TRANSIT ASSIGNMENT MODEL INCORPORATING THE BUS BUNCHING EFFECT

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ABSTRACT

This paper proposes a transit assignment model considering the correlation between vehicles' arrival at stops. The correlation of arrival is represented with a correlation coefficient matrix, and the expected waiting time and the arc split probabilities at stops are calculated with a Monte Carlo simulation-based method where the correlated random variates follow a given distribution function. This function is generated using dependent random variates which follow a normal distribution and a correlation coefficient matrix. Since the correlation coefficients are defined as a function of the number of boarding and alighting passengers, it is possible to consider two sources of correlation in the proposed model; i) increasing boarding and alighting time due to passengers' concentration to a certain vehicle, ii) concentrating vehicles on a certain road segment. The proposed model is formulated as a fixed point problem and the solution approach is illustrated with a toy network.

Keywords: Transit assignment model, Bus bunching, Monte Carlo Simulation

INTRODUCTION

A main reason why not more passengers are using public transport is known to be uncertainty about waiting times at the stops. In particular if passengers are not sure whether lines arrive on time passengers might be frustrated by waiting. Due to the waiting time, the shortest path to the destination is not always the route with minimum in-vehicle time. In case of no information on the platform as to the arrival of the next vehicle, the fastest way to get to the destination might be to take the first vehicle arriving among an "attractive set" of lines. This is referred to as the "common lines problem", which is more formally defined as the problem to find the optimal set of lines among all possible lines that brings the traveller closer to his destination (Chiriqui and Robillard, (1975)). "Optimal" is usally referred to as "shortest expected travel time. Suppose passengers arrive randomly and vehicles also arrive statistically independent with given exponentially distributed headways, then the probability that passengers choose line i, pi, and the expected waiting time at a stop, T, are calculated as follows:

$$p_i = f_i \Big/ \sum_{k \in K} f_k \tag{1}$$

$$WT = 1 \Big/ \sum_{k \in K} f_k \tag{2}$$

where, K and f_k represent the set of attractive transit lines and frequency on transit line $k \in K$ respectively. As a result, the optimal route between origin and destination is not a single path anymore. Nguyen and Pallottino (1988) defined attractive lines as the "hyperpath". Spiess and Florian (1989), refer to it also as "strategy" and expanded this idea to an equiribrium assignment framework. Since then many researchers incorporate the congestion effect into the transit assignment model (Kurauchi et. al. (2003), Cepeda et. al. (2006)) and further expand the assignment model into a dynamic one (Schmöcker et al. (2008)).

The approach by Spiess and Florian (1989) which is utilised in the above mentioned work on crowding effects implies five assumptions; i) vehicles and passengers arrive randomly, ii) passengers have no information as to when the next vehicles arrives iii) arrival times follow an exponential distribution, iv) stopping time at a stop due to passengers' boarding and alighting is neglictable and v) the interarrvial time between lines is constant. In order to relax the second assumption, Shimamoto et. al. (2005) proposed a model which considers the arrival time information provision to passengers when the passengers have the choice between two lines. Billi et al (2004) and Gentile et al (2005) proposed more general approaches which are summarised in Nökel and Wekeck (2007) as "5 cases of passengers' information". Nökel and Wekeck (2009) extend this further comparing passengers' route choice models considering that depending on information provision the optimal alighting point of passengers might vary as well. In order to relax the last assumption, Larrain et. al. (2008) proposed a model in which the travel time between stops is a function of the number of boarding and alighting passengers. Teklu et. al. (2007) proposed a composite frequency-based and schedule-based Markov process model that considers the day-to-day dynamics of transit network. Since the passengers' and buses' movement in each day is based on micro-simulation in their model, it is possible to consider delay of arrival at a stop due to the passengers' boarding and alighting. However, none of the models referred to so far consider the correlation of arrival times due to the network configuration such as lines' overlapping, which is often seen in the urban bus networks.

The two main factors causing the bunching effect between vehicles of the same line as well as between vehicles of different lines are i) increasing the boarding and alighting time due to the passengers' concentration to a certain vehicle, ii) concentrating vehicles to a certain road segment due to line configuration. In their seminal paper Newell and Potts (1964) describe service loads according to "bunching effects" caused by the first factor. If a certain service is for some reason delayed, a larger number of passengers will be boarding this bus, leading in turn to longer dwell times at the stops, hence leading to further delays. As a further consequence the following service will have to carry less passengers, leading to less dwel time and the bus catching up with the previous service which is the well known bunching effect of two buses arriving at the stop shortly after another. Newell and Potts only describe the bunching effects on a single line not considering any assignment effects that passengers might have a choice between several lines. Shimamoto et. al. (2009) propose a transit assignment model considering the effects of bunching effects on line loads in a network. In their paper the bunching effects are assumed to be independent of dwell time but exogenously given with the correlation coefficients which represent for example the service schedule being disturbed due to road congestion, therefore, they only consider the second factor of the bunching effects. This paper further expands this model; here the bunching effects depend on dwell time which is dependent on the number of boarding and alighting passengers. This is achieved by representing the correlation among lines as a function of the number of boarding and alighting passengers. Therefore, the proposed model can consider both of the above mentioned factors causing the bunching effects.

The remainder of this paper is structured as follows. The formulation and solution algorithm of the arc transition probabilities and the expected waiting time for a single OD pair network is shown in the next section. Section 3 explains how correlation of lines can be illustrated in a graph description and the mathematical formulation and solution algorithm for a general network are shown in Section 4. Finally, a case study for a simple network is shown in Section 5.

ARC TRANSITION PROBABILITY AND EXPECTED WAITING TIME UNDER LINE CORRELATION

Assumptions

We adopt following assumptions in the model:

1. Passengers arrive randomly at every stop node, and always board the first arriving vehicle included in their hyperpath;

2. In-vehicle travel times between stops are constant;

3. The service is frequency-based and the headway of all transit lines follow a given probability distribution which depend on each other;

4. This correlation of the headways among vehicles or lines is a function of the number of boarding and alighting passengers.

Note that the first and the second assumption are the same as in the previous model described in Shimamoto et al (2009). The third and forth assumptions are set to describe the vehicles' arrival correlation, which implies that the correlation is generated only on bus stops and the degree of the correlation is a function of the number of boarding and alighting passengers.

Arc Transition Probability Considering Line Correlation

In this section, it is illustrated how the arc transition probabilities and expected travel times depend on the correlation of arrival times. For simplicity a simple network with one OD pair and n lines is included in the hyperpath.

