Using Traffic Flow Data to Predict Bus Travel Time Variability Through an Enhanced Artificial Neural Network

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ABSTRACT

This paper aims at predicting bus travel time and its day-to-day variability using a range of independent variables including traffic flow data. Among many factors impacting bus travel time, existing prediction approaches have not considered a traffic measure making their predictions unresponsive to the time dependent fluctuations in traffic flow and dynamic changes in traffic congestion. In addition, existing methodologies have mainly predicted the average travel time by finding the average value in a range of travel times likely to happen when a certain set of input values is considered. However, little attention has been given to predict the spread of that range created by the stochasticity of determinant factors which reflects travel time variability.

This paper explains how an Artificial Neural Network (ANN) can be modified to predict the variance of a dependent variable. An integrated framework is then proposed which consists of two ANNs to predict both the average and variance of travel times. The proposed framework is developed on GPS based travel time data for a bus route in Melbourne, Australia, traffic flow data collected by the Sydney Coordinated Adaptive Traffic Systems (SCATS) loop detectors, and a measure of schedule adherence. The results demonstrate the value of traffic flow data in the prediction of bus travel time as well as the ability of the proposed method to provide fairly robust prediction intervals.

INTRODUCTION

Accurate predictions of travel time help operators in real time management strategies such as holding and expressing (Osuna & Newell, 1972, Fu & Yang, 2002), and off-line planning including fleet size planning and schedule design (Ceder, 2007). This information also enables passengers to better select departure times to minimize their waiting times.

Information about the variability in travel times also benefits operators and passengers. It assists operators in defining optimal slack times to maximize the on-time arrival performance of buses (Kimpel et al., 2004), and in determining the reliability of systems (Turochy & Smith, 2002). A reduction in travel time variability reduces passengers' anxiety caused by uncertainty in decision making about departure time and route choice (Bates et al., 2001, Lam & Small, 2001), which is why it is found as valuable (Sun et al., 2003), or even more valuable than a reduction in travel time (Bates et al., 2001).

Previous studies have identified a range of determinants of bus travel time primarily through examining data from Advanced Public Transportation Systems (APTS) such as Global Position Systems (GPS), Automatic Vehicle Location (AVL) and Automatic Passenger Counting (APC) systems. They include traffic flow at intersections (Abdelfattah & Khan, 1998, Chien et al., 2002), passenger demand at stops (Shalaby & Farhan, 2004), traffic accidents (Abdelfattah & Khan, 1998), weather conditions (Hofmann & O'Mahony, 2005), different bus and driver characteristics (Mishalani et al., 2008, Strathman & Hopper, 1993), route characteristics (Abkowitz & Engelstein, 1983, Ng & Brah, 1998), and the effect of drivers' timetable compliance on travel time (Lin & Bertini, 2004, Chen et al., 2005, Mazloumi et al., 2008).

Predicting transit arrival/travel times has been the focus for many existing studies. Table 1 summarizes existing studies with respect to their adopted methodologies. As seen, existing methodologies can be generally grouped into four categories: Regression models, Kalman filter models, Artificial Neural Network (ANN) models, and Analytical approaches. Table 1 also shows the variables utilized to predict bus arrival/travel time as well as the data source used. Only two studies (Abdelfattah & Khan, 1998, Chien et al., 2002) have used traffic measures in their predictions. However, in both cases, the traffic data were sourced from simulation modelling. No previous study has adopted a real world traffic flow measure to predict bus travel time; therefore, their predictions may be unresponsive to the time dependent variations in traffic flow and dynamic changes in traffic congestion.

Due to the stochasticity of bus travel time determinants, there may be a range of values corresponding to a certain set of input values. However, existing studies have primarily predicted the average in that range, and no research has been directed to quantify the spread of that range by predicting a measure of variability in travel times. This paper focuses on day-to-day variability which is the variation between the travel times of similar journeys made at the same time on different days.

This paper proposes an integrated framework to predict bus travel time and its variability based on a range of independent variables including traffic flow data collected by the Sydney Coordinated Adaptive Traffic Systems (SCATS) loop detectors. The results are then used to construct a prediction interval corresponding to each input value set. Next section explains the proposed predictive framework, and its application in a case study is then presented. A closing summary, conclusions, and future research directions are included in the final section of the paper.

Table 1: The explanatory variables used in different studies with respect to the adopted methodology.

Study	Route characteristics	Passenger demand/dwell time	Temporal variables/ scheduling	Traffic measures	Schedule adherence	Bus progress data	Historical travel times/speed/ trajectory	weather	Data source
Regression models									
Abdelfattah and Khan (1998)	✓	✓		✓					Simulation
Patnaik et al. (2004)*	✓	✓	✓						APC
Artificial neural network models									
Kalaputapu and Demetsky (1995)			✓		✓				AVL
Jeong and Rilett (2004)		✓			✓	✓			AVL
Park et al. (2004)*						✓	✓		GPS
Chen et al. (2007)		✓	✓					✓	APC
Kalman filter models									
Chien et al. (2002)		✓		✓		✓			Simulation
Shalaby and Farhan (2004)		✓					✓		AVL-APC
Chen et al. (2004)			✓			✓		✓	APC
Dailey et al. (2001)						✓	✓		AVL
Chen et al. (2005)			✓			✓			AVL-APC
Analytical models									
Lin and Zeng (1999)			✓		✓	✓			GPS
Lin and Bertini (2004)						✓	✓		GPS
Sun et al. (2007)						✓	✓		GPS
Mishalani et al. (2008)							✓		GPS

* The study developed a bus arrival/travel time estimation model

MODEL DESCRIPTION

ANN models have shown a promising ability to solve nonlinear and complex problems (Hagan et al., 1996), which is why they have been widely adopted in modelling travel times. The proposed integrated framework in this paper consists of two components of ANN_{avg} and ANN_{var}. Based on a certain set of input values, the former predicts the average and the latter predicts the variance of travel times that may happen between two consecutive timing point stops (Figure 1), where arrival/departure times are recorded to check consistency with timetables. Prior to each bus departs from the upstream timing point stop, traffic flow data collected by the SCATS loop detectors in the last k minutes along with information about other important variables are sent to the framework for prediction purposes. The outcomes of the proposed framework can then be used to obtain a prediction interval and to construct the probability distribution of travel times on the basis of certain input values.

