NETWORK OPTIMIZATION WITH DYNAMIC TRAFFIC INFORMATION AND TOLLING: A PROBABILISTIC MODEL AND ECONOMIC ANALYSIS

LEURENT Fabien[1], NGUYEN Thai Phu,

Université Paris-Est / LVMT / Ecole des Ponts ParisTech, France

ABSTRACT

A model is developed for traffic network subject to recurrent congestion and incidental disturbances on link travel times with the presence of dynamic traffic information, and two classes of users respectively informed and uninformed. This traffic assignment model is bi-layered every informed user chooses his route for the short run in any circumstance, whereas every non-informed user chooses his route on for the long run an average basis.

In addition to to dynamic traffic information service, congestion pricing with various tolling strategies like *no-tolling*, *flat tolling* or *dynamic tolling* is also integrated to the model. This model allows us to analyze the combined operation of two traffic demand management tools as well as the interplays between these tools with recurrent congestion, non-recurrent congestion and users' behaviors.

The model is applied for a two-parallel-link network with one O-D pair and linear travel time functions and additive random disturbances. Analytical formulae and numerical investigation show that the benefits from such combined traffic management system split into three parts: benefit from information and benefit from congestion pricing and benefit from joint operation. The complementarities are highly efficient at high equipment rate with dynamic tolling.

Keywords: Dynamic Traffic Information; Multi-class Traffic Assignment; Bi-layer Equilibrium; User Optimum; System Optimum; Congestion pricing.

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[1] Corresponding author, e-mail: fabien.leurent@enpc.fr

1. INTRODUCTION

1.1. Traffic demand management tools

In a road network, the congestion delays include the recurrent delays due to exceed demand, determined by the flow pattern and the laws of traffic, and the non-recurrent congestion delays due to incidents, accidents or other random events, such weather hazard. To tackle congestion, the network operator can avail itself of a wide set of tools and resources including long run capacity planning, medium run operations planning and short run or even real-time by dynamic adaptation of traffic signals, of the local limit speed, of the fare of HOT lanes etc. Traffic management also includes demand management through quality setting, pricing and information provision which are closely related with traffic information services.

Road guidance by traffic information systems

Traffic information services enjoy constant development in a variety of forms: these include network maps, fixed road signs and variable message signs (VMS), traffic radio etc. Moreover, traffic information services by the telephone or the internet, as well as guidance services based on onboard devices provide advanced information which is customized to the individual needs. Actually the user chooses his route and/or departure time on the basis of experience and the information sources that are available to him (e.g. see Miles and Chen, 2004; Leurent, 2004).

Due to stochastic characteristics of the non-recurrent causes, non-recurrent delays can not be forecasted by the individual user. Thanks to the advancement in technologies, including computer, communication and technical data processing, the network operator has been able to detect disturbances, to measure and even to anticipate traffic conditions so as to better react to occurrences. Furthermore, information about traffic conditions can be communicated to road users. The disposal of sharp information enables the road user to react in real-time to traffic disturbances for improving his decision making. At the network level, it is hoped that benefits from traffic information services are not limited to an individual gain but also extended to system-wide benefits since individually optimal decisions contribute to improve the whole network performance.

Congestion pricing as traffic management tool

Road pricing, as an effective means of both managing road traffic demand and raising additional revenue for road construction, has been studied extensively by both transportation researchers and economists. Thanks to the development of electronic road pricing technologies (ERP), the implementation of congestion pricing has become more and more feasible. Nowadays, various tolling systems are deployed throughout the world, such as: cordon tolling is deployed in some congested urban areas, e.g. London, Singapore, on trail in

Stockholm and on project in New York City, flat tolling or variable-time-step tolling is deployed on numerous highways or freeway sections, e.g. the project EUROTOLL in Europe (OECD, 2002) or HOT-lane application in the United States (see information in the reviews of Lemoine, 2009; Ubbels, 2009). By introducing a toll and influencing the road users' decision (destination, mode, and route or departure time choice), it is hoped that travel demand is better distributed in the time dimension or/and also in the space dimension, so individual gain and the social welfare can be improved

Interplays between traffic information and congestion pricing

A priori, dynamic traffic information services could offer significant benefits in terms of improving individual trip experience, in particular with non-recurrent congestion which cause 30%-50% total delay volume on highway (OECD, 1998). Nevertheless, some studies have demonstrated that as result of drivers' responses to more information, adverse effects and negative externalities may arise on the system, e.g. Ben-Akiva, De Palma *et* al. (1991); Zhang and Verhoef (2006) and confirmed by one of our works, Leurent and Nguyen (2010). This happens because when drivers take their trip decisions don't take into account the external costs that they produce on the other drivers and on the overall system.

It is important to address the joint implementation of both traffic management tools over a common network because traffic information services and congestion tolling have technological complementarities; both tools are used for a similar kind of road infrastructure and based on the information technologies. In addition, a dynamic traffic information service is able to provide information about toll which depends on the actual level of congestion, so that road users are perfectly aware of their private cost including social optimal congestion tolling.

Therefore, it is desirable to analyze the interaction between the two systems because congestion tolling may influence the operation of traveler information as well as the benefits derived from traveler information and *vice versa*

1.2. Related literature review

Analysis of traveler information impacts

Analytical approach, the provision of traveler information makes a complex issue of which objective, contents, target, diffusion medium, equipment type, equipment rate etc. In particular the interplay of the congestion sources and physical laws with the provision of dynamic traffic information calls for a model, useful to analyze and simulate the phenomena hence to gain a better understanding.

The distinction of congestion sources and their interplay with dynamic traffic information has been addressed correctly for the choice of departure time by Noland, Small *et* al. (1998); Leurent (2001, 2004), although the analytical approach does not include the feedback of user

re-timing due to congestion. However, concerning route choice, dynamic simulation in the 2000s, e.g. Lo and Szeto (2004) has taken the same line as static simulation in the 1990s, Al-deek and Kanafani (1989); Van Vuren and Watling (1991); Maher and Hughes (1995) by assuming that informed users would perceive a mean travel time, whereas uninformed users would experience disturbed travel times. In fact, this stands exactly opposite to the essence of dynamic disturbances and information since dynamic traffic information reflects eventual network disturbances.

