}^2} \\ + r_a a_a{}^2 \ln\left\{ \sqrt{\left(\frac{x_a^n}{Q_a} - s_a\right)^2 + a_a{}^2} + \left(\frac{x_a^n}{Q_a} - s_a\right) \right\} + r_a s_a - r_a a_a{}^2 \ln\{1 - s_a\} \end{array} \right\} + z(x_a^n)}$$

where $z(x_a^n)$ is the cost of other type of arcs.

The contribution of the arc in the minimization sub-problem is the following:

Minimize $_{0\le\lambda\le 1}\ z\left[x_a^n + \lambda(y_a^n - x_a^n)\right] = \sum_a \int_0^{x_a^n + \lambda_n (y_a^n - x_a^n)} t_a(\omega)\, d\omega$

It is searched an λ such that a positive term be added to $z(x)$, according to the value of $\left(x_a^n + \lambda(y_a^n - x_a^n) \big/ Q_a \right)$.

If $0 < \left(\left[x_a^n + \lambda(y_a^n - x_a^n)\right] \big/ Q \right) < 1$,

$$\sum_a (y_a^n - x_a^n) \left\{ \begin{array}{l} t_a^0 + \left[\left(\dfrac{C_a}{7200} \right) \left[1 - \left(\dfrac{g_a}{C_a} \right) \right]^2 \dfrac{1}{\left[1 - \left[\dfrac{g_a\left(x_a^n + \lambda(y_a^n - x_a^n)\right)}{Q_a C_a} \right] \right]} \right] \\ + \dfrac{2r_a}{Q_a{}^2} \left\{ \left[x_a^n + \lambda\left(y_a^n - x_a^n\right)\right] - Q_a \right\} \\ + 2\dfrac{r_a}{Q_a} \sqrt{ \left\{ \dfrac{\left[x_a^n + \lambda\left(y_a^n - x_a^n\right) - s_a Q_a\right]}{Q_a} \right\}^2 + a_a{}^2 } \end{array} \right\} + \frac{dz\left[x_a^n + \lambda(y_a^n - x_a^n)\right]}{d\lambda} = 0$$

where the last term corresponds to other type of arcs.

Given that $0 \le \lambda \le 1$, then all of the $(y_a^n - x_a^n)$ factors are positive.

If $\left(\left[x_a^n + \lambda(y_a^n - x_a^n)\right] \big/ Q \right) \ge 1$,

$$\sum_a \left(y_a^n - x_a^n\right) \left\langle \begin{array}{l} t_a^0 + \left[\left(\dfrac{C_a}{7200} \right) \left[1 - \left(\dfrac{g_a}{C_a} \right) \right] \right] + \dfrac{2r_a}{Q_a{}^2} \left\{ \left[x_a^n + \lambda\left(y_a^n - x_a^n\right)\right] - Q_a \right\} \\ + 2\dfrac{r_a}{Q_a} \sqrt{ \left\{ \dfrac{\left[x_a^n + \lambda\left(y_a^n - x_a^n\right) - s_a Q_a\right]}{Q_a} \right\}^2 + a_a{}^2 } \end{array} \right\rangle$$
$$+ \frac{dz\left[x_a^n + \lambda(y_a^n - x_a^n)\right]}{d\lambda} = 0,$$

where the last term corresponds to other type of arcs.

Given that $0 \le \lambda \le 1$, all of the $(y_a^n - x_a^n)$ factors are positive.

Therefore, it is not necessary to limit the value of T, for guarantee a good behaviour of the Frank-Wolfe algorithm.

RESULTS

The network loading processes 1 and 2, were applied to the previously described México City sub-network, considering a demand equal to 20,000 trips/h and an analysis time equal to 2 minutes.

The convex combinations algorithm was programmed in C#. It interacts with the TransCAD files in order to obtain the network characteristics, and executes the network loading process with a known demand. Then, the results are sent to TransCAD files, for displaying them in a GIS (Geographic Information System).

Additionally, a third network leading process was carried out, using the BPR function for all of the arcs, just for comparison reasons. The α and β parameters were calibrated.

The results of the three processes are shown in Figures 5, 6 and 7. Table 2 shows a summary of these results.

In Table 2, the cost is lower is obtained when the BPR function is used; this is because such function ignores the fix and overflow delays produced by traffic lights. Instead, the mixed functions of process 1 and 2 allow obtain fixed and overflow delays produced by traffic lights, then they are more realistic.