Formulation

As we assume that passengers take a line arriving first among n lines, the probability of taking line i equals the probability that the waiting time of line i is minimum, which can be formulated as follows;

$$\begin{aligned} p_i &= \Pr(t_i)\cdot\Pr\left(w_i < t_j\ (j \neq i)\middle| t_i\right) \\ &= \int_{t_i=0}^{\infty}\left\{\int_{t_1=t_i}^{\infty}\cdots\int_{t_{i-1}=t_i}^{\infty}\int_{t_{i+1}=t_i}^{\infty}\cdots\int_{t_n=t_i}^{\infty}\mathbf{g}(\mathbf{t})dt_1\cdots dt_{i-1}dt_{i+1}\cdots dt_n\right\}dt_i \end{aligned} \tag{3}$$

Where

t_i : Waiting time at line i

$g(t)$: Multi-dimensional probability density function representing the headway of each line.

Note that $g(t)$ is introduced to represent the dependency of each line's headway. The expected waiting time at a stop can be formulated as below;

$$WT = \sum_{i=1}^{n}\int_{t_i=0}^{\infty}\left[\left\{\int_{t_1=t_i}^{\infty}\cdots\int_{t_{i-1}=t_i}^{\infty}\int_{t_{i+1}=t_i}^{\infty}\cdots\int_{t_n=t_i}^{\infty}\mathbf{g}(t)dt_1\cdots dt_{i-1}dt_{i+1}\cdots dt_n\right\}\cdot t_i dt_i\right] \tag{4}$$

In summary, if the headway of each line independently follows the exponential distribution, then the arc transition probability and the expected waiting time at a node can be determined as in Eqs. (1) and (2). In contrast Eqs. (3) and (4) describe the more general case that the probability density functions might be correlated. The problem of this more general formulation is, however, the complexity and with it computational expensiveness to solve Eqs. (3) and (4) analytically. Therefore, alternatively a heuristic Monte-Carlo simulation based method is proposed where the waiting time of each line is expressed as a random number. This number is generated as described in the following section. Then, Eqs. (3) and (4) can be approximated with Eqs. (5) and (6).

$$p_i \approx \frac{1}{M}\sum_{m=1}^{M} N(i = \arg.\min_{j\in K} t_j^m) \tag{5}$$

$$WT \approx \frac{1}{M}\sum_{m=1}^{M}\sum_{i\in K} N(i = \arg.\min_{j\in K} t_j^m)\cdot t_i^m \tag{6}$$

where,

$N(*)$: The number of samples which satisfies (*)

t_j^m : Waiting time on line j with the mth random number generated,

M : The number of samples (=random numbers generated).

Generation of Correlated Random Variate

The procedure of generating multivariate, non-normal, correlated random variates is based on Chang et.al (1994). Instead of obtaining the joint distribution, they transform the correlation coefficients of the original distribution (**X**) into the correlation coefficients of the standard normal distribution (**Y**) using Nataf's bivariate distribution model shown as below;

$$\rho_{ij} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{x_i - f_i}{\sigma_i}\right)\left(\frac{x_j - f_j}{\sigma_j}\right)\phi_{ij}(y_i, y_j, \rho_{ij}^*)dy_i dy_j \tag{7}$$

where,

ρ_{ij}^* : The correlation coefficient between the distribution of Y_i and Y_j,

ρ_{ij} : The correlation coefficient between the distribution of X_i and X_j,

f_i : The mean of X_i (service headway of line i),

σ_i : The standard deviation of X_i,

ϕ_{ij} : The bivariate normal PDF of zero means, unit standard deviation, and correlation coefficient ρ_{ij}^*.

Note that from the definition of **X** and **Y**, it follows;

$$Y_i = \Phi^{-1}\left[F_i\left(X_i\right)\right] \tag{8}$$

where

$F_i(X_i)$: The marginal distribution function of X_i

Although it is difficult to obtain ρ_{ij}^* from Eq. (7) with a given correlation coefficient ρ_{ij} and the marginal distributions of X_i and X_j Der Kiureghian and Liu (1985) develop a semi-empirical formulation as below;

$$\rho_{ij}^* = T_{ij} \cdot \rho_{ij} \tag{9}$$

In Eq. (9), T_{ij} is the transformation factor which depends on the marginal distributions of the two random variables X_i and X_j. It takes the value one if both X_i and X_j follow the normal distribution and a constant if both follow the exponential distribution. Using Eq. (9), we can transform the correlation coefficient matrix $\mathbf{R_X}$, which follows a given distribution, into the correlation coefficient matrix $\mathbf{R_Y}$, which follows a normal distribution, where;

$$\mathbf{R_X} = \begin{pmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & \rho_{22} & & \vdots \\ & & \ddots & \\ \rho_{n1} & \rho_{n2} & \cdots & \rho_{nn} \end{pmatrix}, \quad \mathbf{R_Y} = \begin{pmatrix} \rho_{11}^* & \rho_{12}^* & \cdots & \rho_{1n}^* \\ \rho_{21}^* & \rho_{22}^* & & \vdots \\ & & \ddots & \\ \rho_{n1}^* & \rho_{n2}^* & \cdots & \rho_{nn}^* \end{pmatrix} \tag{10}$$

Based on above relationship, the procedures of the correlated random variate generation are as below;

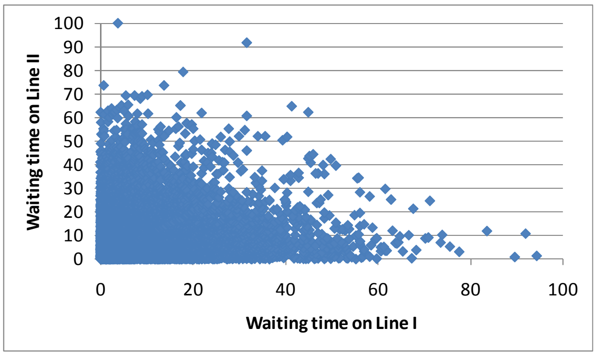
(Step1) Transform the correlation matrix $\mathbf{R_X}$ into $\mathbf{R_Y}$ using Eq. (9).
(Step2) Diagonalise $\mathbf{R_Y}$ as below;

(Step3) Transform the independent standard normal variates **Z** into the correlated standard normal variates **Y** by inverting the orthogonal transformation.

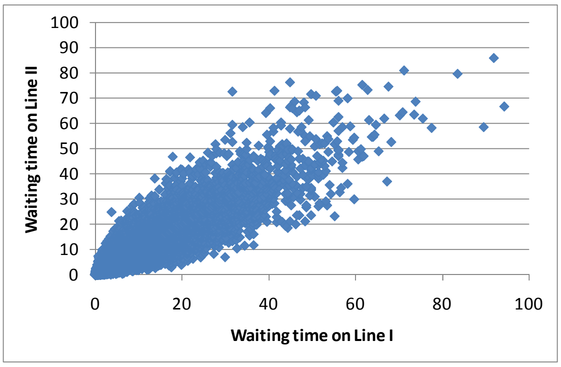
(Step4) Transform the correlated standard normal variates **Y** into the correlated non-normal variates **X** with the inverse transformation of Eq. (8).

