DISCRETE CHOICE AND CONSISTENT MICROECONOMIC TRANSPORT DEMAND MODELING

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ABSTRACT

In transport demand modelling, discrete choice models are common. The major theoretical disadvantage of the current transport demand modelling with discrete choice is that it does not correspond to a full-fledged decision making by a consumer. A typical example is a choice between 'auto' and 'public transport', given a consumer's total transport demand is unity. Why his or her demand is restricted at unity is not explained within discrete choice models. In this sense, discrete choice models describes 'partial' decision making. On the contrary, the standard microeconomic model represents an entire decision making, where all variables are endogenously determined within the model. The approach in this paper overcomes the gap between discrete choice models with exogenous total demand and the standard microeconomics with endogenous total demand. We propose a full-fledged utility maximization framework of a consumer that is consistent with the results of the discrete choice models, and examine various aspects of the characteristics of the utility maximization problem, including the form of the utility function, elasticities, and the measurement of welfare.

Keywords: Discrete choice, transport demand modelling, representative consumer, logit model, generalized extreme value model

1 INTRODUCTION

In transport demand modelling, discrete choice models are common. Discrete choice models have been developed in coordination with econometrics, but their microeconomic foundation is not so strong. In order to clarify the relationship between transport demand modelling and microeconomics, there are two approaches. One approach is to construct the

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travel demand model from the viewpoint of the standard microeconomics, disregarding the fact that discrete choice models are commonly used. This direction was studied by Anas (2007). He tries to formulate travel demand model that is fully consistent with microeconomics, focusing on the fact that transport demand is the derived demand. The other approach is to pursue the microeconomic foundation of the current ordinary transport demand modelling with discrete choice models. This is a fundamental area of research related to discrete choice models. However, perhaps surprisingly, and despite its central importance, this is an area of research that has significantly received less attention than the exploitation of discrete choice modelling in practice[2].

The purpose of this paper is to formulate the transport demand modelling with discrete choice models based on microeconomics and to clarify its implication. In so doing, we resolve a major theoretical disadvantage of the current transport demand modelling with discrete choice; it cannot describe a full-fledged decision making by a consumer, because there is the fixed total demand, which must be determined outside the model. For example, consider a choice between 'auto' and 'public transport'. When we consider this problem in discrete choice models, the total demand for auto and transport is exogenously determined. A typical assumption is that each consumer demands unity. Even if we formulate the nested structure and include the upper-level choice between 'trip' or 'do not trip', this problem remains unsolved. The total number of the upper-level choices must be fixed exogenously; typically, each consumer is assumed to select 'trip' or 'do not trip' once in the first stage and select 'auto' or 'public transport' in the second stage. In this setting, the maximum number of demand for 'auto' or 'public transport' is restricted at unity. On the contrary, the standard microeconomic utility maximization does not have such a restriction; all variables are endogenously determined within the model. For example, a consumer determines the total number of trips as well as the number of trips by auto or public transport.

The approach in this paper overcomes the gap between discrete choice models with exogenous total demand and the standard microeconomics with endogenous total demand. We propose a full-fledged utility maximization framework of a consumer that is consistent with the results of the discrete choice models. The main assumptions are as follows. First, we focus on the GEV model, which is the general form of the logit and nested-logit models, because it has closed-form solutions. Second, we assume consistent aggregation across consumers' preferences. Without this assumption, we cannot aggregate each consumer's demand function to derive the market demand function. This assumption implies that the indirect utility function of a consumer has the Gorman form[3].

The main results we obtain in this paper are as follows.

First, the indirect utility function of a representative consumer includes the log-sum term as an argument, if it yields the transport market demand function consistent with the result of the GEV model. Personalized prices are not allowed; all consumers must face the same prices. The own-price and cross-price elasticities of the market demand for a transport service respectively are the sum of the price elasticity of the total transport demand and the

[2] A few exceptions are Morisugi and Le (1994) and Morisugi et al. (1995). They try to formulate utility maximization problems that are consistent with the logit and GEV models in which the total transport demand is endogenous. However, their analyses lack theoretical consistency and yield unsuccessful results.
[3] See Gorman (1961) and Varian (1992) for a detailed explanation of the Gorman form.

usual own-price and cross-price elasticities in which the total transport demand is assumed fixed. The change in welfare can be measured by using any of the four types of demand: a consumer's demand for a transport service, the market demand for a transport service, a consumer's total transport demand, or the total market transport demand. When we measure the change in welfare by using a consumer's total transport demand or the total market transport demand, the corresponding price is the log-sum term.

Second, the indirect utility function of a representative consumer that is consistent with the mixed GEV model can model the situation where prices are personalized. The corresponding indirect utility function of a representative consumer includes the log-sum term as an argument and quasi-linear. The own-price and cross-price elasticities of the market demand for a transport service are derived by 'mixing' a consumer's elasticities of the logit model by consumer's type. The change in welfare can be measured by using a consumer's demand for a transport service or a consumer's total transport demand. In the latter case, the corresponding price is the log-sum term. Since we allow all consumers to face different prices in the mixed GEV model, there exists no common price index that corresponds to the market demand of a transport service or the total market transport demand. Thus, we cannot calculate the welfare change by using them.

Third, the above results of the GEV and mixed-GEV models can be extended to include the multiple pairs of OD (origin-destination). Each OD transport demand plays a role of a group of good in the standard microeconomics. The relationship between ODs is unrestricted in our analysis; it can be substitute or complement.