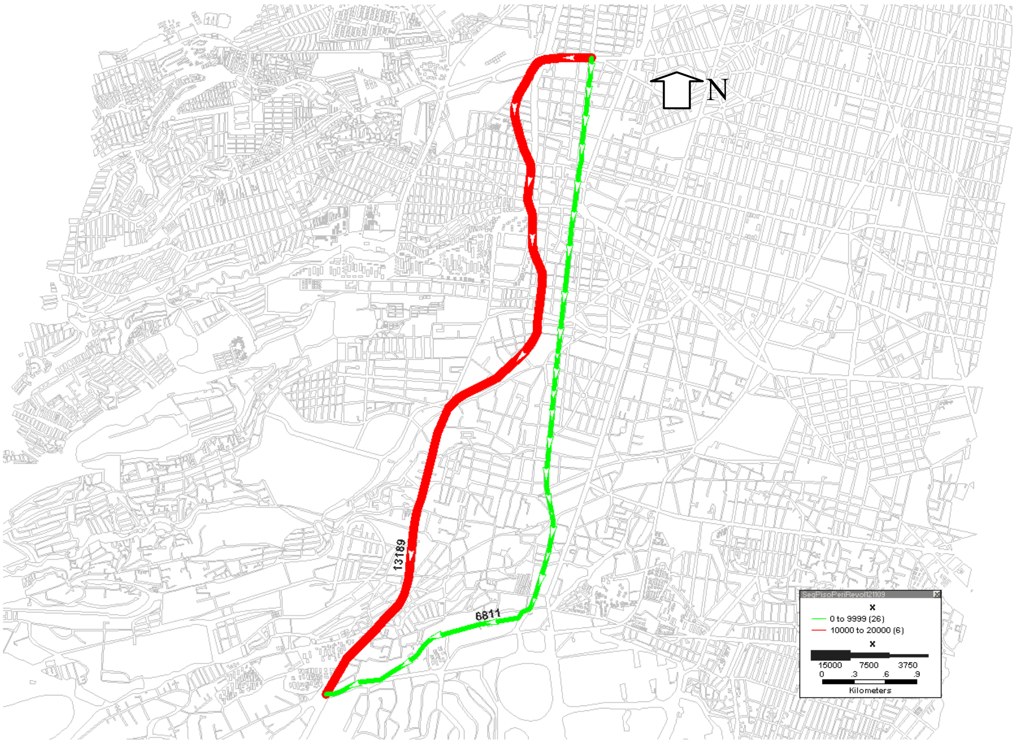

Figure 5. Estimated flow in the Mexico City sub-network, using the Process 1.

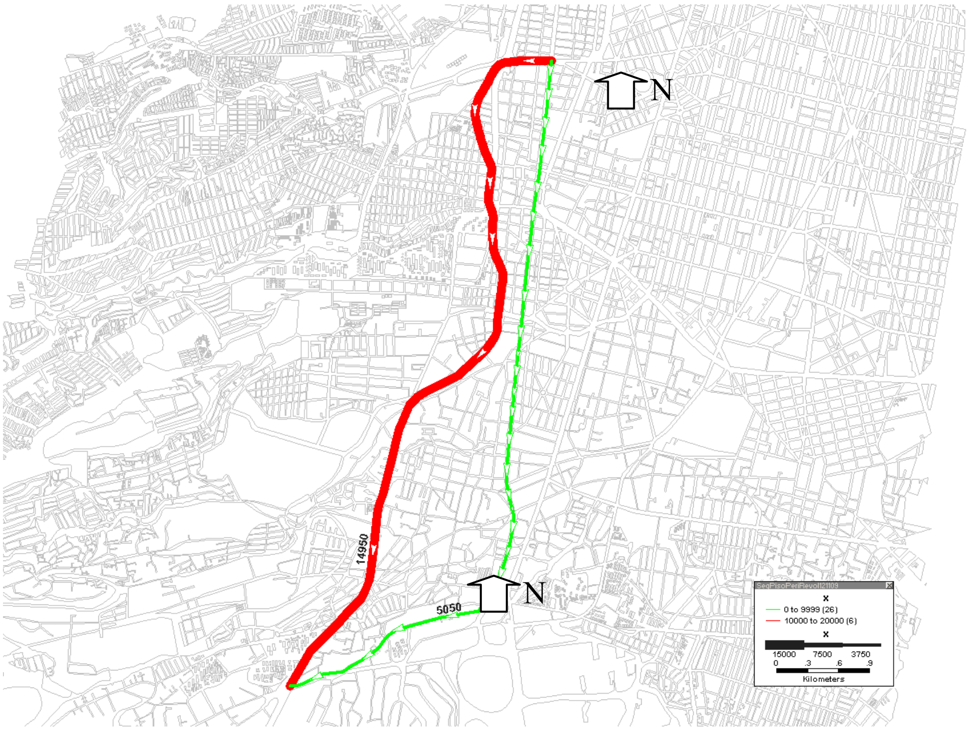

Figure 6. Estimated flow in the Mexico City sub-network, using the Process 2.

