# ON MODELLING MOTORWAYS OF THE SEA

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## ABSTRACT

By now, the continuous increase in freight transport demand has clearly put into evidence that road networks can not constitute the unique way to manage freight transportation. In this framework, an alternative to road transportation, indicated, after the European Union definition, as Motorways of the Sea (MoS), has to be investigated in order to understand its potential performance from the point of view of economic costs, travelling times, environmental impact, and so on.
Then, the aim of this paper is to design a MoS network model, essentially consisting of a graph, able to take into account both ground and maritime links among a set of seaports, and to identify the cost functions characterising the links, as well the non-additive costs characterising the paths, so as to state and solve a shortest path problem on such a complete network. Finally, a case study relevant to the Mediterranean Sea is presented.

*Keywords: Ground Transportation Networks, Motorways of the Sea, Shortest Path Problem, Non-additive Costs*

## INTRODUCTION

The continuous increase in freight transport demand has already made it apparent to everyone that road can not keep being by far the most utilised transport mode to move freight.
In such a framework, when thinking that railway transport can be a valuable alternative to road, it is worth considering that such a transportation mode is affected by the typical capacity problem which rises whenever passenger traffic competes with freight traffic for the limited service. In fact, both railway and road infrastructures, that have to satisfy passenger and freight demand at the same time, are characterised by capacity constraints. On the other hand, the capacity of the maritime transportation of trucks, i.e., the so-called *roll on - roll off* (ro-ro) transport, is larger, significantly under-saturated, and limited, in practice, only by the efficiency of seaports.

Then, such an alternative to road transportation, indicated as *Motorways of the Sea* (MoS), has to be investigated, in order to understand its potential convenience from the point of view of economic costs, travelling times, environmental impact, and so on.
In fact, although some MoS services are already active in the Mediterranean Sea, such a transportation mode appears to be not particularly widespread. Anyway, the existing connections allow evaluating road/rail connections in comparison with them. In this framework, to understand the strategic role played by MoS, note that they are considered, in accordance with the European Union definition in the White Paper on Transport (European Commission, 2001), as an integral part of the *Trans European Transport Network* (TEN-T).
Then, the aim of this paper is to create a MoS network model, essentially consisting of a graph (hereafter indicated as MoS graph), able to take into account both ground and maritime links among a set of origins and destinations, and to identify the cost functions characterising the links, so as to perform a shortest path based cost-benefit analysis. In turn, a real world case, relevant to the Mediterranean Sea and the Countries around it, is presented and discussed. In doing so, only the origin/destination pairs from/towards which both road and maritime links exist, are taken into account.

# DATA COLLECTION AND ANALYSIS

In this section some considerations about the costs assumed in the following sections, as well as on the relevant sources, are provided. In doing so it is worth saying that, being the aim of the paper to provide a model for decision support, rather than a punctual description of MoS dynamics, the attention has been focused mainly on the Italian normative, ferry companies data, highway tolls, fuel and driver costs, and so on.

## Ferry companies data

For each company operating in the considered study area, that is, the Mediterranean Sea, the ferry routes, frequencies, trip durations, cargo capacity of the relevant ships, and finally the ferry fares has been collected. The resulting database represents the source for the data analysis described in the following sections.
It is worth noting that, for the sake of simplicity, the considered tolls are relevant to 16 m articulated lorries (which represent the most common vehicles for transporting containers and most kinds of bulks), with a single driver accompanying the truck. In addition, the most economic fares, and the possibility for drivers to sleep in beds or not, depending on the trip duration, have been assumed.
Evidently such an analysis leads to an estimate of the cost, being their precise computation virtually impossible, due to the fact that fares are often negotiated directly between ferry companies and truckers or logistic enterprises, and not divulgated for competitiveness reasons, and in addition change seasonally.
A particular attention has been dedicated to the effects the subsides provided by some European states. To take into account such a significant effect, the Italian normative (Italian Transportation Department, 2008) has been assumed as a reference, which states that the so called “eco-bonus” is computed on the base of the difference of external costs (mainly congestion, safety and environmental costs) and is, at the maximum, equal to the 20% of the ferry fares for existent routes, or to the 30% of ferry fares for new routes.

At the end, it is worth saying that for what concerns the references for the above data, they mainly consist of the ferry companies websites, or of direct contacts with them.

## Road cost data

As regards the travelling costs of roads, in the following the main costs will be described. In doing so, it is possible to assume that costs are divided into three main categories:

- fixed costs of maintenance and tyres, that may be normalised with respect to the kilometres;
- variable costs proportional to the distance, that is fuel and highway toll costs;
- driver income costs.

### *Fixed costs*

To compute such costs it has been assumed that an average truck is characterised by a mean lifetime of 600,000 km, spanned over 6 years (CSST, 2008). In addition such a report states that a total maintenance cost for truck is equal to 43200€, taking into account oil, spare parts, and labour costs, for the whole lifetime, so as resulting into 0.072 €/km.
Moreover, as regards tyres costs, being they about 900€ for a single wheel, and being their lifetime about 90,000 km, it results a cost per km for a complete set of tyres equal to 0.13 €/km (data source Pirelli and Michelin list of charges).
Then, the fixed costs result to be 0.25€/km.

