SIGCOM3: AN TRAFFIC SIGNAL OPTIMIZATION MODEL FOR ISOLATED INTERSECTIONS

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ABSTRACT

The need of updating the series of SIGCOM programs, now in its third version, is in accordance with the continuous improvements that any program has to have to be valid. Like its predecessors, SIGCOM 3 delivers the optimal cycle length and the optimal green time split for an isolated intersection controlled by a traffic light. The objective function in the optimization process is selected by the user among multiple options that the program offers as performance indicators. As a first approach, the work done by von Mühlenbrock (2005) was revised, to corroborate the important progress contributed, on one side, by incorporating studies done in Chile on estimating capacity and prediction models of lane selection, as well as writing this program in Visual Basic.Net, which offers a much more friendly interface with the user. The most commonly used performance indicators for measuring a intersection are the number of stops, the delays and a combination of the two. In this version of SIGCOM, the expressions that measure these indicators have been modified, replacing Allsop formulas (1971) by Akcelik formulas (1981). The expressions found by Akcelik work even under high level of traffic demand, which was a limitation in the first two versions of this program. The above mentioned Akcelik formulas (1981) for the number of stops and delay consider a term which includes the length of the overflow queue, which corresponds to the number of vehicles that stay at the intersection after a green light has finished. This concept was studied in depth in this study and a correction was made in the way this value was interpreted and calculated. Undoubtedly, this constitutes a relevant contribution of this study. In conclusion, SIGCOM 3 delivers an improved version of this saga, in which the greater versatility and the online help grant a much better experience for the end user.

Keywords: Traffic signal, traffic simulation, traffic signal optimization, isolated intersection

INTRODUCTION

SIGCOM is a computational program developed by Woywood (1989) and capable of finding the optimal cycle length and green times of an isolated intersection. Since its birth, the program has been updated only once (von Mühlenbrock, 2005). The need of updating the last version – known as SIGCMO2 – is in accordance with the continuous improvements that any program must have to be valid.

Goal and scope

The goal of this article is to present the reformulation of the optimization process of the computational program SIGCOM3, a traffic signal optimization model for isolated intersections. This new version updates the objective functions of the program by performance indicators able to work under different traffic conditions, since it was important to widen the validity range of the model for oversaturated conditions. An improvement of the computational characteristics of the program is also desirable.

The article has the following specific goals:

1. To improve the results of the optimization process by including more complex expressions for the delay experienced at the intersection and the number of stops.

2. To analyse the overflow queue problem.

3. To compute green time splits in cycles even under oversaturated conditions

4. To improve the program interface, incorporating helps and hints in real time to get a more user friendly program.

In its current version, the program does not incorporate environmental issues in the optimization process. However, they are somehow implicit by including delays and number of stops.

Content

In addition to this introduction, the paper includes five other sections. Section 2 presents the background of the model. Section 3 describes the model, which includes the objective functions, the set of constraints (logical and operational) and the optimization method use to solve the problem. Section 4 shows the inputs and outputs of the model. Section 5 presents the results of an implementation of the model. Finally, Section 6 lists the main conclusions of this study and provides some recommendations for further research and development.

BACKGROUND

This section presents the foundation of the model. First, some preliminary definitions are explained. Second, new approaches to estimate the saturation flow are described. Third, an expression for the effective green time when vehicles have right of way in two phases is shown. Finally, formulas used in the program to estimate stops and delays are established.

Preliminary definitions

A classical diagram of the discharge process at the stop line of a traffic signal is shown in Figure 1. It is assumed that this discharge process corresponds to phase *i* of a sequence of *m* phases. This figure can be used to define many important concepts used in this paper.