The rest of the paper is structured as follows. In Section 2, we set up a model. In Sections 3 and 4, we examine the logit and mixed logit models for a single OD as the simplest cases, respectively. In Section 5, our analyses are generalized to include multiple ODs and to allow the GEV and mixed GEV models. Section 6 concludes the paper.

2 MODEL

Consumers are heterogeneous with their parameters, θ. The density function of θ is $f(\theta)$. A consumer with his or her parameter θ is called type-θ consumer. Type-θ consumer demands the composite consumer good, $z(\theta)$, and transport service j, $x_j(\theta)$ ($j=1,...,M$). We assume that M kinds of transport service are offered and that a consumer may demand all kinds of transport service. For the time being, we assume that there exists a single OD for the sake of simplicity. This assumption implies that the OD transport demand equals the total transport demand. The multiple ODs will be considered in Section 5-1. The time required and the monetary price of using transport service j for type-θ consumer are $t_j(\theta)$ and $\tau_j(\theta)$, respectively. Similarly, the time required and the monetary price of consuming the composite consumer good are $t_z(\theta)$ and $\tau_z(\theta)$, respectively.

The direct utility function of type-θ consumer is $u(z(\theta), x_1(\theta),..., x_M(\theta), \theta)$. The budget constraint is:

(1) $$w(\theta)L(\theta) = \tau_z(\theta)z(\theta) + \sum_{j'=1}^{M} \tau_{j'}(\theta)x_{j'}(\theta),$$

where $w(\theta)$ and $L(\theta)$ are the wage rate and the working time of type-θ consumer. The time constraint is:

(2) $$\bar{L} = L(\theta) + t_z z(\theta) + \sum_{j'=1}^{M} t_{j'}(\theta) x_{j'}(\theta),$$

where $\bar{L}$ is the fixed available time, which is common for all consumers. Combining (1) with (2) yields:

(3) $$y(\theta) = z(\theta) + \sum_{j'=1}^{M} p_{j'}(\theta) x_{j'}(\theta),$$

where $y(\theta) \equiv \dfrac{w(\theta)\bar{L}}{\tau_Z + w(\theta)t_z}$ and $p_j(\theta) \equiv \dfrac{\tau_j(\theta) + w(\theta)t_j(\theta)}{\tau_Z + w(\theta)t_z}$ are type-θ consumer's full income and the generalized price of transport service j, normalized by the generalized price of the composite consumer good. Maximizing $u(z(\theta), x_1(\theta),..., x_M(\theta), \theta)$ subject to (3) yields the indirect utility function, $v(p_1(\theta),..., p_M(\theta), y(\theta), \theta)$.

In this paper, we assume that each consumer's preference can be aggregated to a representative consumer's preference. Unless this assumption is made, we cannot derive the market demand function by aggregating type-θ consumer's demand function. This assumption is also important in welfare analysis. If a representative consumer cannot be defined, the complete correspondence between the sum of consumers' equivalent variation and the compensation principle are lost, as Blackorby and Donaldson (1990) explain in detail. In such a case, practical welfare analyses are very difficult to implement, because, in order to measure the welfare change for the society, we must derive each consumer's equivalent variation (EV) and aggregate it under a certain rule. Not only we need formidable information in calculating each consumer's EV but we also need to decide a rule under which each consumer's EV is aggregated, which is very unlikely to be unanimously approved.

The necessary and sufficient condition for defining a representative consumer is that the indirect utility function of type-θ consumer has the so-called Gorman form:

(4) $$v(p_1(\theta),..., p_M(\theta), y(\theta), \theta) = A(p_1(\theta),..., p_M(\theta), \theta) + B(p_1(\theta),..., p_M(\theta), \theta) y(\theta).$$

where $B(p_1(\theta),..., p_M(\theta), \theta)$ must be common for all consumers. To satisfy this condition, one of the following must be met:

(5) $B(p_1(\theta),..., p_M(\theta), \theta) = B(p_1,..., p_M)$, or

(6) $B(p_1(\theta),..., p_M(\theta), \theta) = \bar{B}$.

Eq. (5) shows that the generalized prices are independent of consumers' types, that is, common for all consumers. Eq. (6) implies that the indirect utility function of type-θ consumer is quasi-linear if the generalized prices are personalized by consumers' types. As we will see in the following section, the logit (or GEV) model corresponds to (5), while the mixed logit (or GEV) model corresponds to (6).

3 THE LOGIT MODEL

3-1 Derivation

When (5) is met, by aggregating the indirect utility function of type-θ consumer, the indirect utility function of a representative consumer is:

(7)
$$\begin{aligned} V &= \int_\theta (A(p_1,...,p_M,\theta) + B(p_1,...,p_M,)y(\theta)) f(\theta) d\theta \\ &= \int_\theta A(p_1,...,p_M,\theta) f(\theta) d\theta + B(p_1,...,p_M) Y, \end{aligned}$$

where $Y \equiv \int_\theta y(\theta) f(\theta) d\theta$ is an aggregate income. For the sake of simplicity, we focus on the logit model here, and extend our analysis to the GEV model in Section 5-2. The logit model is derived when the indirect utility function of a representative consumer, (7), has the form of:

(8) $V = \int_\theta A(LS, p_1,..., p_M, \theta) f(\theta) d\theta + B(LS)Y$,

where:

(9) $LS \equiv -\frac{1}{\beta} \ln \sum_{j'=1}^{M} \exp(\alpha_{j'} - \beta p_{j'})$ ($\alpha_{j'}$ and β (>0) are constants), and

(10) $\frac{\partial \left(\int_\theta A(LS, p_1,..., p_M, \theta) f(\theta) d\theta \right)}{\partial p_j} = 0$ for any p_j.

