

*A Bayesian model for rail track geometry degradation: a decisive step towards the assessment of uncertainty in rail track life-cycle.*  
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# **A BAYESIAN MODEL FOR RAIL TRACK GEOMETRY DEGRADATION: A DECISIVE STEP TOWARDS THE ASSESSMENT OF UNCERTAINTY IN RAIL TRACK LIFE-CYCLE.**

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## **ABSTRACT:**

A considerably high level of uncertainty in maintenance, renewal and unavailability costs has been pointed out by Infrastructure Managers as one of the major drawbacks of rail track investments. Above all, degradation of rail track geometry is responsible for the greatest part of railway infrastructure maintenance costs. Some approaches have been tried to control the uncertainty associated with rail track geometry degradation at the design stage, though little progress has improved the investors' confidence. Moreover, many studies on rail track life-cycle cost modelling tend to forget the dynamic perspective in uncertainty assessments. In fact, a life-cycle assessment of the uncertainty associated with rail track degradation is needed, quantifying how much the uncertainty in the degradation parameters is reduced as more inspection data is collected after the rail track starts operation.

Therefore, a Bayesian model for rail track geometry degradation is put forward, building up a framework to assess the uncertainty in rail track geometry degradation throughout its life-cycle not only at the design stage, but at all life-cycle phases. The model is run using inspection data from Lisbon-Oporto line: adjusting prior probability distributions to the model parameters at the design stage and updating them as inspection data is processed at the operation stage. Uncertainty reduction in geometry degradation parameters is then assessed by computing their posterior probability distributions each time an inspection takes place.

Finally, the results show that at the design stage, the uncertainty associated with maintenance cycles is considerably high, but it reduces significantly as more inspection data is collected. Significant impacts on the definition of maintenance cost allocation inside railway business models are highlighted, especially for the case of Public and Private Partnerships. Therefore, for the case of a new infrastructure, it is proposed that maintenance costs related to track geometry degradation are no longer assessed at the design stage based only on the prior probability distributions of the degradation model parameters, but renegotiated instead after a 'warm-up' period of operation based on their posterior probability distributions.

**Keywords:** Bayesian model; rail track degradation; uncertainty; life-cycle; maintenance costs

## **INTRODUCTION**

A recent European research project (INNOTRACK) conducting a survey to Infrastructure Managers (IM) concluded that risk analysis is not widely considered in life-cycle cost calculations and identified it as an area of improvement in life-cycle cost calculations (INNOTRACK 2007). Moreover, several Best Practice Studies conducted by the Office of Rail Regulation (ORR) consisting of international visits to IM reported an expected decrease of existing maintenance costs in the order of 20 - 30% through the development of a risk-based approach to infrastructure maintenance (ORR 2008). Considering that maintenance costs for rail track subsystem may represent 55% of total maintenance costs in the case of high-speed line system (López-Pita et al. 2008), more research concerning rail track degradation may bring more cost-effective tools and ideas in rail track management, increasing ultimately railway transport competitiveness.

Previous research works have focused in maintenance strategies to optimize ballast tamping and renewal actions from a life-cycle cost perspective (Zhao et al., 2006), without focusing on the uncertainty in degradation model parameters. A recent work included uncertainty aspects in life-cycle cost estimations for the rail component assigning probability distributions to reliability parameters (Patra et al., 2009). In terms of track geometry degradation, some studies tried to predict deterioration rates at the design stage based on the infrastructure features and operating conditions through multiple linear regression or other data mining technique, though no reasonable model was achieved (Esveld, 2001 on the work carried out by the Office for Research and Experiments (ORE) committee D 161).

Having said that, this paper puts forward a Bayesian model for rail track geometry degradation in order to assess its life-cycle uncertainty. The model is run using inspection data from the main Portuguese rail line (Lisbon-Oporto line).

The Lisbon-Oporto line has a total length of 337 km, and it has been under a renewal process since 1996. The renewal works performed included a thorough improvement of the track bed bearing capacity and a complete renewal of track superstructure, incorporating monoblock concrete sleepers spaced by 60 cm each, rail UIC 60 and Vossloh fastening system with plastic railpads ZW 687 (vertical stiffness 450 kN/mm). The sample analyzed in the present study includes a series of inspection data of 1725 renewed track sections (200 m long). Unfortunately, reliable inspection data is only available from 2001 up to now. In terms of inspection conditions, it is conducted four times a year and in terms of operating conditions, this line has passenger train-sets running at a maximum speed of 220 km/h and freight train-sets running at 80 km/h. Information on infrastructure features such as the location of switches, bridges, stations and plain track, layout percentages in the track section (curves, spiral and tangent), curve radius and cant were collected for each track section.

## **BAYESIAN IDEA**

Before the late 1980s, Bayesian approaches were only considered as interesting alternatives to the 'classical' statistical theory in stochastic modeling. However, as more powerful computers became widely accessible and as two groups of statisticians (re)discovered Markov Chain Monte Carlo (MCMC) methods in the early 1990s, Bayesian statistics suddenly became the latest fashion in modeling throughout all areas of science.

In fact, MCMC methods brought the generalization needed in the calculation of the posterior distribution, in particular for cases with non conjugate priors in which asymptotic methods do not apply. Physicists were familiar with MCMC methodology from the 1950s and, though the atomic bomb project did not involve any simulation technique, the subsequent development of the hydrogen bomb did. Nevertheless, the realization that Markov Chains could be used in Bayesian statistics only came with Gelfand and Smith (1990) and in more practical terms when a dedicated BUGS software (Bayesian Using Gibbs Sampling) became available (Lunn et al. 2000). For more details on the history of MCMC please see Robert and Casella (2008).

