OPTIMAL CONDITION SAMPLING OF INFRASTRUCTURE NETWORKS

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ABSTRACT

In response to the developments in inspection technologies, infrastructure decision-making methods evolved whereby the optimum combination of inspection decisions on the one hand and maintenance and rehabilitation decisions on the other are determined based on an economic evaluation that captures the long-term costs and benefits. Recently, sample size has been included in inspection, maintenance, and rehabilitation (IM&R) decision-making as a decision variable when considering a single facility. While, the question of dealing with a network of facilities in making IM&R decisions has been addressed in the literature, this treatment does not consider condition sampling whereby each facility could require a different set of sample sizes over time. This paper presents a methodology developed to address the network level problem whereby the uncertainty due to condition sampling is captured and its related decision variables included in the IM&R decision-making process.

Keywords: Infrastructure, inspection, condition sampling, uncertainty, maintenance and rehabilitation, networks

INTRODUCTION

Transportation infrastructure systems consist of spatially extensive and long-lived sets of interconnected facilities. Over the past two decades, several new non-destructive inspection technologies have been developed and applied in collecting raw condition data and processing them to produce useful condition input to infrastructure inspection, maintenance, and rehabilitation (IM&R) decision-making aimed at minimizing total expected life-cycle cost. Inspection deals with the gathering of data on the extent of facility damage. The data may be collected by visual inspection, through manual measurements, or by automated sensors. An average of collected damage measurements over a facility (defined as a homogeneous section) is an estimate of the current condition of that facility and, in turn, is one primary input to maintenance and rehabilitation (M&R) decision-making. The developments in non-destructive inspection technologies make it possible to estimate facilities' conditions using large quantities of data. The quality of measurements, the sample size, and the nature of correlation among condition variables at different locations determine the accuracy of condition estimates. Naturally, more accurate estimates have the potential to lead to more effective maintenance and rehabilitation decisions. Consequently, the expected combined user costs and maintenance and rehabilitation costs are reduced over the planning horizon. However, more accurate information requires more resources such as increased inspection frequency, advanced inspection sensor technologies, larger sample sizes, or possibly less correlated observations, as well as data processing methods that appropriately combine all this information.

In response to the developments in inspection technologies, decision-making methods evolved whereby the optimum combination of inspection decisions on the one hand and M&R decisions on the other are determined based on an economic evaluation that captures the long-term costs and benefits. Recently, sample size has been included in IM&R decision-making as a decision variable when considering a single facility. While, the question of dealing with a network of facilities in making maintenance and rehabilitation decisions has been addressed in the literature, this treatment does not consider condition sampling whereby each facility could require a different set of sample sizes over time. Doing so is valuable given the network nature of facilities that most infrastructure agencies are responsible for, the increasing number of inspection technology choices with possible varying degrees of accuracy and cost, and budget constraints agencies have to work within.

In the next section, a methodology for IM&R decision-making at the network level is introduced followed by the presentation of a new methodology developed to address the extension of the original methodology by taking into account recent developments addressing sampling at the single facility level. The result is a methodology that captures the uncertainty due to condition sampling and includes sampling as a decision variables in the IM&R decision-making process at the network level. In the third section, a numerical demonstration of the methodology based on a realistic literature- and practice-derived example network of facilities is discussed and insights regarding condition sampling at the network level are derived. The final section summarizes the study and its findings.

NETWORK-LEVEL APPROACH AND DEVELOPED METHOD

In this section, the network level approach based on a randomized policy is first summarized. Based on this approach the developed methodology is presented. In doing so, the treatment of sampling at the facility level is incorporated in solving the problem at the network level.

Network-level approach without sampling

Some network-level IM&R decision-making methods in the literature adopted randomized policies. Randomized decisions are necessary when certain constraints are imposed. Under such considerations, Smilowitz and Madanat (2000) proposed a linear programming formulation for solving the infrastructure IM&R optimization problem at the network level. Before describing the formulation, certain critical elements are first introduced.