In fact, the market demand function of transport service j, X_j, is derived from (8) as:

(11) $X_j = OD(LS,Y) s_j$,

where:

(12) $OD(LS,Y) \equiv -\frac{1}{B} \frac{\partial V}{\partial LS} \geq 0$, and

(13) $s_j \equiv \frac{\exp(\alpha_j - \beta p_j)}{\sum_{j'=1}^{M} \exp(\alpha_{j'} - \beta p_{j'})}$.

Eq. (11) is the logit-type market demand function which is represented as the total market transport demand, $OD(LS,Y)$, multiplied by the choice probability of transport service j, s_j. Eqs. (8) - (10) are not only sufficient but also necessary to derive the logit-type market demand function from (7). See Appendix A for the proof.

This result alerts us to an important point in econometric estimations of the logit model. $\phi_j \equiv \alpha_j - \beta p_j$ in (13) can be interpreted as the utility gained from consuming a unit of transport service j. Thus, (13) implies that the utility gained from consuming a unit of transport service j must be linear in its generalized price, p_j. In empirical analyses, the unit utility of transport service j can be nonlinearly estimated regarding p_j. In such a case, however, there is no corresponding utility maximizing problem of a representative consumer that is consistent with the logit-type market transport demand function, which has the form of the total market transport demand multiplied by its choice probability. That is, when the unit utility of transport service j is nonlinearly estimated regarding p_j, the microeconomic foundation of the logit-type market transport demand is vague.

Two special cases of (8) are worthy of noting here.

First, (8) includes the ordinary logit model with the fixed total demand as a special case. For example, when $A(LS, p_1, ..., p_M, \theta) = LS\tilde{A}(p_1, ..., p_M, \theta)$ and $B(LS) = 1$, we obtain:

(14) $$OD = \int_\theta \tilde{A}(p_1, ..., p_M, \theta) f(\theta) d\theta,$$

which is a constant from (10).

Second, (8) also includes a direct utility function that corresponds to the logit model derived by Anderson et al. (1988, 1992 Ch. 3),

(15) $$U = Z + h\left(\frac{b}{\beta}\sum_{j'=1}^{M} X_{j'}\right) + \frac{1}{\beta}\sum_{j'=1}^{M}\left[\alpha_{j'} - \ln\left(\frac{X_{j'}}{\sum_{j''=1}^{M} X_{j''}}\right)\right] X_{j'},$$

where Z is the aggregated composite consumer good, b is a constant, $h' > 0$, and $h'' < 0$, as a special case. See Appendix B for the proof.

The indirect utility function of a representative consumer that corresponds to the logit model has important implications in the calculation of the elasticities and in welfare analysis. The following two subsections will explain them in turn.

3-2 Elasticities

From (11), we can derive the own-price and cross-price elasticities. We calculate the elasticities with respect to the monetary price of transport service j, τ_j. The same procedure applies for the calculation of elasticities with respect to the generalized price of transport service j, p_j. The own-price and cross-price elasticities respectively are:

(16) $$\frac{\partial X_j}{\partial \tau_j}\frac{\tau_j}{X_j} = \varepsilon_j - \beta(1 - s_j)\tau_j \text{ and}$$

(17) $$\frac{\partial X_j}{\partial \tau_{j'}}\frac{\tau_{j'}}{X_j} = \varepsilon_{j'} + \beta s_{j'}\tau_{j'},$$

where $\varepsilon_j \equiv \frac{\partial OD}{\partial \tau_j}\frac{\tau_j}{OD}$ is the elasticity of the total market transport demand with respect to the monetary price of transport service j, τ_j.

The results of (16) and (17) represent that ε_j and $\varepsilon_{j'}$ are disregarded in the ordinary logit model in which the total market transport demand is assumed fixed. Accordingly, the estimated elasticities are biased. Especially, the sign of the cross-price elasticity can be reversed; the cross-price elasticity is always positive when the total marked transport demand is fixed, but it can be negative when $\varepsilon_{j'}$ is included. Note also that the cross-price elasticities are the same for all transport services and the independence from irrelevant alternatives (IIA) property holds, even if the total market transport demand is allowed to be endogenous.

3-3 Welfare analysis

We focus on how to calculate EV. The same procedure applies to compensating variation (CV) if V^{WO} and v^{WO} are substituted for V^W and v^W, respectively, in the following analysis. Henceforth, the superscripts WO and W denote without and with a policy, respectively. The results are summarized below. The proof is in Appendix C.

(18)
$$\begin{aligned} EV &= \int_\theta \int_{p_j^W}^{p_j^{WO}} h_j(LS(p_1,...,p_M), p_1,...,p_M, v(\theta)^W, \theta) dp_j f(\theta) d\theta \\ &= \int_{p_j^W}^{p_j^{WO}} H_j(LS(p_1,...,p_M), V^W) dp_j \\ &= \int_\theta \left(\int_{LS^W}^{LS^{WO}} hod(LS(p_1,...,p_M), p_1,...,p_M, v(\theta)^W, \theta) dLS \right) f(\theta) d\theta \\ &= \int_{LS^W}^{LS^{WO}} HOD(LS(p_1,...,p_M), V^W) dLS, \end{aligned}$$

where $h_j(LS, p_1,..., p_M, v(\theta)^W, \theta)$ is type-θ consumer's Hicksian demand function for transport service j, $H_j(LS, V^W)$ is the Hicksian market demand function for transport service j, $hod(LS, p_1,..., p_M, v(\theta)^W, \theta)$ is type-θ consumer's Hicksian total transport demand function, and $HOD(LS, V^W)$ is the Hicksian total market transport demand function.