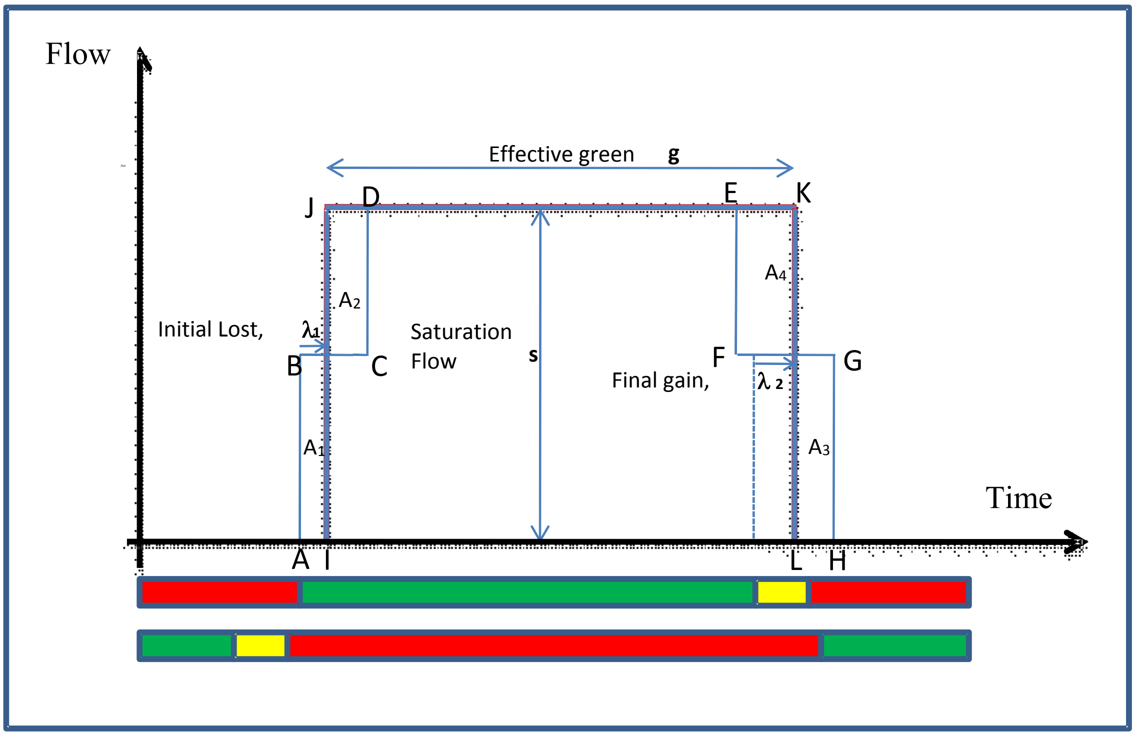

Figure 1: Discharge Diagram

A fully saturated discharge is defined by the polygon ABCDEFGH (blue line). A new rectangle IJKL (red line) can be constructed such that the area under the polygon is equal to the area under the rectangle (i.e. $A_1 = A_2$ and $A_3 = A_4$,). That is, the total number of vehicles discharged is the same for both cases. With the new rectangle the following parameters appear:

1. The height of the rectangle is a fairly constant discharge rate and is called the saturation flow.

2. The base of the rectangle IJKL is a hypothetic green time known as the effective green time. During this time vehicles are discharged at the saturation flow.

3. The offsets that help to define the effective green time correspond to the green start lag or initial lost of phase *i* (λ_1^i) and the green end lag or final gain of phase *i* (λ_2^i).

4. The lost time of phase *i* (l_i) is the time between the end of the effective green of phase *i* and the start of the effective green of the following phase *i+1*. This time may be expressed as follows:

$$l_i = A + Rr + (\lambda_1^i - \lambda_2^i) \tag{2.1}$$

A is the amber period and *Rr* is the simultaneous red for all phases.

5. The lost time of the cycle is defined as:

$$L = \sum_{i=1}^{m} l_i \tag{2.2}$$

6. The cycle time *c* is defined as the elapsed time from the start of a given sequence of phases till the start of the next sequence.

7. The green time split of phase *i* (β_i) corresponds to the proportion of the effective green of phase *i* with respect to the cycle length:

$$\beta_i = \frac{g_i}{c} \tag{2.3}$$

Suppose the intersection has *n* lanes in total. The flow ratio of lane *j* is the ratio between its demand and its saturation flow:

$$y_j = \frac{q_j}{s_j} \tag{2.4}$$

The capacity (Q_j) of lane *j* considers that vehicles can discharge only during a proportion of the cycle, and is defined as:

$$Q_j = \frac{g_j}{c} \cdot s_j \tag{2.5}$$

The term g_j is the effective green time of lane *j*. In case lane *j* has right of way in more than one phase, g_j considers all the green times and the corresponding lost time that is used by the vehicles on this lane.

The intersection capacity (Q_{int}) corresponds to the smallest amongst all the lanes capacities:

$$Q_{\text{int}} = Min(Q_j) \tag{2.6}$$

The degree of saturation of lane *j* (x_j) is the ratio between the demand (q_j) and the lane capacity (Q_j):

$$x_j = \frac{q_j}{Q_j} = \frac{q_j}{\beta_j \cdot s_j} = \frac{y_j}{\beta_j} \quad (2.7)$$

If $x_j < 1$ the lane is theoretically undersaturated. However, due to the fact that values of 0.8 – 0.9 presents long queues, a maximum acceptable degree of saturation (p_j), is defined. This parameter normally fluctuates between 0.8 to 0.95. The degree of saturation of phase *i* corresponds to the biggest one amongst the degrees of saturation of the lanes that have right of way in that phase *i*. The degree of saturation of the intersection (x) is the biggest of all the lanes degrees x_j of the intersection.

New definitions of saturation flows

Previous research done in Chile (Gibson et al, 1998) showed that the saturation flow of lane *j* depends on the proportion of buses in the flow, the period of the day (peak hour and rest of the day), the type of lane (central or lateral) and its width. The above mentioned finding led to a new definition of the basic vehicle: a private car going straight on a lane that only has private cars going straight (ADE for its name in Spanish). As a consequence, new equivalency factors for vehicles were derived, which depend on the factors mentioned above.

Afterwards, Coeymans and Herrera (2003) added to the analysis the effect of turning vehicles. The saturation flow is obtained as follows:

$$s_j^k = \hat{s}_{BASICO,j}^k \cdot f_{ANCHO,j} \cdot \frac{\sum_i q_{i,j}^k}{\sum_i \hat{f}_{i,j}^k \cdot q_{i,j}^k} \quad (2.8)$$

where,

s_j^k : saturation flow on lane *j* in period *k* of the day [veh/hr].

$\hat{s}_{BASICO,j}^k$: basic saturation flow of lane type *j* in period *k* of the day [ADE/hr].

$$\hat{s}_{BASICO,j}^k = 2.141 - 208 \cdot D_{PD} - 149 \cdot D_{PI} + 151 \cdot D_{PMed} - 29 \cdot D_{PD} \cdot D_{PMed} - 22 \cdot D_{PI} \cdot D_{PMed} \quad (2.9)$$

where

D_{PD} : 1 if right lane ; 0 otherwise.

D_{PI} : 1 if left lane ; 0 otherwise.

D_{PMed} : 1 if AM peak period; 0 otherwise.

$f_{ANCHO,j}$: width factor for lane *j*.

$$f_{ANCHO,j} = 1 + \gamma \cdot (A_j - 3{,}0) \cdot D_{PE} \quad (2.10)$$

$$\gamma = \frac{0{,}0568}{1 - 0{,}0568 \cdot \left(A_j - 3{,}0\right)} \approx 0{,}058 \quad (2.11)$$

A_j : lane *j* width [m].

D_{PE} : 1 if external lane; 0 otherwise.