Network disturbances, in particular the relationship between disturbances and dynamic information are still mis-modeled with few analytical contributions. Most of works on this issue are on the basis of numerical simulation, e.g. Al-Deek, Khattak *et* al. (1998); Hu, Wang *et* al. (2005); Levinson, Gillen *et* al. (1999); and Levinson (2003) which provides a limited representation of disturbed cases and also a limited analytical understanding.

Optimal congestion pricing

"*Optimal tolling*" has been largely considered by transportation economists. Three main issues are: which goals should be fixed, i.e. tolling regime, which part of the traffic network should be tolled, i.e. tolling configuration, and what toll must be applied at a given period of time, i.e. the tolling schedule.

The father of congestion pricing is Vickrey who introduced firstly the principal "c*harges should reflect as closely as possible the marginal social cost of each trip in terms of the impacts on others*"- (Vickrey, 1967). The system optimum can be obtained if on each link, every road user is charged at the amount of marginal social cost (first-best).

Since the toll implementation on all road sections is difficultly feasible, a number of works have been targeted contribute to the issues of where and how much toll could be implemented within a limited number of predefined links (second-best). In static simulation, the most comprehensive contributions on second-best tolling should be Vany and Saving (1980); Verhoef, Nijkamp *et* al. (1996); de Palma and Lindsey (2000); Verhoef (2002a, b); and Zhang and Verhoef (2006) among others.

As regards dynamic simulation, a number of works contribute on finding optimal tolling when toll is variable by time-step (third-best) or toll is aimed to maintain no-queue traffic flow (de Palma, Kilani *et* al., 2005; de Palma, Lindsey *et* al., 2008).

Joint analysis of route guidance and congestion pricing

In spite of the numerous works that addressed "traffic information" or "congestion pricing" on a separate basis, there have been few contributions on the interplay between traffic information services and congestion pricing. Let us quote nevertheless the works of Verhoef, Emmerink *et* al. (1996); de Palma and Lindsey (1998); Yang (1999); Zhang and Verhoef (2006), only to mention that they pay no consideration to exogenous disturbances - the reason for what dynamic information services have been developed.

Recently, Fernández, de Cea et al. (2009) attempted to analyze the joint effects of traffic information and congestion pricing in presence of traffic disturbances. However, the representation of disturbances in this modelling framework is limited to a set of four alternative traffic conditions.

1.3. Paper scope

Objective

In a previous work (Leurent and Nguyen, 2008, 2010), the interplays between recurrent congestion, non-recurrent congestion and dynamic traffic information were explicitly modeled by segmenting travel demand into two classes of users respectively informed or not: the informed users have dynamic traffic information about the actual travel time conditions, whereas the non-informed users only know about the average travel times. This is a novel assignment model since it distinguishes two layers of user equilibrium in traffic, associated with two time scales in demand perception and behavior: an upper layer of short run decision-making by informed users as opposed to a lower level of long run decision-making by uninformed users, each layer constraining the other.

The main purpose of this paper is to develop an extension of the basic model integrating various tolling strategies. The extended model allows analyzing the impacts of dynamic traffic information and congestion pricing on traffic as well as splitting individual effects and joint effects from each of these demand management tools. Rather than trying to quantify the costs and profits of a particular system, our focus is on a crucial assumption which has yet been mis-modeled in the evaluation of traffic information systems: namely, the probabilistic structure of dynamic disruptions.

Approach

Our philosophy, throughout a number of works on the issue of traveler information, is explicit and realistic enough in these respects, yet as simple as possible without distorting the physic and behavioral features. Our model makes explicit assumptions about: (i) network; (ii) traffic conditions subject to the phenomena of congestion; (iii) statistical distribution of disturbances; (iv) demand structure consisting of volume, classes of users (informed or non-informed), equipment rate. The individual traffic information device enables the informed user to know about disturbed travel costs while the non-informed user perceives only average travel costs owning to his experience from previous trips.

Then, the modeling framework will be adapted for various tolling strategy, in particular flat-tolling strategy in which toll is fixed for all circumstances and dynamic-tolling strategy in which an optimal toll is designed and tailored to the vary circumstance. The combination of three tolling options (no-tolling, flat-tolling and dynamic-tolling) and two information diffusion

options (with or without dynamic information) makes a set of six management schemes to be compared.

The model is applied to a classroom network of two parallel links, with two classes of users respectively informed or not, in order to simulate the response to disturbances of travelers and the design of traffic management schemes and to evaluate the profit from traffic management tools.

Structure of the paper

Section §2 will set up the modeling framework by recalling the main assumptions and adding a tolling component, including toll variables and tolling strategies, to the basic model presented in (Leurent and Nguyen, 2008, 2010). This setting will bring us to various traffic equilibrium assignment problems which depend on the specified tolling strategy. Since the assignment problem is the same at that in the basic model under no-tolling strategy, section §3 will recall the main steps of the equilibrium analysis which end up with some analytical formulae. Section §4 starts by defining the optimal flat toll and then attempts to convert the new traffic equilibrium assignment problems, including flat-tolling strategy, to a traditional user equilibrium problem solved previously. In the same logic, section §5 starts by defining the optimal dynamic-toll. In fact, the traffic equilibrium assignment problem under dynamic tolling strategy is identical to a system optimum assignment problem which was solved by Leurent and Nguyen (2009). The traffic equilibrium analysis carried out in sections §3,4,5 allows us doing some analytical comparisons as well as some numerical investigations in section §6.

