A Model of Switching Times for rail cars equipped with light engines and remote control

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Abstract

FlexcCargoRail is a technology project aiming to increase efficiency of freight railway operations, particularly, the segment of transport on the last mile and shunting operation. The basic idea is to enhance rail cars with rechargeable battery-powered engines and remote control, thus allowing to switch cars without use of a shunting engine. In this paper we provide an OR model aimed at measuring the savings potential of this technology. We find that this innovation is likely to become profitable. Moreover, we can determine an optimal share of FlexcCargoRail cars in the whole fleet. For several parameter values it turns out that this innovation is likely to be profitable already at a low share, so that the conditions for its introduction seem to be particularly good.

1 Introduction

Providing rail cargo services in a country with a well-developed road infrastructure is a demanding task. This is partly due to the high cost — both in time and money — of car switching. Therefore, innovations that have potential to reduce the cost of switching are certainly interesting. In the context of a research project implemented jointly by university and industry partners, engineers developed the idea of enhancing freight rail cars with rechargeable battery-powered engines that allows them to move autonomously at moderate speed. These engines, called FlexCargoRail (FCR), are particularly useful on switching yard stations where they can be controlled via a wireless control by a human in place. For details see Baier and Enning [1] and Kochsiek [2].

In this paper, we develop an OR model to estimate the magnitude of savings in the switching process that can be achieved by using FCRs. The model builds on a well-known switching time model of Petersen [3]. We then present a simple cost-benefit analysis of the new technology as a function of several parameters.

The cars to be switched are modelled as a random sequence whose members need to be put on different tracks where cars with identical destinations can be moved jointly. A fixed proportion q of all wagons is equipped with FCR features. We calculate the share of locomotive bound switching operations that

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can be saved. The share of FCR in the entire car park, the number of cars that can be pushed or pulled by a FCR, and the complexity of switching are integrated in the calculation as parameters.

For the cost-benefit calculation we design a total cost function consisting of costs for FCR and for locomotive bound switching operations. For FCR we assume one-time conversion costs, which we converted into an annuity. We presume the variable costs for a *deployment* of a FCR car as negligible. In contrast, for each locomotive bound switching operation a constant cost arises.

The resulting total cost function may have different features: Depending on the parameters, the net benefits of FCR in q (the proportion of FCRs) may be both always rising and always falling or it may exist an optimal "saturation level" of FCRs in the car park.

Based on the results from our project partners we determine reasonable estimates of the parameters and calculate how high the investment costs for a conversion to FCR may be in order to be just about profitable.

2 Reduction of switching operations

2.1 Petersens Switching Model

Petersen [3] presents a calculation of the frequency of switching operations depending on characteristics of the yard and the incoming traffic. A series of trains, each consisting of single freight cars (but no FCRs) enters the sorting yard. Those incoming cars need to be sorted into different directions. Let the unconditional probability that a car goes in the same direction as his predecessor be $p \in (0, 1)$ for all cars. The marshalling yard has a switching engine on disposal, which is able to move any number of cars. We want to calculate the *average number of moved cars per switching operation* being denoted as $E(p)$. To simplify matters we assume the sequence of incoming cars has an infinite length so that there is no "last" car.

We assume that incoming locomotives are irrelevant (they cannot be used for the sorting process but also do not disturb the switching process). Furthermore we assume that it is *forbidden* to move cars into the "wrong" direction, i.e. switching back and forth or "presorting" cars does not take place.

Definition 1. *A* rake of wagons *of length $k \geq 1$ is a sequence of k incoming cars, which happen to go into the same direction and which are followed by a car going in a different direction.*