$q_{i,j}^{k}$: flow of vehicles type *i,* on lane *j* in period *k* of the day [veh/hr].

$\hat{f}_{i,j}^{k}$: equivalency factor for vehicles type *i* on lane *j* in period *k* of the day.

This last factor varies depending on the type of vehicle and its manoeuvre. The following parameters are defined:

D_v: 1 if there are turning vehicles on the lane, 0 otherwise.
vir : proportion of turning vehicles in the flow.
ped : pedestrian flow that interacts with turning flow [pedestrians/min].
R_C : adjusted turning radius [m].

$$R_C = 0.980R_A + 0.629W \tag{2.12}$$

R_A: real turning radius given by the curb [m].
W: width of approach where turning manoeuvres are made to [m].
TP : proportion of buses in flow.

$$\varphi(TP) = \frac{0{,}216}{1 + e^{3{,}526 - 20{,}609TP}} \tag{2.13}$$

$\hat{h}_{0,j}^{k}$: mean headway of the basic vehicle on lane *j* and period *k* of the day.

$$\hat{h}_{0,j}^{k} = \left(1{,}676 + 0{,}181 \cdot D_{PD} + 0{,}126 \cdot D_{PI} - 0{,}111 \cdot D_{PMed} + \varphi(0)\right) \tag{2.14}$$

The expressions to compute the equivalency factors of each type of vehicle are the following:

Private car going straight:

$$\hat{f}_{AD,j}^{k} = 1 + \frac{\left[0{,}063 \cdot D_v + \dfrac{1{,}59 \cdot vir \cdot (1 + ped)}{R_c} + \dfrac{0{,}216}{1 + 34 \cdot e^{-20{,}609TP}} - \varphi(0)\right]}{\hat{h}_{0,j}^{k}} \tag{2.15}$$

Turning car

$$\hat{f}_{AV,j}^{k} = 1 + \frac{\left[e^{\frac{-0{,}35 \cdot vir \cdot R_c}{1 + ped}} + \dfrac{0{,}216}{1 + 34 \cdot e^{(-20{,}61 \cdot TP)}} + 0{,}039 - \varphi(0)\right]}{\hat{h}_{0,j}^{k}} \tag{2.16}$$

Bus going straight

$$\hat{f}_{BD,j}^{k} = \begin{cases} f_{ANCHO,j} \dfrac{2.48}{\hat{h}_{0,j}^{k}} & \text{if } j \text{ is left or central lane with no turning vehicles} \\ f_{ANCHO,j} \dfrac{3.13}{\hat{h}_{0,j}^{k}} & \text{if } j \text{ is right lane with no turning vehicles} \\ f_{ANCHO,j} \dfrac{3.16}{\hat{h}_{0,j}^{k}} & \text{if } j \text{ is left or central lane with turning vehicles} \\ f_{ANCHO,j} \dfrac{3.80}{\hat{h}_{0,j}^{k}} & \text{if } j \text{ is right lane with turning vehicles} \end{cases} \tag{2.17}$$

Turning Bus

$$\hat{f}_{BV,j}^{k} = \begin{cases} f_{ANCHO,j}\,\dfrac{3.32}{\hat{h}_{0,j}^{k}} & \text{if left turning from and to a bidirectional road} \\ f_{ANCHO,j}\,\dfrac{4.14}{\hat{h}_{0,j}^{k}} & \text{otherwise} \end{cases} \tag{2.18}$$

For more details on these formulations, the reader is referred to Coeymans and Herrera (2003).

Effective green redefinition

When a lane has right of way or green time in more than one phase, the lost time between both phases (or intergreen time) has to be added to the effective green. Von Mühlenbrock (2005) proposed the following expression for Ω_j , which corresponds to the proportion of effective green of lane *j* with respect to the cycle, when lane *j* has right of way in more than one phase:

$$\Omega_j = \sum_{i=1}^{m}\left[a_{ij} \cdot \left(\beta_i + \alpha_i \cdot \beta_0\right)\right] - \alpha_k \cdot \beta_0 \tag{2.19}$$

Where:

$\alpha_i = \dfrac{l_i}{L}$: proportion of lost time in phase *i* over total lost time of the cycle (sub-index *k* corresponds to the last phase in which the lane has green).

$\beta_i = \dfrac{g_i}{C}$: proportion of effective green time in phase *i* over the cycle time.

$\beta_0 = \dfrac{L}{C}$: proportion of total lost time of the cycle over the cycle time.

a_{ij} = a dummy variable (1 if lane *j* has phase in phase *i*, 0 otherwise)

The overflow queue

It is common in traffic signal operations that certain green lights do not allow all vehicles that are waiting in a queue to cross. These vehicles that must wait for another green light are the overflow queue. This parameter is extremely important, because in different studies (see Miller, 1963; Allsop, 1972; McNeil and Weiss, 1974; Ohno, 1978; Sosin, 1980), is directly correlated with relevant variables such as the delays and stops. This new version of SIGCOM slightly modifies the expression for the overflow queues proposed by Akcelik (1980) in order to incorporate the effect of these queues in the optimization process.

The expression of Akcelik for the overflow queue (deduced after some simplifications) is given by:

$$N_0 = \begin{cases} \dfrac{Q \cdot T_f}{4}\left[(x-1)+\sqrt{(x-1)^2+12\dfrac{x-x_0}{Q \cdot T_f}}\right] & if \;\; x > x_0 \\ 0 & if \;\; x \le x_0 \end{cases} \tag{2.20}$$

Where:
Q = capacity [veh/h]
T_f = period of demand [h]
x = degree of saturation
x_0= threshold of the degree of saturation below which no overflow queue is observed. Its value was obtained by linear regression,

$$x_0 = 0.67 + \frac{s \cdot g}{600} \tag{2.21}$$

s= saturation flow [veh/h]
g= effective green time [h]

It is worthy to mention that in all the analysis reported by the literature, queues are always a function of x (degree of saturation) with a constant ratio between the effective green time and the cycle length. In this model it has been included the analysis of green time, that is in the formula of x. In those studies reported, curve N_d is a right line, producing a wrong estimation of the curve N_0. As it is showed in Figure 2 the result is not the same. This new way of understanding and working with the remaining queue is a relevant finding of this model (see Cortés-Nannig, 2010, for more details).