2. THE BI-LAYER EQUILIBRIUM MODEL

The model will be presented through a set of modelling assumptions that pertain to, respectively: (i) the supply side of the network and its routes, with the local travel times, congestion effects and random disturbances; (ii) different tolling scenarios; (iii) the demand side of the trip-makers whether equipped or non-equipped with a device for dynamic information; (iv) the interaction of supply, demand, toll and information.
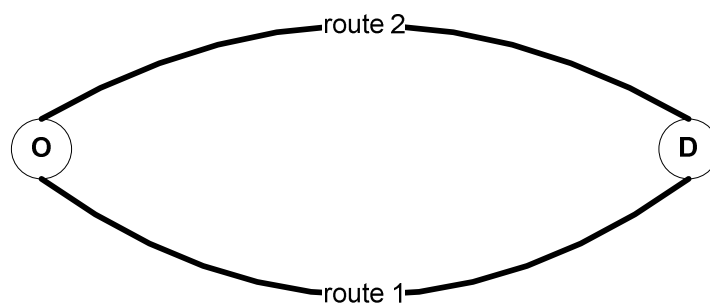
2.1. Supply-side assumption

Figure 1 – Two-parallel-link network with one OD relation

12th WCTR, July 11-15, 2010 – Lisbon, Portugal

Let us consider a transport network made up of arcs $a \in A$ the arc set, with endpoints $n \in N$ the node set. Our application here is restricted to a classroom case of two parallel arcs linking an origin node to a destination node, as in Figure 1: hence $a \in \{1,2\}$.

On each network arc a, the arc flow x_a induces an individual travel time of T_a which is subject to congestion effects on the basis of a travel time function $T_a = \tilde{t}_a(x_a)$, an increasing function of the arc flow. For instance let us take a linear affine function as follows, $\tilde{t}_a(x_a) = \alpha_a + \gamma_a x_a$ in which α_a denotes a free-flow travel time and γ_a the sensitivity of the individual time to the flow (see Figure 2a). This assumption is consistent with the un-queued state of traffic, not with the queued state in which the flow is constrained by a flowing capacity.

Despite our flow-based model is basically a stationary model, we shall consider dynamic effects from period to period, i.e. inter-period variability to be modeled by a random variable (see Figure 2b). To that end, we assume that there is a set Ω of circumstances (or periods) ω, each of which has arc flow $x_{a\omega}$ and arc travel time:

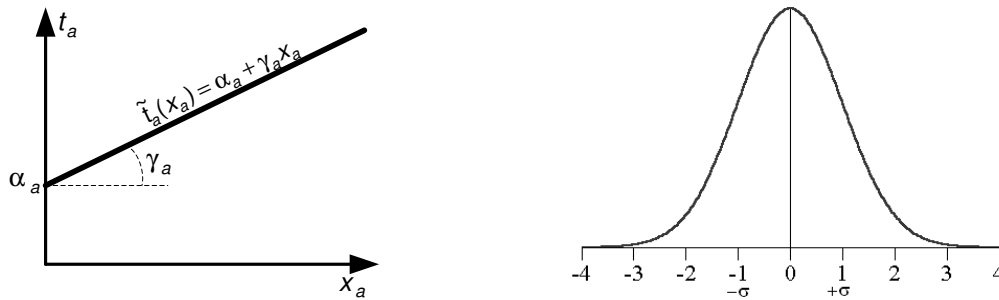$$T_{a\omega} = \tilde{t}_a(x_{a\omega}) + \zeta_a(\omega) \tag{1}$$

Figure 2 – (a) Travel time-flow function; and (b) Link disturbance distribution

The random variable $\zeta_a(\omega)$ models the eventual variation in travel time that may arise due to exogenous disturbances. It is assumed to have variance of σ_a^2 and null mean. For simplicity, it can be assumed that the variables $\zeta_a(\omega)$ are independently distributed.

We have the formula of the average travel time over all occurrences:

$$E_\omega[T_{a\omega} : x_{a\omega} = x] = \tilde{t}_a(x) \tag{2}$$

2.2. Tolling strategies

At a given occurrence ω, on link a, a toll $p_{a\omega}$ (for simplicity, $p_{a\omega}$ is supposed to be converted into time units) computed by the network operator according to traffic conditions under one of the following tolling strategies:

- **No-tolling** strategy, denoted *NT*: toll $p_{a\omega}$ on link a is set at 0 (null) for every link every occurrence.

▪ **Flat-tolling** strategy, denoted *FT*: $p_{a\omega}$ does not depend on occurrence ω: $p_{a\omega} = \overline{p}_a \ \forall \omega$. The value $\overline{p}_a$ is set in order to maximize the average social welfare over all occurrences ω.

▪ **Dynamic-tolling** strategy, denoted *DT*: $p_{a\omega}$ is variable with respect to occurrence ω. For each occurrence ω, $p_{a\omega}$ is set in order to maximize the social welfare for that occurrence.

2.3. Demand-side assumption

Let us analyze the network trips by origin-destination (OD) pair, e.g. from node O to node D, presented in Figure 1. The trips are made on an OD pair by a population of network users who fall into one out of two classes: either class *I* of equipped users who have got a device to receive dynamic information, or class *N* of the non-equipped users that do not receive dynamic information.

Every user is assumed to be homogenous in value of time and to choose his network route from origin to destination under a rational behavior of cost minimization, subject to his knowledge of the costs.

An informed user *I* is assumed to derive perfect knowledge from dynamic information whatever the circumstance, including travel time and toll:

$$c_a^I(\omega) = C_{a\omega} = T_{a\omega} + p_{a\omega} \tag{3}$$

While a non-informed user *N* is assumed to possess only coarse knowledge on the basis of the average travel time and average toll:

$$c_a^N = \mathrm{E}_\omega[T_{a\omega} + p_{a\omega}] = \mathrm{E}_\omega[C_{a\omega}] = \overline{C}_a \tag{4}$$

Here the user cost is restricted to the travel time and travel time-equivalent-unit toll, neglecting comfort and other quality criteria in order to focus on the disturbances which make our primary concern.