### *Variable costs*

For what concerns fuel costs, they are determined by means of the COPERT methodology that gives the fuel consumption on each road link, taking into account both the link slope (by subdividing them into shorter “sub-links” with almost constant slope, if necessary), and the traffic conditions by means of the mean traffic speed provided by the statistics published, every three months, by AISCAT (Italian association of managers of highways and tunnels) and by the other European road network managers. Once estimated the fuel consumptiom per link, the fuel costs has been computed by applying a mean charge of 1.074 € per litre, at the time of the study.
Then, as regards the highway tolls, they have been deteremined, for each highway link, by means of the list of charges provided by the highway managers. In doing so, it is worth noting that the applied charges take into account the kind of vehicle considered in such study and above described.

### *Driver costs*

As regards driver costs, they essentially depend on the European normative (Regulation (EC) 561/2006) that imposes a maximum time for driving and the relevant rest periods. Some details about the effects of such a normative will be given in the following, although it is worth recalling here that the cost for driver income is again computed on the basis of statistical data elaboration provided by CSST (2008).

# THE MODEL OF THE *MOTORWAYS OF THE SEA*

In this section, some definitions are given with the aim of introducing the proposed model. Then, for what concerns the model of a complex network gathering MoS, major roads, and highways, it may be defined as the MoS graph

$$G_{MoS} = \{N, L\}, \tag{1}$$

being $N$ and $L$ the sets of nodes and arcs, respectively. Therefore, such sets are bipartite, that is

$$N = N_g \cup N_s \text{, and } L = L_r \cup L_m \text{,} \tag{2}$$

where:

1. $N_g$ is the set of the so-called *ground nodes* modelling cities, inland terminals, dry-ports, and so on;
2. $N_s$ is the set of seaports;
3. $L_r$ is the set of ground links, that is, the set gathering major roads and highways;
4. $L_m$ is the set of maritime links, that is, the set gathering the ferry routes.

As an instance of a complex transportation network including MoS, consider the graph reported in Figure 1, where the continuous lines represent the ground links $(i, j) \in L_r$, $\forall i, j \in N$, and the dashed lines represent the maritime links $(i, j) \in L_m$, $\forall i, j \in N_s$, whereas the yellow nodes represent the ground nodes $g \in N_g$, and, finally, the cyan nodes represent the seaports $s \in N_s$.

Then, from a general point of view, the costs associated with any generic arc $(i, j) \in L$, $\forall i, j \in N$, consist of:

1. the fixed economic costs $c_{i,j}$ representing, for instance, highway tolls, maritime transportation fares, fuel consumption, and so on;

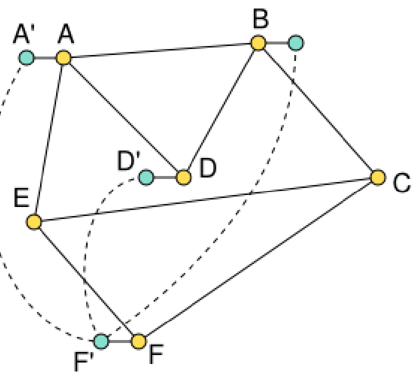


Figure 1 – Example of a MoS graph.

2. the travelling time costs $t_{i,j}$ proper to each link.

In addition to such costs, there are also the so-called *non-additive path costs* which will be introduced in the next sections, where a detailed description of the ground and maritime links will be given.

## Model of maritime links

The first step for building a MoS graph consists of the analysis of the different ferry companies operating in the considered geographic area. In doing so, for the sake of simplicity, and without losing generality, all the ferries of the different shipping companies have been considered together. In other words, ferries are not distinguished by a company criterion.

Then, for what concerns the mean travelling time $t_{i,j}^{m}$ along each maritime link $(i,j) \in L_m$, it may be estimated by means of the equation

$$t_{i,j}^{m} = \frac{1}{H_{i,j}} \sum_{h=1}^{H_{i,j}} t_{i,j}^{h} n_{i,j}^{h} , \qquad (3)$$

where:

1. $H$ is the number of ferries departing from the node $i \in N_s$ towards the destination $j \in N_s$;
2. $t_{i,j}^{h}$ is the travelling time of the $h^{th}$ ferry travelling from the node $i \in N_s$ towards the destination $j \in N_s$;
3. $n_{i,j}^{h}$ is number of weekly trips of the $h^{th}$ ferry from the node $i \in N_s$ towards the destination $j \in N_s$.

Then, for what concerns the mean ticket fare $c_{i,j}^{m}$ along maritime links, it has been estimated by means of the weighted means

$$c_{i,j}^{m} = \frac{1}{H_{i,j}\overline{K}_{i,j}} \sum_{h=1}^{H_{i,j}} K_{i,j}^{h} c_{i,j}^{h} n_{i,j}^{h} , \qquad (4)$$

where:

1. $c_{i,j}^{h}$ is the $h^{th}$ ferry ticket fare;
2. $K_{i,j}^{h}$ is the $h^{th}$ ferry capacity;
3. $\overline{K}_{i,j} = \sum_{h=1}^{H_{i,j}} K_{i,j}^{h}$ is the mean capacity of the link $(i,j) \in L_m$, computed over all the $H_{i,j}$ ferries travelling from $i \in N_s$ towards $j \in N_s$.

Such a formulation allows, in practice, to compute fare costs that do not depend on the capacity of ferries, and, as a consequence, to assume an infinite capacity of the maritime links. In fact, as above mentioned, such an assumption is reasonable in practice, since the real bottleneck of ro-ro transportation results from the level of (in)efficiency of seaports.

