SEAT CAPACITY AND HYPERPATH CHOICE ON-BOARD: ALIGHT OR REMAIN SEATED?

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ABSTRACT

The rationale for the definition of a set of paths referred to as "hyperpath" is that some uncertainty (for example waiting time) means that the choice of a specific option is not being pre-determined. Rather the choice process is assumed to occur on two levels. At a "strategic level" a passenger defines a set of paths any of which might be potentially optimal, and at "a tactic level" a specific path out of these is chosen depending on events occurring en-route. This paper discusses that the same concept of hyperpath might also be true for passengers already on on-board a transit vehicle. The uncertainty in this case arises from the seat availability. The paper extends an earlier version of the frequency-based transit assignment model with seat capacity by Schmöcker et al (2009) to describe that passengers choose their alighting point depending on whether they have obtained a seat or not. This behaviour is compared to the "deterministic" case where passengers have decided their alighting point independent of whether they might obtain a seat or not during the journey.

Keywords: Hyperpath, En-Route Decision, Sitting Capacity, Optimism

INTRODUCTION

Generally, "hyperpath" is an expression borrowed from graph theory, which in transport refers to a set of links that might be attractive from a node. More specifically, in the transit literature hyperpaths are usually utilised to describe the common lines problem at bus stops or stations. This means that to passengers often a number of lines are attractive, depending on, for example, which service arrives next at the bus stop. For instance, if the preferred bus has just left the stop when the passenger arrives, it might be faster for him to choose a slightly longer route rather than to wait for the next arrival of the preferred line.

The rationale for the definition of a hyperpath is that some uncertainty (in the above example waiting time) means that the choice of a specific option is not being pre-determined. Rather the choice process is assumed to occur on two levels. At a "strategic level" a passenger defines a set of paths any of which might be potentially optimal, and at "a tactic level" a specific path out of these is chosen depending on events occurring en-route.

This paper discusses that the same concept of hyperpath might also be true for passengers already on on-board a transit vehicle. The uncertainty in this case arises from the seat availability which is known to influence passengers' travel cost perception. London Underground for example found that passengers perceive the cost for standing in a crush-loaden train 2.7 the value of the actual travel time (LUL, 1988). The U.K.'s Passenger Demand Forecasting Handbook 2001 then reports that this value can even increase significantly depending on length of journey as well as journey purpose (ATOC, 2001). Similarly, in Tokyo it can be frequently observed that some passengers attach significant value to being able to sit. At line terminals passengers frequently do not board the first departing train but rather wait for the following service if they are certain to be able to sit, especially if this service is already available to board.

It is therefore reasonable to assume that seat availability has an influence on passengers' route choice at the stop as well as when on-board and should hence be considered in assignment models. At the station passenger will aim to select less crowded lines and once they have obtained a seat on-board, passengers might transfer later than passengers who are standing. Previous approaches, whether frequency- or schedule-based, have mainly focused on line capacities. In this case obviously on-board choice can be ignored.

There has only been a limited amount of research on modelling the effect of in-vehicle congestion. The main complexity for this problem is the consideration of priority rules between sitting passengers, passengers on-board and newly boarding passengers. Simple crowding cost functions where the cost is depending on the number of passengers have their limitations. They do reflect the inconvenience among standing passengers with growing congestion but do not consider that sitting passengers will be only to a very limited degree affected by the line crowding.

Paper Structure

The following section firstly reviews the existing literature on frequency-based transit assignment considering congestion effects. Then a frequency-based transit assignment model that explicitly considers the likelihood of finding a seat is introduced. In this paper the focus is the explicit distinction of the "deterministic" case in which passengers have decided their alighting point at the time of boarding and the case where passengers choose their alighting point depending on whether they have obtained a seat or not. The network loading procedure is omitted for brevity as it has been described in an earlier version of this work presented in Schmöcker et al (2009). This is followed by a fixed point problem formulation of the resulting assignment problem. The problem is solved with a solution algorithm that embeds the method of successive averages. Finally, the approach is applied to the London network and conclusions and areas of further work as well as model applications are pointed out.

LITERATURE REVIEW

Congested transit assignment has recently attracted significant research attention. Within the group of frequency based transit assignment models the capacity constrained assignment problem has been addressed by several authors either by using various forms of an effective frequency model (see e.g. Cepeda et al (2006) for a recent paper which also summarises previous work starting from De Cea and Fernandez (1993)) or by using "fail-to-board probabilities" (Kurauchi et al, 2003; Schmöcker et al, 2008). Within the group of schedule-based assignment models the departure time as well as route choice under consideration of congestion have been addressed simultaneously. In particular Tian et al (2007) describe a schedule-based transit model that considers congestion effects including seat availability. They formulate an equilibrium model for a many-to-one network applicable for the morning commute into the city centre of large metropolitan areas. Reducing the model to a many-to-one network has the advantage that it avoids the problem of standing passengers being able to find a seat during the journey through alighting passengers. Using a schedule based model allows to model further explicitly the optimal departure time considering schedule delays and the generalised travel cost. The paper illustrates that in an equilibrium situation some long distance commuters will travel before and some will travel after the peak. Tian et al further illustrate that the spread in optimal departure times increases the longer the travel distance as the travel costs of standing gain in importance compared to the early or late arrival penalties.

Sumalee et al (2009) have developed a stochastic transit assignment model that explicitly considers the effect of seat availability on route choice as well as departure time choice. They consider priorities of on-board passengers over newly boarding passengers and further assume that a) passengers who are travelling further and b) passengers who have stood for a longer time have a higher motivation in chasing any free seats. Whereas the first assumption is intuitive the second assumption might be challenged as standing longer might mean that these passengers in fact have a lower motivation to find a seat. The assumption further introduces another complexity as "the past" has to be considered in modelling travellers' behaviour at each decision point. The seat allocation is solved by a simulation type approach at each station allocating firstly all standing and then, in case seats are still vacant, all newly boarding passengers to seats. The model described in Sumalee et al (2009) is tested on a small example network, illustrating the effects of limited seat numbers on the service performance as well as the overall costs for passengers. Questions remain however, whether this model is feasible to be used for transit planning in large scale networks where an equilibrium finding approach that includes simulation might turn out to be too computationally expensive.

Schmöcker et al (2009) propose a simpler model to be used within frequency based assignment for transportation planning when on-board congestion is an issue. The approach considers the priority rules of already seated passengers as well as the priority of standing passengers over newly boarding passengers. It further assumes that all passengers on the platform have the same chance of finding a seat when boarding ("mingling" rather than FIFO). Similarly, it is assumed that passengers on-board have the same motivation finding a seat irrespective of their remaining travel time on this line. The perceived cost of standing is however thought to be distance dependent and hence influences route choice. As part of the perceived cost it is further considered that at each stop passengers might have the chance to find a seat through alighting passengers. An approach with similar assumptions has been developed by Leurent (2008). In their model, however, the loading along a line is based on route sections which can lead to complex network descriptions for long lines as there are arcs from each possible boarding point of the line to each possible alighting point. Each combination of boarding and alighting nodes is transformed into links for all possible stand and sit combinations. In his approach the mean cost for each route section is derived which is then used to find the optimal hyperpath. Leurent and Liu (2009) apply the approach to the Paris network and provide further evidence that considering seat availability can indeed have a significant effect on line loadings and overall passenger cost. They found that line loads change by up to 30% compared to a base case not considering seat availability.

NETWORK DESCRIPTION AND NOTATION

In contrast to the Leurent (2008) approach the model proposed here develops a more compact network description (at least when there are lines with a large number of stops) based on line sections rather than route sections. The model hence also does not rely on the mean or variance in costs over route sections. Instead "fail-to-sit" probabilities are introduced in order to be able to consider the influence of seat availability on passengers route choice in large scale transportation networks. The main idea of the approach is illustrated in Figure 1.