Let us denote by $Q = x_1 + x_2$ the non-elastic total demand volume, assumed constant whatever the circumstance. Let $\beta = q^I/Q$ denote the equipment rate i.e. the ratio of the number of informed users, q^I, to Q, and let $q^N = Q - q^I$ be the number of non-informed users. By arc a, occurrence ω and user class $u \in \{I, N\}$, $x_a^u(\omega)$ denotes the flow on arc a: it holds that:

$$\begin{cases} x_a(\omega) = x_a^I(\omega) + x_a^N(\omega) \\ x_1^I(\omega) + x_2^I(\omega) = q^I \\ x_1^N(\omega) + x_2^N(\omega) = q^N \end{cases} \tag{5}$$

2.4. Interaction supply-demand

Let us chain the assumptions about supply and demand in the following statement:

- In any occurrence ω, every network user chooses his route: his travel along that route makes a piece of flow.

- Considering the population of trip-makers, their individual choices induce the arc flows throughout the network.

- The arc flows determine the arc travel times on the basis of the congestion functions.

- The travel times determine the individual route choice.

Thus there is a cyclical chaining in the interaction of supply and demand. Figure 3 makes the statement more precise by addressing each user class in a specific way: a class I user reacts to any particular occurrence ω by adapting his route choice to the dynamic context – thus leading to $x_a^I(\omega)$ flows that vary with ω hence in the short run, while a class N user makes his route choice only in the long run on the basis of the route performance averaged over the distribution Ω of cases ω, leading to $\overline{x}_a^N$ flows that do not vary with respect to ω. Toll $p_{a\omega}$ is computed on the basis of traffic conditions and of the tolling chosen tolling strategy.
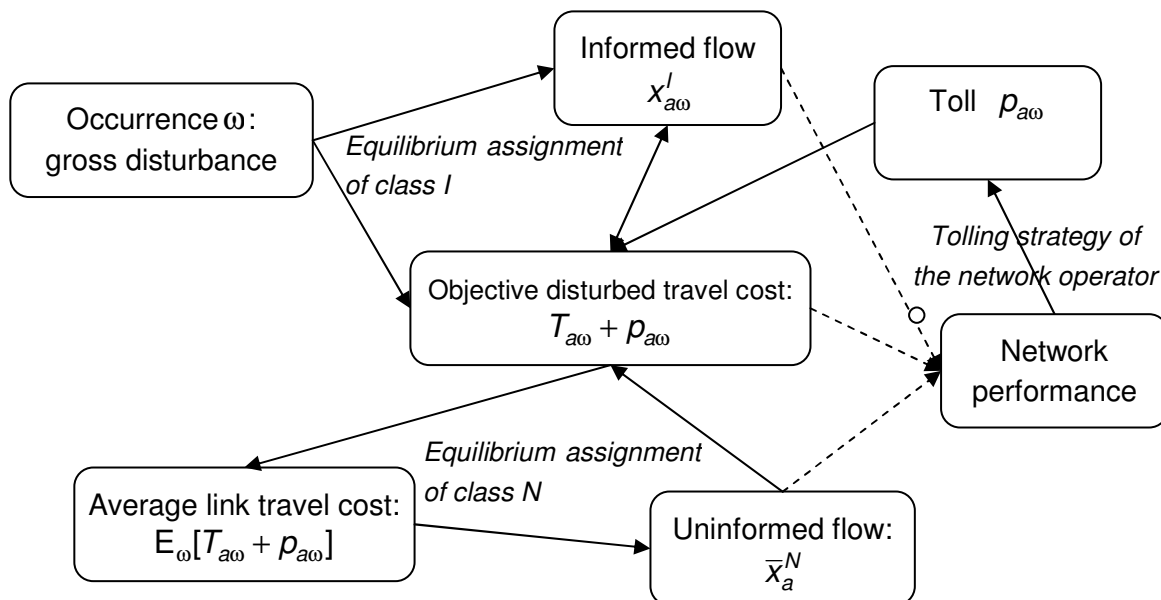

Figure 3: Logic dependencies in the model

3.5. Indicators of utility

Two economic indicators are chosen for analyzing (i) the individual benefit that a road user derives from a personal traffic information device and (ii) the social benefit of the whole system, including dynamic traffic information and congestion pricing system.

Individual gain of being equipped

The individual gain of being equipped is measured by the gap between the average travel cost (per trip) to non-informed user $\overline{C}^N$ and the average travel cost (per trip) to informed user $\overline{C}^I$:

$$\Gamma^P = \overline{C}^N - \overline{C}^I \tag{6}$$

, in which P denote the specific toll strategy in the analysis.

To a non-equipped user of non-informed class N, the average cost is as follows (because the assignment of this class does not depend on ω):

$$\overline{C}^N = c_1^N \frac{x_1^N}{q^N} + c_2^N \frac{x_2^N}{q^N} \tag{7}$$

To an equipped user of class I, the average cost stems from the aggregation of all occurrence costs:

$$\overline{C}^I = \int_\omega (x_{1\omega}^I C_{1\omega} + x_{2\omega}^I C_{2\omega}) d\omega \tag{8}$$

Average travel time per trip undergone by all network users

The social welfare of the whole traffic can be represented by the average travel time per trip undergone by all network users. This average travel time stems from the aggregation of all occurrence travel times:

$$T^P = \int_\omega (x_{1\omega} T_{1\omega} + x_{2\omega} T_{2\omega}) d\omega \tag{9}$$

Note that this indicator does not include toll since toll, which is transferred from users to the network operator, is not a social cost.

3. NO-TOLLING STRATEGY

Under no-tolling strategy, the assignment problem stays a bi-layer user equilibrium problem, denoted, which is treated in (Leurent and Nguyen, 2008, 2010). This section presents only the main steps of the equilibrium analysis which follow the logical structure in Figure 3. We shall first assume a given assignment of the non-informed class and focus on the assignment of the informed class. The average assignment of the informed class (§3.2) will be aggregated from its occurrence assignments (§3.1) over all occurrences ω. Next, the non-informed class will be assigned conditionally on the informed class. Then, the assignment cycle will form the bi-layer equilibrium as a fixed-point problem (§3.4). Lastly, two economic

indicators will be proposed for analysing the effects of dynamic information and congestion pricing on the traffic network (§3.5).