Bayesian approaches diverge from classical statistical theory in the fact that they consider parameters as random variables that follow a prior distribution. This prior distribution is then combined with the traditional likelihood to obtain the posterior distribution of the parameters of interest. This combination of prior and data information is processed using the so-called Bayes' rule, providing an expression for conditional probability of a possible outcome A given outcome B has occurred:

$$P[A|B] = \frac{P[A \cap B]}{P[B]} = \frac{P[B|A].P[A]}{P[B]} \quad (1)$$

The preceding equation offers a probabilistic mechanism of learning from data by considering that outcome B is observing the data and outcome A refers to the likelihood of the parameters of interest. Note again that we have assumed that parameters are also random variables. Therefore, the calculation of the posterior distribution  $f(\theta|x)$  of the parameters  $\theta$  given the observed data  $x$  can be computed according to the Bayes' rule as:

$$f(\theta|x) = \frac{f(x|\theta).f(\theta)}{f(x)} \propto f(x|\theta).f(\theta) \quad (2)$$

The posterior distribution combines the prior distribution  $f(\theta)$  of the parameters  $\theta$  and the likelihood  $f(x|\theta)$ . The denominator in the expression above is the marginal distribution of the data  $f(x)$  and it can be computed by integrating the numerator in the parametric space  $\Theta$ :

$$f(x) = \int_{\Theta} f(x|\theta')f(\theta')d\theta' \quad (3)$$

Usually the target posterior distribution is not analytically tractable, though in some special cases (where priors are conjugate distributions for the likelihood) the resulting posterior distribution belongs to the same distributional family of the prior. In such cases, the parameters that define the posterior distribution can be easily calculated based on the prior

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parameters and some statistics from the data. In the general case (for non conjugate priors) we need MCMC simulation to assess the posterior distribution.

We can assess the posterior distribution  $f(\theta|x)$  by sampling from a target distribution that is equal to  $f(x|\theta).f(\theta)$  up to a normalizing constant  $f(x)$ . MCMC method is the appropriate algorithm to generate samples while exploring the parametric space  $\theta$ . Although for finite parametric spaces, the idea to introduce Markov Chains may seem intuitive, for continuous parametric spaces this idea implies the definition of a Kernel function to represent the conditional density of  $\theta^{(i+1)}$  given the value of  $\theta^{(i)}$ . The idea is to build and simulate a Markov Chain  $\{\theta^{(j)}, j = 1, 2, \dots, N\}$  in a way that it converges in distribution to the posterior distribution  $f(\theta|x)$ , meaning that the equilibrium distribution of the selected Markov Chain is the posterior distribution. Many MCMC algorithms have been developed to perform in such a way: the two most popular MCMC methods are the Metropolis-Hastings algorithm and the Gibbs sampling. We will not cover them in detail and redirect the reader to (Andrieu et al., 2003), (Bernardo 2003) or any introductory Bayesian statistical book (Paulino et al., 2003), or alternatively to a practical insight in WinBUGS (Ntzoufras 2008).

Having introduced the Bayesian idea, we may divide the Bayesian approach into four stages: model building, calculation of the posterior distribution (with the appropriate method of computation), analysis of the posterior distribution and conclusions (inference concerning the problem under consideration). Note that in the first stage (model building), we need to identify the main variable of the problem, find a distribution that adequately describes it (while including other variables that may influence it) and specify the prior distribution and the likelihood of the model. Moreover, a very important step is specifying the prior distribution using a noninformative (or vague prior) or incorporating preceding known information, using old samples from problems under the same boundary conditions or from expert judgment. This process is usually called elicitation of the priors. In the next sections, we will follow as strict as possible, the four stages mentioned above to describe the Bayesian approach.

## **RAIL TRACK GEOMETRY DEGRADATION**

Track geometry degradation is usually quantified by five track defects: the longitudinal leveling defects, the horizontal alignment defects, the cant defects, the gauge deviations and the track twist. Although many infrastructure managers tend to sum up all these defects into a track quality index which is typically function of the standard deviations of each defect and train permissible speed (as reported in El-Sibaie and Zhang 2004 or Zhao et al. 2006), the standard deviation for the short wavelength (3m - 25m) of longitudinal levelling defects is still regarded as the crucial parameter for planned maintenance decisions as it is confirmed by a recent guide on best practices for optimum track geometry durability (UIC 2008). Longitudinal levelling defects are defined as the geometrical error in the vertical plane, measured in millimetres from the rail top in the running surface to the ideal mean line of the longitudinal profile.

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In the past, many experimental studies have validated a linear relationship between the standard deviation of longitudinal levelling defects and the accumulated tonnage. The accumulated tonnage is the sum of all axle load (in tons) of all trains that have run in a given track section since renewal or last maintenance operation (tamping operation), usually quantified in Million Gross Tons (MGT). Using accumulated tonnage instead of time is a more convenient way to assess the evolution of the standard deviation of longitudinal levelling defects as it allows to distinguish track sections heavily operated from others. Other ways to compute the accumulated tonnage have been used (UIC 2009) separating passenger from freight train sets, though these formulations have been under large criticism. Therefore, accumulated tonnage is here considered as the above-mentioned definition.

For optimum use of information and control of maintenance and renewal processes, track inspection records should be condensed, usually referring to track sections 200 m long (Esveld, 1990). Therefore, the evolution of the standard deviation of longitudinal levelling defects for each 200-m section can be estimated using the following linear relationship:

$$\sigma_{LD} = c_1 + c_0 \cdot T \quad (4)$$

In which:  $\sigma_{LD}$  is the standard deviation of longitudinal levelling defects (mm);  $c_1$  is the initial standard deviation measured after renewal or tamping operations (mm);  $c_0$  is the deterioration rate (mm/100MGT);  $T$  is the accumulated tonnage since renewal or tamping operations (100MGT).

Although other models may capture the nonlinear characteristics of track quality deterioration (Riessberger 2001, Ubalde et al 2005 or Veit 2007), the linear function is widely used and is employed in the following analysis.

## REQUIRED GEOMETRIC QUALITY LEVELS

For safety and passenger comfort reasons, the standard deviation of longitudinal levelling defects should not be higher than a certain limit depending on the train speed. The UIC standard 518 (UIC 2005) and the European Standard EN 13848-5<sup>1</sup> (EN 2008) set three geometric track quality levels:

- QN1 quality level (AL – Alert Limit), which refers to the value which necessitates monitoring or taking maintenance actions as part of regularly-planned maintenance operations;
- QN2 quality level (IL – Intervention Limit), which refers to the value that requires short term maintenance action;

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<sup>1</sup> The European Standard EN 13848-5 set also three quality levels with similar meaning of QN1, QN2 and QN3 but with different names: AL (Alert Limit), IL (Intervention Limit) and IAL (Immediate Action Limit) respectively. As next tables confirm, the limits of train speeds are a little bit different from UIC 518 for classes in higher speed and EN 13848-5 only recommends an interval to Alert limits (AL).

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- QN3 quality level (IAL – Immediate Action Limit), which refers to the value above which is no longer desirable to maintain the scheduled traffic speed.

Although maximum values for defects are also limited in both standards, the scheduling of planned maintenance actions are made based only on the standard deviation of longitudinal levelling defects.