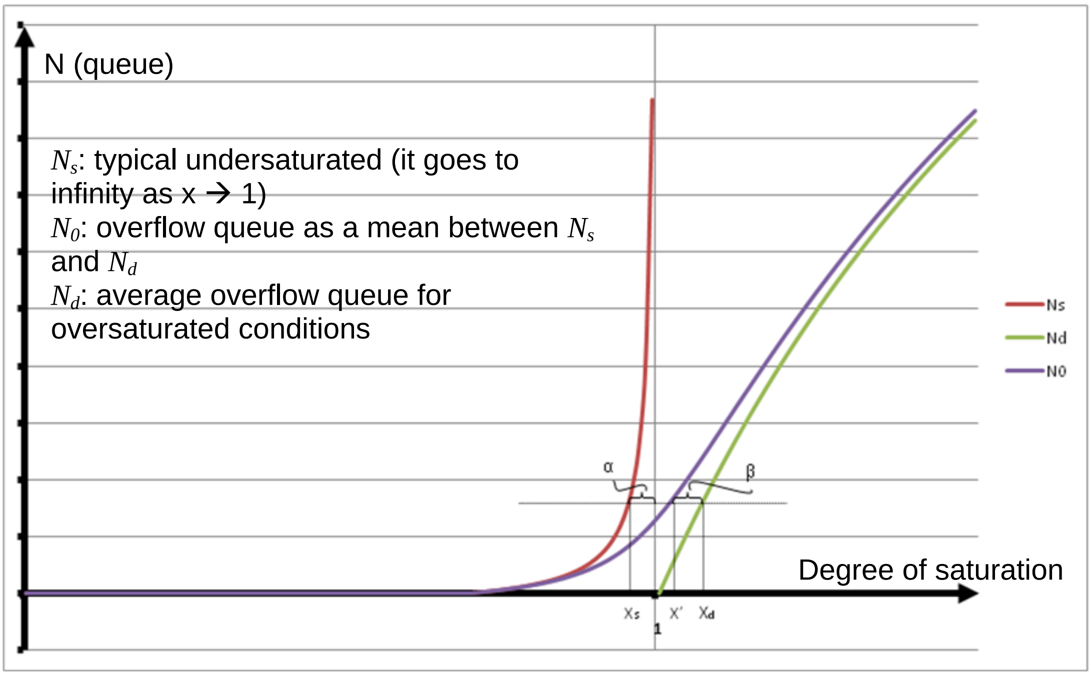

Figure 2: Transition Function of overflow queue

Akcelik and other authors assume N_d as a straight line, which is true if the ratio between the effective green time and the cycle length remains constant. When this ratio changes, N_d is a curve (see Figure 2).

Stops and Delays

Stops

A vehicle is said to be stopped if it is forced to drive at 0 km/h during certain period of time (regardless of the length of the period of time). Akcelik (1981) proposed and expression for estimating the proportion of all vehicles that have to stop on lane *j*, which is also known as the stop rate for lane *j*. In his formulation, Akcelik includes the case of oversaturated conditions:

$$h_j = \delta \cdot \left[\frac{1-\beta_j}{1-y_j} + \frac{N_0}{q \cdot c} \right] \tag{2.22}$$

Where h_j is the stop rate on lane *j*. All other parameters are the same as explained before.

Factor δ takes into account partial stops. That is, vehicles that do not necessarily stop because they join the queue when the discharging process of the queue was already started. Allsop (1971) estimated this factor as 0.9 and Woywood (1989) calibrated the same factor for the Chilean case as 0.95. The first term inside the parenthesis is the same as the one proposed by May (1965), which provides the proportion of all the vehicles arriving in one cycle that are stopped (assuming undersaturation). The second term includes the overflow queue, and takes into account the multiple stops suffered by some vehicles at the intersection.

By multiplying the flow on lane *j*, q_j, and the stop rate for lane *j*, h_j, the total number of stops on lane *j*, H_j, are obtained. Because the flow q_j may have right of way in more than one phase, the proportion β_j is replaced by Ω_j.. As a consequence, the total number of stops on a lane *j* is as follows:

$$H_j = q_j \cdot h_j \rightarrow H_j = \delta \cdot q_j \cdot \left[\frac{1-\Omega_j}{1-y_j} + \frac{N_0}{q_j \cdot c} \right] \tag{2.23}$$

Finally, the total number of stops at a given intersection with *n* lanes is the sum of the number of stops for all the *n* lanes:

$$H(\beta) = \sum_{j=1}^{n} \delta \cdot q_j \cdot \left[\frac{1-\Omega_j}{1-y_j} + \frac{N_0}{q_j \cdot c} \right] \tag{2.24}$$

Delays

A vehicle experiences delay when it takes longer to cross the intersection than what it would take if no traffic light or queue would have been there. Most optimization models of traffic signal intersections use Webster's (1958) and Allsop's (1972) formulas to compute the delay. These formulas, however, do not consider the oversaturation case.

Akcelik (1980) includes the overflow queue N_0 (equation 2.20) in the estimation of delay on lane j, d_j. By doing so, his expression is considering the oversaturation case

$$d_j = \frac{c \cdot (1-\beta_j)^2}{2 \cdot (1-y_j)} + \frac{N_0 \cdot x_j}{q_j} \tag{2.25}$$

Where d_j is the average delay per vehicle on lane j, and all the other parameters have the same meaning as before.