Assessed facility condition is assumed to fall into one of a finite number of discrete condition states. Considering that inspection is not perfect, it is assumed that the measurement of condition states does not produce the true condition state. In order to infer the true condition states based on measurements, the nature of measurement error has to be considered. To do so, the concept of the information vector is introduced. This vector is a probability mass function on all possible condition states summarizing both condition measurements as well as prior information. This extended state consists of the initial condition state before any decisions are made, the M&R actions applied up to the current point in time, and all the condition measurements made including the most recent inspection.

In the formulation developed by Smilowitz and Madanat, two types of actions are considered. One represents the M&R actions to be performed and the other represents whether to inspect condition or not. The various M&R actions have different costs and result in different condition state transition probabilities over time. The inspection action space includes taking a condition measurement or not. Inspection improves the understanding of the true condition state. That is, the information vector will be more concentrated around the true condition state.

Transition probabilities specify how facility condition evolves during the next time period, given the current condition state, age, and M&R action applied. More specifically, a transition probability represents the likelihood of a facility to transition to a certain state in one period given the state it is currently in. Therefore, transition probabilities can be organized into a matrix representing all combinations of transitions from state to state. Two facilities in the same condition state to which the same M&R action is applied will have different transition probabilities if their age is different. Age is defined as the number of years since the most recent rehabilitation action was applied. The facility with lower age has a smaller probability of deteriorating to a poorer state during the next period. Therefore, it is important to allow for non-stationary transition probabilities conditional on the age of a facility.

Finally, two cost components are considered. The first component is the cost incurred by IM&R and depends on the specific actions performed. The other component is the user cost which only depends on the condition state.

Given the above elements, the problem of making the best decision can then be formulated as a linear programming problem with the following components.

1. <u>Decision variables</u>: $W_t(\mathrm{I},y,a,r)$ denotes the number of facilities at time t whose information vector is I and age is □ on which M&R action a will be performed, and which will be inspected if □ = 1 and not if □ = 0. The nature of the decision variables when solving the network problem is that of a randomized policy. That is, facilities associated with the same information vector could receive different actions (optimally determined). Such a result is possible due to the presence of budget and condition constraints (discussed in more detail below). It is important to note that

which of the specific facilities associated with the same information vector receive which actions is not part of the solution outcome.

2. Objective function: For each feasible realization of the decision variables, the expected total discounted cost includes the user and IM&R costs and is a linear function of the decision variables.

3. Constraints: (i) Non-negativity: These constraints guarantee that each decision variable is non-negative. (ii) Conservation: These constraints ensure the conservation of facilities over time. That is, the information vectors must transition from one period to the next in a manner consistent with the condition state transition probabilities. (iii) Initial state: The initial distribution of facilities as defined by a set of information vectors is assumed known. (iv) Condition states: These constraints require that the proportion or number of facilities in the condition states considered to be poor are bounded by a maximum value each year. (v) Budget: These constraints require that the IM&R cost is bounded by a maximum and possibly a minimum value each year.

At this point, the problem is reduced to finding the values of the decision variables that satisfy all the constraints and achieve the minimal objective function value. Since the objective function and all the constraints are linear with respect to the decision variables, the problem can be solved using linear programming.

Developed network-level approach incorporating sampling

The formulation developed by Smilowitz and Madanat (2000) described above generalizes the facility level framework developed by Madanat and Ben-Akiva (1994) to a network level one by introducing randomized decisions and solving a linear programming problem. It includes many realistic elements. However, an important decision variable and an associated modeling element are not captured in this formulation, namely, the sample size and spatial correlation among measurements of condition of the same facility. The role of sampling is to increase the accuracy of the information regarding facility condition state. On the one hand, more samples result in higher accuracy. On the other hand, more samples will introduce more cost. Thus, sample sizes for each facility are important decision variables. Effectively, in the formulation developed by Smilowitz and Madanat, for a facility either one sample is taken or not, which is quite limiting. Therefore, the extended formulation developed in this paper includes sample sizes as decision variables. In addition, in the application two condition inspection technologies are considered rendering the set of inspection decisions more flexible. Once multiple samples are taken from the same facility, the spatial correlation among these condition observations must be considered in quantifying the combined measurement and sampling uncertainty. Therefore, a spatial correction function (Mishalani and Koutsopoulos 2002) is adopted in determining this uncertainty.