## *Driver costs on maritime links*

Truck drivers are paid not only when driving on roads, but also when travelling on ferries, although with a different hourly remuneration. Nevertheless, drivers embarked on ferries are paid up to 13 hours per day, in compliance with regulatory norms.

Then, the costs of truck drivers travelling on maritime link $(i,j) \in L_m$ may be computed by means of the Euclidean division defined by the relation

$$t_{i,j}^{m} = 24 n_d + \tau , \qquad (5)$$

where $n_d$ is the number of whole days required for the trip on $(i,j) \in L_m$, whereas the remainder $\tau$ represents the remaining hours in the $(n_d + 1)^{th}$ day.

Then, the driver cost associate with the link $(i,j) \in L_m$ is given by the relation

$$C_{i,j}^{m,driver} = 13 \cdot c_{hour}^{m,driver} \cdot n_d + \begin{cases} 13 \cdot c_{hour}^{m,driver} , & \text{if } \tau > 13 \\ \tau \cdot c_{hour}^{m,driver} , & \text{if } \tau < 13 \end{cases} , \qquad (6)$$

where $c_{hour}^{m,driver}$ is the driver cost per hour when travelling on a ferry.

## Model of road links

For what concerns the model of road links in MoS networks, in this framework only the major road and highway links between nodes are taken into account. Then, the costs of the road links consists of the highway tolls, when due, but also of all the other costs depending on the mileage, such as truck maintenance costs and fuel costs. Hence, each arc can be associated with an economic cost

$$c^f_{i,j}\ ,\ \forall(i,j)\in L_r\ , \tag{7}$$

summarizing the above contributions.

Then, as regards the costs of truck drivers, they depend on the norms regulating freight transportation, which limit the maximum number of driving hours per month to $n_{hpm} = 180$, or to $n_{hpy} = 1980$ hours per year (see, as a reference norm, the European Regulation (EC) 561/2006). As a consequence, knowing the mean driver cost per year $C_{dpy}$, it is possible to estimate the cost per hour as the ratio

$$c_{dph} = \frac{C_{dpy}}{n_{hpy}}\ . \tag{8}$$

Note that, the working hours of drivers include any period between the beginning and the end of their duties, up to 13 hours per day. In other words, they include the mandatory resting pauses of 45 minutes, imposed for safety reasons.

Then, taking into account such constraints, a typical trucker working day can be represented as in Figure 2, where it is easy to note that each trip requiring a driving time greater than 4.5 hours, has to "pay" an extra travelling time due to the above pauses. In this framework, it is of great importance to underline that the time spent in pauses can be computed only when the path from an origin $o$ towards a destination $d$ is defined.

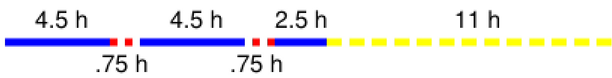


Figure 2 – Graphical representation of a mean driving day.

Hence, the real travelling time of a generic path $k$ between $o$ and $d$, hereafter indicated as $P_{o,d}(k)$, can to be computed by means of the equation

$$t_{TOT}[P_{o,d}(k)] = t_{tt}[P_{o,d}(k)] + T[P_{o,d}(k)], \tag{9}$$

where:

- $t_{tt}[P_{o,d}(k)]$ indicates the actual driving time on path k, given by the relation

$$t_{tt}[P_{o,d}(k)] = \sum_{(i,j)\in P_{o,d}(k)} t^r_{i,j}, \tag{10}$$

  being the terms $t^r_{i,j}\ ,\ \forall(i,j)\in L_r$, computed as

$$t^r_{i,j} = \frac{l_{i,j}}{v_{i,j}}, \forall(i,j)\in L_r \tag{11}$$

  where $l_{i,j}$ and $v_{i,j}$ are the length of the road arc $(i,j)\in L_r$, and the mean truck speed travelling on it, respectively;
- $T[P_{o,d}(k)]$ indicates the extra travelling time deriving from the above described resting pauses.

Note that, due to the above constraints, the travelling time $t_{TOT}[P_{o,d}(k)]$ may differ from the paid travelling time $t_{PAID}[P_{o,d}(k)]$. As an example, consider a truck making a journey requiring a trucker to actually drive for $t_{tt}\left[P_{o,d}(k)\right]=15$ hours. Then, due to the rest periods, the resulting travelling time is $t_{PAID}\left[P_{o,d}(k)\right]=27.5$ hours, whereas the driver salary is due for only $t_{PAID}\left[P_{o,d}(k)\right]=16.5$ hours (15 driving hours plus two breaks of 45 minutes).

Then, to conclude, while the travelling time on road paths can be computed by means of Eq. (9), the cost of truck drivers travelling on path $P_{o,d}(k)$ results to be

$$C^{r,driver}\left[P_{o,d}(k)\right]=c_{dph}\cdot t_{PAID}\left[P_{o,d}(k)\right], \tag{12}$$

where $c_{dph}$ is the above defined driver cost per hour, whereas $t_{PAID}\left[P_{o,d}(k)\right]$ is obtained by subtracting the time amount of the unpaid pauses from $T\left[P_{o,d}(k)\right]$, that is,

$$t_{PAID}\left[P_{o,d}(k)\right]=T\left[P_{o,d}(k)\right]-11n_d\left[P_{o,d}(k)\right], \tag{13}$$

where $n_d\left[P_{o,d}(k)\right]$ is the number of whole days required for the trip on path $P_{o,d}(k)$.