The Relationship Between the Correlation Coefficient and the Punctuality of the Transit Service

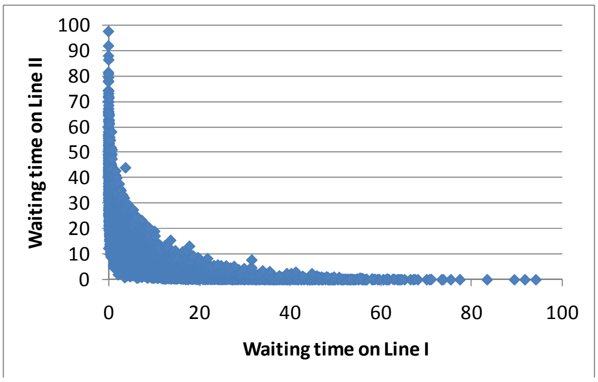
In this model, the correlation of arrival among lines is represented with ρ_{ij}. In order to confirm the relationship between ρ_{ij} and the punctuality of the service, we generate two correlated random variates whose marginal distribution follows an exponential distribution with the average being the service frequency in this case assumed to be 6 trains per hour (in other words the expected waiting time is 10 minutes). Figure 1 illustrates the waiting time for different correlation between the two lines. ρ_{ij} being positive is equivalent to the case that, if the waiting time of line i is small, then that of line j is also small. Similar, if the waiting time of line i is large, then that of line j is also large. This means the arrival time difference between the vehicles of line i and j is also short; hence, a positive ρ_{ij} represents for the bunching effect among lines i and j. Contrary, in case ρ_{ij} is negative, if the waiting time of line i is small, then that of line j is large and vice versa. If passengers are assumed to have both lines in their attractive set of lines, the expected waiting time becomes hence smaller and therefore a negative ρ_{ij} represents co-ordianted services. The expected waiting time calculated from Eq. (6) is 5.0, 8.6 and 3.1 respectively for the three cases illustrated in Figure 1 (ρ_{ij} =0.0, 0.9 and -0.9). Figure 2 illustrates the probability distribution of waiting time for each correlation coefficient. As seen the figure, in case ρ_{ij} =-09, the probability that the waiting time is greater than 10 miniutes is almost 0 whereas it is still positive in case ρ_{ij} =0.0 and 0.9. Note that this only empirically demonstrates the relationship between the correlation coefficient and the punctuality of the service, hence in future work one should theoretically prove this relationship.



(a) ρ=0.0



(b) ρ=0.9



(c) ρ=-0.9

Figure 1 – Relationship Between Correlation Coefficient and Waiting Time

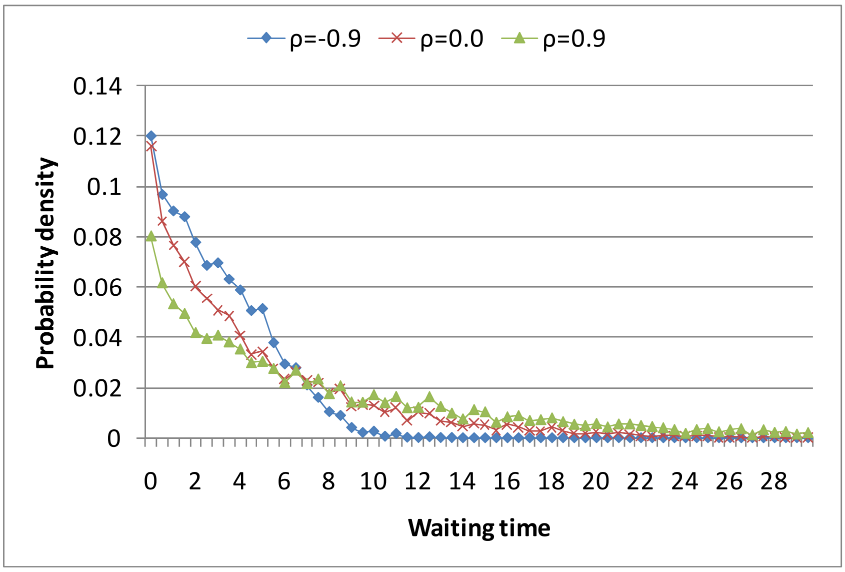


Figure 2 – Probability Density of the Expected Waiting Time for Each Correlation Coefficient

NETWORK TRANSFORMATION AND NOTATIONS

Section 2 only considers the simple case with one OD pair. In order to load passengers to a general network, we transform a transit network into the graph model shown in Figure 3 following the idea in Kurauchi et. al (2003). An origin node represents a trip start node. It has no predecessors and at least one successor. A destination node represents a trip end node. It has no successors and at least one predecessor. A stop node represents a platform at a station. Any one or more transit lines stopping at the same platform are connected via boarding arcs. The arc transition probabilities and the expected waiting time is calculated at stop nodes and passengers are assigned to any of the attractive lines in proportion to the arc transition probabilities. A boarding node is a line-specific node at the platform where passengers' board and similarly an alighting node is a line-specific node at the platform where passengers alight.