Elements similar to those of the formulation discussed above, are first noted. The assessed facility condition is assumed to fall into one of a finite number of discrete condition states. Given the current condition assessment through measurement and sampling and historical information including previous measurements and IM&R actions, the posterior probability mass function of the true condition state of each facility (i.e., the information vector) can be determined. Also, it is important to keep track of age in addition to time because the transition probabilities (which take the same form discussed above) depend on age.

The variance of the assessed condition is a critical element of the new formulation and constitutes a major departure from the formulation developed by Smilowitz and Madanat. This variance is determined as a function of the measurement technology, sample size, and the characteristics of the facility (in terms of its inherent variability in condition and the spatial correlation). The determination of this variance is based on the formulation developed by Mishalani and Gong (2009) for a single facility. Another important departure from the formulation discussed above is the introduction of facility lengths, *h*, in representing the network. This variable influences the value of the determined variance of the assessed condition depending on the sample size, inherent variability, and spatial correlation. Finally, in addition to the user and IM&R costs discussed above, the hypothetical terminal cost incurred at the end of the time horizon represents the cost of bringing the facility back to the best condition state for the purpose of equalizing the service life from that point onward.

In light of the elements described above, the problem of making the best decision can similarly be formulated as a linear programming problem with the following components.

1. <u>Decision variables</u>: $W_t(I,h,y,a,r,n)$ denotes the number of facilities at time *t* whose information vector is I, its length is *h*, and of age *y* on which M&R action *a* will be performed, and inspection technology *r* will be used with *n* samples. As in the case of the formulation developed by Smilowitz and Madanat, the nature of the decision variables when solving the network problem is that of a randomized policy. That is, facilities associated with the same information vector could receive different actions (optimally determined). Again, such a result is possible due to the presence of budget and condition constraints. And, as in the case of the study by Smilowitz and Madanat, which of the specific facilities associated with the same information vector receive which specific action is not part of the solution outcome.

2. <u>Objective function</u>: For each feasible realization of the decision variables, the expected total discounted cost includes the user, IM&R, and terminal costs. Of course, the decision variables now include the sample size *n* and the original measurement uncertainty is extended to include measurement and sampling uncertainty taking into account the sample size, facility length, inherent variability, and spatial correlation. The objective function remains a linear function of the decision variables.

3. <u>Constraints</u>: The constraints are similar to those of the formulation developed by Smilowitz and Madanat.

As in the case of the study by Smilowitz and Madanat, the problem is reduced to finding the values of the decision variables that satisfy all the constraints and achieve the minimal objective function value, which can be solved using linear programming.

NUMERICAL EXAMPLE

In this section an example application is described and results and insights are presented. First, the example scenario is specified. Second, the effect of measurement error is explored. Third, the effect of the budget constraint is investigated.

Scenario development

The parameters of the example of interest are determined by drawing upon the literature to arrive at a realistic scenario for analysis. The specification is for the most part based on the scenarios developed by Gong (2006). The parameter values are shown in Table I.