## Non-additive costs

As previously mentioned, a MoS network can be modelled as an oriented, weighted graph. Nevertheless, due to the above considerations about the travelling times and the cost of truck drivers, it is not easy to assign weights to the arcs. In fact, MoS networks are characterised by two classes of costs:

1. *additive costs*, i.e., costs depending on the mileage of arcs, such as the highway toll costs, the fuel costs, as well as the truck maintenance costs;
2. *non-additive path costs*, i.e., costs depending on the particular path from an origin $o$ to a destination $d$ , and can be computed only after defining the path.

In this framework, it is well known that solving problems on graphs with non-additive path costs is a difficult task to tackle with (Carlyle et al., (2008), Skriver (2000), Van der Zijpp, and Fiorenzo Catalano, (2005)). On the other hand, all traffic assignment problems require the computation of some shortest path, so as to apply deterministic or stochastic user equilibrium criteria to determine the traffic flows among links.

Hence, in the following section, the Shortest Path (SP) problem for MoS is addressed, and a heuristic solution algorithm is also provided.

# THE SHORTEST PATH PROBLEM

## Problem formulation

Consider the problem of finding the Shortest Path (SP) from an origin $o$ and a destination $d$ in a graph (Christofides, (1975)). From the most general point of view, such a problem may be formulated as the Linear Integer Programming (LIP) problem

$$\min_{(i,j)\in L}\sum_{(i,j)\in L} w_{i,j}x_{i,j}\ , \tag{14}$$

where the terms $x_{i,j}$ are the problem variables indicating whether the link $(i,j)\in L$ belongs to the shortest path ($x_{i,j}=1$), or not ($x_{i,j}=0$), and the terms $w_{i,j}$ are the costs the links $(i,j)\in L$.

In addition, the minimization problem in Eq. (14) is subject to the constraints

$$\begin{aligned}&\sum_{(j,h)\in\delta^-(h)} x_{j,h}-\sum_{(h,i)\in\delta^+(h)} x_{h,i}=\begin{cases}-1 & \text{if } i\equiv o\\ 1 & \text{if } i\equiv d\\ 0 & \text{otherwise}\end{cases}\qquad \forall h\in N\\ &x_{i,j}\in\{0,1\}\qquad \forall (i,j)\in L\end{aligned} \tag{15}$$

where $\delta^{-}(h)$, $\delta^{+}(h)$ are the so-called "forward star" and the "backward star" of node $h$, and which imposes the existence of a path from origin $o$ to destination $d$. Note that, due to the formulation in Eq. (14), the weighting terms $w_{i,j}$ represent the additive costs. On the contrary, in the considered framework, the cost function in Eq. (14) has to be modified in

$$\min_{(i,j)\in L} \sum_{(i,j)\in L} w_{i,j} x_{i,j} + W(P_{o,d}), \qquad (16)$$

which expresses, by means of the term $W(P_{o,d})$, the SP problem with non-additive costs. Then, by substituting into Eq. (16) the economic costs and the travelling times, it is possible to state the minimization problem

$$\min_{(i,j)\in L} \sum_{(i,j)\in L} \left[\alpha t_{i,j} + \beta c_{i,j}\right] x_{i,j} + \alpha t_{TOT}(P_{o,d}) + \beta C^{r,driver}(P_{o,d}), \qquad (17)$$

where

$$c_{i,j} = \begin{cases} c_{i,j}^{m} + C_{i,j}^{m,driver} & \text{if } (i,j)\in L_m \\ c_{i,j}^{f} & \text{if } (i,j)\in L_r \end{cases}, \text{ and } t_{i,j} = \begin{cases} t_{i,j}^{m} & \text{if } (i,j)\in L_m \\ t_{i,j}^{r} & \text{if } (i,j)\in L_r \end{cases}, \qquad (18)$$

and $\alpha$ and $\beta$ are weighting terms that equalise the contributions of the economic costs and of travelling times, respectively, whereas the other terms have been introduced in Eq. (9) and Eq.(12).

Then, due to the above considerations about the non-additive costs, such a formulation of the SP problem constitutes a non-linear minimization problem with non-additive costs, which is known to be hard to solve.

## Solution algorithm

The proposed solution approach consists of a two-phase algorithm that:

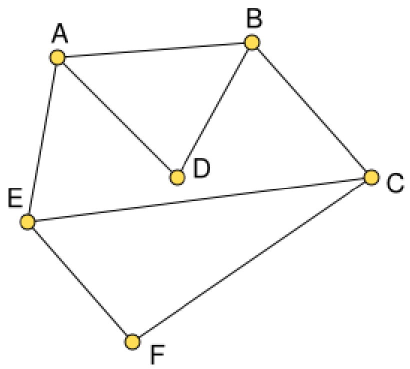


Figure 3 – Graph in **Error! Reference source not found.** without maritime links and seaports.

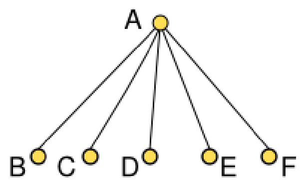


Figure 4 – SPT of the graph in **Error! Reference source not found.**.

1. in the first phase, computes the shortest paths between all the pairs of ground nodes $o,d$, $o \in N_g$, $d \in \{N_g - o\}$, without taking into account the maritime links $(i,j)\in L_m$;
2. in the second phase, compares the shortest paths computed in first phase with those including maritime links, too.