3.1. Occurrence assignment of informed users

Let $\alpha_a^I = \alpha_a + \gamma_a \bar{x}_a^N$: then $T_{a\omega} = \alpha_a^I + \gamma_a x_a^I(\omega) + \zeta_{a\omega}$ is the case travel time of arc a under occurrence ω. If there were only one informed user, he would choose the route of minimum $T_{a\omega}(0) = \alpha_a^I + \zeta_{a\omega}$. However the dynamic reassignment of informed users will tend to increase the travel time of that route due to its congestion function: this effect may yield partial compensation i.e. $T_{a\omega}$ remains less than $T_{b\omega}$, or total compensation i.e.

$$\alpha_a^I + \gamma_a x_a^I(\omega) + \zeta_{a\omega} = \alpha_b^I + \gamma_b x_b^I(\omega) + \zeta_{b\omega} \tag{10}$$

In the latter case, since $x_a^I(\omega) + x_b^I(\omega) = q^I$ it holds that:

$$x_a^I(\omega) = \frac{\gamma_b q^I + \alpha_b^I + \zeta_b - \alpha_a^I - \zeta_a}{\gamma_a + \gamma_b} \tag{11}$$

$$T_{a\omega} = \frac{\gamma_a \gamma_b q^I + \gamma_a \alpha_b^I + \gamma_b \alpha_a^I}{\gamma_a + \gamma_b} + \frac{\gamma_b \zeta_a + \gamma_a \zeta_b}{\gamma_a + \gamma_b} \tag{12}$$

In the former case, $x_{a\omega}^I = q^I$ and $x_{b\omega}^I = 0$, hence $T_{a\omega} = \alpha_a^I + \gamma_a q^I + \zeta_{a\omega}$ and $T_{b\omega} = \alpha_b^I + \zeta_{b\omega}$, with $T_{a\omega} \le T_{b\omega}$ hence:

$$\zeta_{b\omega} - \zeta_{a\omega} \ge \alpha_a^I - \alpha_b^I + \gamma_a q^I \tag{13}$$

3.2. Conditional and average assignment of informed users

Let $B \equiv \alpha_1^I - \alpha_2^I + \gamma_1 q^I$ and $A \equiv \alpha_1^I - \alpha_2^I - \gamma_2 q^I$: conditional on $z = \zeta_2 - \zeta_1$ it holds that

- if $z > B$ then $T_{1\omega}(q^I) \le T_{2\omega}(0)$ hence $x_1^I(\omega) = q^I$ and $x_2^I(\omega) = 0$

- if $z < A$ then $T_{1\omega}(0) \ge T_{2\omega}(q^I)$ hence $x_1^I(\omega) = 0$ and $x_2^I(\omega) = q^I$

- if $z \in [A,B]$ then $T_{1\omega} = T_{2\omega}$ at flows $x_1^I(\omega) = \frac{z - A}{\gamma_1 + \gamma_2}$ and $x_2^I(\omega) = \frac{B - z}{\gamma_1 + \gamma_2}$.

Denoting by F the distribution function of $Z = \zeta_2 - \zeta_1$ over the set Ω of cases ω, and by $\tilde{F}$ the truncated moment function $\tilde{F}(x) = \int^x z\,dF(z)$, by aggregation

$$\bar{x}_1^I = 0 . \int_{-\infty}^{A} dF(z) + \int_{A}^{B} \frac{z - A}{\gamma_1 + \gamma_2} dF(z) + q^I . \int_{B}^{+\infty} dF(z) = q^I - \bar{x}_2^I \tag{14}$$

in which:

$$\bar{x}_2^I = \frac{G(B) - G(A)}{\gamma_1 + \gamma_2} \quad \text{where } G(x) \equiv x F(x) - \tilde{F}(x) \tag{15}$$

The independence of ζ_1 and ζ_2 makes Z randomly distributed with mean value $\mu = \mu_2 - \mu_1 = 0$ and variance $\sigma^2 = \sigma_1^2 + \sigma_2^2$

3.3 Assignment of non-informed users

Let $\alpha_a^N \equiv \alpha_a + \mu_a + \gamma_a \overline{x}_a^I$ and compare α_1^N with α_2^N :

- if $\alpha_1^N \geq \alpha_2^N + \gamma_2 q^N$ then $\overline{x}_2^N = q^N$ and $\overline{x}_1^N = 0$

- if $\alpha_1^N + \gamma_1 q^N \leq \alpha_2^N$ then $\overline{x}_1^N = q^N$ and $\overline{x}_2^N = 0$

- if $-\gamma_1 q^N \leq \alpha_2^N - \alpha_1^N \leq \gamma_2 q^N$ then $\overline{x}_a^N = \dfrac{\alpha_b^N - \alpha_a^N + \gamma_b q^N}{\gamma_a + \gamma_b}$

The last condition stems from $\overline{t}_a = \overline{t}_b = \theta$ with $\overline{t}_a = \alpha_a + \gamma_a \overline{x}_a^I + \gamma_a \overline{x}_a^N$, when the assignment of non-informed users to the routes of minimal cost subject to the congestion functions yields a traffic equilibrium with equilibrium time of θ.

3.4. Fixed-point Characterization of Supply-Demand Equilibrium

Linking together the previous formulae, we obtain that the non-informed flows $(\overline{x}_a^N)$ induce the reference informed travel times (α_a^I), which in turn determine the average informed flows $(\overline{x}_a^I)$, which in turn determine the reference non-informed travel times (α_a^N), which in turn determine the non-informed flows $(\overline{x}_a^N)$. This cycle states that any variable set out of $(\overline{x}_a^N)$, $(\overline{x}_a^I)$, (α_a^I), (α_a^N) solves a specific problem of fixed point. In the two-link case that kind of problem is easy to solve since it involves only one real unknown; a relaxation algorithm would be appropriate, for instance a convex combination algorithm in the non-informed flows.