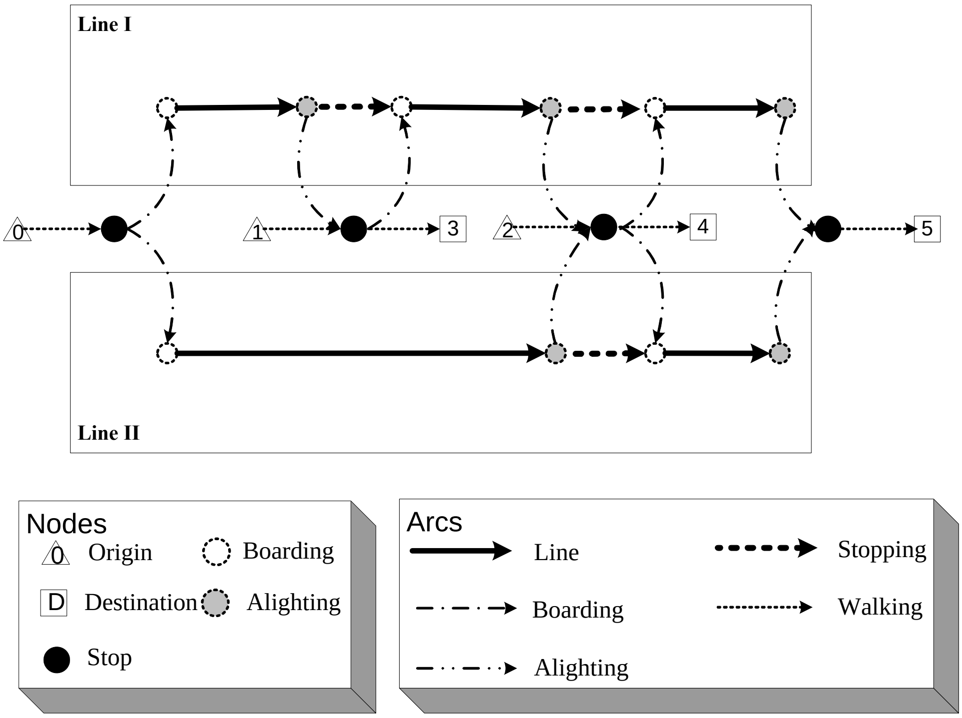


Figure 3 – Network Representation

A line arc represents a transit line connecting two stations. A boarding arc denotes an arc connecting the stop node to the boarding node. The flow on this arc represents the boarding for the transit line from a specific platform. An alighting arc denotes an arc from an alighting node to a stop node. A stopping arc denotes a transit line stopping on a platform after the passengers alight and before new passengers' board. Creating this arc is also needed to express the available capacity and the priority rules on the line explicitly. A walking arc connects an origin to a platform or a platform to a destination.
Hereafter, we use below notations in this paper.

c_a	:	Arc cost on arc $a \in A$,
λ_{lh}	:	Probability of choosing any particular elementary path l of hyperpath h,
p_{ah}	:	Arc transition probability on hyperpath h,
α_{ah}	:	Probability that traffic traverses arc a,
β_{ih}	:	Probability that traffic traverses node i,
δ_{al}	:	Takes 1 if arc a is included in l, otherwise 0,
ε_{il}	:	Takes 1 if elementary path l traverses node i, otherewise 0,
I	:	Set of nodes,
S	:	Set of stop nodes,
A_h	:	Set of arcs on hyperpath h,
V_h	:	Set of elementary paths on hyperpath h,
S_h	:	Set of stop nodes on hyperpath h,
I_h	:	Set of nodes on hyperpath h,
O	:	Set of origins,
D	:	Set of destinations,
$In(i)$	:	Set of arcs going into node i,
$Out(i)$	:	Set of arcs going out from node i,

Q_{od} : Travel demand between *od*,

Ω : The set of feasible flow satisfying below flow conservation,

$x^k_{al(l)}$: The number of alighting passengers from stop node *k* on line *l*,

$x^k_{b(l)}$: The number of boarding passengers from stop node *k* on line *l*,

ρ : Matrix of correlation of lines from the same stops (with $\rho^k_{ml} \in \rho$ where *k* denotes the stops, *m* and *l* denotes the lines departing from stop node *k*)

κ^k_{ml} : Scale parameter related to the number of boarding and alighting passengers,

ν^k_{ml} : Scale parameter related to factors except the number of boarding and alighting passengers,

δ^k_{ml} : Dummy variable which takes 1 if line *m* and line *l* belongs to the same route, otherwise takes 0

MATHEMATICAL FORMULATION AND SOLUTION ALGORITHM

Incorporating of Correlation Coefficients

The correlation coefficient matrix is required to generate the set of correlated random numbers as shown in Chapter 2. It is assumed that the correlation coefficient is a function of the number of boarding and alighting passengers in order to describe the increase in dwell time as the number of boarding and alighting passengers' increases, which causes a reduction of punctuality. Since a larger correlation coefficient means bunching effects between lines *m* and *l* are more likely to occur, the correlation coefficient between lines *m* and *l* at bus stop *k* is defined as below;

$$\rho^k_{ml} = \begin{cases} 2 \cdot \dfrac{1}{1+\nu^k_{ml}\exp\left\{\left(-\kappa^k_{ml}\left(x^k_{al(l)}+x^k_{al(m)}+x^k_{b(m)}+x^k_{b(l)}\right)\right)\right\}} - 1 & (\delta^k_{ml}=1) \\ 0 & (\delta^k_{ml}=0) \end{cases} \quad (11)$$

Note that above formulation is also an assumption which should be verified with observed bus operation data. δ^k_{ml} is introduced to consider the arrival correlation only for the same line, which implies that different bus lines run on different roads. The first term of Eq. (11) is defined as twice of the logistic function which takes values from 0 to 1, therefore, ρ^k_{ml} takes values from -1 to 1 and closer to 1 as the number of boarding and alighting passengers increases. Also, we introduce two positive scale parameters, κ^k_{ml} and ν^k_{ml}. κ^k_{ml} is a scale parameter related to the number of boarding and alighting passengers which takes a small value if the boarding and alighting time per passenger decreases due to, for example, introducing an IC card fare collection system. ν^k_{ml} is assumed to be related to other factors, such as the bus network configuration or the quality of the facilities, i.e., ν^k_{ml} takes a larger value if many lines gather in the same road section and a smaller value if the accelerating performance of the buses is good. Therefore with these two scale parameters, it is possible to consider both factors of the bunching effects; i) increasing the boarding and alighting time due to the passengers' concentration to a certain vehicle, and ii) concentrating vehicles to a certain road segment.

The Cost of Hyperpath

Let us formulate the cost of the hyperpath h, g_h, in the general network as follow;

$$g_h = \sum_{a \in A_h} \alpha_{ah} c_a + \sum_{k \in S_h} \beta_{kp} WT_{kh} \tag{12}$$

Where

$$\lambda_{lh} = \prod_{a \in A_h} p_{ah}^{\delta_{al}} \ , \forall l \in V_h \tag{13}$$

$$\sum_{a \in OUT(i)} p_{ah} = 1 \, , \ \forall i \in I_h \tag{14}$$

$$p_{ah} \geq 0 \, , \ \forall a \in A_h \tag{15}$$

$$\alpha_{ah} = \sum_{l \in V_h} \delta_{al} \lambda_{lh} \ , \ \forall a \in A_h \tag{16}$$

$$\beta_{ip} = \sum_{l \in V_p} \varepsilon_{il} \lambda_{lp} \ , \ \forall i \in I_p \tag{17}$$