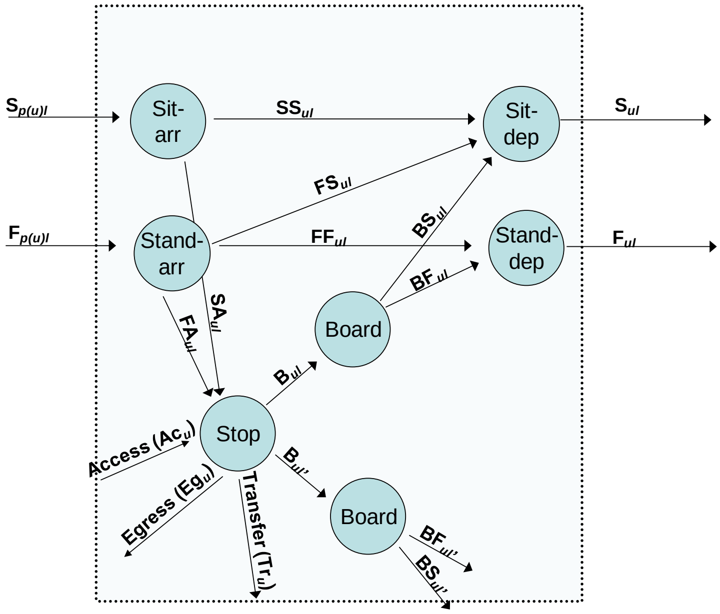

Figure 1: Network representation of a single platform

The stop node (Stop) represents the bus stop or platform at which passengers wait for the service to arrive. From each stop node passengers might be able to board several services. It is assumed that the common lines problem applies so that the hyperpath minimising passengers expected travel cost might contain non-zero boarding probabilities for several boarding nodes. Besides the stop node in total 5 nodes and 8 arcs are associated with each line. The names of the nodes are self-explanatory. All nodes except those $\in$ Stop are platform and line specific, i.e. nodes $\in$ Sit-Arr, Sit-Dep, Stand-Arr, Stand-Dep, Board can be uniquely identified by their platform u and their line l. For simplicity Figure 1 only illustrates all nodes and links associated with a single line at one platform.

The S within the arc names stands for success or sitting whereas the F is used to describe failing (to sit). The eight arcs types are hence, SS: “keep sitting” (success + success), FF: “keep standing”, FS: “previously standing getting a seat”, BS: “board and sit”, BF: “Board and stand” and SA: “Sit and alight”, FA: “Stand and alight” and B: “Boarding”. Besides these line specific arcs at each stop node there might be a number of further walking arcs for passengers transferring to other platforms Tr as well as access arcs Ac and egress arcs Eg for passengers starting or ending their journey at this station.The second stop node in Figure 1 indicates that a second line serves the same platform. The passenger’s choice is therefore to decide which line(s) to include in his hyperpath from each stop node. Alternatively the passenger might decide to take a walking link.

Note that the network description with the “dual sitting and standing line” allows for a second choice not much discussed in the literature but which is a focus of this paper. Depending on whether the passenger has obtained a seat he might choose to stay on-board or alight. This choice behaviour might be observed among commuters who can transfer at a number of stations. Depending on whether they obtained a seat they might decide to transfer at an earlier or later possibility. Alternatively one might assume that passengers determine their alighting point independent of whether they obtained a seat or not. It will be discussed in this paper that this assumption is mathematically more convenient as it more likely leads to a unique solution.

Flow independent variables:

O:	Set of origins (with $o \in$ O)
D:	Set of destinations (with $d \in$ D)
L:	Set of transit lines (with $l \in$ L and $l(a)$ denoting the line of arc a)
$seat_l$:	Number of seats on line l
f_l:	Frequency of transit line l
U_l:	Set of platforms served by line l (with $u \in U_l$ and $p(u)l$ denoting the platform of the previous station of line l and u(a) denoting the platform of the head node of arc a)
I:	Set of nodes (with $i \in I$) with subsets *Sit-Arr, Sit-Dep, Stand-Arr, Stand-Dep, Board, Stop* $\in I$ nodes as illustrated in Fig.1
A:	Set of arcs (with $a \in$A and A_{ih} denoting the forward star of node i that are included in hyperpath h) with subsets *SS, FS, FF, BS, BF, SA, FA, B, S, F, Ac, Eg, Tr* $\in A$ as illustrated in Fig. 1
c_a:	Cost of travelling on arc a connecting nodes i and j
SP:	Standing penalty, indicates the increase in perceived cost if standing compared to being seated.
$Out(a)$:	The tail node of arc a
$In(a)$:	The head node of arc a
y_{od}:	Demand from origin o to destination d
H_{od}:	Set of all feasible hyperpaths between OD pair od (with $h \in H_{od}$)
P_h:	Set of elementary paths within hyperpath h (with $p \in P_h$)
g_h:	Cost of travelling hyperpath h from origin o to destination d
w_{ih}:	Expected waiting time at node i when travelling on hyperpath h

Flow dependent variables:

x_a:	Passenger flow on arc a (with x_a^{od} denoting the OD specific arc flow)
v_i:	Passenger flow vector traversing node i (v_{id}: and destined for d)
q_i^{BS}:	Fail probability to get a seat for newly boarding passengers at node $i \in$ *Board*

q_i^{FS} : Fail probability to get a seat for on-board standing arriving at node $i \in$ *Stand-Arr*

sp^a_i: Vacant seats at node *i* after passenger alighted (before new passengers boarding)

sp^r_i: Remaining vacant seats at node *i* (after all passengers boarded)

z_h: Flow on hyperpath *h* within the set of used hyperpaths to destination *d*

α_{ah}: Probability of using arc *a* when travelling on hyperpath *h*

β_{ih}: Probability of traversing node *i* when travelling on hyperpath *h*

π_{ah}: Probability of choosing arc *a* when traveller is at *Out*(*a*) and travelling on hyperpath *h*. Similarly, π_{ad} denotes the probability of choosing arc *a* from *Out*(*a*) and travelling to destination *d*. Let $\boldsymbol{\pi}_h$ and $\boldsymbol{\Pi}_d$ denote the corresponding arc and node transition matrices.

Note that the above network description ensures that the Markov property holds. This means that arc split probabilities only depend on the traveler's current node position which can be expressed as πh = πd for the shortest hyperpath h from all origins o with the same destination d. This important property is utilised in the following to establish priority rules and for route choice.