In addition, in the first phase, the shortest paths are computed by means of the Dijkstra algorithm, and by neglecting the path cost in the objective function. In doing so, only the additive costs are considered. Hence, the found solutions are sub-optimal approximations for the original problem.

Then, in the second phase, after building the so-called "graph of shortest paths" a PLI formulation of the SP problem is applied with the aim of taking into account some additional constraints.

## *Phase 1*

In this section, the first phase of the proposed algorithm is described. In doing so, to the end of focusing on a particular example, consider the graph depicted in Figure 1.

Then, the algorithm consists of the following steps:

- Step 1.1. consider the reduced graph in Figure 3, where only ground nodes ($\in N_g$) and ground links ($\in L_r$) of the network in Figure 1 are depicted. In such a graph, each arc is associated with the cost $w_{i,j} = \alpha t_{i,j} + \beta c_{i,j}$, $(i,j) \in L_r$;
- Step 1.2. apply the Dijkstra algorithm to all the pairs $o,d$, $o \in N_g$, $d \in \{N_g - o\}$, so as to compute the shortest path $P^*_{o,d}$ and the relevant optimal additive cost $c^{ADD}_{o,d}\left(P^*_{o,d}\right)$;
- Step 1.3. for each path $P^*_{o,d}$, $\forall o,d$, $o \in N_g$, $d \in \{N_g - o\}$, compute the real cost by adding the non-additive cost, thus obtaining

$$C_{o,d}\left(P^*_{o,d}\right) = c^{ADD}_{o,d}\left(P^*_{o,d}\right) + \alpha t_{TOT}\left(P^*_{o,d}\right) + \beta C^{r,driver}\left(P^*_{o,d}\right); \quad (19)$$

- Step 1.4. stop.

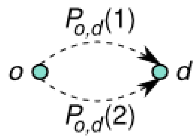


Figure 5 – Example.

The result of such a phase may be represented as a tree of sub-optimal paths, as the one represented in Figure 4.

## *Sub-optimality of the solution*

To understand the sub-optimality of the results provided by the first phase, consider the simple network depicted in Figure 5, where two alternative paths between an origin $o$ and a destination $d$ are reported.

Then, suppose that path $P_{o,d}(2)$ is characterised by the minimum additive cost but, from the point of view of the travelling time, it requires more than 4.5 hours. On the contrary, suppose that the path $P_{o,d}(1)$ is characterised by a greater additive cost, and by a travelling time shorter than 4.5 hours. As an example, the additive costs and the travelling times of such paths are reported in the second and in the third column of Table , respectively. Therefore, the non-additive costs resulting from the different travelling times, reported in the fourth column of Table , appear to be significantly different. In fact, while $P_{o,d}(2)$ requires a driving pause, $P_{o,d}(1)$ does not. As a consequence, the total cost of $P_{o,d}(2)$ results greater than the total cost of $P_{o,d}(1)$, so that the real shortest path results to be $P_{o,d}(1)$, as reported in the fifth column of Table .

Table I – Additive and non-additive costs for the paths in Figure 5.

| Path | Costs $\forall k = 1,2$ | | | |
|---|---|---|---|---|
| | $\sum_{(i,j) \in P_{o,d}(h)} \alpha t_{i,j} + \beta c_{i,j}$ | $\sum_{(i,j) \in P_{o,d}(h)} t_{i,j}$ | $\alpha t_{TOT}[P_{o,d}(k)] + \beta C^{r,driver}[P_{o,d}(k)]$ | $C_{o,d}[P_{o,d}(k)]$ |
| $P_{o,d}(1)$ | 46 | 4.2 hours | 0 | 46 |
| $P_{o,d}(2)$ | 44 | 4.7 hours | 3 | 47 |

Note that such a case is feasible in real cases, when paths including highways links (shorter travelling times - higher fix costs due to the presence of tolls), are compared with paths consisting only of major roads (greater travelling times - lower fix costs due to the absence of tolls). Anyway, while such a phenomenon is possible, it results to be not frequent in practice since both additive costs and non-additive costs increase as the travelling time increases, although linearly with it the first ones, and non-linearly the second ones. In fact, different, ad-hoc performed simulations showed that, in most cases, the higher are the additive costs of paths, the higher are the relevant non-additive costs. As a consequence, the error derived from considering only additive costs when looking for the shortest path may be considered negligible.

### *Phase 2*

In the second phase of the algorithm, the above determined sub-optimal shortest paths are compared to those including maritime links. In doing so, for the sake of simplicity and without

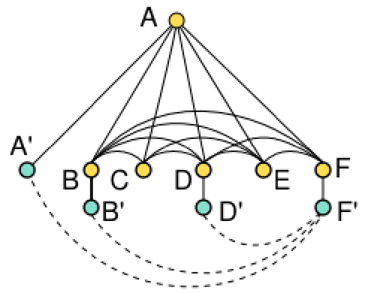


Figure 6 – SPG of the graph in **Error! Reference source not found.**.