An *elementary cycle* is defined to be the process of the switching engine picking up as many cars as possible, that is the complete rake of wagons it just encounters. The probability of the rake of wagons being exactly k cars long is $p^{k-1}(1-p)$. Summing over all possible lengths of rake of wagons k, we obtain the following expression for $E(p)$:

$$E(p) = \sum_{k=1}^{\infty} kp^{k-1}(1-p) \tag{1}$$

$$= (1-p)\left(\sum_{k=1}^{\infty} p^{k-1} + \sum_{k=2}^{\infty} p^{k-1} + \dots\right) \tag{2}$$

$$= (1-p)\left(\frac{1}{1-p} + p\frac{1}{1-p} + \dots\right) \tag{3}$$

$$= (1-p)\frac{1}{1-p}\left(1+p+p^2+\dots\right) \tag{4}$$

$$= (1-p)\left(\frac{1}{1-p}\right)^2 \tag{5}$$

$$= \frac{1}{1-p} \tag{6}$$

Note that for the case $p = 0$ it follows $E(0) = 1$: If two consecutive cars never go into the same direction, exactly one switching operation for every car is needed. In contrast $\lim_{p\to 1} E(p) = \infty$ holds as expected: Assuming infinite locomotive capacity, all cars can be moved with a single switching if they all go into the same direction.

More generally, $E(p)$ is increasing. It is illustrative to interpret p as an inverse measure of the "degree of complexity" of the switching situation.

2.2 Extension to FCR

Now we want to extend the basic model presented above to the presence of FlexCargoRail (FCR) in the car park. Let the probability that a car is an FCR be denoted by $q \in [0, 1]$. The event "car X is an FCR" is independent of the event "car X goes into the same direction as its predecessor" and the event "car X goes into the same direction as its successor". Wagons that are not equipped as FCR are called N-cars (meaning normal or non-FCR).

2.2.1 Capacity of an FCR

Let f be the total number of cars that can be moved by one FCR, including itself, $f = 1, 2, \dots, \infty$. An FCR can pull or push other cars alike. Therefore, the position of the FCR within the rake of wagons does not matter for its capacity. For simplicity we assume that the capacity of FCR is *not* increased by using more than one in combination.

2.2.2 Optimal Switching Behaviour

We now define rules describing the switching possibilities and then determine an optimal switching strategy minimizing the number of switching operations by the locomotive.

Without FCR only the locomotive can move cars. But the incoming sequence of cars also contains some FCRs which are able to move themselves and (for $f > 1$) some other cars. There are thus exactly two possible types or "rules" of car movement:

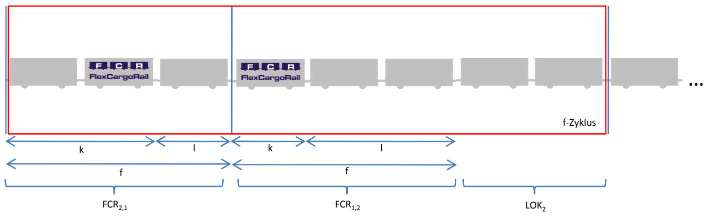


Figure 1: f-cycle for $f = 3$. The partition of the sequence of wagons (going to the left) into two rakes is illustrated by the two boxes. The consecutive application of rules $\mathrm{FCR}_{2,1}$, $\mathrm{FCR}_{1,2}$, LOK_2 is depicted. The complete f-cycle is the union of the two boxes.

$\mathrm{FCR}_{k,l}$ for every $k \leq f$ and $k + l \leq f$: If the k-th waiting car is an FCR this car is pushing all cars in front of it in one direction and takes l following cars with it

LOK_k : A locomotive pulls k cars into one direction

The rule LOK_k can only be applied to a rake of waggons of the length k that is heading the sequence. The rule $\mathrm{FCR}_{k,l}$ cannot be applied if $k > 1$ (FCR is not in front) and a car in front of the FCR is not going into the same direction. The rule can always be applied if $k = 1$ (FCR at front); then the FCR can pull additional l cars going into the same direction with a maximum of $l + 1 \leq f$. If $k > 1$ and in front of the FCR are only cars going into the same direction as the FCR, it can push those cars (as long as $k < f$) and, if applicable, pull some more. We now define a certain switching strategy and show subsequently that this strategy is optimal:

Definition 2. FCR-Switching *describes the application of the following rules to a sequence of wagons:*