PRIORITY RULES

For simplicity the model used here assumes that all passengers wishing to board a service are able to do so, meaning that the capacity of standing arcs is not limited. Therefore, once a service that is within the set of attractive lines has arrived the passenger only faces uncertainty whether it is possible to find a seat. It is assumed that all passengers prefer to sit. This is expressed as the path split between successful transferring to the *Sit-dep* node or (unsuccessfully) transferring to the *Stand-dep* node. The passengers who are already on board are assumed to have priority over the newly boarding passengers in two ways: Firstly, passengers arriving at a station sitting (*Sit-arr*) are guaranteed a seat, so that they either alight or remain sitting. Secondly, passengers arriving standing (*Stand-arr*) who do not alight are further assumed to have priority over the passengers newly boarding, i.e. these passengers have a prior chance to occupy any seat that might become vacant through alighting passengers. These priority rules can be expressed as follows:

$$seat_l \geq x_{S_{ul}} \quad , \ \forall u \in U_l,\ l \in L \tag{1}$$

$$x_{S_{ul}} = x_{SS_{ul}} + x_{FS_{ul}} + x_{BS_{ul}} \quad , \ \forall u \in U_l,\ l \in L \tag{2}$$

$$\text{with} \quad x_{FS_{ul}} = \left(1 - q_{FS_{ul}}\right)\left(x_{F_{p(u)l}} - x_{FA_{ul}}\right) \tag{3}$$

$$\text{and} \quad x_{BS_{ul}} = \left(1 - q_{BS_{ul}}\right)\left(x_{B_{ul}}\right) \tag{4}$$

$$x_{F_{ul}} = x_{FF_{ul}} + x_{BF_{ul}} = q_{FS_{ul}}\left(x_{F_{p(u)l}} - x_{FA}\right) + q_{BS_{ul}} x_{B_{ul}} \quad , \ \forall u \in U_l,\ l \in L \tag{5}$$

with $x_{FF_{ul}} = q_{FS_{ul}}\left(x_{F_{p(u)l}} - x_{FA}\right) \quad , \forall u \in U_l,\ l \in L$ (6)

and $x_{BF_{ul}} = q_{BS_{ul}} x_{B_{ul}} \quad , \forall u \in U_l,\ l \in L$ (7)

$$q_{ul}^{FS} := \begin{cases} 0 & \text{if } x_{F_{p(u)l}} = x_{FA_{ul}} \\ 1 - \min\left(\dfrac{\left(seat_l - x_{SS_{ul}}\right)}{\left(x_{F_{p(u)l}} - x_{FA_{ul}}\right)}, 1\right) & \text{otherwise} \end{cases} \quad , \forall u \in U_l, l \in L \tag{8}$$

$$q_{ul}^{BS} := \begin{cases} 0 & \text{if } x_{B_{ul}} = 0 \\ 1 - \max\left(0, \min\left(\dfrac{\left(seat_l - x_{SS_{ul}} - x_{FS_{ul}}\right)}{x_{B_{ul}}}, 1\right)\right) & \text{otherwise} \end{cases} \quad , \forall u \in U_l, l \in L \tag{9}$$

Where x_a denotes the flow on arc *a*.. Eq. (1) ensures that the overall seat availability of the service is not exceeded. Eq. (2) and (3) describe the flows of the passengers leaving the station sitting and standing respectively. To ensure the seat capacity constraints are kept the probabilities describing the chance of not getting a seat for those who were already on board but are standing and those newly attempting to board – $\mathbf{q}^{\mathbf{FS}}$ and $\mathbf{q}^{\mathbf{BS}}$ respectively – need to be adjusted. These adjustments are done with Eq. (4) and (5), which imply that

- q_{ul}^{FS} is non-zero or all passengers who boarded line *l* before it arrives at station *u* have found a seat already which means also that there might be seats available for passengers boarding the service at this station.
- q_{ul}^{BS} is non-zero or all passengers boarding the service anew at this stop have found a seat which means that there might be still empty spaces after the departure of the service from current station *u*.
- Fail probabilities are by default set to zero when there is no flow, i.e. when all passengers alight q_{ul}^{FS} is zero, or, when no one is boarding then q_{ul}^{BS} is zero.

Note that from the priority rules two useful propositions can be established.

<u>Proposition 1</u>: $q_i^{FS} > 0 \Rightarrow q_i^{BS} = 1$

<u>Proof:</u> The proof of this lemma follows directly from the priority rules. If not all passengers already on-board can obtain a seat, none of the passengers newly attempting to board will be able to obtain a seat.

<u>Proposition 2</u>: $0 < q_i^{BS} < 1 \Rightarrow q_i^{FS} = 0$

<u>Proof:</u> This proof follows in the same way as Lemma 1. If at least some passengers newly boarding can obtain a seat, all of the passengers with higher priority must have obtained a seat.

ROUTE CHOICE

Generalised Cost function

The waiting time at a node is given by (10) which assumes an exponential distribution of the vehicle arriving times with the mean being their nominal frequency, but alternative assumptions are also possible as discussed in Nökel and Wekeck (2009). A_{ih} denotes the hyperpath specific set of arcs included among the outgoing arcs from node i.

$$w_{ih} = \frac{1}{\sum_{a \in A_{ih}} f_a} \quad \forall i \in S_h \qquad (10)$$

Following assumption (10) and Spiess and Florian (1989) the split between lines that are part of the optimal hyperpath is given by (11).

$$\pi_{ah} = \frac{f_a}{\sum_{a \in A_{ih}} f_a} \quad \forall a \in B_h \qquad (11)$$

For boarding nodes as well as all other nodes flow conversation is observed by (12). In particular for Stand-Arr nodes this means that the sum of the transition probabilities for alighting (FA), keep standing (FF) and sit-down (FS) arcs must add up to 1.

$$\sum_{a \in A_{ih}} \pi_{ah} = 1 \qquad (12)$$

As in Nguyen and Pallentino (1988) let the probability of choosing any particular path p of a hyperpath h, λ_p, be denoted as

$$\lambda_p = \prod_{a \in A_h} \pi_{ah}^{\delta_{ap}}, \quad \forall p \in P_h. \qquad (13)$$

with δ_{ap} equal to 1 if arc a is an element of path p and 0 otherwise. It follows therefore that

$$\sum_{p \in P_h} \lambda_p = 1 \qquad (14)$$

Further β_{ih} is defined as the "probability of traversing node i when travelling hyperpath h" and ε_{ip} is equal to 1 if node i is an element of path p and 0 otherwise so that

$$\beta_{ih} = \sum_{p \in P_h} \varepsilon_{ip} \lambda_p, \quad \forall i \in I_h \qquad (15)$$

α_{ah} is defined as the probability of using arc a when travelling hyperpath h, so that

$$\alpha_{ah} = \sum_{p \in P_h} \delta_{ap} \lambda_p \quad , \quad \forall a \in A_h \tag{16}$$

Using the definitions for β_{ih} and α_{ah}, the notation of the generalised cost for travelling on hyperpath g_h can be described as:

$$g_h = \sum_{i \in Stop_h} \beta_{ih} w_{ih} + \sum_{a \in S_h} \alpha_{ah} c_a + SP \sum_{a \in F_h} \alpha_{ah} c_a \tag{17}$$

Costs occur at the stop nodes Stoph that are part of the hyperpath as well as on sitting line arcs S_h and standing line arcs F_h. In (10) w_{ih} is hence the expected waiting time at boarding node *i* and ca the travel time on sitting and standing links. Standing is further penalised by the factor SP. A standing penalty $SP > 1$ means that the probability of getting a seat is one factor in passengers' route choice and hence needs to be reflected in the search for the optimal hyperpath. Note that $SP < 1$ is unreasonable as it would imply that people are more willing to stand than to get a seat and that SP = 1 indicates the same perceived cost for travelling standing or sitting which implies that finding a seat is not a factor for route choice. β_{ih} and α_{ah} represent the probabilities of traversing node *i* and link a respectively when travelling on hyperpath *h*.

Kurauchi et al (2003) show that for a cost function with the same first two cost elements as in (17) plus a nonlinear third term depending on capacity constraints the node costs are separable as in (18). Since (17) is identical except that the third term is replaced by a linear cost function similar to the second term the proof can be easily repeated and is omitted for brevity. The applicability of the Bellmann principle is also the basis for the hyperpath search described in the following.

$$g^*_{h^i} = \begin{cases} 0 & if \ = d, \\ c_a + g^*_{h^j} & if\ i \in \{\text{Sit} - \text{Dep}\} \\ SP \cdot c_a + g^*_{h^j} & if\ i \in \{\text{Stand} - \text{Dep}\} \\ \min\limits_{A^* \subseteq A_i} \dfrac{1 + \sum\limits_{a \in A^*} f_a g^*_{h^j}}{\sum\limits_{a \in A^*} f_a} & if\ i \in \{\text{Board}\} \\ g^*_{h^j} & \text{otherwise} \end{cases} \tag{18}$$

Finding the optimal path set

The following algorithm determines the optimal hyperpath for a passenger travelling to destination *d* from all origins considering fail probabilities $\mathbf{q}^{\mathbf{FS}}$ and $\mathbf{q}^{\mathbf{BS}}$. The cost g^i hence denotes the cost of a node *i* to destination *d* and at termination the cost g^i is equivalent to $g_{h(id)}$ under current flow conditions.