Table 1 – Standard deviation limits of the longitudinal levelling defects for different train speeds (V) and quality levels according to UIC standard 518.

Standard deviation limits of the longitudinal levelling defects	QN1 (mm)	QN2 (mm)
$V \leq 80$ km/h	2,3	2,6
$80 < V \leq 120$ km/h	1,8	2,1
$120 < V \leq 160$ km/h	1,4	1,7
$160 < V \leq 200$ km/h	1,2	1,5
$200 < V \leq 300$ km/h	1,0	1,3

Table 2 – Standard deviation limits of the longitudinal levelling defects for different train speeds (V) and quality levels according to European standard EN 13848-5.

Standard deviation limits of the longitudinal levelling defects	AL (mm)
$V \leq 80$ km/h	2,3 – 3,0
$80 < V \leq 120$ km/h	1,8 – 2,7
$120 < V \leq 160$ km/h	1,4 – 2,4
$160 < V \leq 230$ km/h	1,2 – 1,9
$230 < V \leq 300$ km/h	1,0 – 1,5

Note that the limit values for quality level QN1 are exactly equal to the inferior bound of the recommended interval for the Alert Limit (AL), though the two classes with higher train speed are a little bit different (230 km/h splits them instead of 200 km/h). Therefore, the UIC standard 518 is more demanding than standard EN 13858-5 in terms of the standard deviation of longitudinal levelling defects. In this paper, we will assume as limit values the ones given by the UIC standard 518.

For example, for train speeds between 200 and 300 km/h, QN1 quality level for standard deviation of longitudinal levelling defects is 1.0 mm and QN2 quality level is 1.3 mm. QN3 quality level is 130 per cent of the QN2. Table 1 shows how these limits vary depending on train speed.

Therefore, maintenance needs (tamping actions) for a given track section can be estimated by inverting the degradation model expression:

$$T_{\text{tamping cycle}} = \frac{\sigma_{\text{lim}} - c_1}{c_0} \quad (5)$$

In which:  $\sigma_{lim}$  is a specified target limit for the standard deviation of longitudinal levelling defects. Note that this limit is often assumed as a quality level (QN1 or QN2) depending on the train speed.

Having estimated the tamping cycle for a given rail track section, maintenance costs can be assessed by the framework found in (Andrade 2008), where a Monte Carlo simulation allowed to assess the uncertainty related to the series of tamping cycles for each 200 m track section.

## PRIOR UNCERTAINTY ASSOCIATED WITH DEGRADATION PARAMETERS

Uncertainty associated with rail track geometry parameters  $c_1$  and  $c_0$  at the design stage has been assessed in (Andrade and Teixeira 2010) using inspection data from the Portuguese Main line (Lisbon-Oporto line). It was found that the hypothesis that the deterioration rate  $c_0$  followed a Log-Normal distributions was not rejected at the 5% significance level. Next table reproduces similar findings presented there but for a larger sample (N = 1725 track sections of 200 m length), representing inspection data series of 15 to 20 inspections depending on track sections. Four groups of track sections were defined in a sequential way based on infrastructural features, so that they represent a track partition of the set of all track sections.

Table 3 – Goodness-of-fit tests to Log-Normal distributions for deterioration rates and initial standard deviation for all track sections and each track section group.

	$c_0$ – Deterioration rate (mm / 100 MGT)					$c_1$ – Initial standard deviation (mm)				
	All	Groups				All	Groups			
		Switches	Bridges	Stations	Plain Track		Switches	Bridges	Stations	Plain Track
Scale parameter ( $\mu$ )	1,183	1,615	1,560	1,295	1,062	0,415	0,571	0,410	0,417	0,394
Shape parameter ( $\sigma$ )	0,895	0,837	0,923	0,868	0,883	0,407	0,474	0,373	0,428	0,370
N	1725	200	170	175	1185	1725	200	170	175	1180
K-S test (p-value)	0,3291	0,5437	0,1251	0,3702	0,0947	0,0005	0,1522	0,7926	0,8567	0,0015

The classification of all track sections into these four groups were based on infrastructural features. The first track section group was constituted by all track sections that had switches within them. The second group corresponded to track sections that were not included in the first group and were located on bridges or had bridge transitions. The third group had all the remaining track sections that were in station areas and finally, the last group were all the remaining track sections that were in plain track. As these four groups were defined in a sequential way, they were mutually exclusive, defining all together a track partition. This exploratory analysis was based on a similar assessment of track geometric quality for high speed lines (López-Pita et al. 2007), in which it was found that the presence of switches and bridge transitions strongly influence deterioration.



As it can be seen in table 3, the hypothesis that deterioration rate ( $c_0$ ) follows a Log-Normal distribution is not rejected at the 5% significance level for all track sections and inside each track section group (K-S test p-value is higher than 5%). The same does not apply to the initial standard deviation ( $c_1$ ), though for all groups except 'plain track', the hypothesis is not rejected at the 10% significance level. It is important to mention that other distributions (such as: Uniform, Normal, Gamma, Exponential and Weibull) were fitted to both variables, but none has shown a higher p-value than the Log-Normal distribution for the Kolmogorov-Smirnov (K-S) goodness-of-fit test.

Moreover, linear correlation between  $c_1$  and  $c_0$  was also explored inside each track section group, so that a Bivariate Log-Normal distribution could be defined as a prior distribution for the degradation parameters for each track section group. Next table presents the linear correlation between deterioration rate ( $c_1$ ) and initial standard deviation ( $c_0$ ) for each track section group:

Table 4 – Linear correlations between deterioration rate ( $c_0$ ) and initial standard deviation ( $c_1$ ) and their transformation  $\log(c_0)$  and  $\log(c_1)$  for each track section group.

		Linear correlation between $c_1$ and $c_0$	Linear correlation between $\log(c_1)$ and $\log(c_0)$
Track section group	Switches	0,377	0,424
	Bridges	0,258	0,129
	Stations	0,436	0,423
	Plain Track	0,508	0,400
All		0,460	0,397

All correlations are statistically significant at the 1% significance value (2-tailed).