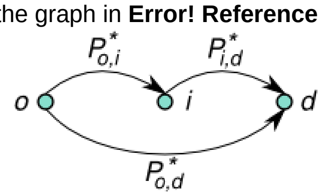


Figure 7 – Example of SPG.

losing generality, the origin $o$ is now considered to be fixed. In particular, with reference to the example in Figure 1, the origin is fixed in node $A$, that is, $o \equiv A$.Then, the second phase consists of the following steps:

Step 2.1. consider a tree where the root coincides with node $o$ and the leaves are represented by all the nodes $d \in \{N_g - o\}$;

Step 2.2. associate the cost $C_{o,j}\left(P^*_{o,j}\right),\ \forall j \in \{N_g - o\}$, computed in the first phase with all the arcs $(o,j)$ of the tree, thus obtaining the so-called *Shortest Path Tree (SPT)*;

Step 2.3. add to the SPT:

- all the nodes in $N_s$;
- the road links connecting seaports and ground nodes, as well as the relevant costs;
- the maritime links $(i,j) \in L_m$, as well as the relevant costs $c_{i,j} = c^m_{i,j} + C^{m,driver}_{i,j}$;
- the arcs between all the nodes $i \in N_g$, representing the SP among them, and the relevant costs $C_{i,j}\left(P^*_{i,j}\right),\ \forall i,j \in N_g,\ i \neq j$, computed in the first phase.

The resulting graph is the so-called *Shortest Path Graph (SPG), which is defined as*

$$SPG = \{N_{SPG}, L_{SPG}\}, \qquad (20)$$

where $N_{SPG}$ is the set of the nodes, defined as $N_{SPG} = N_g \cup N_s$, whereas, $L_{SPG}$ is the arc set gathering the shortest paths between all the pairs $i,j \in N_g$, $i \neq j$, the maritime links, and the links connecting seaports and ground nodes. As an example, consider the SPG of the MoS network in Figure 1 reported in Figure 6;

Step 2.4. for a given destination $d$ in the SPG, solve the generic shortest path problem in Eq. (14), where the costs are:

$$w_{i,j} = \begin{cases} C_{i,j}\left(P^*_{i,j}\right) & \text{if } i,j \in N_g, i \neq j \\ c^m_{i,j} + C^{m,driver}_{i,j} & \text{if } i,j \in N_s\ i \neq j \\ c^f_{i,j} & \text{if } i \in N_g, j \in N_s \text{ or } i \in N_s, j \in N_g \end{cases}, \qquad (21)$$

and subject to

$$\begin{aligned} &\sum_{(j,h)\in\delta^-(h)} x_{j,h} - \sum_{(h,i)\in\delta^+(h)} x_{h,i} = \begin{cases} -1 & \text{if } i = o \\ 1 & \text{if } i = d \\ 0 & \text{otherwise} \end{cases} \quad \forall h \in N_{SPG} \\ &x_{ih} + x_{hj} \leq 1, \quad \forall\ i,j,h \in N_g \\ &x_{i,j} \in \{0,1\}, \quad \forall\ i,j \in N_g \cup N_s \end{aligned}, \qquad (22)$$

where the second constraint expresses the fact that the shortest path can not gather a sequence of two road paths connecting ground nodes, as it will be discussed in the following. In other words, it "forces" the problem to consider only solutions consisting of sequences like

$$P_{o,i_1} - (i_1,i_2) - \ldots - (i_{n-1},i_n) - P_{i_n,d}, \qquad (i_{h-1},i_h) \in L_m, \forall h = 1,\ldots,n-1, \qquad (23)$$

where $P_{o,i_1}$ and $P_{i_n,d}$ are the shortest paths computed in the first phase, and gathering only road links.

Step 2.5. stop.

To fully understand the need of the second constraint in Eq. (22), consider the simple SPG depicted in Figure 7, where, apparently, there are two possibilities to reach the destination $d$ from the origin $o$. Suppose now that the travelling times on links $(o,i)$ and $(i,d)$ are shorter than 4.5 hours. Then, the non-additive costs on such arcs result to be null. As a consequence, the solution consisting of the sequence $P_{o,i} - P_{i,d}$ would not take into account the non-additive costs resulting of the whole sequence $P_{o,i} - P_{i,d}$ which, indeed, may correspond to a whole travelling time greater than 4.5 hours. On the other hand, the direct path $P_{o,d}$, computed in the first phase, takes into account non-additive costs, and, in addition, is the unique feasible solution since, in fact, non-additive costs have to be taken into account. For what concerns the computational effort, solving the SP problem by means of a LPI formulation leads to a high complexity, although the number of variables of the problem, i.e., the number of arcs, is generally limited (about some hundreds variables in real cases). By the way, such a formulation is necessary to take into account the above constraint.

# A CASE STUDY

In this section, to the aim of showing the capabilities of both the proposed model and the SP solution algorithm, a case study is reported. To this end, a brief description of the considered geographic area is first given. Then, the shortest paths between some significant origin/destination pairs will be introduced and discussed.