- *For $k = 1, 2, \ldots, f$ we first try to apply rule $FCR_{k,l}$ for the smallest possible k, where l is chosen so that the maximum possible number of N-wagons but no FCR will be pulled. This procedure is repeated as often as possible.*
- *If rule $FCR_{k,l}$ cannot be applied (any more) for any $k \leq f$, rule LOK_k will be applied with maximum possible k. That means all N-wagons and FCR going in the same direction as the first car will be pulled by the locomotive.*

An f-cycle *is the sequence of cars arising from applying the above rules of FCR-switching until rule LOK_k is being applied for the first time for some k.*

Figure 1 illustrates the application of FCR-switching according to definition 2. In this example, it happens that two FCRs can make use of their full capacity $f = 3$ (i.e., the adjacent N-wagons go in the same direction; it doesn't matter whether the two rakes go in the same direction or not). The second rake is longer than f, so the locomotive has to be used for the remaining cars.

We make the following two general observations:

Lemma 1.
- *Any random sequence of wagons can always be interpreted as a sequence of an f-cycle followed by a random sequence of wagons.*

- *To switch an f-cycle* exactly one *operation of the switching engine is necessary.*

After having applied rule LOK_k with maximum k indeed the remaining cars again have a probability of q for being an FCR and starting with the second remaining car each remaining car has a probability of p for going in the same direction as its predecessor. Thus the second point of the lemma is obvious as well as the following

Corollary 1. *With FCR-switching the average number of cars being moved per locomotive operation equals the average length of an f-cycle.*

We can now state a basic qualitative result.

Theorem 1. *Among all possible switching rules FCR-switching is minimizing the average number of required locomotive operations.*

Proof. We show that a deviation from the FCR-switching approach can at most decrease the length of cycles.

- Let the rule $\text{FCR}_{k,l}$ be applicable. If instead the LOK-rule is being applied, then all cars going in the same direction as the first one will be removed (afterwards the cycle ends). If rule $\text{FCR}_{k,l}$ is applied additionally before rule LOK (no matter what k and l) no fewer cars can be removed.
- Let the rule $\text{FCR}_{k,l}$ be applicable but so be the rule $\text{FCR}_{k',l'}$ for $k' > k$, i.e. cars k and k' are FCRs going into the same direction. But then the $\text{FCR}_{k,l}$ can be followed by $\text{FCR}_{k,l'+k'-k}$, and these two movements would lead to the same outcome as $\text{FCR}_{k',l'}$. Therefore the application of $\text{FCR}_{k',l'}$ is no improvement.
- Rule $\text{FCR}_{k,l}$ has been been applied but with l being "too small", i.e. N-wagons are left behind that could be taken along. For every following application of the FCR-rule this can only imply a decrease of the number of moved cars. But if the LOK-rule is used to move the cars left behind the length of the cycle will likewise be decreased.
- Rule $\text{FCR}_{k,l}$ has been applied but with l being "too big", i.e. an FCR is being pulled. Then the rake of all moved cars could be seperated before that FCR. This can never reduce the total number of cars moved in the cycle (but could increase it).

□

2.2.3 Expected Number of Locomotive Operations for $f = 1$

As in Pertersen's basic model, we are interested in the average number of cars that can be switched by one move of the locomotive or, what is the same thing by Corollary 1, the average length of an f-cycle. That will now also depend on f and q, besides p, and will be denoted by $E_f(p, q)$.

Let us first consider the case $f = 1$, in which the FCR cannot move any other cars but itself.[1]

[1] At the moment the developers of FCR hold the view that an FCR will be able to move approximately one to two other cars. However, it is considered to always keep FCRs in a *fixed combinations* with as many cars as it can move. Such a fixed group of cars may be considered as one single "large" FCR with capacity $f = 1$.