As the multivariate Log-Normal distribution is not available at the distribution library in WinBUGS software, we will need to do a simple transformation in order to run the MCMC simulation. Therefore, we know that if a given random variable  $X$  follows a Log-Normal distribution<sup>2</sup> with probability density function:

$$f(x|\mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\log(x)-\log(\mu))^2}{2\sigma^2}} \quad (6)$$

It can be shown that  $Y = \log(X)$  is normally distributed with mean and variance:

$$\mu_Y = \log\left(\frac{\mu_X^2}{\sqrt{\mu_X^2 + \sigma_X^2}}\right); \sigma_Y^2 = \log\left(1 + \frac{\sigma_X^2}{\mu_X^2}\right). \quad (7)$$

<sup>2</sup> Mean ( $\mu_X$ ) and Variance ( $\sigma_X^2$ ) of a Log-Normal distribution can be computed based on the scale ( $\mu$ ) and the shape ( $\sigma$ ) parameters:  $\mu_X = e^{\log(\mu) + \frac{1}{2}\sigma^2}$  and  $\sigma_X^2 = (e^{\sigma^2} - 1) \cdot e^{2\log(\mu) + \sigma^2}$ . (9)

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Moreover, in the case of two correlated Log-Normal variables  $X_1$  and  $X_2$  with linear correlation equal to  $\rho_{X_1X_2}$ , the linear correlation ( $\rho_{Y_1Y_2}$ ) between two normally distributed variables  $Y_j = \log(X_j)$  for  $j = 1, 2$ . can be assessed by inverting the expression that can be found in Johnson and Kotz (1972):

$$\rho_{X_1X_2} = \frac{\exp(\rho_{Y_1Y_2} \cdot \gamma_{X_1} \gamma_{X_2})}{\sqrt{\{\exp(\gamma_{X_1}^2 - 1)\} \cdot \{\exp(\gamma_{X_2}^2 - 1)\}}} \quad (8)$$

Note that  $\gamma$  is the skewness of the Log-Normal marginal distribution and can be computed based on the shape parameter  $\sigma$  as  $\gamma = (e^{\sigma^2} + 2) \cdot \sqrt{e^{\sigma^2} - 1}$ .

Although we could have used the procedure above to elicitate  $\rho_{Y_1Y_2}$  without any further information than the scale and shape parameters contained in table 3 and  $\rho_{X_1X_2}$ , we will use the Pearson correlation from the raw data after the transformation  $Y = \log(X)$  for each observation of deterioration rates ( $c_0$ ) and initial standard deviation ( $c_1$ ). In fact, some evidence suggests that for bivariate Log-normal variables the sample correlation may be quite different from the population correlation coefficient (Lai et al. 1999), that is mainly the reason why it is preferable to estimate it after the transformation.

Having said that, we are now able to specify the model and prior distribution for the track degradation phenomenon:

$$\sigma_{LD_i} = c_1 + c_0 \cdot T_i \quad i = 1, \dots, n \quad \sim N(c_1 + c_0 \cdot T_i; \sigma^2) \quad (10)$$

$$f(c_0, c_1, \sigma^2) = f(c_0, c_1) \cdot f(\sigma^2) \quad (11)$$

$$\{\log(c_0), \log(c_1)\} \sim N_2(\mu, \Sigma) \quad (12)$$

$$\sigma^2 \sim IG(a, b) \quad (13)$$

We will assume that  $(c_1, c_0)$  and  $\sigma^2$  are a priori independent, and therefore their joint prior density function factorizes into their marginal prior density functions as formulated above. Therefore, we will assume that  $(c_1, c_0)$  will follow a-priori a bivariate Log-Normal distribution, whereas  $\sigma^2$  will follow an inverse-gamma distribution. Note that as the conjugate prior for the normal model (equation (10)) is not the Log-Normal distribution, the MCMC simulation is needed to assess the posterior distributions.

Having built the model, we need to find values for the parameters of the prior distributions, meaning using proper values for  $\mu$ ,  $\Sigma$ ,  $a$  and  $b$ . It is important to mention that though it is not explicitly specified above, this model can be applied considering no partition of track section groups or making use of prior information available in table 3 and 4 for each track section group. Note that the latter option implies considering different parameters  $\mu$  and  $\Sigma$  for each track section. Therefore, we can compute both parameters for each track section group based on the shape and scale parameters in table 3 and using equations (9) in the last

page's footnote to compute the mean and variance and substitute those values in equations (7):

$$\begin{aligned}\mu_{\text{switches}} &= \begin{Bmatrix} 0,496 \\ -0,557 \end{Bmatrix}; \mu_{\text{bridges}} = \begin{Bmatrix} 0,449 \\ -0,899 \end{Bmatrix}; \mu_{\text{stations}} = \begin{Bmatrix} 0,266 \\ -0,875 \end{Bmatrix}; \mu_{\text{plain track}} = \begin{Bmatrix} 0,061 \\ -0,938 \end{Bmatrix} \\ \Sigma_{\text{switches}} &= \begin{bmatrix} 0,711 & 0,169 \\ 0,169 & 0,223 \end{bmatrix}; \Sigma_{\text{bridges}} = \begin{bmatrix} 0,876 & 0,045 \\ 0,045 & 0,137 \end{bmatrix}; \\ \Sigma_{\text{stations}} &= \begin{bmatrix} 0,757 & 0,158 \\ 0,158 & 0,184 \end{bmatrix}; \Sigma_{\text{plain track}} = \begin{bmatrix} 0,790 & 0,131 \\ 0,131 & 0,135 \end{bmatrix}.\end{aligned}$$

Or alternatively, without considering any partition:

$$\mu_{\text{all}} = \begin{Bmatrix} 0,171 \\ -0,885 \end{Bmatrix}; \Sigma_{\text{all}} = \begin{bmatrix} 0,814 & 0,145 \\ 0,145 & 0,164 \end{bmatrix}$$

For the non diagonal values of matrices  $\Sigma$ , those values were not computed based on equation (8) but instead they were calculated based on the linear correlation between  $\log(c_1)$  and  $\log(c_0)$  in table 4 and the computed variances in equation (7) by the expression:

$$\text{COV}[\log(c_1), \log(c_0)] = \rho_{\log(c_1)\log(c_0)} \sqrt{\sigma_{\log(c_1)}^2 \cdot \sigma_{\log(c_0)}^2} \quad (14)$$

Finally, for the inverse-gamma prior distribution the parameters were chosen so that it can be considered a noninformative or vague prior distribution. Although there are robust methods to define a vague or noninformative prior distribution (e.g. Bayes-Laplace method or Jeffreys method), we will follow a typical approach in normal models for the parameter  $\sigma^2 \sim \text{IG}(a, b)$ . The precision  $\tau = \sigma^{-2}$  is used instead, and it can be shown that  $\tau \sim G(a, b)$ . Therefore, as the precision need to be vague, parameters were set as  $a = b = 0,01$ , so that  $E[\tau] = \frac{a}{b} = 1$  and  $\text{VAR}[\tau] = \frac{a}{b^2} = 100$ .