The first and the second lines of (18) show how to calculate EV from each consumer's transport demand function and the market transport demand function for transport service j. As a result of the assumption of the Gorman form, each consumer's transport demand function can be aggregated to the market transport demand function exactly. Thus, either transport demand function for transport service j, whose price is p_j, can be used in calculating EV. The third and the forth lines of (18) show logit-specific calculation methods, where the log-sum term plays a role of the price index; in the logit model, we can calculate EV from type-θ consumer's total transport demand or the market transport demand, in which case, the log-sum term acts as their prices. This result is a natural extension of Small and Rosen (1981, Eq. (5.9)) to allow endogenous total demand[4]. For a small price change, we can approximate EV by applying the 'rule-of-half':

(19) $$EV \approx \frac{1}{2}\left(LS^{WO} - LS^W\right)\left(HOD(LS^{WO}, V^W) + HOD(LS^W, V^W)\right).$$

In (18), we ignore the possibility that a change in the price of transport service j affects the price of transport service j', by changing the degree of congestion, for example. However, this possibility is easily dealt with by applying the method of Kidokoro (2004). Specifically, the first and the second lines in (18) are modified to:

[4] The well-known method of Small and Rosen (1981, Eq. (5.9)), under which EV is calculated as the change in the log-sum term multiplied by the number of consumers, N, is derived as a special case of the forth line of (18) by assuming $HOD(LS, V^W) = N$.

(20)
$$EV = \int_{\theta}\int_{p_j^W}^{p_j^{WO}} \left(\sum_{j'=1}^{M} h_{j'}(LS, p_1, ..., p_M, v(\theta)^W, \theta) \frac{\partial p_{j'}}{\partial p_j} \right) dp_j f(\theta) d\theta$$
$$= \int_{p_j^W}^{p_j^{WO}} \left(\sum_{j'=1}^{M} H_{j'}(LS, V^W) \frac{\partial p_{j'}}{\partial p_j} \right) dp_j .$$

The third and fourth lines remain unchanged, because the log-sum term can include the price changes of all transport services.

4 THE MIXED LOGIT MODEL

4-1 Derivation

In the case of (6), by aggregating the indirect utility function of type-θ consumer, the indirect utility function of a representative consumer is:

(21)
$$V = \int_{\theta} (A(p_1(\theta), ..., p_M(\theta), \theta) + \overline{B} y(\theta)) f(\theta) d\theta$$
$$= \int_{\theta} A(p_1(\theta), ..., p_M(\theta), \theta) f(\theta) d\theta + \overline{B} Y.$$

Train (2003, pp.138-139) defines the mixed logit model as a model which has the following choice probability, s_{ML_j}:

(22) $$s_{ML_j} = \int_{\theta} s_{ML_j}(p_1(\theta), ..., p_M(\theta), y(\theta), \theta) f(\theta) d\theta .$$

Extending this definition to include the case of endogenous total demand, we define the mixed logit model as the model in which the market transport demand function has the form of:

(23) $$X_j = \int_{\theta} OD(\theta) s_{ML_j}(p_1(\theta), ..., p_M(\theta), y(\theta), \theta) f(\theta) d\theta ,$$

where the total transport demand function of type-θ consumer is denoted by $OD(\theta)$. For the sake of simplicity we focus on the mixed logit model here; the analysis is extended to the case of the mixed GEV model in Section 5-2. The mixed logit model is derived when the indirect utility function of a representative consumer, (21), has the following form:

(24) $$V = \int_{\theta} A(LS_{ML}(\theta), \theta) f(\theta) d\theta + \overline{B} Y ,$$

where:

(25) $$LS_{ML}(\theta) \equiv -\frac{1}{\beta} \ln \sum_{j'=1}^{M} \exp(\alpha_{j'} - \beta p_{j'}(\theta)) .$$

From (24), the market demand function of transport service j, X_j, is derived as:

(26) $$X_j = \int_{\theta} OD(LS_{ML}(\theta), \theta) s_{ML_j}(\theta) f(\theta) d\theta ,$$

where:

(27) $$OD(LS_{ML}(\theta), \theta) \equiv -\frac{1}{\overline{B}} \frac{\partial V}{\partial LS_{ML}(\theta)} \geq 0 \text{, and}$$

(28) $$s_{MLj}(\theta) \equiv \frac{\exp(\alpha_j - \beta p_j(\theta))}{\sum_{j'=1}^{M} \exp(\alpha_{j'} - \beta p_{j'}(\theta))}.$$

Eqs. (24) and (25) are not only sufficient but also necessary to derive the mixed logit-type market demand function from (21). The proof is almost the same procedure as in the case of the logit model, and then omitted.

A major difference from the logit model is that the indirect utility function of the representative consumer that is consistent with the mixed logit model is quasi-linear[5]. The generalized price differs among consumers, and consequently, the log-sum term also differs among consumers. Given the Gorman restriction, the coefficient of income must be identical for all consumers. Thus, the coefficient of income cannot include the log-sum term as an argument. By contrast, in the case of the logit model, the coefficient of income may depend on the log-sum term, which is the same for all consumers.

As is the case with the logit model, $\phi_j(\theta) \equiv \alpha_j - \beta p_j(\theta)$ in (28) can be interpreted as type-θ consumer's utility gained from consuming a unit of transport service j, and must be linear in its generalized price, $p_j(\theta)$. If it is nonlinear, there is no corresponding utility maximizing problem of a representative consumer for the mixed-logit-type market transport demand function, and the microeconomic foundation of the mixed-logit model is vague.