The total delay on lane j, D_j, is obtained by multiplying the average delay, d_j, and the flow on that lane, q_j. As before, β_j is replaced by Ω_j.

$$D_j = d_j \cdot q_j = \left[\frac{c \cdot q_j \cdot (1-\Omega_j)^2}{2 \cdot (1-y_j)} + N_0 \cdot x_j \right] \tag{2.26}$$

The total delay is calculated as the sum of total delays on each lane. If a weighting factor, w_j, is considered for each lane (for instance, to take into account the fact that lanes are used by vehicles with different number of persons), the total weighted delay $D^w(\beta)$ is computed as follows:

$$D^w(\beta) = \sum_{j=1}^{n} d_j \cdot q_j \cdot w_j \rightarrow D(\beta) = \sum_{j=1}^{n} w_j \cdot \left[\frac{c \cdot q_j \cdot (1-\Omega_j)^2}{2 \cdot (1-y_j)} + N_0 \cdot x_j \right] \tag{2.27}$$

OPTIMIZATION MODEL

The definitions and expressions presented in Section 2 are used in the optimization process of SIGCOM3. This section describes the optimization problem that SIGCOM3 tries to solve. First, the decision variables of the problem are defined. Second, all the possible objective functions are presented. Third, the set of constraints of the problem is discussed. Finally, the optimization method use to solve the problem is mentioned.

Decision variables

The decision variables are the variables that the program controls and modifies in order to improve the value of the objective function. In this case, the decision variables are the cycle length c and the proportion of effective green time for each phase i, β_i.

Objective functions

The program is able to optimize according to four different performance indicators:

1. Total number of stops (*H*): the objective function, in this case, is given by equation (2.24), which provides an estimate of the total number of stops in an isolate intersection during one hour.

2. Total weighted delay (D^w): the objective function is given by equation (2.27), which provides an estimate of total delay experienced in an intersection.

3. Fuel consumption rate (*F*): the fuel consumption rate for a vehicle depends on the status of the vehicles. The rates are different if the vehicle is accelerating/decelerating, cruising, or stopped. The program assumes that the fuel consumption has only two components, one related to the consumption when the vehicle is idle or stopped (FC_{idle}) and the other one related with the extra consumption during acceleration/deceleration process due to a stop (FC_{extra}).

$$F(\beta) = \sum_{j=1}^{n} \left(FC_{idle} D_j + FC_{extra} H_j \right) \qquad (2.28)$$

4. Total social cost (TSC): the TSC includes the cost related to the time spent by drivers at the intersection and the cost of operating a vehicle. The social cost related to the time is computed using the delays experienced at the intersection. The social cost related to the operation of a vehicle is assumed to be twice the cost associated to the fuel consumption experienced by the vehicle.

$$TSC = SVT\ D(\beta) + 2\ SVF\ F(\beta) \quad \text{[CL\$/h]} \qquad (2.29)$$

where SVT and SVF are the social value of time and fuel, respectively. $D(\beta)$ corresponds to the total delay according to equation (2.27) and and $F(\beta)$ corresponds to the fuel consumption rate as in equation (2.28).

Since the performance of the intersection is high if these indicators are low, the goal is to minimize them.

Set of constraints

There are four sub-sets of constraints:

1. Length of the cycle: the cycle can be set as fixed or as a decision variable of the problem. In the first case, the cycle length is given. In the second case, a maximum cycle length is defined in order to avoid unrealistically long cycle lengths. Mathematically, this constraint is expressed as follows:

$\beta_0 \geq \frac{L}{C_{\max}}$ → if maximum cycle C_{max} is defined

$\beta_0 = \frac{L}{C_0}$ → if fixed cycle C_0 is defined

2. Minimum green time (V_i^{min}): it is desirable to set a minimum green time in order to give pedestrians enough time to cross the side approach that is stopped. Therefore, there are as many of these constraints as phases, and they are expressed as follows:

$$\beta_i \cdot c \geq V_i^{\min} \rightarrow L \cdot \beta_i - V_i^{\min} \cdot \beta_i \geq 0 \quad \forall i = 1...m$$

3. Maximum acceptable degree of saturation (p_j): this sub-set of constraints tries to limit the saturation experienced on some lanes, because of the exponential growth of the delay when the saturation degree approaches to 1 or more (see Section 2.1). There are as many of these constraints as lanes, and they are expressed as follows:

$$x_j \leq p_j \rightarrow \Omega_j - \frac{y_j}{p_j} \geq 0 \qquad \forall j = 1...n$$

4. Green time split: this constraint imposes the all the time to be allocated among the *m* phases is actually used (no more, no less). This constraint is expressed as follows:

$$\sum_{i=1}^{m} \beta_i = 0$$

5. Sign constraints: all the decision variables have to be positive. Therefore:

$$\beta_i > 0 \qquad \forall i = 1...m$$

The total number of constrains is $2m+n+2$, where *m* is the number of phases at the intersection and *n* is the number of lanes. In a real problem, *m*=3 and *n*=10 are reasonable values. The total number of constraint would be 18 in this case, which can be easily handled.

Optimization method

All the possible objective functions presented in Section 3.2 are nonlinear, continuous and differentiable in the domain. On the other hand, all the constraints in Section 3.3 are linear. Therefore, the optimization problem can be solved using the gradient method (Cortés-Nannig, 2010)

INPUTS AND OUTPUTS OF THE PROGRAM

The software has been developed using Windows platform. It has a friendly interface with the user, which allows a more efficient and easy way to input the required information, and to look at the results.

Inputs

The inputs can be grouped into five groups:

1. General:

    a. Social value of time and social value of fuel: these values are used to compute the total social cost as per equation (2.29)

    b. Fuel consumption of cars/buses when they are idle and extra consumption rate during acceleration/deceleration: these values are used to compute the fuel consumption rate as per equation (2.28).