3.5. Case of average link cost equality

For the case of equal average link costs (the most popular case, see Leurent and Nguyen (2008, 2010)). The indicators can be formulated analytically:

- The individual gain of being equipped

$$\Gamma^{NT} = G(-\tfrac{1}{2}.\gamma.\beta.Q) \qquad (16)$$

, in which: γ is the sum of sensitivity-to-congestion parameters: $\gamma \equiv \gamma_1 + \gamma_2$.

- The average travel time (per trip) undergone by all users amounts to:

$$T^{NT} = \theta - \beta.G(-\tfrac{1}{2}.\gamma.\beta.Q) \tag{17}$$

, in which $\theta \equiv \dfrac{\gamma_1\gamma_2 Q + \alpha_1\gamma_2 + \alpha_2\gamma_1}{\gamma_1 + \gamma_2}$

4. FLAT-TOLLING STRATEGY

4.1. Optimal flat toll

Under flat-toll strategy, denoted *FT*, toll rates are invariable with regard to ω: $p_{a\omega} = \overline{p}_a \;\forall \omega$. $\overline{p}_a$ is set in order to minimize the average travel time (per trip) of all users:

$$\min_{\overline{p}_a}\left[T^{FT}\right] \tag{18}$$

$\overline{p}_a$ will be calculated in the procedure of traffic assignment (sub-section §4.2) when T^{FT} is explicitly formulated.

4.2. Traffic assignment

Assuming that the $\overline{p}_a$ are known, we will search a solution to the traffic assignment problem. When the traffic assignment and the indicators are formulated, the average travel time per trip T^{FT} can be used as an objective function to determine $\overline{p}_a$.

Under fixed $\overline{p}_a$, the the link travel cost function becomes:

$$C_{a\omega} = T_{a\omega} + p_{a\omega} = \alpha_a + \overline{p}_a + \gamma_a x_{a\omega} \tag{19}$$

Denoting $\alpha_a^* = \alpha_a + \overline{p}_a$, the formula can be rewritten as:

$$C_{a\omega} = \alpha_a^* + \gamma_a x_{a\omega} \tag{20}$$

The traffic assignment problem becomes a traditional traffic assignment problem as in section §3 when the demand with volume Q and equipment rate β is assigned to a network with parameter set: α_a^*, γ_a (instead of α_a, γ_a)

4.3. Case of average link cost equality

For the case of equal average link costs, the indicators can be formulated analytically as below:

▪ The individual gain of being equipped:

$$\Gamma^{FT} = G\left[-\tfrac{1}{2}.\gamma.\beta.Q\right] \tag{21}$$

▪ The average travel time (per trip) undergone by all users amounts to:

$$\overline{T}^{FT} = \theta^* - \beta.G(-\frac{1}{2}.\gamma.\beta.Q) - [\frac{\overline{x}_1}{Q}.\overline{p}_1 + \frac{\overline{x}_2}{Q}.\overline{p}_2] \tag{22}$$

, in which: $\quad \theta^* \equiv \dfrac{\gamma_1\gamma_2 Q + (\alpha_1 + \overline{p}_1)\gamma_2 + (\alpha_2 + \overline{p}_2)\gamma_1}{\gamma_1 + \gamma_2}$

Since $\overline{C}_1 = \overline{C}_2$, the formula of $\overline{T}^{FT}$ can be reduced to:

$$\overline{T}^{FT} = \theta - \beta.G(-\frac{1}{2}.\gamma.\beta.Q) + \frac{1}{\gamma Q}\Delta p(\Delta p + \Delta \alpha) \tag{23}$$

, in which: $\Delta \overline{p} = \overline{p}_2 - \overline{p}_1$ and $\Delta \alpha \equiv \alpha_2 - \alpha_1$

Let us consider $\overline{T}^{FT}$ as a function of a unique variable $\Delta \overline{p}$. $\Delta \overline{p}$ must be set at $-\dfrac{\Delta \alpha}{2}$ for $\overline{T}^{FT}$ to be minimal. At this state:

$$\overline{T}^{FT} = \theta - \frac{\Delta \alpha^2}{4.\gamma.Q} - \beta.G(-\frac{1}{2}.\gamma.\beta.Q) \tag{24}$$

5. DYNAMIC-TOLLING STRATEGY

5.1. Optimal dynamic toll

Under dynamic-tolling strategy, toll $p_{a\omega}$ are set in order to minimize the total travel time undergone all user at the occurrence ω:

$$\min_{p_{a\omega}}[x_{1\omega}T_{\omega1} + x_{2\omega}T_{\omega2}] \tag{25}$$

In the literature, it is well-known that: traffic assignment can achieve system optimum if "*user is charged so that travel cost reflects the marginal social cost in term of impact on others*". e.g. link travel cost introduced in (3) should amount to:

$$C_{a\omega} = T_{a\omega} + \frac{dT_{a\omega}}{dx_{a\omega}} \tag{26}$$

In the other word, toll on link *a* at occurrence ω must be set to:

$$p_{a\omega} = x_{a\omega}\frac{d\tilde{t}_a}{dx_{a\omega}} \tag{27}$$

At this state, the traffic assignment problem coincide with a bi-layer system optimum assignment as addressed in (Leurent and Nguyen 2009).

5.2. System optimum traffic assignment

Thanks to the linear assumption in travel time function, the formula (25) can be rewriten as: $p_{a\omega} = \gamma_a.x_{a\omega}$. The link travel cost function becomes:

$$C_{a\omega} = T_{a\omega} + p_{a\omega} = \alpha_a + 2\gamma_a x_{a\omega} + \zeta_a = \alpha_a + \gamma_a^{\#} x_{a\omega} + \zeta_a \tag{28}$$

, in which $\gamma_a^{\#} = 2\gamma_a$.