Thus the first part of a 1-cycle consists of all the leading FCR cars, and the second part of a rake of wagons with a leading N-wagon pulled by the locomotive. The lengths of the two parts are independent. The expected length of the rake of wagons pulled by the locomotive is $E(p) = \frac{1}{1-p}$, in accordance with formula (6). With probability $q^l(1-q)$ the sequence of leading FCR cars has the length $l = 0, 1, \ldots$. Similar to the derivation of formula (6) the average length l_{FCR} of the leading FCR cars is

$$
\begin{aligned}
l_{\mathrm{FCR}} &= \sum_{l=0}^{\infty} l q^l (1-q) && (7)\\
&= (1-q)\left(\sum_{l=1}^{\infty} q^l + \sum_{l=2}^{\infty} q^l + \ldots\right) && (8)\\
&= (1-q)\frac{q}{1-q}\left(1 + q + q^2 + \ldots\right) && (9)\\
&= (1-q)\frac{q}{1-q}\frac{1}{1-q} && (10)\\
&= \frac{q}{1-q} && (11)
\end{aligned}
$$

and $E_1(p,q)$ results as

$$E_1(p,q) = \frac{q}{1-q} + \frac{1}{1-p} \tag{12}$$

2.2.4 General f

For the general case the best way to model this problem is a markov chain in discrete time. We introduce the following set of states:

$$Z = \{I, F_1, \ldots, F_f, N_1, \ldots, N_{f-1}, L, E\} \tag{13}$$

I is the initial state. State F_i stands for a rake of wagons of i cars containing one FCR. State N_i $(i < f)$ stands for a rake of wagons of the length i without any FCR that could be pushed by an arriving FCR. L stands for more than f N-cars, which cannot be moved by a following FCR and therefore need (at least in some portion) to be switched by an engine sooner or later. E is the final state.

Each car out of an infinite random sequence of wagons now triggers a transition of state. Each state transition, except the first one and the one to E, represents one switched car. Only for the transition to E a switiching engine is needed — thus the trajectory from I to E corresponds to one f-cycle.

Figure (2) is a graphical representation of the transition of states. The matrix of transition probabilities can be written as follows:

$$
M(f,p,q) = \left(\begin{array}{lllllllllll}
 & I & F_1 & F_2 & F_3 & \ldots & F_f & N_1 & N_2 & L & E \\
I & 0 & q & 0 & 0 & \ldots & 0 & 1-q & 0 & 0 & 0 \\
F_1 & 0 & q & p(1-q) & 0 & \ldots & 0 & (1-p)(1-q) & 0 & 0 & 0 \\
F_2 & 0 & q & 0 & p(1-q) & \ldots & 0 & (1-p)(1-q) & 0 & 0 & 0 \\
\ldots \\
F_f & 0 & q & 0 & 0 & \ldots & 0 & 1-q & 0 & 0 & 0 \\
N_1 & 0 & 0 & p\,q & 0 & \ldots & 0 & 0 & p(1-q) & 0 & 1-p \\
N_2 & 0 & 0 & 0 & p\,q & \ldots & 0 & 0 & 0 & p(1-q) & 1-p \\
L & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & p & 1-p \\
E & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 1
\end{array}\right) \tag{14}
$$