The path costs are calculated taking into account the chance of having to stand initially as well as the probabilities to find a seat at subsequent stations. In other words, moving upstream from the destination, the node costs at *Board* as well as *Stand-Arr* node are updated taking into account the given fail probabilities. This search continues until the origin is reached. Finally, Step 5 is added which forces passengers to add the standing path to their set of paths if the parallel sitting arc is part of their hyperpath.

Algorithm 1: Hyperpath Set Determination
(Input: Network description, link costs, SP, $\mathbf{q}^{\mathbf{BS}}$, $\mathbf{q}^{\mathbf{FS}}$)

For each $d \exists o$ with $X^{od} > 0$

Step 1 (Initialisation)

Set: $M_1 := A$, $M_2 := \varnothing$, $M_3 := \varnothing$;
$g^i := \infty$, $\forall i \in \{I - d\}$, $g^d := 0$;

Step 2 (Finding a node of minimum cost from the destination)
Find a^* **such that** $a^* = \arg\min_{a \in M_1} \left\{ c_a + g^{In(a^*)} \right\}$;
Set $M_1 := M_1 - \{a^*\}$, $M_2 := M_2 + \{a^*\}$;

Step 3 (Updating node labels)
if $Out(a^*) \in Stop$ **and** $In(a^*) \in Board$ **then**
Find a set of arcs, **such that**

$$g^{Out(a)} := \min_{A^* \subseteq \{A_i \cap M_2\}} \frac{1 + \sum_{a \in A^*} f_{l(a)} g^{In(a)}}{\sum_{a \in A^*} f_{l(a)}}; \quad \forall a \in \{A_i \cap M_2\},$$

if $a \notin A^*$ **then** **Set** $M_2 := M_2 - \{a\}$, $M_3 := M_3 + \{a\}$;

else if $Out(a^*) \in Board$
if $g^{Out(a^*)} \geq g^{Stand-dep_{u(a^*)}} \cdot q^{BS}_{Out(a^*)} + \left(1 - q^{BS}_{Out(a^*)}\right) \cdot g^{Sit-dep_{u(a^*)}}$ **then**

$$g^{Out(a^*)} := g^{Stand-dep_{u(a^*)}} \cdot q^{BS}_{Out(a^*)} + \left(1 - q^{BS}_{Out(a^*)}\right) \cdot g^{Sit-dep_{u(a^*)}}$$

else
Set $M_2 := M_2 - \{a^*\}$, $M_3 := M_3 + \{a^*\}$;
else if $Out(a^*) \in Stand - arr$ and $In(a^*) \notin Stop$
if $g^{Out(a^*)} \geq g^{Stand-dep_{u(a^*)}} \cdot q^{FS}_{Out(a^*)} + \left(1 - q^{FS}_{Out(a^*)}\right) \cdot g^{Sit-dep_{u(a^*)}}$ **then**

$$g^{Out(a^*)} := g^{Stand-dep_{u(a^*)}} \cdot q^{FS}_{Out(a^*)} + \left(1 - q^{FS}_{Out(a^*)}\right) \cdot g^{Sit-dep_{u(a^*)}}$$

else
Set $M_2 := M_2 - \{a^*\}$, $M_3 := M_3 + \{a^*\}$;

else

if $g^{Out(a^*)} \geq c_{a^*} + g^{In(a^*)}$ **then**

$g^{Out(a^*)} := c_{a^*} + g^{In(a^*)}$

else

Set $M_2:=M_2-\{a^*\}$, $M_3:=M_3+\{a^*\}$;

Step 4 (Iteration, Termination of Arc Search Loop)

Repeat (2) to (4) until $M_1=\varnothing$

Step 5 (Add *BF* and *FF* arcs to hyperpath)

$\forall a \in M_3 \cap BF$

Find $a' \in$ BS with platform $u(a') = u(a)$ and line $l(a') = l(a)$

if $a' \in M_2$ then

Set $M_2:=M_2+\{a\}$, $M_3:=M_3-\{a^*\}$;

$\forall a \in M_3 \cap FF$

Find $a' \in$ FS with platform $u(a') = u(a)$ and line $l(a') = l(a)$

if $a' \in M_2$ then

Set $M_2:=M_2+\{a\}$, $M_3:=M_3-\{a^*\}$;

Transition Probabilities

Once the set of optimal links has been determined, the optimal hyperpath for each OD pair can be determined. The split among paths is defined in (11) and the split among the sitting and standing lines considers the fail probabilities in order to observe the priority rules.

Algorithm 2: **Arc Split Determination** (Input: Line frequencies, M_2, $\mathbf{q}^{\mathbf{BS}}$, $\mathbf{q}^{\mathbf{FS}}$)

For each $d \exists o$ with $y_{od} > 0$

$\forall a \notin M_2$ **set** $\pi_a = 0$,

$\forall a \in M_2$,

if $Out(a) \in Stop$ **then**

$$\pi_{ah} = f_{l(a)} \Big/ \sum_{a \in A_1 \cap M_2} f_{l(a)}$$

else if $Out(a) \in Board$ **then**

$$\pi_{ah} = \begin{cases} 1 - q^{BS}_{Out(a)} & \text{if } In(a) \in Sit - dep \\ q_{Out(a)} & \text{otherwise} \end{cases}$$

else if $Out(a) \in Stand - Arr$ **then**

$$\pi_{ah} = \begin{cases} 1 - q^{FS}_{Out(a)} & \text{if } In(a) \in Sit - dep \\ q_{Out(a)} & \text{otherwise} \end{cases}$$

else

$\pi_{ah} = 1$;

Note that for a single hyperpath at arrival nodes either the alighting links or the remain on board links (*SS* arc from Sit-Arr and {*FS*, *FF*} arcs from Stand-Arr) are attractive. In the equilibrium solution presented in the following, however, several hyperpaths might be attractive, so that the split between alighting and remaining on board is not binary anymore. Further, if the passenger's choice of transfer point is assumed to possibly depend on having obtained a seat or not, this means that a standing passenger possibly prefers to alight earlier whereas a sitting passenger prefers to alight later. To assume the opposite, i.e. that a traveler decides his alighting point at the time of boarding requires that (19) is fulfilled. In the hyperpath search this "predetermined alighting" might be introduced by the adjustment to the arc split probabilities described in Algorithm 3. This guarantees that both standing and sitting passengers alight with a probability ζ. This probability might be interpreted as conditional pessimism or risk-aversion, in case the shortest route differs between sitting and standing passengers. The pessimistic passenger with $\zeta = 1$ had decided to alight from this station when boarding (following the shortest hyperpath from the Stand-Arr node), whereas the optimistic passenger with $\zeta = 0$ had decided to remain on board (following the shortest hyperpath from the *Sit-Arr* node).