2. Supply:

    a. Number and type of lanes per approach: if needed, a weighting factor for the delays can be also defined (see Section 2.5.2).

    b. Saturation flow per lane: this value can be directly entered to the program or it can be estimated using the procedure described in Section 2.2. In the later case, the width of the lane and information about traffic composition, among others, are also required to perform the estimation.

3. Demand:

    a. Flow of cars, buses, and turning proportions: these values are defined per lane, associating one movement per lane. The flows can also be defined by approach, in which case the program distributes the flow on all the lanes of the approach such that all of them are equally saturated.

    b. Average speed on the approach and vehicle's acceleration: these values are used in the computation of the fuel consumption.

4. Control:

    a. Define phases and lanes that have right of way on each of them: Phases can also be automatically generated. To this end, the compatibility matrix (which specifies which movements/lanes are compatible, i.e. movements/lanes that can have right of way during the same phase) must be defined first.

   b. Maximum or fixed cycle length, minimum green times, and maximum acceptable degree of saturation: these values are related to the constraints described in Section 3.3.

   c. Lost time per phase

   d. Effective green time (optional): these values are provided only if the user wants to know the performance indicators of a given program of the traffic light. If the user want to optimize green time split, these values are obviously not provided.

5. Optimization:

   a. Maximum number of iterations and convergence criteria: these two parameters are related to the convergence criteria of the algorithm that solves the optimization problem.

   b. Objective function to be optimized.

As an example, Figure 3 shows a snapshot of the module use to enter information about the lanes (capacity estimator, flow, weighted factor for delay, etc).

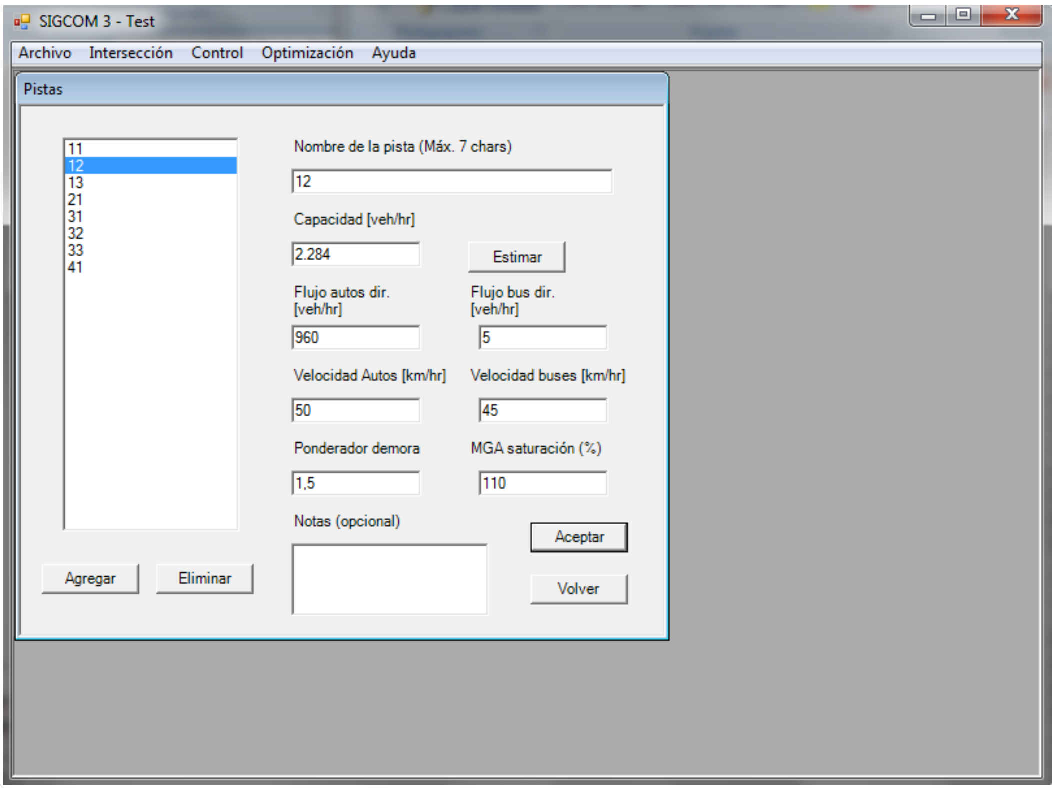

Figure 3 – Snapshot of the lane module

Outputs

The outputs are delivered at both the junction level and movement/lane level. At the junction level, the optimization process gives the cycle length, which is the optimal cycle length if a maximum cycle length was defined as a constraint. The performance of the junction in term of all the indicators described in Section 3.1 is computed: total and weighted total delay per hour (h), total number of stops per hour, total fuel consumption per hour (lt), and total social cost per hour (CL$). That is, regardless of the objective function chosen, the program also computes the other performance indicators. The effective green times for all the phases are also provided.

At the movement/lane level, the program computes the degree of saturation, overflow queue, total/weighted/average delays, and total/average number of stops.

Figure 4 is a snapshot of the output file.