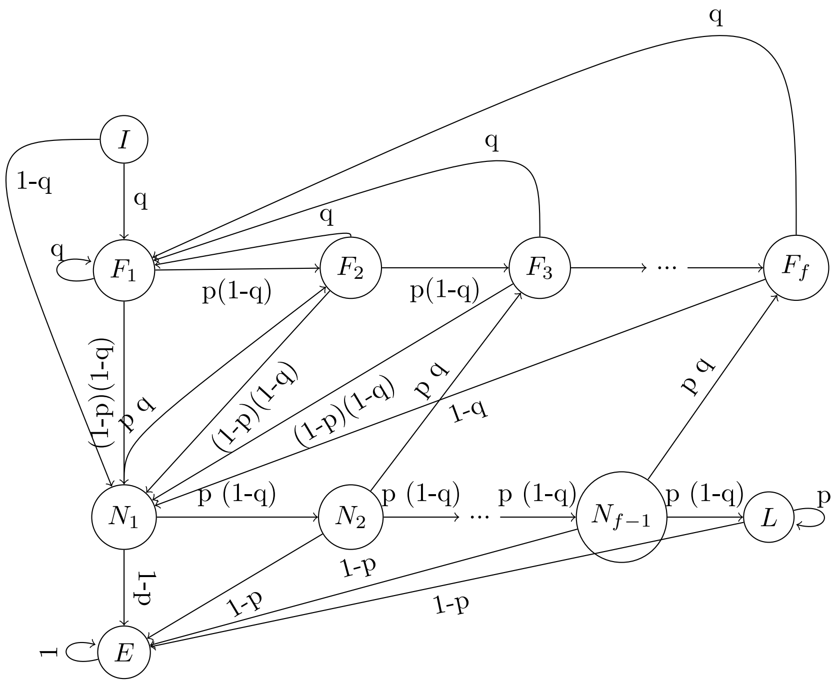


Figure 2: State diagram for a general f

The vector of the initial distribution is

$$\vec{z}_0 = (1, 0, \ldots, 0) \tag{15}$$

and the distribution of states following the k-th transition of state is

$$\vec{z}_k = \vec{z}_0 M(f, p, q)^k \tag{16}$$

$(\vec{z}_k)_{f+2}$ (meaning the $f+2$-th component of the vector $\vec{z}_k$) represents the probability that the system resides in the final state after k transitions of state, so that at most $k-1$ cars can be switched in one cycle. According to this the probability for a cycle to have *exactly* the length k is $(\vec{z}_k)_{f+2} - (\vec{z}_{k-1})_{f+2}$. The average length of a cycle can be stated as

$$E_f(p, q) = \sum_{k=2}^{\infty} (k-1)(\vec{z}_k - \vec{z}_{k-1})_{f+2} \tag{17}$$

For $f = 2$ this expression can be reduced as follows:

$$E_2(p, q) = \frac{1 - p\,q\,\left(1 + p\,(1-q)\,(2 - p\,q)\right)}{(1-p)\,(1-q)\,(1-p\,q)^2} \tag{18}$$

For $f > 2$ no cohesive form can be specified. In that case the function needs to be examined numerically.

2.3 Discussion of the function $E_f(p, q)$

The avarage length of a cycle, $E_f(p, q)$, is monotonously increasing in q and f, but not in p. We now discuss the influence of the different variables in detail.

2.3.1 Dependency on q

Even for $f = 1$, and so even more for $f > 1$, it holds that $\lim_{q\to\infty} E_f(p, q) = \infty$. How can we interpret that? In our model the set of cars to be switched is depicted as a potentially infinite sequence of wagons. For any q in the vicinity of 1 a switching engine will be used only very rarely since FCRs are abundant to to all the work. For such large values of q one might want to refine the model by considering a maximum length of cycles, a minimum deployment of switching engines, or even a capacity constraint for switching engines. However, our study is concerned with the profitability of the *introduction* of the FCR technology, so that we focus on relatively low levels of q.

2.3.2 Dependency on p and f

The *comparative cost savings* of FCR can be expressed as the ratio of the cycle length for a given q to the cycle length at $q = 0$: $\frac{E_f(p,q)}{E(p)}$.

Figure 3(a) shows that the comparative cost savings of FCR is, for $f > 1$, initially rising in p, reaching a maximum and then falling in p. This can be explained as follows. Generally, with an increasing p the probability increases that a rather long rake of wagons evolves that can be switched in one single move. This also tends to increase the value of FCR as switching engines. However, the capacity of FCR as switching engines is limited. If p approaches 1, FCR will very rarely be sufficient as an engine so that its contribution to the switching process is vanishing.

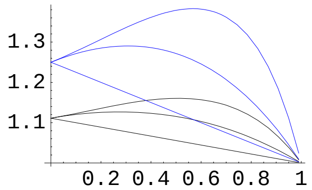
When q is increasing it will happen more often that rakes of wagons will contain FCRs. Moreover, very long rakes may contain several FCRs so that there is a chance that the capacity limit f of each FCR is compensated by the existence of a next FCR that can move the remaining cars. As a consequence, if q is higher, the comparative cost savings of FCR should be (i) higher and (ii) increasing in p for a larger range (i.e. the maximum in p should be at a larger p). Figure 3(a) also illustrates these effects.

Figure 3(b) shows that the effect of an increase in the capacity f drops dramatically with decreasing p: If there are only short rakes of wagons an increase of FCR capacity will contribute only little.