4-2 Elasticities

The own-price and cross-price elasticities with respect to the monetary price of transport service j, respectively, are:

(29) $$\frac{\partial X_{MLj}}{\partial \tau_j}\frac{\tau_j}{X_{MLj}} = \int_\theta \left\{ \frac{OD(LS_{ML}(\theta),\theta) s_{ML_j}(\theta)}{X_{MLj}} \Big(\varphi_j(\theta) - \beta(1 - s_{ML_j}(\theta))\tau_j \Big) \right\} f(\theta) d\theta, \text{ and}$$

(30) $$\frac{\partial X_{MLj}}{\partial \tau_{j'}}\frac{\tau_{j'}}{X_{MLj}} = \int_\theta \left\{ \frac{OD(LS_{ML}(\theta),\theta) s_{ML_j}(\theta)}{X_{MLj}} \Big(\varphi_{j'}(\theta) + \beta s_{ML_{j'}}(\theta)\tau_{j'} \Big) \right\} f(\theta) d\theta,$$

where $\varphi_j(\theta) \equiv \dfrac{\partial OD(LS_{ML}(\theta),\theta)}{\partial \tau_j}\dfrac{\tau_j}{OD(\theta)}$ is the elasticity of type-θ consumer's total transport demand with respect to the monetary price of transport service j, τ_j.

Eqs. (29) and (30) are 'mixed' elasticities of those of the logit model by consumer's type. As we can easily see, (29) and (30) coincide with (16) and (17) respectively, if consumer's type is fixed. Otherwise, both elasticities become more flexible. In particular, the IIA property

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[5] In the case of the mixed logit model, the market transport demand function, (23), requires that type-θ consumer's transport demand function be written as the logit form, that is, his or her total transport demand multiplied by the choice probability. This condition implies that type-θ consumer's indirect utility function incorporates the price of each transport service only through the log-sum term like (24). By contrast, in the case of the logit model, type-θ consumer's transport demand function may not be consistent with the logit model; the requirement is that the market transport demand function is consistent with the logit model. In this case, we only need (10), which implies that we cannot exclude the case where type-θ consumer's transport demand function depends directly on the price of each transport service.

does not hold, because the cross-price elasticities depend on $\dfrac{OD(LS_{ML}(\theta),\theta)s_{ML_j}(\theta)}{X_{MLj}}$ in (30) and accordingly differ by transport services.

4-3 Welfare analysis

The indirect utility function of the representative consumer that yields the mixed logit model, (24), is quasi-linear. Thus, the Hicksian and the Marshallian demand functions coincide, and EV, CV, and the change in consumer surplus are also equivalent. EV can be calculated from either type-θ consumer's demand for transport service j or type-θ consumer's total transport demand; that is:

$$\text{(31)} \quad \begin{aligned} EV &= \int_\theta \left(\int_{p_j^W(\theta)}^{p_j^{WO}(\theta)} x_{ML_j}(LS_{ML}(\theta),\theta)dp_j(\theta) \right) f(\theta)d\theta \\ &= \int_\theta \left(\int_{LS_{ML}^W(\theta)}^{LS_{ML}^{WO}(\theta)} OD(LS_{ML}(\theta),\theta)dLS_{ML}(\theta) \right) f(\theta)d\theta, \end{aligned}$$

where $x_{ML_j}(LS_{ML}(\theta),\theta) \equiv OD(LS_{ML}(\theta),\theta)s_{MLj}(\theta)$ is type-θ consumer's demand function for transport service j. The derivation procedure is the same as in the case of the logit model, and then is not repeated here.

If a change in the price of transport service j affects those of other transport services (through a change in the degree of congestion, for example), we must also calculate the welfare changes relating to other transport services. In this case, the first line of (31) is modified to:

$$\text{(32)} \quad EV = \int_\theta \left(\int_{p_j^W(\theta)}^{p_j^{WO}(\theta)} \sum_{j'=1}^{M} \left(x_{ML_{j'}}(LS_{ML}(\theta),\theta) \frac{\partial p_{j'}(\theta)}{\partial p_j(\theta)} \right) dp_j(\theta) \right) f(\theta)d\theta .$$

(The second line is unchanged because the log-sum term can include the price changes of all transport services.)

In the case of the mixed logit model, EV cannot be calculated by using the market demand function for transport service j or by using the total market transport demand function. This represents a contrast from the logit model. It arises because the price and the log-sum term differ among consumers in the mixed logit model, and consequently, there exists no price index that corresponds to either the market demand for transport service j or to the total market transport demand. This implies that we cannot calculate a total welfare change by using 'averaged' price or log-sum term, which integrates out individual consumers' type, θ.

5 EXTENSIONS

The analyses in Sections 3 and 4 focus on the simplest cases. We consider two extensions here. One is an extension to include multiple ODs, and the other is an extension to the GEV model.

5-1 The model with multiple ODs

The analysis of Sections 3 and 4 can be easily extended to the more realistic case in which there are multiple ODs. (More generally, the analysis of multiple ODs is a variant of that of multiple groups of goods.) We extend the single-OD logit model to the multiple-ODs logit model here; the procedure is the same for the mixed logit model. Suppose that the number of ODs is k $(k=1,...,K)$[6]. We denote the generalized price of transport service j and the log-sum term for k th OD by p_{kj} and LS_k respectively. In the case of the multiple ODs, the indirect utility function of a representative consumer can be written as:

(33) $V=\int_{\theta} A(LS_1,...,LS_K, p_{11},..., p_{KM}, \theta) f(\theta) d\theta + B(LS_1,...,LS_K)Y$ [7].