## LIFE-CYCLE ASSESSMENT OF UNCERTAINTY

Having specified the model and conducted proper elicitation of the priors, we are able to perform an MCMC simulation to obtain the posterior distribution of the parameters of interest, which in this case are mainly the deterioration rate ( $c_0$ ), the initial standard deviation ( $c_1$ ) and  $\sigma^2$  (or precision ( $\tau = \sigma^{-2}$ )). Therefore, in order to simulate it, the model specified above was built in WinBUGS software (Lunn et al. 2000). Nevertheless, as the main purpose of the model was to capture the dynamic component of uncertainty throughout the rail track life-cycle, four main phases were distinguished:

- 1) Assessing uncertainty at the design stage:

At the design stage, the only prior information that is available concerns to which group a given track section belongs. In practical terms, for a given line we will know a-priori which

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group a certain 200-m long track section belongs and we can assess the uncertainty associated with the parameters of interest (exclusively given by the prior distributions). Therefore, comparisons between different design options in terms of uncertainty associated with track geometry degradation are possible.

In fact, a simple exercise in terms of assessing the uncertainty associated with the tamping cycle for different groups and for distinct quality levels (QN1 and QN2) was conducted using Monte Carlo simulation in Andrade and Teixeira (2010) that we reproduce in the figure below:

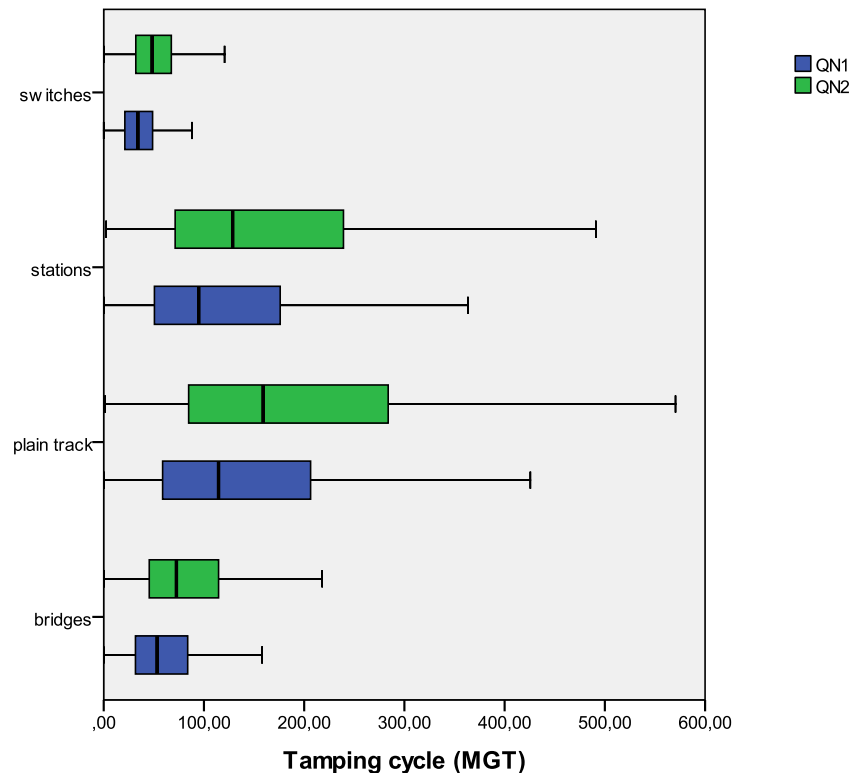


Figure 1 – Box and whisker plot for the Tamping cycle in MGT (Million Gross Tonnes) with related uncertainty at the design stage for different track sections and quality levels (for train speed  $200 < V \leq 300$  km/h and according to table 1 QN1 = 1,0 and QN2 = 1,3).

Note that the tamping cycle is a function of the deterioration rate, the initial standard deviation and the limit value depending on the quality level as expressed in equation (5). Therefore, the uncertainty associated with tamping cycle at the design stage can be assessed by running a Monte Carlo simulation, assigning probability distributions to the deterioration rate and initial standard deviation for each track section group, and the respective linear correlation. Nevertheless, as we are only interested in assessing the evolution of uncertainty associated the parameters (particularly the deterioration rate), we will not focus again in uncertainty associated with tamping cycle, though at each inspection ( $i$ ) it could be calculated using the same procedure mentioned before.

Figure 1 suggests higher values for tamping cycle for track sections belonging to the groups of stations and plain track, whereas for track sections with switches or in bridges, the

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tamping cycle is considerably lower. This implies higher maintenance costs associated with tamping actions for track sections with switches and bridges. Nevertheless, the uncertainty associated with tamping cycles is very large at the design stage even inside each group.

Moreover, note that using this information to predict maintenance costs should always be regarded as an approximate approach for new or upgraded lines in similar boundary conditions of the analyzed sample. Therefore, the analyst should have in mind the infrastructure features and the operating conditions of Lisbon-Oporto line described in the introduction.

2) Assessing uncertainty after the first inspection ( $i = 1$ )

As soon as the first inspection values are collected, the initial standard deviation ( $c_1$ ) is known and there is no longer uncertainty associated with it. Therefore, the prior distribution at this stage is no longer the bivariate Normal<sup>3</sup> distribution, but it is replaced with a conditional Normal distribution:

$$\log(C_0)|\log(C_1) = x \sim N(\mu, \sigma^2) \quad (15)$$

$$\mu = \mu_{\log(C_0)} + \frac{\text{COV}[\log(C_1), \log(C_0)]}{\sigma_{\log(C_1)}^2} \cdot (x - \mu_{\log(C_1)}), \sigma^2 = (1 - \rho_{\log(C_1)\log(C_0)}^2) \cdot \sigma_{\log(C_0)}^2 \quad (16)$$

For details in the demonstration of the conditional distribution see Bertsekas and Tsitsiklis (2002). Therefore, after the first inspection, the initial standard deviation is no longer a parameter, but it is assumed to be known. Although this phase has been separated from the next stage for conceptual reasons, the simulated values ( $i = 1$ ) were included in the next section in figures 2, 3, 4 and 5.