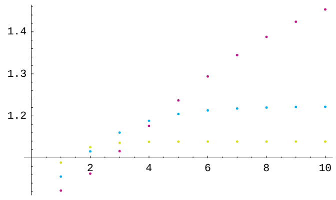
3 Cost benefit analysis

We model the costs of upgrading a wagon to become an FCR as an annuity A_{FCR}. Furthermore we assume that a car has to be switched r times per year on average. Let K_L be the costs of the locomotive per switching operation. Let X be the number of all wagons in the system. We are interested in the total switching cost per year of the existing car fleet, denoted $C_f(p, q, r, K_L, A_{\mathrm{FCR}}, X)$, since it will depend on all parameters.

The number of FCRs is qX causing annual costs of $A_{\mathrm{FCR}}\, q\, X$. The number of switching operations per year is rX. The costs of locomotive operation is



(a) Comparative savings $\frac{E_f(p,q)}{E(p)}$ of switching operations for $q = 0.1$ (black) and $p = 0.2$ (blue) respectively and $f = 1, 2, 3$ for $0 \le p \le 0.9$



(b) Comparative savings of switching operations for $q = 0.1$ and $p = 0.2, 0.5, 0.8$ for $f = 1, \ldots, 10$

Figure 3: Comparative savings by FCR

inversly proportional to the average length of an f-cycle:

$$C_f(p, q, r, K_L, A_{\mathrm{FCR}}, X) = K_L \frac{rX}{E_f(p,q)} + A_{\mathrm{FCR}}\, q\, X \tag{19}$$

Therefore the costs are linear in X. The discussion of the cost function is simplified by relating total costs to X and to the anuity and thus integrating the cost parameters into one variable. We define $c_f := \frac{C_f}{X\, A_{\mathrm{FCR}}}$ and $z := r \frac{K_L}{A_{\mathrm{FCR}}}$ and get

$$c_f(p, q, z) = q + \frac{z}{E_f(p,q)} \tag{20}$$

The *relative profitability* of FCR can be defined as quotient $\frac{c_f(p,q,z)}{c_f(p,0,z)}$. The introduction of a share q of FCRs is profitable exactly if the relative profitability exceeds 1. We argue below that, realistically, z assumes values between 1 and 2.

3.1 Case $f = 1$

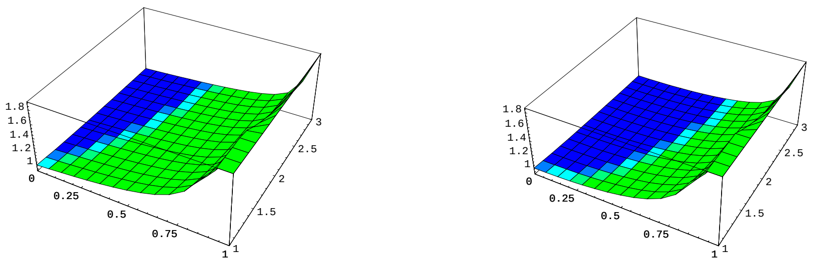
Let us first consider the case that $f = 1$. With formula (12) we get

$$c_1(q) = q + \frac{z}{E_1(p,q)} \tag{21}$$

$$= q + z \frac{(1-p)\,(1-q)}{1 - p\,q} \tag{22}$$

$$c_1'(q) = 1 - z \frac{(1-p)^2}{(1-p\,q)^2} \tag{23}$$

$$c_1''(q) = 2z \frac{p(1-p)^2}{(pq-1)^3} < 0 \tag{24}$$



(a) $f = 1$: FCR is profitabel for all $p < 0.1$.

(b) $f = 2$: For $p < 0.25$, FCR is profitable for all $z > 1.1$.

Figure 4: Relative profitability $\frac{c_f(p,0.1,q)}{c_f(p,0,q)}$ of FCR for $q = 0.1$ compared to $q = 0$, as function of p (x-axis) and z (y-axis). Profitable areas are printed dark (blue/purple), unprofitable ones light (green).