A distinct feature of (33) is that the resulting OD transport demands are endogenous in all ODs and the relationship between ODs is unrestricted. For example, the familiar nested logit model can deal with multiple ODs, but the relationship between ODs is limited to the logit model. Eq. (33) does not imply such a restriction.

An example of the transport demand forecast that is consistent with (33) is as follows. i) In each OD, we construct the logit model to derive the choice probability, s_{kj}, and the log-sum term, LS_k. ii) Based on the log-sum terms in all ODs, we derive the transport demand function for k th OD as $OD_k(LS_1,...,LS_K,Y)$. iii) The transport demand function for transport service j in k th OD is derived as $X_{kj}=OD_k(LS_1,...,LS_K,Y)s_{kj}$. iv) The market transport demand function for transport service j is derived as $\sum_{k=1}^{K} X_{kj}$.

The elasticities and the welfare analysis in the case of multiple ODs are simple extensions of those in the case of the single OD. Thus, we only note the differences.

First, the cross-price elasticity depends on whether transport services j and j' belong to the same OD or not. When they belong to the same OD, the cross-elasticity is the same as (17) except for the notation. When they belong to the different ODs, the corresponding cross-price elasticity is:

(34) $$\frac{\partial X_{kj}}{\partial \tau_{kj'}}\frac{\tau_{kj'}}{X_{kj}}=\varepsilon_{kj'},$$

where $\varepsilon_{kj'} \equiv \frac{\partial OD_k}{\partial \tau_{kj'}}\frac{\tau_{kj'}}{OD_k}$. Eq. (34) shows that the cross-elasticity depends only on the price elasticity of the OD transport demand.

Second, if a change in the generalized price of transport service j in k th OD does not affect those of other transport services, EV can be calculated in the same way as (18). If it does, (20) is modified to:

[6] We do not distinguish between inbound transport and outbound transport for the sake of simplicity. The analysis is essentially the same if we treat them differently.
[7] Although we assume that the number of available transport services is the same at M regardless of ODs, the analysis remains unchanged even if we allow the number of available transport services to differ among ODs.

(35)
$$
\begin{aligned}
EV &= \int_{\theta} \left(\int_{p_{kj}^{W}}^{p_{kj}^{WO}} \left(\sum_{k'=1}^{K} \sum_{j'=1}^{M} h_{k'j'}(LS_1,...,LS_K, p_{11},..., p_{KM}, v(\theta)^W, \theta) \frac{\partial p_{k'j'}}{\partial p_{kj}} \right) dp_{kj} \right) f(\theta) d\theta \\
&= \int_{p_{kj}^{W}}^{p_{kj}^{WO}} \left(\sum_{k'=1}^{K} \sum_{j'=1}^{M} H_{k'j'}(LS_1...,LS_K, V^W) \frac{\partial p_{k'j'}}{\partial p_{kj}} \right) dp_{kj} \\
&= \int_{\theta} \left(\int_{LS_k^{W}}^{LS_k^{WO}} \sum_{k'=1}^{K} hod_{k'}(LS_1,...,LS_K, p_{11},..., p_{KM}, v(\theta)^W, \theta) dLS_k \right) f(\theta) d\theta \\
&= \int_{LS_k^{W}}^{LS_k^{WO}} \sum_{k'=1}^{K} HOD_{k'}(LS_1...,LS_K, V^W) dLS_k .
\end{aligned}
$$

A difference from (20) is that a change in the price of a transport service may affect transport services in other ODs, and consequently, calculations are required for all ODs.

5-2 The GEV model

The analysis of Sections 3 and 4 can be extended to the GEV model, which is a general form of the logit model. This extension is useful when modelling hierarchical structures within ODs. For example, consider a model in which, in the first stage, a consumer chooses between auto and public transport, and in the second stage, a consumer who chooses public transport in the first stage selects bus or rail. A typical model to include such a hierarchical structure is the nested logit model, which is a special case of the GEV model. For the sake of simplicity, we consider an extension from the logit model to the GEV model. The same procedure applies for an extension from the mixed logit model to the mixed GEV model.

Following McFadden (1978, Theorem 1), the GEV model can be described by using the function $G(z_1,\cdots,z_M)$ ($z_j \equiv \exp(u_j(p_j,Y))$), which satisfies the following conditions of (GEV-1) - (GEV-4):

(GEV-1) $G(z_1,\cdots,z_M)$ is nonnegative;

(GEV-2) $G(z_1,\cdots,z_M)$ is homogenous of degree *n*;[8]

(GEV-3) $\lim_{z_j \to \infty} G(z_1,\cdots,z_M) = \infty$;

(GEV-4) The ς th partial derivative of $G(z_1,\cdots,z_M)$ with respect to any combination of distinct z_j s is nonnegative if ς is odd, and nonpositive if ς is even. That is, $\frac{\partial G}{\partial z_j} \geq 0$ for all j, $\frac{\partial^2 G}{\partial z_j \partial z_{j'}} \leq 0$ for all j and j', where $j' \neq j$, $\frac{\partial^3 G}{\partial z_j \partial z_{j'} \partial z_{j''}} \geq 0$ for any distinct j, j', and j'', where $j'' = 1,...,M$ (and so on for higher-order derivatives). The logit model is a special case of the GEV model by assuming $G = \sum_{j'=1}^{M} \exp(\alpha_{j'} - \beta p_{j'})$.

The following are the different points from the analysis in Section 3.