Ensuring (19) through Algorithm 3 allows establishing Proposition 3. The following section will then utilise this proposition to establish proofs on the convergence.

$$\pi_{FA_{ul}h} = \pi_{SA_{ul}h} \qquad \forall u \in U_l,\ l \in L,\ h \in H \qquad (19)$$

<u>Algorithm 3:</u> Pre-determined alighting adjustment (Input: π_{FA}, π_{SA}, π_{SS}, ζ)

$$\begin{aligned}
&\forall u \in U_l,\ l \in L,\ h \in H \\
&\quad \text{if } \pi_{FA_{ul}h} = 1 \text{ and } \pi_{SA_{ul}h} = 0 \\
&\qquad \pi_{FA_{ul}h} \leftarrow \zeta \\
&\qquad \pi_{SA_{ul}h} \leftarrow \zeta \\
&\qquad \pi_{SS_{ul}h} \leftarrow 1 - \zeta
\end{aligned}$$

<u>Proposition 3</u>: If (19) is fulfilled, then, for a given set of hyperpaths, the demand for a line at each station is independent of the fail probabilities.

<u>Proof:</u> The boarding demand B_{ul} is determined by the arc split at stop nodes which are fixed according to (11) and independent of fail probabilities. Further, the incoming arc flows at stop nodes from walking arcs are determined by the transition probabilities in Algorithm 2 and independent of fail probabilities. Finally, Eq. (19) implies that the demand at stop nodes from alighting passengers is independent of the proportion of passengers that are sitting or standing. In conclusion, the sum of the incoming (and outgoing) demand at stop nodes and hence for lines is independent of $\mathbf{q}^{\mathbf{FS}}$ and $\mathbf{q}^{\mathbf{BS}}$. Qed.

NETWORK LOADING AND DUE SOLUTION

The arc split probabilities can be converted into network flows by the simple method in Spiess and Florian (1989) where the demand is loaded from its origin in the order of decreasing link costs plus node potentials. Though the method is feasible it is computationally expensive as it requires a loop over all OD pairs. Kurauchi et al (2003) or Schmöcker et al (2008) utilise instead the Markov property of the network to assign demand, which is the approach also utilised in this research. As this approach has been published and is not focus of this paper, for brevity only the complementary slackness conditions that must apply at the equilibrium are given:

Network Equilibrium Conditions

Let H^* be defined as the set of optimal hyperpaths to destination d. Firstly, the user equilibrium implies that for all destinations H^* is empty or the cost difference $\boldsymbol{g}'$ between the used hyperpaths h and all other (unused) hyperpaths $h' \in H$ is zero (Wardrop principle). This can be expressed with (27) where the cost difference $\boldsymbol{g}'$ is defined as in (28) and z_h^* is the flow on the hyperpath h in the set of optimal hyperpaths H^*. The cost of the minimum cost hyperpath in H^*_{od} is denoted as $g_{od}^{\min}$. The problem becomes a fixed point problem as route costs depend on the failure probabilities, which themselves depend on the route flows which in turn depend on route costs. For simplicity only in the first line of (27) the functional dependencies of $\mathbf{q^{BS}}$ and $\mathbf{q^{FS}}$ are denoted.

$$z_h^* \cdot g_h'\left(\mathbf{z}^*, \mathbf{q}^{\mathbf{BS}^*}(\mathbf{x}^*, \mathbf{seats}), \mathbf{q}^{\mathbf{FS}^*}(\mathbf{x}^*, \mathbf{seats})\right) = 0, \quad \mathbf{g}'\left(\mathbf{z}^*, \mathbf{q}^{\mathbf{BS}^*}, \mathbf{q}^{\mathbf{FS}^*}\right) \geq \mathbf{0}, \ \forall h \in H_{od} \ \forall d \in D \ \forall o \in O \tag{20}$$

$$g_h'\left(\mathbf{z}^*, \mathbf{q}^{\mathbf{BS}^*}, \mathbf{q}^{\mathbf{FS}^*}\right) = g_h - g_{od}^{\min}, \ \forall h \in H_{od} \ \forall d \in D \ \forall o \in O \tag{21}$$

Secondly, for each boarding point of each line it must be true that either q_i^{FS} (the fail-to-sit probability for passengers already on-board) is zero or all seats must be filled before any newly boarding passengers can attempt to find a seat. Thirdly, q_i^{BS} (the fail-to-sit probability for newly boarding passengers) must be zero or there must be no spaces left when the vehicle is leaving the stop. These latter two complementary slackness conditions are expressed with (29) for stand-arrival nodes (*Stand-Arr*) and (31) for boarding nodes (*Board*); $\mathbf{sp^a}$ denotes the available seats after passengers have alighted and before new passengers are boarding and $\mathbf{sp^r}$ denotes the seats remaining empty after the service has left the platform.

$$\mathbf{q}^{\mathbf{FS}^*} \cdot \mathbf{sp^a}\left(\mathbf{z}^*\left(\mathbf{q}^{\mathbf{FS}^*}\right)\right) = 0, \mathbf{sp^a}\left(\mathbf{z}^*, \mathbf{q}^{\mathbf{FS}^*}\right) \geq \mathbf{0}, \ \forall i \ 0 \leq q_i^{FS^*} \leq 1 \tag{22}$$

with

$$sp_{\mathrm{ul}}^a = seats_l - x_{SS_{ul}} - x_{FS_{ul}}, \forall u \in U_l, l \in L \tag{23}$$

$$\mathbf{q}^{BS^*} \cdot \mathbf{sp}^{\mathbf{r}}\left(\mathbf{z}^*\left(\mathbf{q}^{BS^*}\right)\right) = 0\,,\ \mathbf{sp}^{\mathbf{r}}\left(\mathbf{z}^*, \mathbf{q}^{BS^*}\right) \geq \mathbf{0}\,,\ \ \forall i\ 0 \leq q_i^{BS^*} \leq 1 \qquad (24)$$

with

$$sp^r_{\mathrm{ul}} = seats_l - x_{sit_{ul}}\ , \forall u \in U_l, l \in L \qquad (25)$$

In summary, the assignment has to find ($\mathbf{z}^*$,$\mathbf{q}^{FS*}$,$\mathbf{q}^{BS*}$) such that (20), (22) and (24) are fulfilled. The existence of a solution to this fixed point problem is guaranteed as any demand exceeding the seat capacity can be simply assigned to the uncapacitated standing arcs with non-zero failures to sit. However, Leurent (2008) show that in general multiple solutions to the assignment problem might exist that fulfill (20). Uniqueness can only be guaranteed for specific cases. Firstly, for SP = 1 uniqueness is guaranteed as only a single hyperpath for each OD pair will be used. Secondly, for very large SP multiple solutions become less likely.
This is because with increasing SP the relative importance of standing in the generalized cost function increases. Therefore, only that split of hyperpaths on which passengers experience least standing and other hyperpath combinations are less likely to be attractive. Further, the following section shows that for each set of hyperpaths used a unique solution exists that fulfills (22) and (24).

Correction algorithm and solution uniqueness

To fulfill conditions (22) and (24) for a given set of hyperpaths for each OD pair an iterative procedure is developed (Algorithm 4). In Step 1 of the first iteration the transition probabilities of all arcs are obtained. In Step 1 of subsequent iterations it is sufficient to update only the transition probabilities for the arcs that have a boarding node (*Board*) or a Stand Arrival node (*Stand-Arr*) as outgoing nodes.