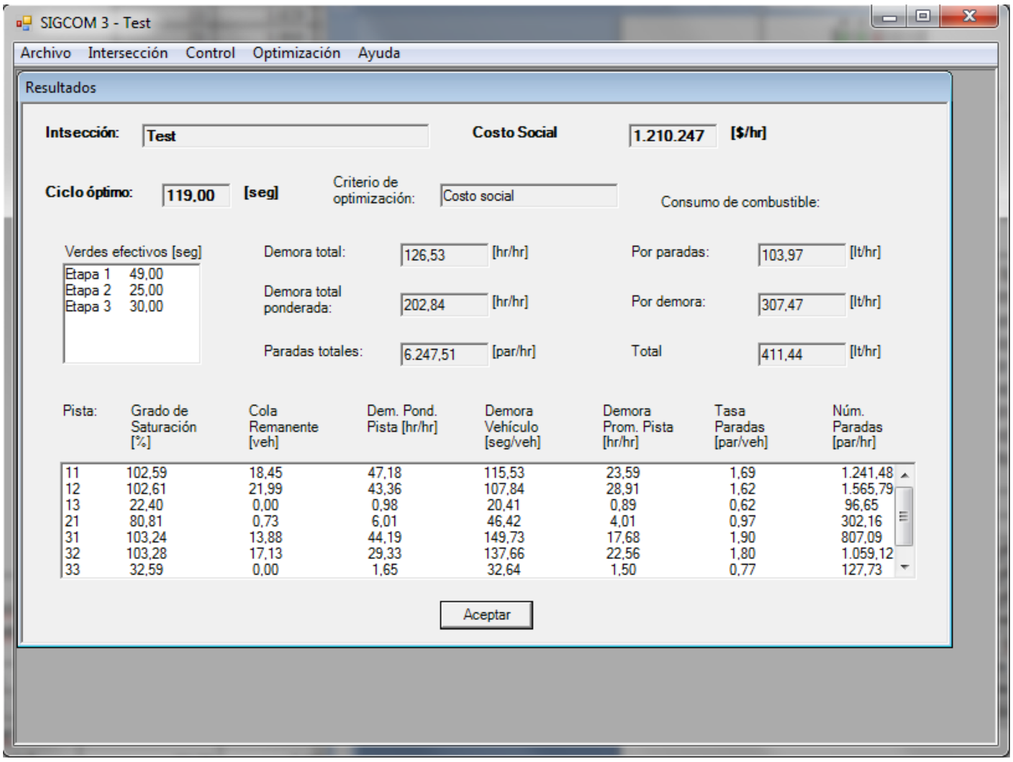

Figure 4: Snapshot of the output of the program.

RESULTS

The program was tested on the fictitious intersection showed in Figure 5. The information for each lane is shown in Table 1. Note that *i)* the saturation flow is not provided, and *ii)* in some cases only the flow on the approach is provided, and not the flow per lane. In these cases the program will determine the flow per lane as described in Section 4.1. Table 2 shows the capacities estimated by the program for each lane and the flows determined for each lane.

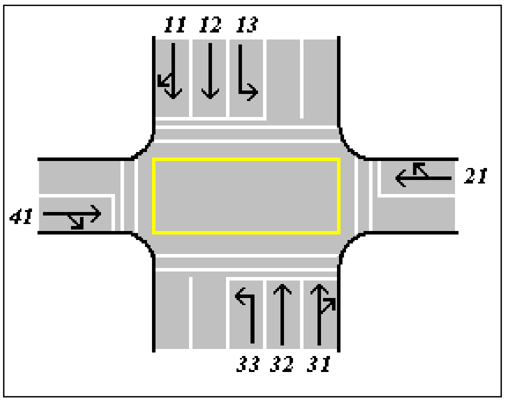

Figure5: Intersection layout

Table 1: Summary of Inputs

Lane	Width	Flow of cars (vph)	Flow of buses (vph)	Weighted factor.	Radius (m)	Approach width (m) (turning)	Flow of turning buses (vph)	Flow of turning cars (vph)	Pedestrians (ped/min)
11	3.50	1500	25	2.0	6	4	5	165	1.0
12	3.25		5	1.5	n/a	n/a	n/a	n/a	n/a
13	3.25	0	0	1.1	6	4	20	137	0.0
21	4.00	253	0	1.5	4	10	0	58	0.5
31	3.50	800	30	2.5	6	4	5	170	0.7
32	3.25		10	1.3	n/a	n/a	n/a	n/a	n/a
33	3.25	0	0	1.1	4	10	n/a	165	n/a
41	4.00	305	0	1.5	4	10	0	97	1.1

Table 2 Capacity estimated and flow of cars assigned per lane (when flow on the approach was specified)

Lane	Capacity (vph)	Flow of cars (vph)
11	1740	540
12	2284	960
13	1702	-
21	1832	-
31	1633	220
32	2266	580
33	2008	-
41	1741	-

In this case, the compatibility matrix was defined in order to obtain possible phase sequences. Figure 6 depicts the two sequences automatically generated by the program. For comparison purposes, both sequences are optimized respect to the four objective functions mentioned in Section 3.2.