## The considered network

Considered the geographic area depicted in Figure 8, and consisting of the Countries around the Mediterranean Sea. In such a figure, the existing ferry routes are also reported. Then, it is worth underlining that, in the considered case study, only origins and destinations reachable by both paths including or excluding maritime links, have been taken into account. As a consequence, nodes in Northern Africa, Turkey, as well as in the islands, have been neglected, since, in these cases, road alternatives do not exist. Nevertheless, a special case is represented by Sicily, which has been, on the contrary, taken into account. In fact, due to

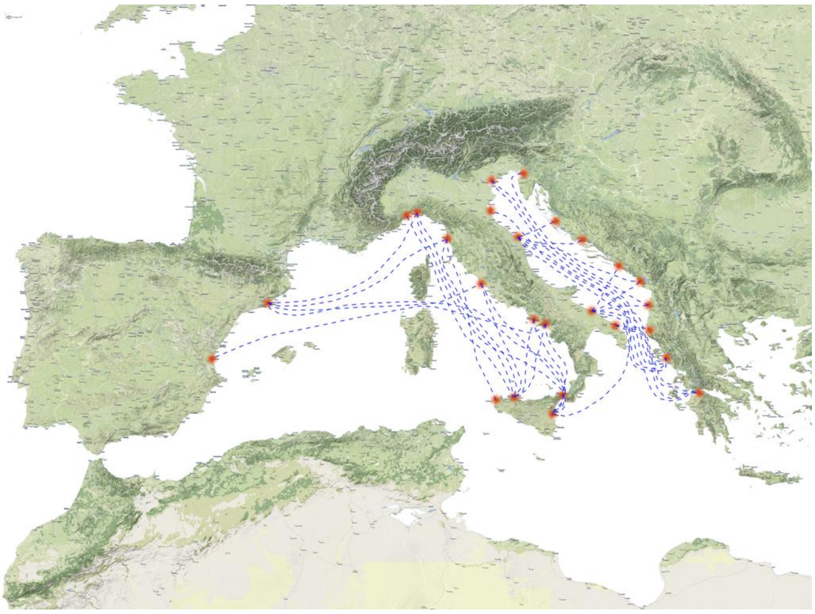

Figure 8– Main maritime links in the Mediterranean Sea.

Figure 9 – Highway network in the considered area.

the very high frequency of ferries (approximately about every 30 minutes in both the directions) and to its small width (approximately 3 km), the so-called Strait of Messina, separating the Italian peninsula from Sicily, can be considered, in practice, as a particular road link simply characterised by additional delay and costs.
For what concerns the considered ground transportation network, it consists of the network gathering the major roads and highway links depicted in Figure 9. To conclude, the set of the considered nodes is reported in Table II.

## Analysis of significant origin-destination pairs

In this section the shortest path computed for some significant origin/destination pairs, by means of the proposed algorithm, will be described. In doing so, it is worth saying that the fixed costs per kilometre of road arcs have been estimated taking into account the national mean, whereas the non-additive costs of truck drivers, both on road and maritime arcs, have been computed as described in the above sections.
Then, it is worth saying that, for the sake of comprehension, the shortest paths described in the following are depicted with different colours in Figure 10.

### *Milan – Palermo (green line)*

The first origin/destination pair taken into account consists of the trip Milan-Palermo. The first city is in the North of Italy, about 150 km far from Genoa, which represents the nearest seaport. On the contrary, Palermo is in Sicily and has its own seaport. As mentioned above, since the Strait of Messina may be assumed to represent a particular road arc, the two cities are linked both by a road path and by a path including maritime links.
Then, by applying the described solution algorithm, it resulted that the cost of the road shortest path (first phase of the algorithm) is 1788€. On the other hand, the path gathering maritime links results to be:

- Milan (node 14);
- Genoa (node 12);

- Genoa seaport (node 97);
- Palermo seaport (node 99);
- Palermo (node 51).

As regards its costs, it resulted to be equal to 1154€. Then, the maritime alternative is more convenient with respect to the direct road path, which is affected, in this case, by a great contribution of the non-additive costs.

## *Turin – Thessaloniki (red line)*

The second trip here considered is Turin-Thessaloniki. The first city is in the North-Western Italy, whereas the second one in the North-Eastern Greece. For such a pair, the best path is represented by the road path:

- Turin (node 7);
- intermediate ground nodes (10, 65);
- Milan (node 14);
- intermediate ground nodes (25, 17, 21, 26, 70);
- Trieste (node 27);
- Thessaloniki (node 88),

which is characterised by a cost equal to 2155€. In this case, the ground path results to be the more advantageous with respect to any other alternative including maritime links.

## *Barcelona – Dubrovnik (yellow line)*

The third analysed case study consists of the trip Barcelona-Dubrovnik. In this case, the two cities are located at extreme West and at the extreme East of the considered geographic area, respectively. Then, as regards the shortest path, it consists of the node sequence:

- Barcelona (node 78);
- Barcelona seaport (node 114 );
- Civitavecchia seaport (node 93);
- Civitavecchia (node 34);

Figure 10 – Computed Shortest Paths.

- Ancona (node 22);
- Ancona seaport (node 102 );
- Split seaport (node 106);
- Split (node 81);
- Dubrovnik (node 82),

with a relevant cost equal to 2265€, whereas the shortest direct road path is characterised by a cost equal to 2650€. It is interesting to note that, in this case, the best path consists of a sequence maritime link – road – maritime link.

*Genoa – Patra (blue line)*

The last presented case study consists of the trip Genoa-Patra. In this case the best path provided by the proposed algorithm, is composed by the sequence of nodes:

- Genoa (node 12);
- intermediate ground nodes (13, 64, 31, 69, 20, 18, 15, 23);
- Ancona (node 22);
- Ancona seaport (node 102 );
- Patra seaport (node 113);
- Patra (node 87),

with a relevant cost equal to 1723€. The road cost is, in this case, equal to 2790€, which is very expensive with respect to the best one.