Table I – Base Case Values of Example Parameters (all costs in ($/m^2))

Condition State	4	3	2	1
Routine maintenance	0.34	1.63	4.4	14.79
User cost	18.47	56.735	75.155	113.43
Terminal cost	2.675	22.485	55.695	64.87
Rehab Cost	64.87			
Additional user cost due to M&R	1.45 for routine maintenance and 10.13 for rehabilitation			
Additional User cost due to insp.	0.09 for inspection technology 1 and 0.0015 for inspection technology 2			
Fixed inspection cost	0.0012 for inspection technology 1 and 0.009 for inspection technology 2			
Unit inspection cost	0.00023 for inspection technology 1 and 0.000085 for insp. technology 2			
Intrinsic variance	A function of the true condition state			
Variance of insp. technology	Technology 1: 7.99, technology 2: 17.95			

Effect of measurement error

To test the effect of measurement error, an important element of the formulation especially in light of sampling, different pairs of values of the standard deviations of the two measurement technologies are specified as shown in Table II reflecting a wide spectrum ranging from perfection measurements to very poor measurements. Scenario 2 represents the base case while scenarios 1 and 7 represent the two extreme cases of perfect and poor measurements, respectively.

Table II – Standard Deviations of Measurement Errors

	Standard Deviation	
Scenario	Technology 1	Technology 2
1	0	0
2 (base case)	7.99	17.95
3	30	60
4	80	160
5	150	300
6	500	1000
7	1000	2000

Figure 1 shows the total inspection cost (again in $/m^2) at optimality as a function of the standard deviation of measurement technology (of technology 1 in this case). Notice that the inspection cost first increases as the measurement error increases reflecting the situation where larger samples are taken to compensate for the deteriorating measurement accuracy. At some point, however, the inspection cost starts to decrease reflecting the loss of value of information as measurement errors becoming particularly large where sample size cannot even compensate for the degradation in accuracy.

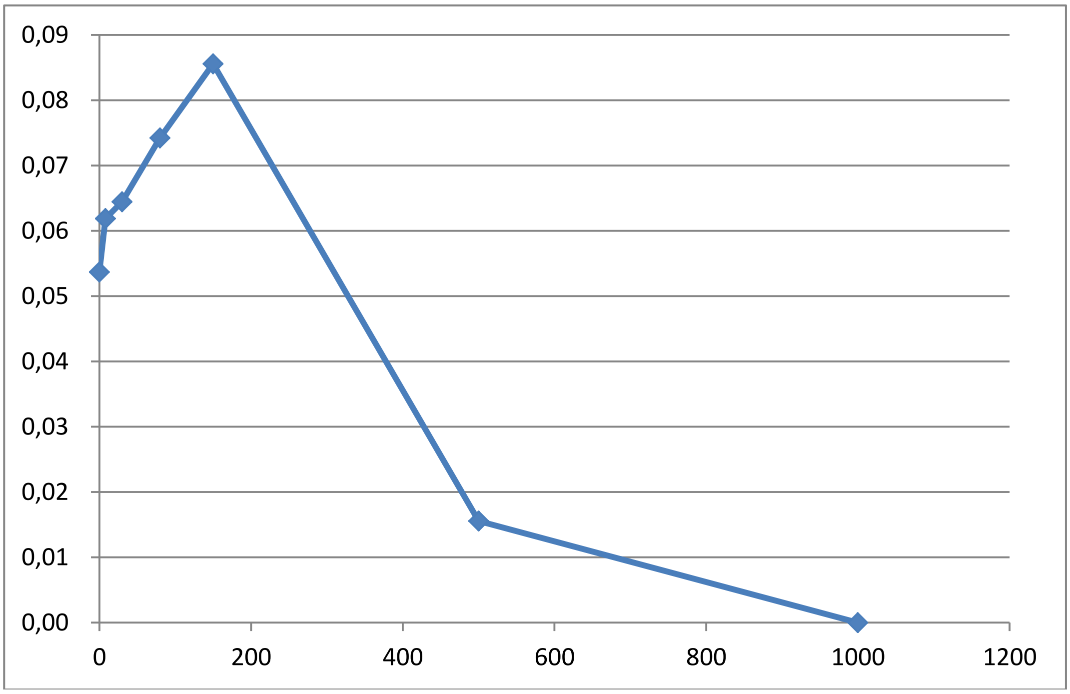

Figure 1 – Inspection Cost versus Standard Deviation of Measurement Error

Effect of budget constraint

The budget constraint is another important element of the formulation. Practically, the infrastructure agency budget is limited and often predetermined. In this application the condition state is not restricted. However, the worse the condition state, the larger the user cost. As such the budget constraint plays a critical element in the optimization.