[8] McFadden (1978, Theorem 1) originally assumed homogeneity of degree one. Ben-Akiba and Francois (1983) demonstrate that G can be homogeneous of degree n. See also Ben-Akiba and Lerman (1985, p. 126).

i) The log-sum term is modified from (9) to:

(36) $$LS_{GEV} \equiv -\frac{1}{n\beta}\ln G(\exp(\alpha_1-\beta p_1),\cdots,\exp(\alpha_M-\beta p_M))$$

ii) The choice probability for transport service j is modified from (13) to:

(37) $$s_{GEVj} = \frac{\frac{\partial G}{\partial z_j}\exp(\alpha_j-\beta p_j)}{nG}$$

iii) As in the logit model, the own-price and cross-price elasticities in the GEV model are the sum of the elasticities of the OD transport demand and the standard own-price and cross-price elasticities in which the OD transport demand is assumed fixed. The own-price and cross-price elasticities respectively are:

(38) $$\frac{\partial X_{GEVj}}{\partial \tau_j}\frac{\tau_j}{X_{GEVj}} = \varepsilon_{GEVj} - \beta(1-ns_{GEVj})\tau_j + \eta_{jj}\text{, and}$$

(39) $$\frac{\partial X_{GEVj}}{\partial \tau_{j'}}\frac{\tau_{j'}}{X_{GEVj}} = \varepsilon_{GEVj'} + \beta n s_{GEVj'}\tau_{j'} + \eta_{jj'}\text{,}$$

where $\eta_{jj} \equiv \frac{\partial\left(\frac{\partial G}{\partial z_j}\right)}{\partial \tau_j}\frac{\tau_j}{\left(\frac{\partial G}{\partial z_j}\right)}$. In the logit model, $\eta_{jj}=0$ from $\frac{\partial G}{\partial z_j}=1$.

iv) Welfare analysis is the same as that for the logit model, except that the log-sum term is modified from (9) to (36).

v) Extending to the case of multiple ODs in the GEV model is analogous to the analysis of the logit model in Section 5-1.

6 CONCLUDING REMARKS

In this paper, we have studied the microeconomic foundation of the transport demand modelling with the GEV model. The main result is that the transport demand modelling with the GEV or mixed GEV models is consistent with a consumer's full-fledged utility maximization problem (or the aggregated utility maximization problem of a representative consumer) under certain conditions. Before concluding our analysis, we comment on three issues in relation to an application to empirical analysis.

First, the true own-price and cross-price elasticities differ from those derived in the ordinary GEV or mixed GEV models in which the total demand is assumed fixed, because the elasticity of the total demand is not taken into account. As Oum et al. (1992, p. 145) point out[9], the derived elasticities in the ordinary GEV or mixed GEV models should be considered as "conditional" elasticities in which the total demand is assumed fixed.

[9] They state "Some discrete choice models are concerned solely with user's mode choice decisions given a fixed volume of traffic. Many studies of urban work trips fall into this category. The demand elasticities computed from these models are more appropriately interpreted as mode-choice elasticities rather than regular demand elasticities since the effect of a price change on aggregate traffic is not taken into account."

Second, the total market transport demand function, (12), is a function of the log-sum term, if the transport demand modelling including the GEV model is consistent with a utility maximization of a representative consumer whose indirect utility function is (8). This implies that the total transport demand function in the GEV model has a consistent microeconomic foundation, as long as it is estimated as the function of the log-sum term. Even in the case of multiple ODs, this result holds true, as long as the total transport demand function is estimated as the function of the log-sum terms in all ODs. This implication is very useful in empirical research, because it gives us a clear condition under which any total transport demand function can be considered as the derived result from a consumer's utility maximization. The same implication holds for the mixed GEV model.

Third, to be consistent with our analysis, the following two condition must be met; i) a consumer's indirect utility function is the Gorman form (or quasi-linear in the mixed GEV model) [10], and ii) the utility obtained from a transport service is linear in its generalized price. These conditions seems restrictive, but the condition i) is required for consistent aggregation across consumers and the condition ii) is necessary to derive the demand function for a transport service as a form of the total demand function, which is common for all transport services, multiplied by the choice probability of a transport service. When these conditions are not satisfied, we cannot derive the exact relationship between the transport demand function in the GEV or mixed GEV forms and each consumer's utility maximization. The next step would be to focus on the approximate relationship between them in more general settings. This direction merits future research.

[10] Herriges and Kling (1999), McFadden (1999), De Palma and Kilani (2003), and Dagsvik and Karlstrom (2005) analyze welfare measurement in discrete choice models with indirect utility functions that are nonlinear in income, i.e., non-Gorman form. These analyses are appropriate given that they focus on a 'one-consumer' economy in which there is only one consumer or there are many consumers who all have the same indirect utility function.

APPENDICES

Appendix A Necessity for Logit-type Demand Functions

The logit-type market demand function is represented as:

(A.1) $$X_j = OD(p_1,..., p_M, Y)\frac{\exp(\phi_j(p_j, Y))}{\sum_{j'=1}^{M}\exp(\phi_{j'}(p_{j'}, Y))}.$$

By Roy's Identity, (A.1) can be rewritten as:

(A.2) $$\frac{\partial V}{\partial p_j} = \frac{\partial V}{\partial Y} OD(p_1,..., p_M, Y)\left(-\frac{\exp(\phi_j(p_j, Y))}{\sum_{j'=1}^{M}\exp(\phi_{j'}(p_{j'}, Y))}\right).$$

Noting that $\frac{\partial\left(-\ln\sum_{j'=1}^{M}\exp(\phi_{j'}(p_{j'}, Y))\right)}{\partial\phi_j} = -\frac{\exp(\phi_j(p_j, Y))}{\sum_{j'=1}^{M}\exp(\phi_{j'}(p_{j'}, Y))}$, the indirect utility function of a representative consumer includes $-\ln\sum_{j'=1}^{M}\exp(\phi_{j'}(p_{j'}, Y))$ as its argument. That is, from (7), the indirect utility function of a representative consumer is:

(A.3) $$V = \int_{\theta} A(LS, p_1,..., p_M, \theta) f(\theta)d\theta + B(LS, p_1,..., p_M)Y,$$

where:

(A.4) $$LS \equiv -\ln\sum_{j'=1}^{M}\exp(\phi_{j'}(p_{j'})),$$

which is independent of Y because of the Gorman restriction.