Note that WT_{kh} in Eq. (12) represents the expected waiting time at stop node k, which is calculated with Eq. (4) or Eq. (6) at every stop node. The arc transition probabilities at stop node k correspond to Eq. (3) or Eq. (5). The first and the second term of Eq. (12) represent the in-vehicle time and the expected waiting time, respectively. As Eq. (12) is node separable, Bellman's principle can be applied to find the minimum-cost hyperpath from every destination node (See Appendix A)., The arc transition probabilities and the expected waiting time at stop nodes (Eq. (5) and (6)) can then be defined as below;

$$p_{ah}^{i} \approx \frac{1}{M} \sum_{m=1}^{M} N(a = \arg. \min_{a' \in Out(i) \cap A_h} t_{a'}^{m}) \quad , \forall i \in S_h \tag{19}$$

$$WT_{ih} \approx \frac{1}{M} \sum_{m=1}^{M} \sum_{a \in Out(i) \cap A_h} N(a = \arg. \min_{a' \in Out(i) \cap A_h} t_{a'}^{m}) \cdot t_a^m \quad , \forall i \in S_h \tag{20}$$

Formulation and Solution Algorithm

In Eq. (3), the arc transition probabilities $\mathbf{p} \in R^{|I|} \times R^{|D|}$ are a function of the probability density function representing the waiting time of each line, $\mathbf{g}(\mathbf{t}) \in R^{|S|}$ whereas in Eq. (11), ρ^{k}_{ml} in $\mathbf{g(t)}$ is a function of the number of boarding and alighting passengers $\mathbf{x}$ which in turn is a function of arc transition probabilities $\mathbf{p}$. Hence, the right hand side of Eq. (3) is also a function of $\mathbf{p}$. In addition, if travel demand and destination specific arc transition probabilities are given, one can uniquely obtain arc flows $\mathbf{x}$ applying the destination-specific Absorbing Markov Chains assignment (See Appendix B). Therefore, the proposed transit assignment model can be formulated as a fixed point problem defined as below;

$$\mathbf{x} = f(\boldsymbol{\rho}(\mathbf{x}), \mathbf{p}(\mathbf{x})) \quad \forall \mathbf{x} \in \Omega \tag{21}$$

Note that Eq. (11) is not continuous with regards to arc flow $\mathbf{x}$, therefore we can not theoretically prove the existence of the solution. The proposed model is solved using the method of successive average as below;

(Step 0) Set n:=0, $x^{(n)}$:=0;

(Step 1) Calculate hyperpath cost g_h from Eq. (12) and arc transition probability $\mathbf{p}$ using Eq. (19) at every stop node.

(Step 2) Loading passengers to the network using Absorbing Markov Chains according to the destination specific arc transition probabilities $\mathbf{p}$ and then obtain arc flows x_{temp}.

(Step 3) Update $\mathbf{x}$ by $\mathbf{x}^{(n+1)}$:= $(1\text{-}1/n)*x^{(n)}+ 1/n *x_{temp}$

(Step 4) If is $\mathbf{x}^{(n+1)}$ is close enough to $\mathbf{x}^{(n)}$, stop the calculation, otherwise set n: $=n+1$ and go to (Step 1)

Note that (Step 1) and (Step 2) are excuted destination specific. By summing up all destination specific arc volumes, the total arc volume is obtained;,

$$\mathbf{x} = \sum_{d \in D} \mathbf{x}^d \tag{22}$$

where,

$\mathbf{x}^d$: Destination specific arc volume vector.

NUMERICAL EXAMPLE

Case Setting

Since the vehicle capacity has not been considered in the proposed model so far, the passengers' flows from upstream do not affect the route choice behaviour at downstream stations. Therefore, the proposed model is applied to a single OD network shown in Figure 4 in order to capture the fundamental features of the model. The headway of each line is assumed to follow an exponential distribution and average headway and the travel time of each line is shown in the figure. We assume that no correlation occurs between Line I and Line II because they are on the different roads. In order to describe the correlation of two subsequent services of the same line, each line is virtually converted into double lines. Since there is only one stop node in a converted network and the correlation is generated only in the same line in this network, the correlation coefficients and their parameters is represented as σ_l, κ_l, ν_l $(l=1,2)$ for simplicity.

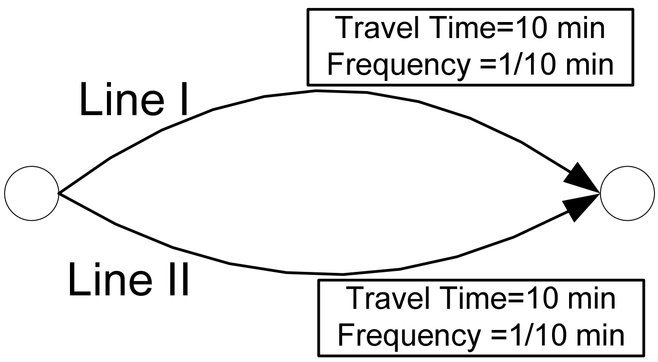


Figure 4 – Single OD network

Convergence Check

Figure 5 shows the relationship between the number of iterations and the distance to fixed point solution ("flow error") assuming κ_1=0.05, ν_1=10, κ_2=0.1, ν_2=10 and travel demand=100 passenger/ minute. Note that the flow error is defined as below;

$$FE^{(n)} = \sum_{i=1}^{N} \sqrt{\left(x_i^{(n+1)} - x_i^{(n)}\right)^2} \tag{23}$$

where,

N : The number of arcs in the converted network
$x^{(n)}_i$: The flow on the ith arc in the nth MSA iteration

By looking at Figure 5, the flow error becomes small as the iteration number increase, therefore it can be said that the proposed model converges.

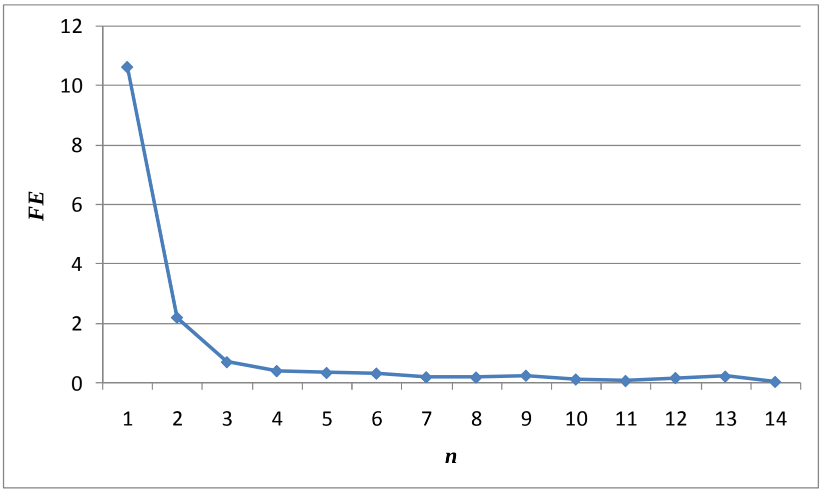


Figure 5 – Relationship Between Iteration and Flow Error

Comparison of Line Usage Rate and Correlation Coefficient of Each Line

Firstly, we compare the relationship between ν_1 or κ_1 and the usage rate of Line I while fixing the demand as 100 and parameters in Eq. (11) regarding to Line II as κ_2=0.1, ν_2=10. Figure 6 shows the relationship between ν_1 and the usage rate of Line I under the condition of κ_1 as 0.1. From the definition of Eq. (11) ρ_1 decreases as ν_1 increases, as a result, the usage rate of Line I slightly increase. At the same time, although the parameters regarding to Line II is fixed, ρ_2 also slightly decreases due to the reduction of the usage rate of Line II. Figure 7 shows the relationship between κ_1 and the usage rate of Line I assuming $\nu_1 = 10$. From the definition of Eq. (11) ρ_1 increase as κ_1 increases, as a result, the usage rate of Line I decreases. Also, ρ_1 is close to 1.0 if κ_1 is greater than 0.2, which means the two buses run continuously (bunching effect).