3) Assessing uncertainty between the first inspection and the first tamping action.  
( $i = 2, \dots, n$ )

In this period, new inspection data will be collected as time goes by and we will have less uncertainty associated with the deterioration rate ( $c_0$ ) for a given 200-m long track section. This associated uncertainty can be easily assessed by computing sequentially the successive posterior distributions of deterioration rate at each inspection till the tamping operation or alternatively by computing the corresponding 90% Credibility Interval<sup>4</sup>.

Some details regarding the simulation process to assess the successive posterior distributions should be mentioned. As MCMC algorithms only reach the equilibrium distribution, which is equivalent to the target distribution (posterior distribution), if the algorithm has converged, monitoring the convergence of the algorithm is essential for

<sup>3</sup> Note that in terms of  $\{\log(c_1), \log(c_0)\}$ , it follows a bivariate Normal distribution, which is equivalent to say that  $\{c_1, c_0\}$  follows a bivariate Log-Normal distribution.

<sup>4</sup> The Credibility Interval (CI) is the Bayesian Confidence Interval. In this case, the 90% CI was computed based on the 95<sup>th</sup> and the 5<sup>th</sup> Percentiles (P 95% and P 5% in the next figures).

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producing proper results. Other relevant aspects is the choice of initial values. These initial values may influence the posterior distribution if they are far away from the highest posterior probability areas and the simulated sample is not large enough to eliminate its effect. Of course, it is always possible to mitigate the influence of initial values by running the algorithm a few times and eliminate afterwards all iterations before (burning period). Moreover, note that the final MCMC generated sample is not independent. Therefore, we need to monitor the autocorrelations of the generated values so that each iteration of the simulated sample is considered independent. In practical terms, we should choose a lag  $L$  (or thin interval) after which the corresponding autocorrelation is low and keep only the first generated values in every group of  $L$  iterations.

Having said that, MCMC simulations were run for four representative track sections belonging to each track section group (plain track, bridges, stations and switches). Their location is properly identified in the figure captions. An MCMC simulation was run at each inspection  $i = 1, 2, \dots, n$  using all inspection data collected till that  $i^{\text{th}}$  inspection. This procedure was repeated for the four representative track sections belonging to each group. In each MCMC simulation, convergence was monitored and guaranteed, using a burning period of 2000 values and a simulated sample of 10000 values with a lag  $L = 500$  (in order to assure that autocorrelations between simulated values for each parameter were negligible). In this way, all Monte Carlo errors reported were less than 1% of mean values for each parameter simulated.

Next figures will exemplify the reduction of uncertainty associated with the parameters deterioration rate ( $c_0$ ) and  $\sigma$  (or precision ( $\tau = \sigma^{-2}$ )) for the four track sections belonging to each group:

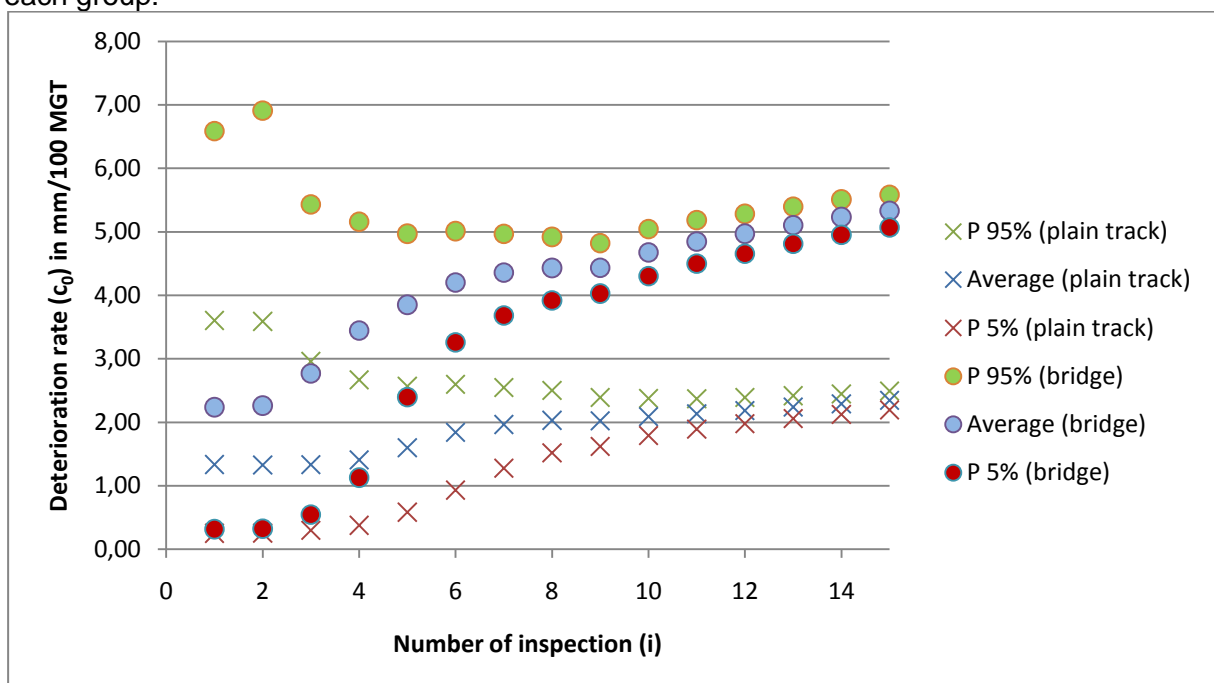


Figure 2 – Evolution of uncertainty associated with deterioration rate ( $c_0$ ) in mm/100 MGT as more inspection data is available for two track sections belonging to different groups: plain track (PK 266,4-266,6 VA) and bridge (PK 268,6-268,8 VA).