So the function $c_1(q)$ is concave. The value of q minimizing $c_1(q)$, that is, the optimal share of FCR, is therefore either $q = 0$ or $q = 1$. However, as pointed out above, the value $q = 1$ is neither realistic nor captured particularly well by our model. We are concerned with the profitability of the *introduction* of FCR. Figure 4(a) shows the relative profitability of FCR for $q = 0.1$: For all $p < 0.25$ and $z > 1.1$, the upgrade of 10% of the cars to become FCRs is profitable as compared to no introduction of the technology.

3.2 Cases $f = 2$ and $f = 3$

The case $f = 2$ yields

$$c_2(q) = \frac{z}{E_2(p,q)} + q \tag{25}$$

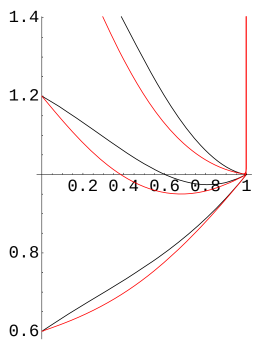
$$= q + z\frac{(1-p)(1-q)(1-pq)}{1-p\,q(1+p-p\,q)} \tag{26}$$

The function $c_2(q)$ is not concave in general, and different shapes are possible in q, as shown in figure 5(a). Figure 5(b) shows where the function increases or decreases in q, here for the case $z = 2$.

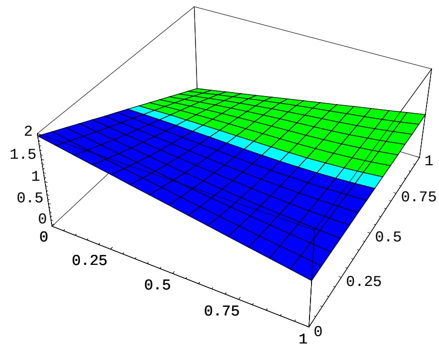
We did not work out a closed form for $f = 3$. However, numerical analyses show that these cost functions have similar shapes as in the case $f = 2$, see the red curves in figure 5(a).

3.3 Optimization of car circulation

The benefit of FCRs increases if they can be used more intensively. Suppose that it would be possible to optimize the use of the FCR cars in particular. While an N-car has to be switched r times per year on average, an FCR can be used βr times per year on average, with a factor $\beta \geq 1$. With q the share of FCR in the total car park, the probabilty that a random car to be switched is



(a) Cost function $c_2(q)$ (black) and $c_3(q)$ (red) for different values of z, with $p = 0.4$ fixed. For $z = 1$ costs grow monotonously, for $z = 2$ there is an inner cost minimum and for $z = 3$ costs decrease monotonously in q.



(b) Cost c_2 as function of q (x-axis) and p (y-axis), for constant $z = 2$. Costs decrease in q for small p and increase in q for large p.

an FCR becomes

$$q^*(\beta) = \frac{\beta rqX}{\beta rqX + r(1-q)X} \tag{27}$$

$$= \frac{\beta q}{\beta q + 1 - q} \tag{28}$$

For $\beta = 1$ we have $q^* = q$. For $\beta > 1$ it follows that $q^* > q$. The derivative of q^* is

$$q^*(\beta)' = \frac{1-q}{(1-q(1-\beta))^2} \tag{29}$$

$$\lim_{q\to 0,\, \beta\to 1} q^*(\beta)' = 1 \tag{30}$$

This means that while β and q are small, $q^*(\beta)$ grows *approximately linear* in β. Therefore, for small q, an increase of β has about the same impact as an increase of q.

3.4 Numerical investigation

Equation (19) is the key to get an estimate of the cost benefit ratio of FCR. But what are realistic values for the parameters of the calculation? Some data can be found in Kochsiek [2].

The yearly mileage of a car in single freight car transport is around 20,000 km. Assuming that there is need for switching once in 50km, we conclude that a car has to be switched around $r = 400$ times per year.

For the costs of using a locomotive we assume a mean value of 100 € per hour including variable and fixed costs. For time consumption of one switching process we assume a value between 30 and 45 minutes. Multiplying costs per hour and duration of switching yields $50 \leq K_L \leq 75$.