Algorithm 4: Correction of fail probabilities (Input: Network description, A_h, $\mathbf{q}^{\mathbf{BS}}$, $\mathbf{q}^{\mathbf{FS}}$)

Repeat Steps 1 to 3 until $\mathbf{q}^{\mathbf{FS}}$ and $\mathbf{q}^{\mathbf{BS}}$ cease to change

Step 1: Obtain transition probabilities (Alg. 2 and 3)	π_d (A_h, $\mathbf{q}^{\mathbf{FS}}$, $\mathbf{q}^{\mathbf{BS}}$)
Step 2: Network loading (as in Section 6.1)	$\mathbf{x}$ (π_d)
Step 3: Update fail probabilities (with Eqs. 8 and 9)	$\mathbf{q}^{\mathbf{BS}}(\mathbf{x},\mathbf{seats})$, $\mathbf{q}^{\mathbf{FS}}(\mathbf{x},\mathbf{seats})$

Proposition 4: The correction algorithm (Alg. 4) finds a unique solution for a given set of hyperpaths that fulfills the capacity constraints (22) and (24) in case (19) is fulfilled (pre-determined alighting).

Proof: Since A_h is assumed to be fixed a unique solution is obtained if $\mathbf{q}^{\mathbf{BS}}$ as well as $\mathbf{q}^{\mathbf{FS}}$ are unique. The uniqueness of $\mathbf{q}^{\mathbf{FS}}$ follows from Lemma 1 and that of $\mathbf{q}^{\mathbf{BS}}$ from Lemma 2.

Lemma 1: The fail to sit probabilities $\mathbf{q}^{\mathbf{FS}}$ are unique for a given set of hyperpaths if (19) is fulfilled.

Proof: Suppose the fail-to-sit probabilities for passengers already on board are not unique, i.e. suppose that there are two values $q_i^{FS}{}' > q_i^{FS} > 0$. This implies that $x'_{FF} = x_{FF} + \delta$ and $x'_{FS} = x_{FS} - \delta$. Further, $x'_{BS} = x_{BS} = 0$ according to Proposition 1 and hence also $x'_{FS} = x_{FS}$ since the boarding demand $x_{BS} + x_{FS}$ must be constant according to Proposition 3. Since $q_i^{FS}{}' > 0$ means that the capacity constraints must be fulfilled this implies further that (1) must be an equality. Therefore (2) implies that $x'_{SS} = x_{SS} - \delta$. This leads to a contradiction as Proposition 3 and (13) imply that the total demand for passengers continuing to travel on the line at a station is constant, i.e. $x'_{Sp(u)l} + x'_{Fp(u)l} = x_{Sp(u)l} + x_{Fp(u)l}$. However, $x'_{SS} + x'_{FF} + x'_{FS} = (x_{SS} + \delta) + (x_{FF} + \delta) + (x_{FS} - \delta) > x_{SS} + x_{FS} + x_{BS}$.
The proof for $q_i^{FS}{}' < q_i^{FS}$ can be derived in the same way leading to $x'_{SS} + x'_{FF} + x'_{FS} = (x_{SS} - \delta) + (x_{FF} - \delta) + (x_{FS} + \delta) < x_{SS} + x_{FS} + x_{BS}$. Finally $q_i^{FS} = 0$ for all stations implies that no capacity constraints are active, so that any $q_i^{FS}{}' > 0$ implies that passengers are standing though seats are available which contradicts the definition of the fail probabilities in (8). Qed.

Lemma 2: The fail to sit probabilities $\mathbf{q}^{\mathbf{BS}}$ are unique for a given set of hyperpaths if (19) is fulfilled.

Proof: Suppose the fail-to-sit probabilities for passengers newly boarding are not unique, i.e. suppose that there are two values $q_i^{BS}{}' > q_i^{BS}$. This implies that $x'_{BS} = x_{BS} - \delta$ and $x'_{BF} = x_{BF} + \delta$. Since less passengers obtain a seat this must mean that more passengers of the already on-board standing passengers are able to sit. This means that $q_i^{FS}{}' < q_i^{FS}$ which leads to a contradiction according to Lemma 1. Alternatively, if no on-board passengers are standing, this leads to a direct contradiction of the capacity constraints and the definition of the of the fail probabilities in (9). The proof for $q_i^{BS}{}' > q_i^{BS}$ can be derived in the same way leading to $q_i^{FS}{}' > q_i^{FS}$ and a contradiction according to Lemma 1. Qed.

Proposition 5: If uniqueness is guaranteed, the solution of the correction algorithm is found in a maximum of 2*n - 3* iterations where *n* is the number of stations of the longest line in the network.

Proof: In the first iteration of the correction algorithm the final value for q_i^{BS} of the starting node *i* of the line can be found as both *seats* and demand x_{B_i} are unique and x_{SS_i} as well as x_{FS_i} must be zero. This means that in Step 2 of the second iteration the final values for x_{S_i}, x_{F_i} as well as $x_{SS_{i+1}}$ are found where *i*+1 is the downstream station of *i*. Therefore in Step 3 of the second iteration the final value q_{i+1}^{FS} is found. Hence in the third iteration the final value for q_{i+1}^{BS} are found and so on until the end of the line. Since q_i^{BS}, q_n^{FS} and q_n^{BS} are all necessarily zero the unique solution can be found in a maximum of 2*n - 3* iterations. Qed.

Note that in practice the correction algorithm terminates much faster. Whenever the final value of a node at any location on the line is found in subsequent iteration also the fail-to-board probability of the downstream node can be finalized. Therefore, especially in networks with low level of congestion with large numbers of nodes having zero fail-probability, often final values of several line sections are found within the same iteration. Therefore, though in Step 2 network loading can be computationally expensive, the above correction algorithm is in most cases faster than sequential loading of each line similar to Leurent (2008) which would be an alternative way to ensure the capacity constraints.

Solution algorithm for a DUE Solution

From (22) follows that a gap function to assess the distance of a solution to an equilibrium solution is

$$G = \max_{\forall od} \max_{\forall h \in H_{od}} \left(g_h - g_{od}^{\min} \right) \qquad (26)$$

Assessing (26) is however not suitable for large scale problems as it would require calculation of costs on each hyperpath used. In analogy to Cepeda et al (2006) a feasible gap function based on destination specific arc flows can be derived. Cepeda et al`s proof is based on the Bellmann property of node cost separability as in (18). If at all nodes, costs on all outgoing arcs that are part of the set of optimal hyperpaths plus node costs of the downstream nodes are equal then equilibrium conditions are fulfilled. According to (17) and considering the network in Figure 1 this condition is trivial for all nodes in the model proposed here except for stop nodes, since either all outgoing arcs carry no cost or there is only one outgoing arc which hence must be part of all hyperpaths. This leaves a proof for the stop nodes at which the split between optimal hyperpaths occurs. Whereas in the Cepeda et al (2006) problem the frequencies of lines are dependent on line flows, frequencies are assumed constant here which reduces the complexity. In analogy to Section 3.2 in Cepeda et al (2006) following theorem can be posed and proven:

Eq. (27) must hold for all feasible flows and the inequality will become an equality at equilibrium.

$$\sum_{a \in Ai} g^{In(a)} x_a^d + \max_{a \in A_i} \frac{x_a^d}{f_{l(a)}} \geq g^i \sum_{a \in Ai} x_a^d \quad \forall Stop \qquad (27)$$

The general proof for flow dependent frequencies can now be simplified as follows: In (11) flows at stop nodes are split according to frequency, hence

$$x_a^d = \sum_{h \ni a} z_h \frac{f_{l(a)}}{\sum_{a' \in h} f_{l(a')}} \qquad (28)$$

Since the shortest arc will be an arc which belongs to every used optimal strategy, it follows for the second term in (27) that the maximum will be

$$\max_{a \in A_i} \frac{x_a^d}{f_{l(a)}} = \sum_{h \ni a} \frac{z_h}{\sum_{a' \in h} f_{l(a')}} \qquad (29)$$

And therefore (27) can be restated as

$$\sum_{h \in H_{id}} z_h \frac{g_{j(a)} f_{l(a)} + 1}{\sum_{a \in h} f_{l(a)}} \geq g^i \sum_{a \in A_i} x_a^d \qquad (30)$$

At equilibrium this becomes an equality since the left hand side of (30) is simply the sum over the node costs of the downstream nodes plus the waiting time (10) which completes the proof. From this theorem it follows directly that minimization of a gap function based on network flows leads to equilibrium conditions.