The traffic assignment problem becomes a traditional traffic assignment problem presented in section §3 when the demand with volume Q and equipment rate β is assigned into a network of the parameter set: $\alpha_a, \gamma_a^{\#}$ (instead of α_a, γ_a).

By replacing $2\gamma_a$ by $\gamma_a^{\#}$, this assignment problem is identical to the bi-layer user equilibrium assignment presented in §3. This assignment problem called system optimum assignment is also treated in (Leurent and Nguyen 2009).

5.3. Case of link cost equality

For the case of equality of average link cost, the indicators can be formulated analytically as below:

- The individual gain of being equipped:

$$\Gamma^{DT} = G[-\gamma.\beta.Q] \tag{29}$$

- The average travel time (per trip) undergone by all users amounts to:

$$\overline{T}^{DT} = \theta - \beta.G(-\frac{1}{2}.\gamma.\beta.Q) - \frac{\Delta\alpha^2}{4.\gamma.Q} - \left[\frac{\sigma^2}{2.\gamma.Q}[\frac{1}{2} - F(-\gamma.\beta.Q)] + \beta[\frac{1}{2}.G(-\gamma.\beta.Q) - G(-\frac{1}{2}.\gamma.\beta.Q)]\right] \tag{30}$$

6. COMPARING DIFFERENT MANAGEMENT SCHEMES

In the three previous sections, the analytical formulae of average travel cost per trip undergone by all users are identified. The comparison of the different tolling strategies start with a setting of management scheme (§6.1), then an analytical comparison (§6.2) and a numerical investigation (§6.3) will enable us to gain a better understanding, in particular on the sensitivity of the effects with regard to network configuration or to information diffusion level.

6.1. Traffic management schemes

The combination of two options with regard to dynamic traffic information and three options with regard to congestion pricing makes totally six available management schemes:

Table 1 – Management schemes by combining dynamic information and congestion pricing

Information ⟍ Congestion pricing	Without dynamic information	With dynamic information
No-toll	(1)	(2)
Flat-toll	(3)	(4)
Dynamic-toll	(5)	(6)

6.2. Analysis

Under the assumption of equal average travel costs, the average travel time per trip without any regulation measure is θ. The benefit given by the traffic information service and the congestion pricing system at a tolling strategy *P*, is the different between θ and T^P.

Under the no-tolling strategy, the benefit from dynamic traffic information is: $\beta.G(-\frac{1}{2}.\gamma.\beta.Q)$ which is a positive function of β. But it achieves its maximum at a relatively low value of β

Under flat-tolling strategy and traffic information service, the benefit is $\frac{\Delta\alpha^2}{4\gamma Q} + \beta.G(-\frac{1}{2}.\gamma.\beta.Q)$ which is composed of two parts:

- The benefit from dynamic traffic information: $\beta.G(-\frac{1}{2}.\gamma.\beta.Q)$

- The benefit from flat tolling: $\frac{\Delta\alpha^2}{4\gamma Q}$.

Under dynamic-tolling and traffic information service, the benefit is still higher:
$\beta.G(-\frac{1}{2}.\gamma.\beta.Q) + \frac{\Delta\alpha^2}{4.\gamma.Q} + \left[\frac{\sigma^2}{2.\gamma.Q}[\frac{1}{2} - F(-\gamma.\beta.Q)] + \beta[\frac{1}{2}.G(-\gamma.\beta.Q) - G(-\frac{1}{2}.\gamma.\beta.Q)]\right]$ which can be divided into three parts:

- The pure benefit from dynamic traffic information: $\beta G(-\frac{1}{2}\gamma\beta Q)$ which depends on network parameters, demand volume and equipment rate.

- The pure benefit from congestion pricing (both flat and dynamic toll) $\frac{\Delta\alpha^2}{4\gamma Q}$ which depends only on network parameters and the demand volume

- The benefit from the joint operation of dynamic traffic information and congestion pricing: $\left[\frac{\sigma^2}{2.\gamma.Q}[\frac{1}{2} - F(-\gamma.\beta.Q)] + \beta[\frac{1}{2}.G(-\gamma.\beta.Q) - G(-\frac{1}{2}.\gamma.\beta.Q)]\right]$ which is a positive and increasing function of β.

At the individual level, the gain of being equipped is identical under two strategies *NT* and *FT*: $\Gamma^{NT} = \Gamma^{FT} = G[-\frac{1}{2}.\gamma.\beta.Q]$ Under *DT* strategy, the gain of being equipped $\Gamma^{DT} = G[-\gamma.\beta.Q]$ is a somewhat lower than under the other strategies.

6.3. Numerical investigation

Numerical setting

We set up two configurations for a two link network:

- ▪ "Motorway versus City Arterial" in which one route dominates the other in both link capacity and free flow link travel time. The numerical values of the parameters are set to $\alpha_1 = 40$, $\gamma_1 = 1$ and $\alpha_2 = 80$, $\gamma_2 = 2$.

- ▪ "City Arterial versus City Road" in which each of the two routes has its own advantage either in capacity or free flow travel time. The numerical values of the parameters are set to $\alpha_1 = 40$, $\gamma_1 = 2$ and $\alpha_2 = 80$, $\gamma_2 = 1$.

The random disturbances on link travel times are taken as independent, centred Gaussian variables, each one with variance of $\sigma^2 / 2$ to ensure that the difference variable $Z = \zeta_2 - \zeta_1$ has variance σ^2. The value of σ is set to 40, which supplies a link time with a relative dispersion of $\sigma / \bar{t}\sqrt{2}$ that decreases from 2/3 to 1/5 as the link flow increases.
The travel demand Q is varied from 5 to 150 by step of 5 in order to examine a range of traffic conditions. The equipment rate β is varied from 0% to 100% by increment of 1%.