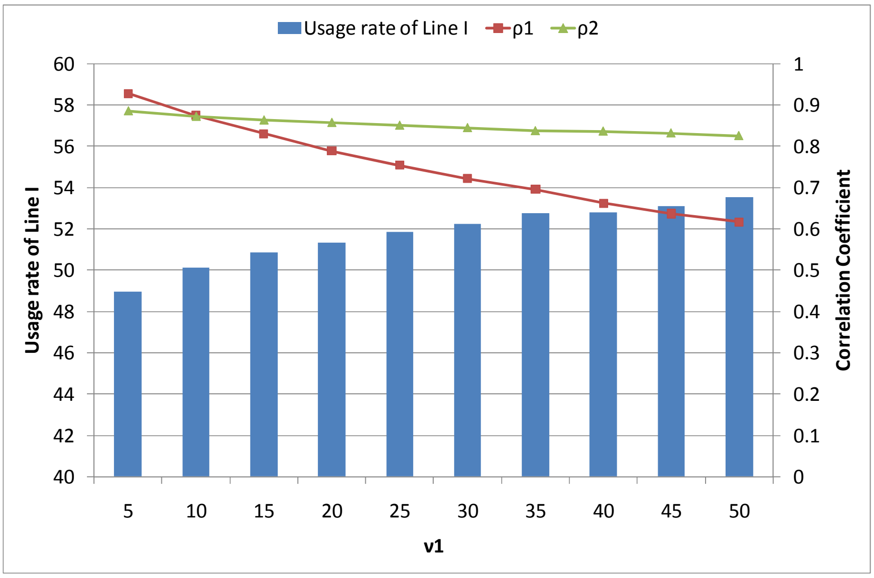


Figure 6 – Relationship Between ν_1 and Usage Rate of Line I

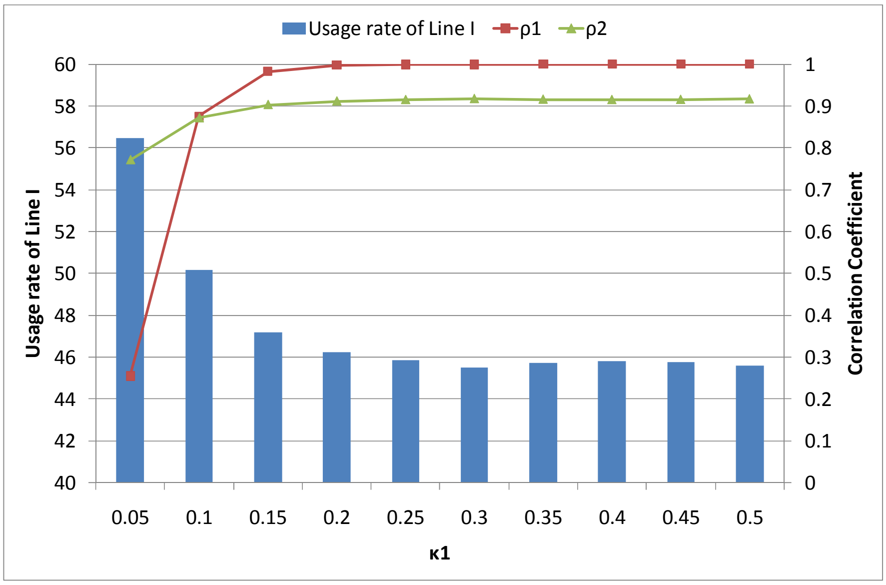


Figure 7 – Relationship Between κ_1 and Usage Rate of Line I

Secondly, we set the parameter κ_l, ν_l $(l=1, 2)$ in Eq. (11) as $\kappa_1=0.1$, $\nu_1=50$, $\kappa_2=0.1$, $\kappa_2=10$ and compare the relationship between the arc transition probabilities and the travel demand shown in Figure 8. Note that ρ_1 is always smaller than ρ_2 under the same travel demand, which means that Line I is less likely to suffer from the bunching effects than Line II under the same travel demand. When the travel demand is small, the difference of ρ_1 and ρ_2 is small, therefore, the usage rate of Line I is close to 50%. For low demand, as the travel demand increases, ρ_1 increases less rapidly than ρ_2, and as a result, the usage rate of Line I increase. However, as the travel demand exceeds 60 passengers / minute, ρ_1 increases smoothly while ρ_2 increases rapidly, which causes the reduction of the usage of Line I. Finally, as the travel demand becomes enough large, both ρ_1 and ρ_2 are close to 1.0, meaning that the two buses run continuously, and as a result, the usage rate of Line I is close to 50 % again.

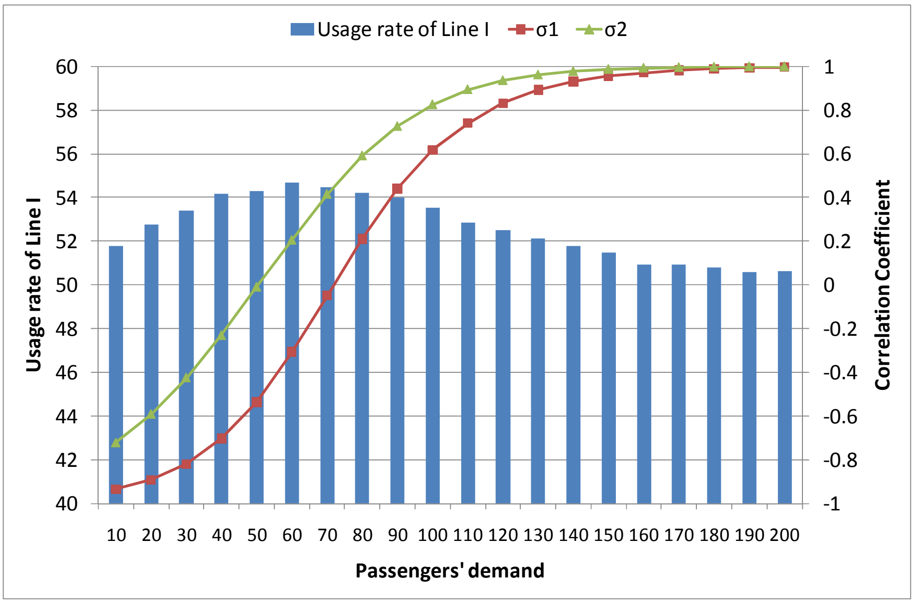


Figure 7 – Relationship Between Passengers' Demand and Usage Rate of Line I
(κ_1=**0.1**, ν_1=**50**, κ_2=**0.1**, ν_2=**10**)