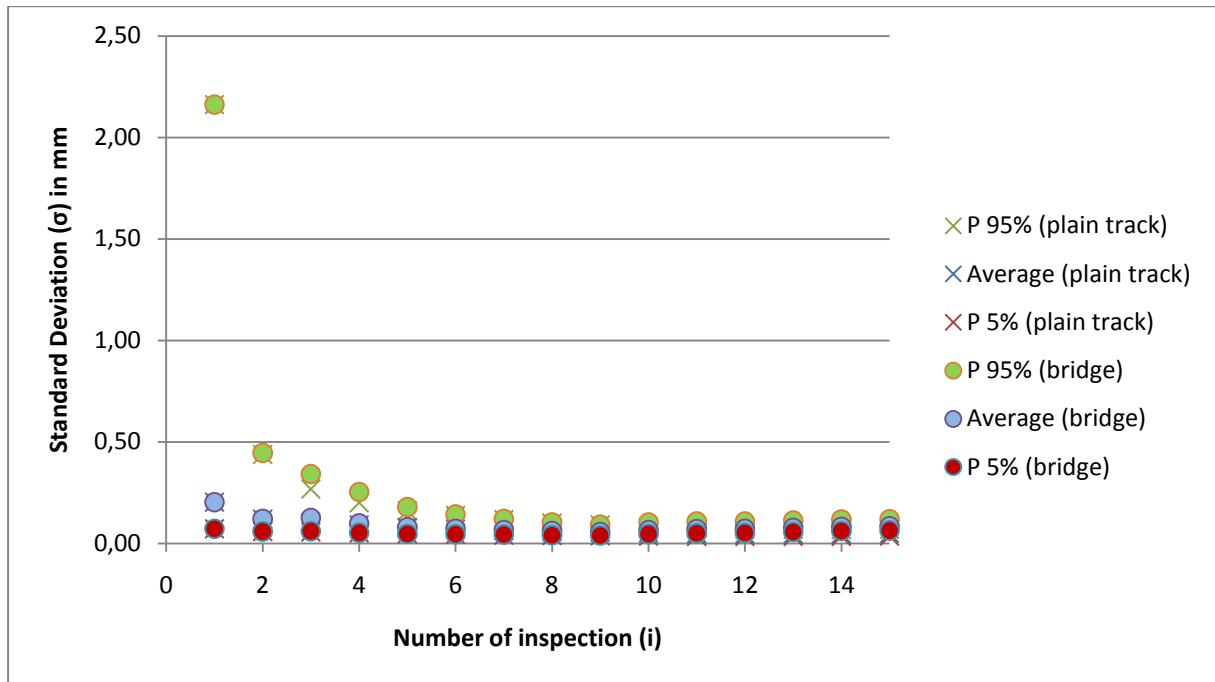


Figure 3 - Evolution of uncertainty associated with the standard deviation ( $\sigma$ ) in mm as more inspection data is available for two track sections belonging to different groups: plain track (PK 266,4-266,6 VA) and bridge (PK 268,6-268,8 VA).

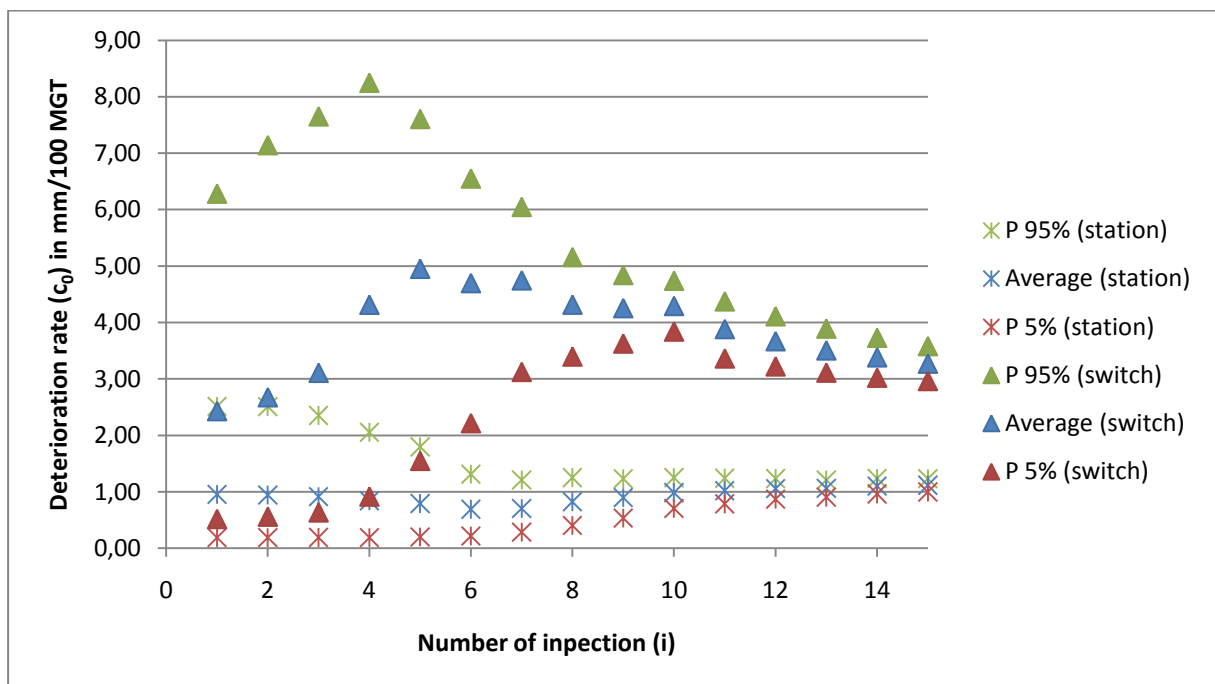


Figure 4 - Evolution of uncertainty associated with deterioration rate ( $c_0$ ) in mm/100 MGT as more inspection data is available for two track sections belonging to different groups: station (PK 277,6-277,8 VA) and switch (PK 287,0-287,2 VA).