# CONCLUSIONS

In this paper, the problem of evaluating the convenience of MoS with respect to the road alternative has been addressed. To such an aim, a SP problem for complex network characterised by non-additive non-linear costs has been stated, and a heuristic solution algorithm has been provided.

Table II – Nodes of the considered network.

| Node | Identifier | Node | Identifier | Node | Identifier |
|---|---|---|---|---|---|
| 1 | Chamonix (FR) | 41 | Pescara (IT) | 81 | Split (HR) |
| 2 | White Mount Tunnel (IT-FR) | 42 | L'Aquila (IT) | 82 | Dubrovnik (HR) |
| 3 | Switzerland (SW) | 43 | Turin (IT) | 83 | Bar (YU) |
| 4 | Gran San Bernardo Tunnel (IT-SW) | 44 | Naples (IT) | 84 | Durres (AL) |
| 5 | Modane (FR) | 45 | Caserta (IT) | 85 | Vlora (AL) |
| 6 | Bardonecchia (IT) | 46 | Nola (IT) | 86 | Igoumenitsa (GR) |
| 7 | Turin (IT) | 47 | Salerno (IT) | 87 | Patra (GR) |
| 8 | Aosta (IT) | 48 | Canosa (IT) | 88 | Thessaloniki (GR) |
| 9 | Ivrea (IT) | 49 | Bari (IT) | 89 | Marseille (FR) |
| 10 | Santhià (IT) | 50 | Taranto (IT) | 90 | Savona seaport (IT) |
| 11 | Savona (IT) | 51 | Palermo (IT) | 91 | Genoa seaport (IT) |
| 12 | Genoa (IT) | 52 | Messina (IT) | 92 | Leghorn seaport (IT) |
| 13 | Serravalle Scrivia (IT) | 53 | Catania (IT) | 93 | Civitavecchia seaport (IT) |

| | | | | | |
|---|---|---|---|---|---|
| 14 | Milan (IT) | 54 | Reggio Calabria (IT) | 94 | Napoli seaport (IT) |
| 15 | Bologna (IT) | 55 | Varese (IT) | 95 | Salerno seaport (IT) |
| 16 | Brenner Tunnel (IT-AU) | 56 | Varese motorway junction (IT) | 96 | Messina seaport (IT) |
| 17 | Verona (IT) | 57 | Lainate (IT) | 97 | Palermo seaport (IT) |
| 18 | Modena (IT) | 58 | Como (IT) | 98 | Trapani seaport (IT) |
| 19 | La Spezia (IT) | 59 | Chiasso (IT) | 99 | Catania seaport (IT) |
| 20 | Parma (IT) | 60 | Alessandria (IT) | 100 | Brindisi seaport (IT) |
| 21 | Padua (IT) | 61 | Genoa Voltri (IT) | 101 | Bari seaport (IT) |
| 22 | Ancona (IT) | 62 | Gravellona Toce (IT) | 102 | Ancona seaport (IT) |
| 23 | Ravenna motorway junction (IT) | 63 | Brindisi (IT) | 103 | Ravenna seaport (IT) |
| 24 | Ravenna (IT) | 64 | Turin-Piacenza and Milan-Genoa highway intersection (IT) | 104 | Venice seaport (IT) |
| 25 | Brescia (IT) | 65 | Turin-Piacenza and Greavellona Toce/Genoa Voltri - Milan-Genoa highway joint (IT) | 105 | Trieste seaport (IT) |
| 26 | Mestre (IT) | 66 | Predosa (IT) | 106 | Split seaport (IT) |
| 27 | Trieste (IT) | 67 | Novi Ligure (IT) | 107 | Dubrovnik seaport (IT) |
| 28 | Belluno (IT) | 68 | Sestri Levante (IT) | 108 | Zadar seaport (HR) |
| 29 | Udine (IT) | 69 | Fiorenzuola (IT) | 109 | Bar seaport (YU) |
| 30 | Tarvisio (IT) | 70 | Palmanova - Udine highway junction(IT) | 110 | Durrës seaport (AL) |
| 31 | Piacenza (IT) | 71 | Viareggio (IT) | 111 | Vlata seaport (AL) |
| 32 | Ventimiglia (IT) | 72 | Lucca (IT) | 112 | Igoumenitsa seaport (YU) |
| 33 | Leghorn (IT) | 73 | Lanciano (IT) | 113 | Patra seaport (GE) |
| 34 | Civitavecchia (IT) | 74 | Bonfornello (IT) | 114 | Barcelona seaport (ES) |
| 35 | Florence (IT) | 75 | Vercelli (IT) | 115 | Valencia seaport (ES) |
| 36 | Pisa (IT) | 76 | Castelvetro (IT) | | |
| 37 | Rome South (IT) | 77 | Trapani (IT) | | |
| 38 | Rome North (IT) | 78 | Barcellona (ES) | | |
| 39 | Rome East (IT) | 79 | Valencia (ES) | | |
| 40 | Rome West (IT) | 80 | Zadar (HR) | | |

Then, some real world case studies have been analysed with the aim of showing, on one hand, that the proposed algorithm is able to provide reasonable solutions and might be implemented, in a future framework, as a step of a more sophisticated traffic assignment. On the other hand, the selected case studies also showed the convenience of some paths including maritime links, with respect to the traditional road transportation.
To conclude, it is worth saying that work is in progress to quantify the sub-optimality of the proposed heuristic approach, as well as to state the above cited traffic assignment problem for MoS networks.

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