CONCLUSION

This paper proposes a transit assignment model considering the correlation between vehicles' arrival at stops. Following the afore-constructed model (Shimamoto et. al.(2009)), the correlation of arrival is represented with a correlation coefficient matrix, and the expected waiting time and the arc split probabilities at stops are calculated with a Monte Carlo simulation-based method where the correlated random variates follow a given distribution function. This function is generated using dependent random variates, which follow a normal distribution, and a correlation coefficient matrix. The previous model assumed the correlation coefficient exogenously given, whereas in this paper it is derived endogeneously by assuming that it is a function of the number of passengers boarding and alighting. As a result, it is possible to consider both sources of correlation in the proposed model; i) increasing the boarding and alighting time due to the passengers' concentration on a specific vehicle, ii) concentration of vehicles to a certain road section. The proposed model is formulated as a fixed point problem. The solution procedure and the influence of the model parameters are demonstrated with an application to a toy network.

In future work, one should verify the relationship between the correlation of vehicles' arrival and the number of boarding and alighting passengers using observed data. Also, since the calculation cost of the proposed model is expensive, there is room to develop more effective algorithms to apply for the approach to larger size networks. Furthermore, the congestion effect should be considered by introducing the vehicles' capacity.

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APENDIX

Separability of the Cost Function

The separability of the cost function (Eq. (12)) can be shown in the same way as in Kurauchi et. al. (2003). By substituting Eq. (13), (17) and (18) into Eq. (12), we obtain

$$g_h = \sum_{l \in V_h} \lambda_l \left(\sum_{a \in A_h} \delta_{ah} c_a + \sum_{k \in S_h} \varepsilon_{kl} WT_{kh} \right) \tag{24}$$

Above result suggests that the cost for hyperpath h is a sum of the costs of the elementary paths weighted by their choice probabilities. Let g'_{ih} denote the cost of the sub-hyperpath from i to s on hyperpath h, and define $\lambda_{lh}(i)$ as:

$$\lambda_{lh}(i) = \prod_{a \in A_{ih}} p_{ah}^{\delta_{al}} \tag{25}$$

$$\sum_{l \in V_h} \lambda_{lh}(i) = 1 \tag{26}$$

Then, clearly,

$$\lambda_{lh}(i(a)) = \tau_{ah} \lambda_{lh}(j(a)), \ \forall a \in A_{ih} \tag{27}$$

where $i(a)$ and $j(a)$ respectively are a head node and a tail node of arc a. Using $\lambda_{lh}(i)$, g'_{ih} can be written as follows.

$$\begin{aligned} g'_{ih} &= \sum_{l \in L_h} \lambda_{lh}(i) \left(\sum_{a \in A_{ih}} \delta_{al} c_a + \sum_{k \in S_{ih}} \varepsilon_{kl} \cdot WT_{kh} \right) \\ &= \sum_{l \in L_h} \lambda_{lh}(i) \left(c_{a(i,l)} + \varphi_{iS} \cdot WT_{ih} \right) + \sum_{a \in OUT_h(i)} p_{ah} g'_{j(a)h} \\ &= \sum_{a \in OUT_h(i)} p_{ah} c_a + \varphi_{iS} \cdot WT_{ih} + \sum_{a \in OUT_h(i)} p_{ah} g'_{j(a)h} \end{aligned} \tag{28}$$

where φ_{iS} is 1 if node i is a stop node and 0 otherwise, and $a(i,l)$ denotes an arc on elementary path l leading out from node i. Eq. (28) gives the cost of the hyperpath from the intermediate i to the destination node s as the sum of the cost at node i, a cost for arcs connecting to other nodes weighted by the respective arc split probabilities, and a total cost from node j to s also weighted by arc split probabilities. Since the costs of subsequent nodes are separable, Bellman's principle shown by Eq. (28) can be applied for finding the minimum cost hyperpath.

$$\gamma_{is}^{*}=\begin{cases}0 & (i=s)\\ \min\limits_{a\in OUT(i)}(c_a+\gamma_{j(a)s}^{*}) & (i\in\{I-S-\{s\}\})\\ \min\limits_{K\subseteq OUT(i)}(WT_{iK}+\sum\limits_{a\in K}p_a^i\gamma_{j(a)s}^{*}) & (i\in S)\end{cases} \tag{29}$$

Note that WT_{ik} in Eq. (29) represents the expected waiting time at stop node i if passengers choose lines among line set K and $p_a{}^i$ represents arc split probability at stop node i. Therefore, by finding the arc sets which minimise the third column in Eq. (28) at each stop node, we can find the minimum cost hyperpath without enumerating all possible hyperpaths.

Traffic Assignment by Markov Chain

By using the recursive equation shown in the previous Appendix, the optimal arc split probability, p^*as explaining passenger choices can be found. To assign the traffic to the network, a Markovian loading process is applied. A Markov chain is characterised by a transition matrix defining the probability of an entity (in this case a traveller) moving from one state to another. Conservation requires the rows of this matrix to sum to one. The states of the Markov chain represent the origins, the intermediate vertices of the graph and a destination. The destination constitutes absorbing states (Bell et al. (2002), Akamatsu (1996)). In this study, all traffic is absorbed into their destinations. Defining $a = (i(a), j(a))$, p^*as can be rewritten as $p^*_{i(a)j(a)s}$, which is equivalent to the outcome of an optimal hyperpath search. Let $\mathbf{P}_s$ denote a transition probability matrix for trips destined for s, whose size is n x n (where n is the number of nodes). Then $\mathbf{P}_s$ can be written as follows:

$$\mathbf{P}_s=\left(\begin{array}{c|c}1 & \mathbf{0}\\ \hline \mathbf{r}_s & \mathbf{Q}_s\end{array}\right)\begin{matrix}1\\ n-1\end{matrix} \qquad \begin{matrix}1 & n-1\end{matrix} \tag{30}$$