The cost of a hyperpath in (17) is determined by flow independent costs c_a, w_{ih} and SP as well as the flow dependent probability of finding a seat which are determined by $\mathbf{q}^{\mathbf{BS}}$ and $\mathbf{q}^{\mathbf{FS}}$. Minimising the gap function means hence to find the set of optimal $\mathbf{q}^{\mathbf{BS}}$ and $\mathbf{q}^{\mathbf{FS}}$ or, equivalently, arc split probabilities at boarding and stand-arrival nodes. For an iterative solution algorithm that ensures convergence to (at least locally optimal) equilibrium flows, the variation in fail probabilities between iterations k and k-1 can hence approximately describe the distance to an equilibrium solution. In summary, an even simpler approximate gap function can be described as:

$$G \approx \max\left(\max_{\forall i \in \text{Board}} \left| \left(q_i^{BS}\right)^k - \left(q_i^{BS}\right)^{k-1} \right|, \max_{\forall i \in \text{Stand-Arr}} \left| \left(q_i^{FS}\right)^k - \left(q_i^{FS}\right)^{k-1} \right| \right) \qquad (31)$$

A general procedure to solve fixed point problems is the Method of Successive Averages (MSA) applied in the following with the correction algorithm embedded to update the sitting probabilities according to the line flows. In each MSA the shortest hyperpath is found for each destination as discussed in Section 5. In the embedded correction algorithm (Algorithm 4) the averaged flows from this and previous iterations are then taken into account to find fail probabilities that fulfill the latter two constraints of the complementary slackness conditions (22) and (24) of the equilibrium problem. Only the split between hyperpaths found needs to be averaged which are determined by the arc split at boarding nodes as well alighting probabilities. Note that the correction algorithm itself does not include an MSA like averaging as the iteration counter m is only updated in the main MSA loop. The arc splits between sitting and standing arcs are updated in each iteration as in Algorithm 4. All other arcs are either part of the hyperpath or not, which corresponds to binary arc split probabilities.

The process is repeated until convergence. To explore a possible variety in local optima the below MSA should be repeated with different initial values for $\left(\mathbf{q}^{\mathbf{BS}}\right)^k$ and $\left(\mathbf{q}^{\mathbf{FS}}\right)^k$.

<u>Solution algorithm</u> (Input: Network description, SP, ζ)

<u>Initialisation</u>

Iteration counter $m = 0$

Fail probabilities Set q_i^{BS} $\forall i \in Board$ and q_i^{FS} $i \in Stand\text{-}Arr$ by random number between 0 and 1.

Arc splits $\boldsymbol{\pi}_d^m = \mathbf{0}\ \forall d$

Repeat until G < ε

Update iteration counter	$m \leftarrow m+1$
Hyperpath search (Alg. 1)	A_h ($\mathbf{q}^{BS}$, $\mathbf{q}^{FS}$, link costs $\mathbf{c}$, SP)
Obtain transition probabilities (Alg. 2)	$\boldsymbol{\pi}_d^{aux}$ (A_h, $\mathbf{q}^{BS}$, $\mathbf{q}^{FS}$)
Predetermine alighting prob. (Alg. 3)	Adjust $\boldsymbol{\pi}_d^{aux}$ if required
Add arcs to hyperpath	$\pi_{a,d}^m \leftarrow 1$ if $\pi_{a,d}^{aux}=1$
(sitting and standing arcs)	$\forall a \in S, F$
Average path splits	$\pi_{a,d}^m \leftarrow (\ \pi_{a,d}^{m-1} + (m\text{-}1) * \pi_{a,d}^{aux}) / m$
(boarding, alighting and walking arcs)	$\forall a \in B, FA, SA, SS, Ac, Tr, Eg$

Repeat until $\mathbf{q}^{FS}$ and $\mathbf{q}^{BS}$ cease to change (Embedded Alg. 4, Correction algorithm)

Update board and sit proportion	$\pi_{a,d}^m \leftarrow \pi_{a,d}^{aux}$
	$\forall a \in BS, BF$
Update stand and sit down proportion	$\pi_{a,d}^m \leftarrow \pi_{a,d}^{aux}\left(1-\pi_{FA_{u(a)l(a)},d}^m\right)$
	$\forall a \in FS, FF$
Network loading (as in 6.1)	$\mathbf{x}$ ($\boldsymbol{\pi}_d^m$)
Update fail probabilities (Eqs. 8 and 9)	$\mathbf{q}^{BS}$ (**x, seats**); $\mathbf{q}^{FS}$ (**x, seats**)

LONDON CASE STUDY

In order to illustrate the feasibility of the concept to be applied to larger networks the model has been applied to the inner part of the London Underground network as shown in Figure 4, which includes 74 stations and 11 lines. Different branches of District and Northern lines have been modeled as different lines; the resulting network consisting of 14 transit lines (corresponding to 28 oriented lines) and 297 transit arcs. With the network description as in Figure 1 this results in a total of 1751 nodes and 3233 arcs being created.

Figure 4: London Underground network modeled (taken and adjusted from Transport for London)

The model has been run for demand and supply of transit services in the morning peak hours (7 – 10 am) with data on run times, walking times between platforms and demand data obtained from Transport for London. All demand and supply data correspond to timetables and observations made in 2001. In the OD matrix 2,830 non-zero elements generate a total of 636,904 trips for the 3hour period.

The hyperpath search for this network size is still very fast requiring less than a second. A single iteration of the correction algorithm including assignment requires around 1min on an Intel® Core™ Duo CPU with 2.99 GHz, 1.93 GB RAM. Simulations have been carried out with SP equal to 1, 2, 3 and 5. Initialisation with random seeds has been tested and the model has further been run with and without predetermined alighting. For low SP the alighting behaviour assumption does not make a significant difference in the convergence behaviour but for SP=5 no solution to the correction algorithm can be found in case the adjustment of Algorithm 3 is omitted. This is because of the above discussed effect that for large SP the correction itself influences demand at stop nodes preventing convergence. The following figures therefore show the results assuming predetermined alighting, with ζ as indicated.

Convergence of destination specific arc flows and the approximate gap measure (31) for SP=2 is illustrated in Figure 5. It appears that depending on the initialization of the fail probabilities the MSA converges to slightly different solutions, though the differences are not significant. For SP = 3 on 10 arcs (out of 596 sitting and standing arcs) the difference between flows is above 100 passengers. On all these links this is, however, less than 3% of the total flow. For SP = 2 the difference on all links is less than 100 passengers. Figure 6 illustrates the convergence behaviour of the correction algorithm. The longest line in the network is the circle line with 28 stations, so that according to Proposition 5 at most 53 correction algorithms are required. The figure shows that this condition is fulfilled. Increasing ζ also leads to a slight increase in the average number of correction algorithms per MSA. This is because with more pessimism the total number of boardings and alightings increases.