The expected total cost at optimality along with each of its elements – IM&R, user, and terminal costs – in $/m^2 are plotted against the budget constraint applied on an annual basis (also in $/m^2) in Figure 2. As intuitively anticipated, a stricter budget constraint will results in reduced IM&R cost and larger user and terminal cost. And, overall, an increase in the total cost is expected.

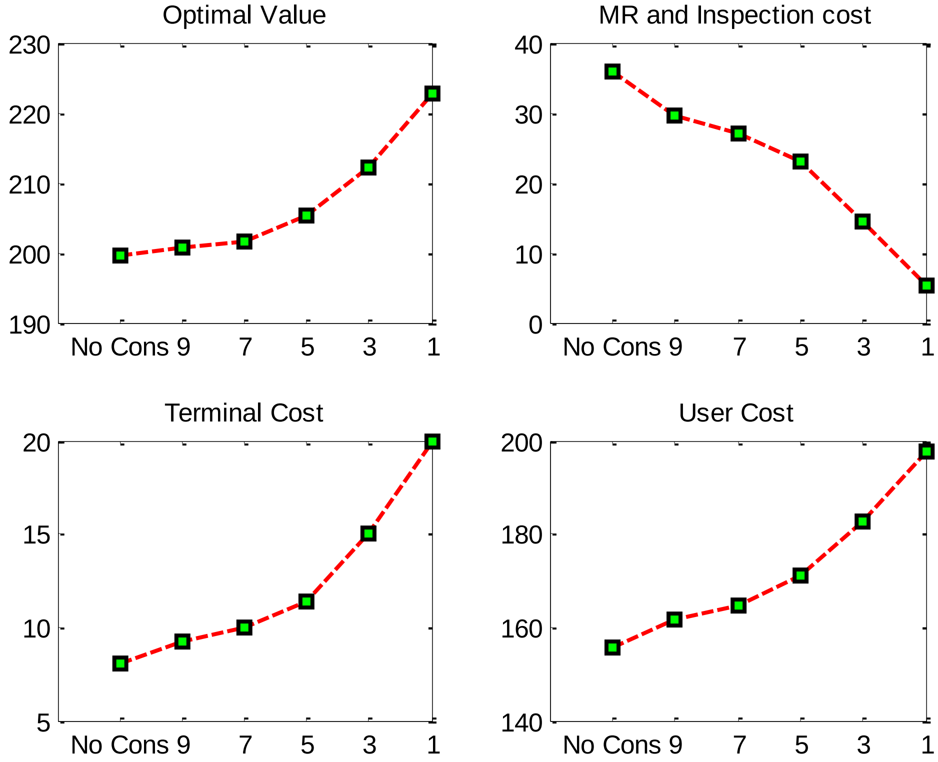

Figure 2 – Expected Total, IM&R, User, and Terminal Cost versus Budget Constraint

SUMMARY

This paper presents a methodology developed to address the IM&R decision-making at the network level whereby the uncertainty due to condition sampling is captured and its related decision variables included in the optimization. An application to a hypothetical yet realistic example is discussed. In this example, it is clear that larger sample sizes can compensate for decreasing inspection accuracy up to a point where the degrading accuracy is so large, increasing the sample sizes does not offer much if any value. In addition, and no surprisingly, a stricter budget constraint will results in reduced IM&R cost and larger user and terminal costs. And, overall, an increase in the total cost is expected.

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