where $\mathbf{r}_s$ is a vector of probabilities that traffic is absorbed to s from node i, and Qs is a matrix of transition probabilities between intermediate nodes. By definition, r_s corresponds with transition probabilities of arcs leading into s. When we multiple $\mathbf{P}$ by itself k times, we obtain:

$$\mathbf{P}_s^k=\left(\begin{array}{c|c}1 & \mathbf{0}\\ \hline \mathbf{r}_{ks} & \mathbf{Q}_s^k\end{array}\right) \tag{31}$$

where $\mathbf{r}_{ks}$ is the first column (without the first element). When the transition is repeated ad infinitum, all traffic should be absorbed into the destination, i.e. $\lim\limits_{k\to\infty}\mathbf{Q}_s^k=0$. By using this relationship, the probability that traffic destined to s traverses from i to j can be calculated as follows:

$$\mathbf{Q}_s^0+\cdots+\mathbf{Q}_s^k+\cdots=(\mathbf{I}-\mathbf{Q}_s)^{-1} \tag{32}$$

Note that $\mathbf{Q}_s$ can be written as

$$\mathbf{Q}_s=\begin{pmatrix}\mathbf{0} & \mathbf{Q}_{1s}\\ \mathbf{0} & \mathbf{Q}_{2s}\end{pmatrix}\begin{matrix}g\\ n-g-1\end{matrix} \qquad \begin{matrix}g & n-g-1\end{matrix} \tag{33}$$

Where g is the number of origins. Then,

$$\left(\mathbf{I}-\mathbf{Q}_s\right)^{-1}=\begin{pmatrix}\mathbf{I} & \mathbf{Q}_{1s}\left[\mathbf{I}-\mathbf{Q}_{2s}\right]^{-1}\\ \mathbf{0} & \left[\mathbf{I}-\mathbf{Q}_{2s}\right]^{-1}\end{pmatrix} \tag{34}$$

Let $\mathbf{o}_s$ be the vector of traffic produced at origin i destined to s. Then the traffic traversing intermediate node j ($j \notin R, j \notin S$), $\mathbf{b}_s$, can be obtained as follows:

$$\mathbf{b}_s=\mathbf{o}_s{}^t\mathbf{Q}_{1s}\left[\mathbf{I}-\mathbf{Q}_{2s}\right]^{-1} \tag{35}$$

Finally, arc traffic volumes $\mathbf{x}$ are calculated using $\mathbf{b}_s$ as:

$$\mathbf{x}=\sum_s \mathbf{x}_s=\sum_s\left(\mathbf{Q}_{1s} \quad \mathbf{Q}_{2s}\right)\begin{pmatrix}\mathbf{o}_s\\ \mathbf{b}_s\end{pmatrix} \tag{36}$$

The above discussion implies that once we obtain a set of transition probabilities between nodes by the minimum cost hyperpath search, the arc traffic volumes are given by Eq. (34) to (36).

REFERENCE

Akamatsu, T. (1996) Cyclic Flows, Markov Process and Stochastic Traffic Assignment, Transportation Research, 30B, 369-386.

Bell, M.G.H., Schmöcker, J.-D., Iida, Y. and Lam, W.H.K. (2002) Transit Assignment: An Application of Absorbing Markov Chains. In. Transportation and Traffic Theory in the 21st Century (Ed. Taylor, M.A.P.), 43-62.

Billi, C., Gentile, G., Nguyen, S. and Pallottino, S (2004) Rethinking the wait model at transit stops, Proceedings of TRISTAN-Workshop, Guadelupe

Cepeda, M., R. Cominetti, and M. Florian (2006) A Frequency-based Assignment Model for Congested Transit Networks with Strict Capacity Constraints: Characterization and Computation of Equilibria, Transportation Research, 42B, 437-459

Che-Hao, Chang, Yeou-Koung Tung and, Jinn-Chuang Yang (1994) Monte Carlo Simulation for Correlated Variables with Marginal Distribution, Journal of Hydraul., ASCE, 120(3), 313-331.

Chiriqui, C. and Pallotino, S. (1975) Common Bus Lines, Transportation Science, 9, 115-121

Der Kiureghian, A., and Liu, P. L. (1985) Structuaral reliability under incomplete probability information, Journal of Engrg. Mech., ASCE, 112(1), 85-104.

Fearnside, K. and Draper, D.P. (1971) Public Transport Assignment – A New Approach, Traffic Engineering and Control, 12, 298-299.

Gentile, G., Nguyen, S. and Pallottino, S. (2005) Route Choice on Transit Networks with Online Information at Stops, Transportation Science, 39(3), 289-297.

Kurauchi, F., Bell, M.G.H. and Schmöcker, J.-D (2003) Capacity Constrained Transit Assignment with Common Lines, Journal of Mathematical Modelling and Algorithms, 2-4, 309-327.

Larrain, H. and Munoz, J. C. (2008) Public Tranist Corridor Assignment Assuming Congestion Due to Passenger Boarding and Alighting, Networks and Spatial Economics, 8, 241-256.

Nguyen, S. and Pallotino, S. (1988) Equirlibrium Traffic Assignment for Large Scale Transit Networks, European Journal for Operation Research, 37, 176-186.

Newell, G. F. and R. B. Potts (1964) Maintaining a Bus Schedule, Proceedings of the 2nd ARRB Conference, 1, 388-393.

Nökel, K. and Wekeck, S. (2009) Boarding and Alighting in Frequency-based Transit Assignment, Proceedings of the 88th Annual Meeting of TRB, Washington, D.C., CD-ROM.

Schmöcker, J.-D, Bell, M.G.H. and Kurauchi, F. (2008) A Quasi-dynamic Capacity Constrained Frequency-based Transit Assignment Model, Transportation Research, 42B, 925-945.

Shimamoto, H., Kurauchi, F. and Iida, Y. (2005) Evaluation on Effect of Arrival Time Information Provision Using Transit Assignment Model, International Journal of ITS Research, 3, 11-18

Shimamoto, H., Kurauchi, F., Schmöcker, J.-D and Bell, M.G.H. (2009) Transit Assignment Model Considering the Inter-dependent of Each Line's Arrival, Proceedings of the 11th International Conference on Advanced Systems for Public Transportation, CD-ROM.

Spiess, H. and Florian, M. (1989) Optimal Strategies: A New Assignment Model for Transit Network, Transportation Research, 23B, 83-102.

Teklu, F., Watling, D. and Connors, R. (2007) A Markov Process Model for Capacity Constrained Transit Assignment, Transportation and Traffic Theory 17, edited by. R. E. Allsop, M. G. H. Bell and B. G. Heydecker, 483-505, Elservier.