Applying Roy's Identity to (A.3) yields the following market demand function for transport service j:

(A.5) $$X_j = \frac{1}{B}\left(\frac{\partial V}{\partial LS}\frac{\exp(\phi_j(p_j))}{\sum_{j'=1}^{M}\exp(\phi_{j'}(p_{j'}))}\frac{\partial\phi_j(p_j)}{\partial p_j} - \frac{\partial V}{\partial p_j}\right).$$

To obtain the logit-type market demand function, (A.1), we need:

(A.6) $\frac{\partial\phi_j(p_j)}{\partial p_j}$ is a constant, and

(A.7) $$\frac{\partial V}{\partial p_j} = 0.$$

From (A.6), we have:

(A.8) $$\phi_j(p_j) = \alpha_j - \beta p_j.$$

Defining:

(A.9) $LS \equiv \frac{1}{\beta} LS$,

we obtain (8) - (10) from (A.3), (A.7), (A.8), and (A.9).

From (A.5), (A.8), and (A.9), the total market transport demand can be rewritten as:

(A.10) $OD(p_1, ..., p_M, Y) = -\frac{1}{B} \frac{\partial V}{\partial LS}$.

Since the indirect utility function is nonincreasing in price and nondecreasing in income, we obtain:

(A.11) $\frac{\partial V}{\partial p_j} = \frac{\partial V}{\partial LS} \frac{\exp(\alpha_j - \beta p_j)}{\sum_{j'=1}^{M} \exp(\alpha_{j'} - \beta p_{j'})} \leq 0$ and

(A.12) $\frac{\partial V}{\partial Y} = B \geq 0$.

Eqs. (A.11) and (A.12) assure that the total market transport demand is nonnegative in (A.10).

Appendix B The direct utility function, (15), corresponds to a special case of (8)

Maximizing (15) subject to the following budget constraint,

(B.1) $Y = Z + \sum_{j'=1}^{M} p_{j'} X_{j'}$,

we obtain the logit-type market demand function:

(B.2) $X_j = \frac{\beta}{b} h'^{-1}\left(\frac{\beta}{b} LS\right) \frac{\exp(\alpha_j - \beta p_j)}{\sum_{j'=1}^{M} \exp(\alpha_{j'} - \beta p_{j'})}$.

Substituting (B.2) into (15) yields the following representative consumer's indirect utility function in terms of LS and Y:

(B.3) $V = h\left(h'^{-1}\left(\frac{\beta}{b} LS\right)\right) - \frac{\beta}{b} LSh'^{-1}\left(\frac{\beta}{b} LS\right) + Y$.

We know that (B.3) is a special case of (8), by setting:

$\int_\theta A(LS, p_1, ..., p_M, \theta) f(\theta) d\theta = h\left(h'^{-1}\left(\frac{\beta}{b} LS\right)\right) - \frac{\beta}{b} LSh'^{-1}\left(\frac{\beta}{b} LS\right)$ and $B(LS) = 1$.

Appendix C Derivation of (18)

From (7), the expenditure functions of type θ consumer and the representative consumer are, respectively:

(C.1) $e(LS, p_1, ..., p_M, \theta) = \frac{v(\theta) - A(LS, p_1, ..., p_M, \theta)}{B(LS)}$, and

(C.2) $$E(LS,V)=\frac{V-\int_{\theta}A(LS,p_1,...,p_M,\theta)f(\theta)d\theta}{B(LS)}.$$

From (C.1) and (C.2), we derive:

(C.3) $$\begin{aligned}EV&=\int_{\theta}\int_{p_j^W}^{p_j^{WO}}\frac{\partial e(LS,p_1,...,p_M,\theta)}{\partial p_j}dp_j f(\theta)d\theta\\&=\int_{p_j^W}^{p_j^{WO}}\frac{\partial E(LS,V)}{\partial p_j}dp_j\\&=\int_{\theta}\left(\int_{LS^W}^{LS^{WO}}\frac{\partial e(LS,p_1,...,p_M,\theta)}{\partial LS}dLS\right)f(\theta)d\theta\\&=\int_{LS^W}^{LS^{WO}}\frac{\partial E(LS,V)}{\partial LS}dLS.\end{aligned}$$

Eq. (18) is derived by defining:

(C.4) $$h_j(LS(p_1,...,p_M),p_1,...,p_M,v(\theta)^W,\theta)\equiv\frac{\partial e(LS,p_1,...,p_M,\theta)}{\partial p_j},$$

(C.5) $$H_j(LS,V^W)\equiv\frac{\partial E(LS,V)}{\partial p_j},$$

(C.6) $$hod(LS,p_1,...,p_M,v(\theta),\theta)\equiv\frac{\partial e(LS,p_1,...,p_M,\theta)}{\partial LS}, \text{ and}$$

(C.7) $$HOD(LS,V^W)\equiv\frac{\partial E(LS,V)}{\partial LS}.$$

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