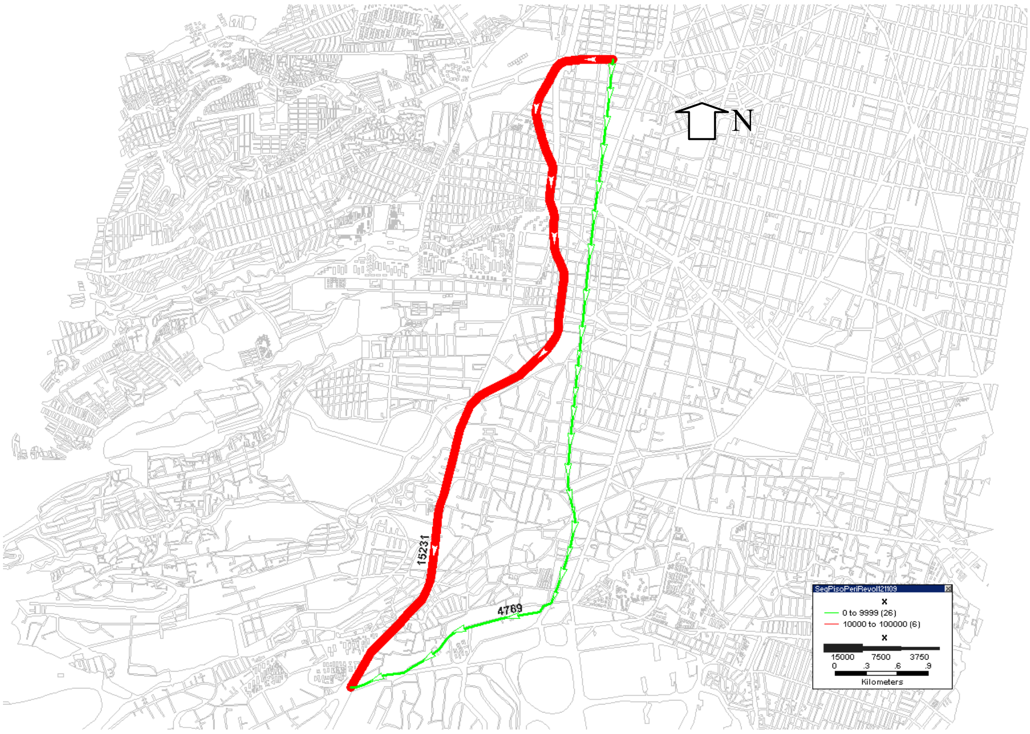

Figure 7. Estimated flow in the Mexico City sub-network, using the Process 3.

Tabla 2. Summary of the results of the three process application

Process	$z(x_a)$(h)	Flow on Revolución (veh/h)	Flow on Periférico (veh/h)	Travel time on the Revolución path (h)	Travel time on the Periférico path (h)
1: Mixed function including Webster and BPR functions	3,417	5,155	14,844	0.34	0.32
2: Mixed function including Webster and Akcelik functions	4,499	4,942	15,057	0.35	0.36
3: BPR function	3,166	6,814	13,186	0.26	0.26

The objective of this analysis was to prove the algorithm's performance, by means of the use of combined functions, which has been satisfactorily made.

The convergence routine of the C# program will be improved to obtain equal times on the paths, for processes 1 and 2.

CONCLUSION

A review of the state of the art on dynamic function in DTA problems was presented, considering an analytical and macroscopic approach, emphasizing on the travel time on the arc, the functions' characteristics and their potential for making realistic estimations.

Any DTA model must have desirable properties as causality and correspondence with the static problem. The DTA problem can be formulated as a bi-level problem. In the lower level, the flow is estimated under static case considerations, generating an initial value for the dynamic variable x_a, which feeds the upper problem, where the dynamic variables x_a, u_a, v_a have a relation.

This paper deals with process for the lower level, incorporation travel time functions, which are flexible to changes in the supply and adaptable to different type of arcs.

Mixed travel time functions are proposed, which include flow dependent delays, overflow delays and delays due to traffic lights. Such functions are propagated, by means of the Frank-Wolfe algorithm, and their behaviour as well as their parameters' constraints, are analyzed.

Some functions allow obtain realistic estimations and accept changes on the arc characteristics. They are widely used in microscopic analysis. However, their incorporation to macroscopic analysis, even in the static case, constitutes a powerful resource for the transportation planning, and it is a contribution to the research on traffic assignment on urban networks.

The application to a real network generates interesting results and proves that the algorithm can incorporate mixed functions, with a possible small limitation to the analysis time.

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