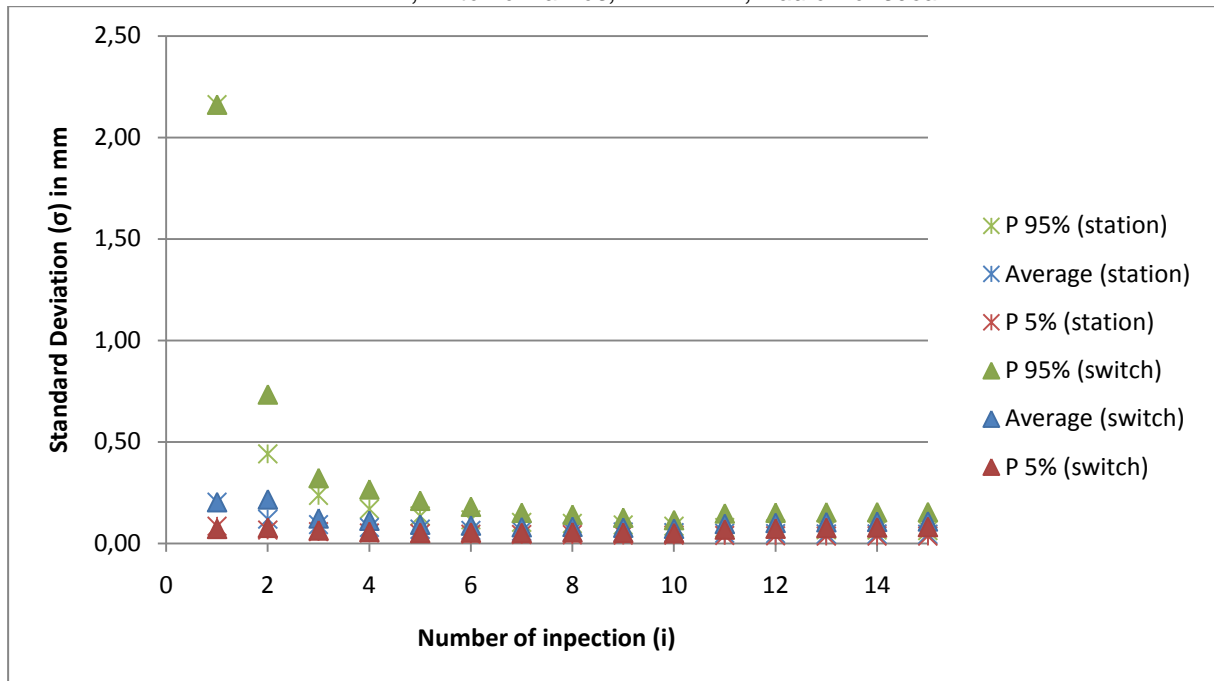


Figure 5 - Evolution of uncertainty associated with the standard deviation ( $\sigma$ ) in mm as more inspection data is available for two track sections belonging to different groups: station (PK 277,6-277,8 VA) and switch (PK 287,0-287,2 VA).

As figures 2 and 4 show, the 90% Credibility Interval associated with the deterioration rate ( $c_0$ ) predominantly reduces its length as more inspections are made, showing that the uncertainty associated with them is significantly reduced. Another important aspect concerns to the fact that the average values seem to stabilize, especially for the cases of track sections belonging to plain track and stations groups. Concerning the evolution of the uncertainty associated with the standard deviation ( $\sigma$ ), the four track sections exhibit a similar significant reduction after the second inspection. In fact, the standard deviation seem to converge to a small value ( $< 0,1 \text{ mm}$ ) as more inspections are made, which gives a certain confidence on how the model fits the data.

Therefore, in general the uncertainty associated with the parameters of interest reduces significantly as more inspection data is available and as operation starts.

#### 4) Assessing uncertainty for the second and remaining tamping cycles

To assess the evolution of uncertainty in rail track degradation for the remaining life-cycle period, information on the maintenance strategies should be given. Alternatively, a budget restriction or any optimization criteria should be established in order to assess the remaining life-cycle uncertainty.

Moreover, more information on the track quality improvement after tamping operations is needed to apply to this model. The exploratory analysis done so far with the inspection and maintenance data from the Portuguese lines have not reached yet a reliable way to simulate this improvement and try to correlate it with infrastructure data or deterioration rates.



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Moreover, literature usually do not cover this aspect in great detail, in fact only some references have been found (Lichtberger, 2005) and (Veit, 2007).

Nevertheless, some references suggest that deterioration rates remain a constant parameter for a track section regardless the quality achieved by the tamping machines throughout the infrastructure life-cycle (Esveld, 2001 on the work carried out by the Office for Research and Experiments (ORE) committee D 161). This suggestion makes the authors believe that uncertainty associated with deterioration rates and with the standard deviation may increase a little bit when tamping actions occur, but their effect will diminish more rapidly than in the first tamping cycle.

## **INTEREST OF THE METHODOLOGY TOWARDS ENHANCED RAIL INFRASTRUCTURE MANAGEMENT AND CONTRACTS**

As it was shown the uncertainty associated with deterioration rates for each 200-m track section is extremely large at the design stage, but it reduces significantly as operation starts and more inspection data is collected. A significant impact that these results suggest is the fact that medium and long-term planning of maintenance and renewal actions should be promoted by the IM organizations after this 'warm up' period, detailing predicted logistic and availability impacts in them, as less uncertainty in degradation would contribute to reduce the uncertainty associated with these aspects. Therefore, the information on degradation obtained with this 'warm up' period should be regarded as a major contribution to the definition of a cost-effective maintenance and renewal strategy based on the continuous learning process obtained from inspection data.

Therefore, the authors believe that a 'warm up' period would contribute tremendously to increase investors' confidence. This 'warm up' period would consist in a 2 to 3 years period after which maintenance costs concerning tamping actions would be re-estimated based on the assessment of the uncertainty associated with degradation parameters as it was conducted above.

In the context of Public and Private Partnerships in railway, the allocation of maintenance costs and respective risk is currently undertaken by the private sector. As a result, many investors tend to assume a very pessimistic point of view in deterioration rates, leading to overestimate maintenance costs in the procurement stage, so that their future risk exposures are reduced. This attitude often results into unnecessary increase of public financing. Therefore, the authors believe that this proposed 'warm up' period would permit to increase considerably investors' confidence, while transferring for that period degradation risks to the public entity or other third entity (e.g. risk seekers). Real options would be a proper financing tool to model this approach. Note that after that 'warm up' period, life-cycle maintenance costs would be predicted based on the posterior probability distributions assessed before and the maintenance cost risks would return to the private sector. In fact, this approach could be viewed as a real option that other third entity (risk seeker investors) would prefer to take

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after each large scale renewal cycle (where the impact of the new subsystem is still uncertain).

Therefore, not only IM planned maintenance and related impacts (availability and its uncertainty) will benefit from this assessment, but also investors' confidence in rail investment if we introduce the idea to negotiate life-cycle maintenance costs not at the design stage but after some operation period (e.g. 8-12 inspections (2-3 years)).

## **CONCLUSIONS**

In this paper, a Bayesian model for rail track geometry degradation is put forward. It allows to assess the evolution of uncertainty associated with degradation parameters throughout the rail track life-cycle. Prior probability distributions were fitted to track geometry degradation, showing that Log-Normal distribution is the most suitable distribution to model deterioration parameters. The results show that at the design stage, the uncertainty associated with maintenance cycles is considerably high, but it reduces significantly as more inspection data is collected. These results suggest the possible interest of the methodology towards enhanced infrastructure contracts, introducing some impacts in the negotiation and planning of maintenance costs. Maintenance costs related to track subsystems degradation would no longer be assessed at the design stage based only on the prior probability distributions of the degradation model parameters, but renegotiated instead after a 'warm-up' period of operation based on their posterior probability distributions.

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