	Conservative scenario		Optimistic scenario	
	$K_L = 50$	$K_L = 75$	$K_L = 50$	$K_L = 75$
$I = 50,000$	0.5%	4.0%	4.2%	7.8%
$I = 60,000$	-1.5%	2.6%	2.2%	6.4%
$I = 80,000$	-5.6%	0.0%	-1.9%	3.6%
$I = 100,000$	-10.0%	-3.0%	-6.1%	-2.0%

Table 1: Per cent cost saving from upgrading 10% of all cars to become FCR, assuming switching complexity $p = 0.4$. Conservative scenario: $f = 2$ and $\beta = 1$. Optimistic scenario: $f = 3$ and $\beta = 1.1$.

We estimate the annuity costs to be yearly 25% of the investment, including maintenance. The costs of an FCR would currently be about 100,000 €. Economies of scale let it seem realistic to bring costs down to 80,000 €. If there is a price reduction for lithium cells, costs could reduce even to 60,000 €. As low-cost scenario we will also include the case of 50,000 €.

The reduced cost function (20) depends on the parameter $z = \frac{rK_L}{A_{\text{FCR}}}$. With $r = 400$, the values $K_L = 50$ and $K_L = 75$, and the four values for the investment cost we derive eight values for z between 0.8 and 2.4, with mean $z = 1.48$. Omitting the two extremal values, z assumes values between 1 and 2.

Table 1 summarizes our results. It contains the percentage cost savings from upgrading 10% of the cars with the FCR technology, in two different scenarios. It turns out that for investment cost of $I = 100,000$, the technology is never profitable, while for a cost of $I = 50,000$, FCR is profitable in both scenarios.

But what about FCR shares different from 10%? To answer this question, another look at figure 5(a) will be helpful. There, the average cost of car switching is shown in relation to the FCR share q ($p = 0.4$ and black for $f = 2$, red for $f = 3$). The shape of the curve depends on z whose values realistically should range between 1 and 2. The higher z, the more profitable is FCR. For $z = 1$, the average switching costs have a global minimum at $q = 0$: FCR is absolutely unprofitable. For $z = 2$, on the other hand, the average costs *decrease* until $q = 0.8$, that means that the optimal share of FCR would be around 80%. For $z = 1.5$ and $f = 2$ (not pictured), the average switching costs are almost constant in q until $q = 0.5$, then they increase in q.

The two inner curves in figure 5(a) for $z = 2$, are at least partially decreasing in q with concave shape. This means that the average switching costs have *decreasing* economies of scale in q. In contrast to many other innovations in the railway industry, FCR is *not* a system solution that would be only profitable if introduced on a large scale. Rather, FCR is a technology that can earn revenue in conjunction with the traditional single freight car transport service.

4 Conclusion

The idea that rail cars equipped with light engines and remote control can overcome many of the difficulties encountered in railyard switching is around for several years. Recently, a project centered at RWTH Aachen, see Baier and Enning [1], has come up with concrete development of such car called FlexCargoRail. In this paper we try to evaluate such concept from an economic or OR perspective.

The switching model of Petersen [3] nicely extends to car parks where a share of cars is enhanced with the FCR technology. The extension, based on an application of queueing theory, allows the computation of the savings in switching operations induced by FCR, depending on the number of cars that an FCR can move, the share of FCR-equipped cars and a parameter (p) that can be understood as an inverse measure of "switching complexity". We think that it would be interesting to model the switching complexity more in-depth to find out for which switching situations FCR can be used particularly well.

The results on the switching operation savings can be used to perform a cost-benefit analysis for the FCR technology, aiming at the optimal share of cars to be equipped as FCR. Interestingly, it turns out that depending on the parameters mentioned above, there may be an optimal FCR share strictly between 0 and 1. This implies that FCR technology may *not* be characterized by economies of scales (for the full range). This is rather atypical for the railway industry where many other innovations require a large-scale implementation in order to be profitable. Rather, FCR is a technology that can earn revenue in conjunction with the traditional single freight car transport service.

Finally, a calculation based approximative price and cost data for Germany shows that the FCR technology is likely to be profitable, or to become so rather soon.

References

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