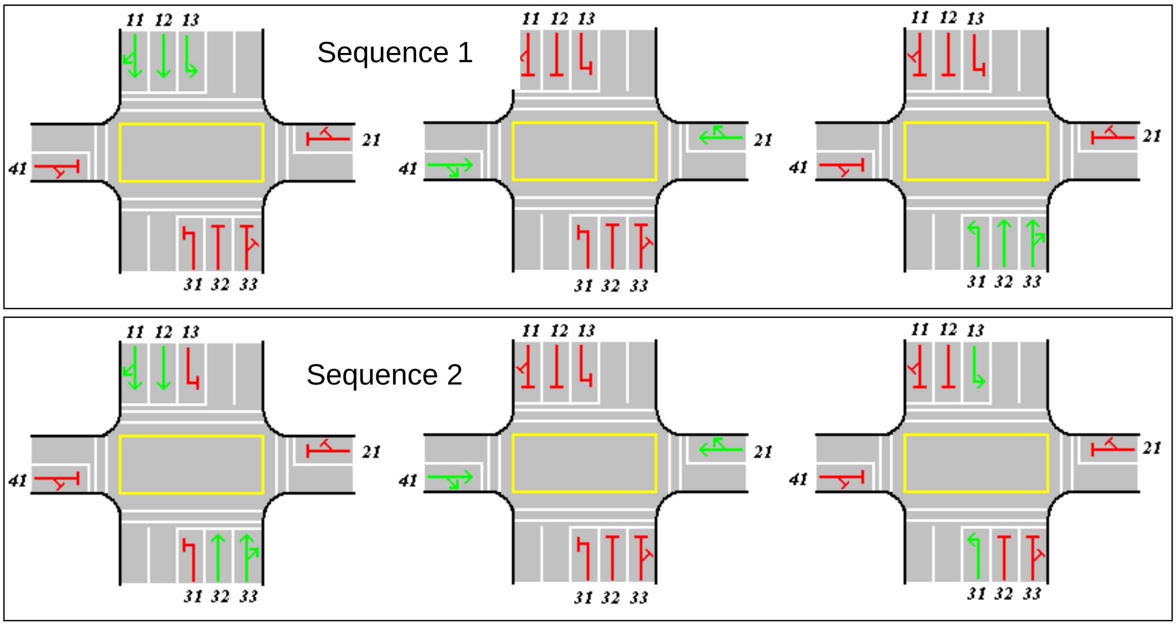

Figure 6: Possible sequences of phases

Table 3 shows the results for the minimization of the total social cost (TSC) for both sequences. In terms of the TSC, sequence 2 provides significant savings compared to sequence 1. This difference is also observed when any of the other objective functions is used, confirming the convenience of using sequence 2 over sequence 1.

Table 3: Results comparison between sequence 1 and sequence 2

	Sequence 1	Sequence 2
Cycle length (s)	119	107
Effective green time phase 1 (s)	49	53
Effective green time phase 2 (s)	25	27
Effective green time phase 3 (s)	30	12
Maximum degree of saturation (%)	109.9 (lane 41)	91.5 (lane (41)
Total weighted delay (veh-h/h)	202.84	45.52
Total number of stops per hour	6247.5	3222.2
Fuel consumption rate [lt/h]	411.44	122.75
Total social cost [CL$/h]	1210247	338514

Results for sequence 1 are exactly the same when the objective function is the total weighted delay, the fuel consumption rate, or the total social cost, suggesting that the intersection would be highly congested in this case. In fact, only lanes 13, 21 and 33 have a degree of saturation less than 1 and do not exhibit overflow queues. Given the congestion level of the intersection, the previous version of the program would have not been capable of providing results for this case. A snapshot of the output of the minimization of the TSC for sequence 1 is shown in Figure 4.

For sequence 2, Table 4 shows the differences observed in the results when different objective functions are used.

Table 4: Results for sequence 2

Objective function	D^w (veh-h/h)	H (stops/h)	F (lt/h)	TSC (CL$/h)	x_{max} (%)	Cycle (s)
D^w (veh-h/h)	44.4	3397.3	124.1	339827	93.4	93
H (stops/h)	47.2	3110.2	123.5	342613	92.4	120
F (lt/h)	45.8	3191.7	122.7	338776	90.7	110
TSC (CL$/h)	45.5	3222.2	122.8	338514	91.5	107

Note first that the minimum value of each indicator is obtained when the same indicator is the objective function, which is somewhat expectable (note, however, that the difference between the minimum and next larger value are very small in some cases). Because of approximations made in the optimization process, this is not always necessarily the case. The difference in those cases should be small.

Results obtained for different objective functions are similar. The difference between the maximum and the minimum value for *F* and *TSC* is 1% of the corresponding minimum value. This percentage goes up to 6% and 9% for *H* and D^w, respectively. Lane 41 is always the one with the largest degree of saturation. In terms of the cycle length, results suggest a cycle length of the order of 100 – 110 seconds. If the intersection is part of a bigger network with common cycle, this information would be very useful to determine whether the intersection should work with simple or double cycle. As expected, the maximum cycle length is used when the goal is to minimize the total number of stops.

FINAL REMARKS

Conclusions

SIGCOM is a program to optimize traffic signal timings for an isolated intersection. The third version of this program has been presented in this article.

Compared to its previous version, the scope of the program was increased by incorporating the capability of optimizing traffic signal timings at oversaturated intersections. For this purpose, Akçelik's formula (1980) for the overflow queue was slightly modified by considering that capacity is not constant but depends on the degree of saturation, which in turns depends on the duration of the effective green time. Although the differences produced are not big, this new interpretation of the formula is more consistent analytically.
The effect of overflow queues through the modified formula was incorporated using delays and stops formulae of Akcelik.. Results are consistent and in oversaturated conditions of traffic give to SIGCOM a bigger versatility.

The program can also be used to compare the performance of the intersection under different sequences of phases. As shown in the example presented in Section 5, the difference between two sequences could be significant.

The interaction of the program with the user was also improved by providing more and clearer information for a better comprehension.

In summary, SIGCOM3 is an improved version of the series, with a wider scope and a friendly and simple interface with the user.

Further research

This new version of the program incorporates Akcelik's formula for delays and stops. It would be interesting to test whether these expressions are a good approximation for the Chilean case or not. It has happened before that some parameters or expressions need to be slightly modified to better represent the Chilean reality. This is an important step, because the program assumes that the expressions for delay and stops are accurate.

Regardless of the objective function used, results were similar. The incorporation of a new objective function is also desirable. For instance, future versions of the program could incorporate the maximization of capacity as another objective function. On one hand, it will enhance the actual version that allows only four optimization objective functions, providing – eventually – more variability on the results. On the other hand, it will provide an output easy to validate in practice. Another aspect to include in the optimization could be the environmental issue, which is an important issue in many developed and developing big cities. For instance, vehicle emissions can be added to the problem.

Finally, this tool could be extended to assess the operation of not only traffic light intersections, but also priority intersections and roundabouts. This idea is being currently studied.

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