Low travel demand condition

The average travel time per trip undergone by all users over all occurrences is representative to the social welfare of the system. At low travel demand Q=10 (Figure 4), dynamic traffic information contributes significantly to improve network performance. However, congestion pricing has no impact to the system. It is quite logic since congestion pricing is not needed if traffic conditions are not congested.
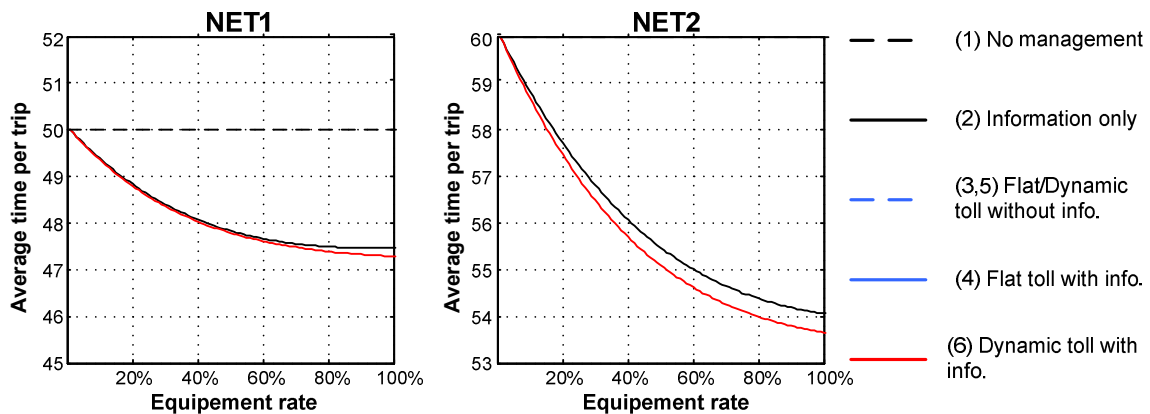

Figure 4 - Average travel time per trip at high demand Q=10 under different management schemes

High travel demand condition

More interesting the in the case with high travel demand, Q=100, depicted in Figure 5. The effects of demand management on the average travel time per trip are much more significant.
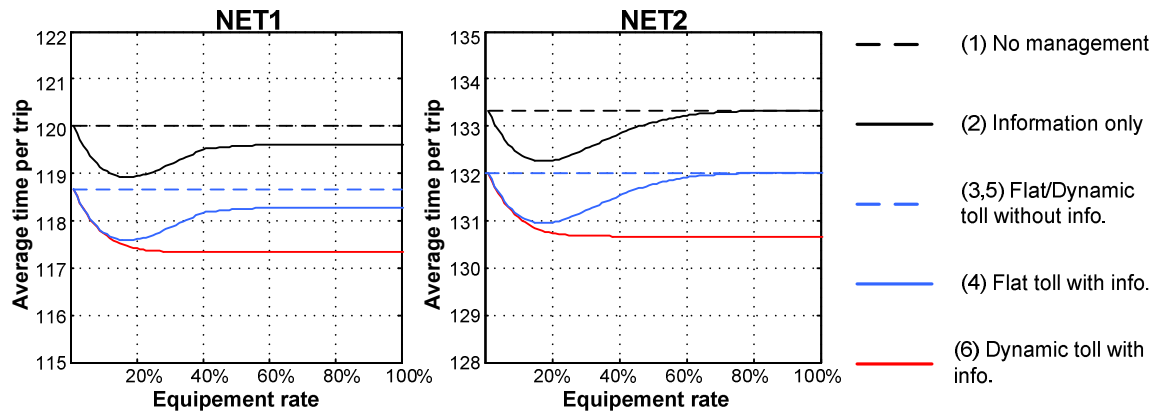

Figure 5 – Average travel time per trip at high demand Q=100 under different management schemes

The difference between the top line (1) and the continuous black line (2) represents the pure value of selfish dynamic traffic information (only dynamic information). An exciting point must be remarked is that the benefit from dynamic information achieves its maximum at a relatively low value of equipment rate.

The difference between the continuous black line (2) and the continuous blue line (4), as well as between the top line (1) and the dash blue line (3,5) represents the pure value of congestion pricing (only congestion pricing). It is constant with regard to the equipment rate.

The difference between the continuous red line (6) and the continuous blue line (4) represents the benefit from the dynamic synchronisation between two demand management tolls. This benefit increases with respect to the equipment rate.

7. CONCLUSION

In this paper, the effects of dynamic traffic information and of congestion pricing are jointly modeled with the presence of recurrent and non-recurrent congestion. The benefits from the two management tolls are formulated analytically. Further, analytical formulae allow us slit the total benefit to different benefit components: the pure benefit from information, the pure benefit from congestion pricing and the benefit from the joint operation and the synchronization of the two management tolls.

Apart from simplifying the network structure to two parallel links, our treatment is subject to several limitations. First, recurrent congestion is modelled in a static way by assuming that the link travel time is a linear function of the link volume: this corresponds to the light-to-medium range of congestion effects, not to the medium-to-severe range. Second, the

dynamic disruptions in link travel times are modelled as random variables, involving specific assumptions of, respectively: (i) independence between the two links; (ii) Gaussian distribution; (iii) homosketasticity. It is easy to deal with dependency and heteroskedasticity under the Gaussian assumption, by specific adaptation of the difference in link travel times, our Z variable. Furthermore, the Gaussian assumption was shown not to differ significantly from the assumption of Bernoulli-Exponential disturbances by network link if the property of a symmetric distribution is maintained for the Z variable.

Further work about our model and the associated economic analysis may be targeted at the following topics:

- Numerical investigation of dependent, heteroskedastic disturbances, for instance in the case of Bernoulli-Exponential variables.

- Analytical study of the interplay of congestion recurrent and incidental, DTO and capacity management.

- Analytical study of the interplay of congestion recurrent and incidental, DTO and toll setting, in order to asses the complementariness and eventual redundancy of the two tools to improve on the system state.

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