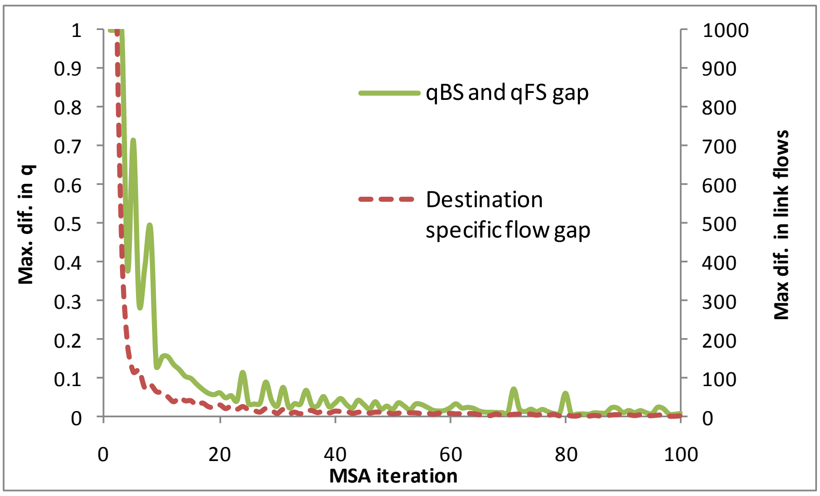

Figure 5: Convergence of the MSA (SP= 2, ζ=0.5)

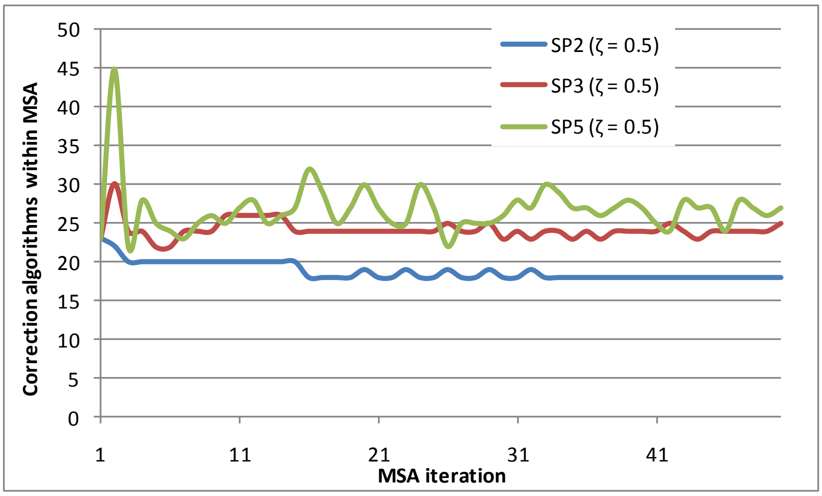

Figure 6: Number of correction algorithms needed per MSA iteration for different SP

Figure 7 illustrates the overall congestion in the London network. In all modeled scenarios at least 155 out of the 298 arcs are crowded, i.e. all seats are taken. Interestingly with higher SP the total number of congested links is increasing rather than decreasing, illustrating the priority effects as well as that a large number of passenger do not have attractive options to avoid crowded lines. Therefore also the calculation time increases for higher SP as more iterations for the correction algorithm are needed within each MSA to fulfill equilibrium conditions (Figure 7). The most congested line sections can be found on the Victoria line as well as the Central line which corresponds to observations. In particular some sections of the fast Victoria Line loose passengers with higher SP as some transfer to slower but less crowded alternatives (e.g. Victoria Line section between Euston and Warren Street: 45974 passengers for SP1 and 41446 passengers in SP2). The effect of ζ on the number of congested arcs is minimal, though the loads on single arcs varies by up to 7800 passengers for SP=5, comparing solutions of ζ=0 and ζ=1.

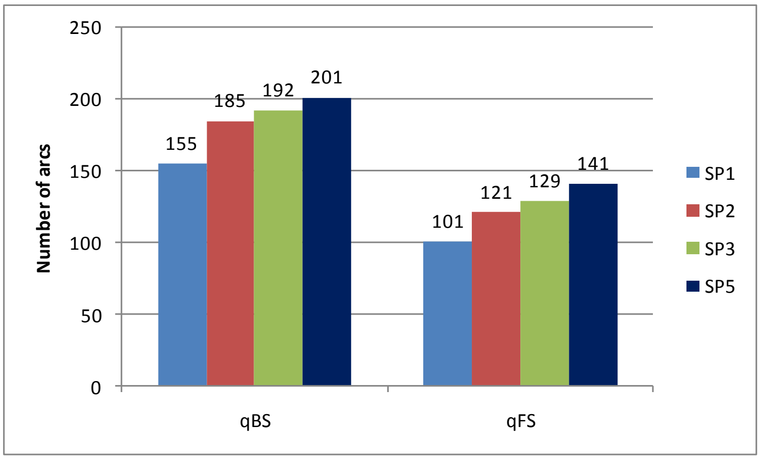

Figure 7: Number of arcs with fail probability > 0 (ζ = 0.5)

SUMMARY AND APPLICATIONS

This paper proposes an equilibrium transit assignment approach that explicitly considers seat availability and the resulting priority rules for on-board and newly boarding passengers. The network is represented as a sitting and a standing line with standing passengers being able to "upgrade" at each station to the "sitting line" because of seated passengers alighting. Though this approach requires a large number of nodes and arcs for the network description a solution to the hyperpath search and user equilibrium assignment problem is presented that is feasible to be applied to large networks.

Besides a general desire to sit expressed as "standing penalty" in the generalised cost, the approach further allows to distinguish passengers with and without predetermined alighting point decisions. Those without predetermined alighting point might transfer later if they have obtained a seat than those not having obtained a seat. To model the behaviour of those with predetermined alighting point a second parameter ζ, referred to as coefficient of optimism, is introduced. The higher ζ, the more passengers expect to obtain a seat and follow the shortest route suggested under these "optimistic conditions". The assumption of en-route alighting point decisions might be more realistic, at least for passengers who are familiar with the network and its various transfer options. In the proposed model, it is however also shown that only under predetermined alighting conditions, convergence can be proven.

The approach has been tested with an application to the inner part of the London Underground network. When the importance of being able to travel seated increases (higher SP) passengers tend to split among more lines resulting in rather different equilibrium solutions. Our results hence confirm the importance of considering standing penalties for transit assignment problems. It is also found that ζ can have a significant influence on the line split.

The approach presented here using "fail-to-sit probabilities" clearly has similarities to previous work by the same authors on fail-to-board probabilities to consider capacity constraints. A straightforward extension of this work is therefore to include "fail-to sit" as well as "fail-to-board" probabilities in the same model. In this way the total capacity as well as the seat capacity of lines can be considered. Further, one could add a BPR type function for the costs on standing links in order to reflect the increasing inconvenience by standing in crowded trains or buses. Schmöcker et al (2008) extend the fail-to-board approach to consider dynamic effects. The same ideas can also be transferred to the approach presented in this paper to consider the changing availability of seats during the day. The main complexity of the Schmöcker et al (2008) approach is how to reassign passengers who have to wait at the platform for the next service because of insufficient total service capacity. In the seat capacity model "failed passengers" can be simply assigned to the standing links, hence avoiding these problems.

The model can be used in several ways. Firstly, a direct application is for transport planning purposes to better understand the influence of providing more seats either through service frequency variations or through changed vehicle configurations. Further, the effect of turning vehicles short or, mainly for bus services, changing the route of services will have an effect on the equilibrium as in particular passengers from the new terminals will have an increased chance to obtain a seat. Secondly, the model could be used to provide customers with information about the "average" seat availability from stops. For some passengers groups this information might be a decision criterion before deciding on a particular route. Whereas passengers at bus stops expect information about loads of the next arriving service, a static model as presented here could be used at the journey planning stage. Information could be given, for example, when passengers are selecting their route via a "journey planner" webpage. Currently often expected travel times, number of interchanges and walking times between platforms are given as information to choose between different route options. Adding information on the probability of finding a seat, especially at the boarding point, could be a useful addition.

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