Among various available structures for ANNs, the study selects fully connected feedforward ANN which has proven its capability to solve complex transportation problems (Hagan et al., 1996). The adopted networks comprise three layers: Input, Hidden and Output layers. Only one hidden layer is used since it is shown sufficient to closely map any relationship (Jain & Nag, 1997). Sigmoid activation functions are used in the hidden layer neurons, whilst linear ones are used in the input and output layer neurons.

During the training of an ANN_{avg}, the model parameters are adjusted to minimize a predefined objective function, which can be defined as the sum of squared errors of the predictions over the set of N training examples as formulated in Equation 1:

$$\frac{1}{2}\sum_{n=1}^{N}[\hat{\mu}_y(x^n;w)-y^n]^2 \qquad (1)$$

Where

$\hat{\mu}_y(x^n;w)=E(y \mid x^n)$: the estimated average of y conditioned on x^n,

y^n: the n^{th} example of dependent variable,

x^n: a vector including the values of input variables, which is corresponding to y^n and

w: a vector containing the values of the model adjusted parameters.

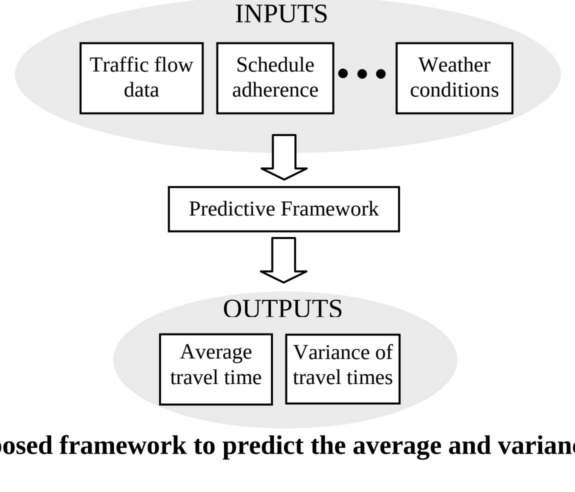

Figure 1: The proposed framework to predict the average and variance of travel times.

In the problem of bus travel time prediction, y is bus travel time, and the vector of input values (x^n) may contain the data of variables believed to affect bus travel time such as traffic flow, weather conditions, passenger demand, etc.

ANNs might be trapped in an overfitting problem, where they generalize poorly given unseen examples. This generally occurs when the model parameters are too large. This study adopts the Bayesian Regularization training method (Bishop, 1995), which minimizes a combination of prediction error (Equation 1) and parameters, and then determines the correct combination so that the model generalizes well.

The variance of y conditioned on input vector x^n , $Var(y \mid x^n)$, can be calculated using $Var(y \mid x^n) = E[(E(y \mid x^n) - y)^2 \mid x^n]$. Considering that $E(y \mid x^n)$ is estimated by an ANN with an error function presented in Equation 1, replacing y in Equation 1 with $(E(y \mid x^n) - y)^2$ gives rise to $E[(E(y \mid x^n) - y)^2 \mid x^n]$ instead of $E(y \mid x^n)$. Therefore, assuming $\hat{\mu}_y(x^n; w)$ to be equal to $E(y \mid x^n)$, the ANN_{var} estimates the variance using Equation 2 as the error function (Bishop, 1995).

$$\frac{1}{2}\sum_{n=1}^{N}[\hat{\sigma}_y^2(x^n;u) - [\hat{\mu}_y(x^n;w) - y^n]^2]^2 \tag{2}$$

Where

$\hat{\sigma}_y(x^n;u)$: the estimated variance of y conditioned on input x^n , and

u : a vector containing the values of the ANN_{var} model adjusted parameters.

Assuming that travel times follow a Normal distribution and using $E(y \mid x^n) \pm Z_\alpha \times \hat{\sigma}_y(x^n;u)$, the prediction interval for y can now be constructed. For example, to construct the 95% prediction interval, Z_α equals to 1.96. In our problem, this prediction interval is interpreted as a range where travel times may occur with 95% probability when certain values of input variables are considered. This range takes into account the fact that travel times vary, even if the same values for input values are taken, as a result of uncertainties in factors such as dwell time and signal delay experienced by different buses.

CASE STUDY

The study data were supplied from bus route 246 in inner Melbourne, Australia. Figure 2 is a schematic presentation of this test bed, which is about eight kilometres in length, and comprises four sections demarcated by five timing point stops including Clifton Hill, Johnston St., Bridge Rd., Toorak Rd., and St Kilda junction. The sections are relatively equal in length, and experience high levels of passenger demand and traffic flow especially in peak hours. Bus headways vary from ten minutes in peak hours to about half an hour in the off peak. Buses operate in mixed traffic and there is no separate lane allocated to buses.

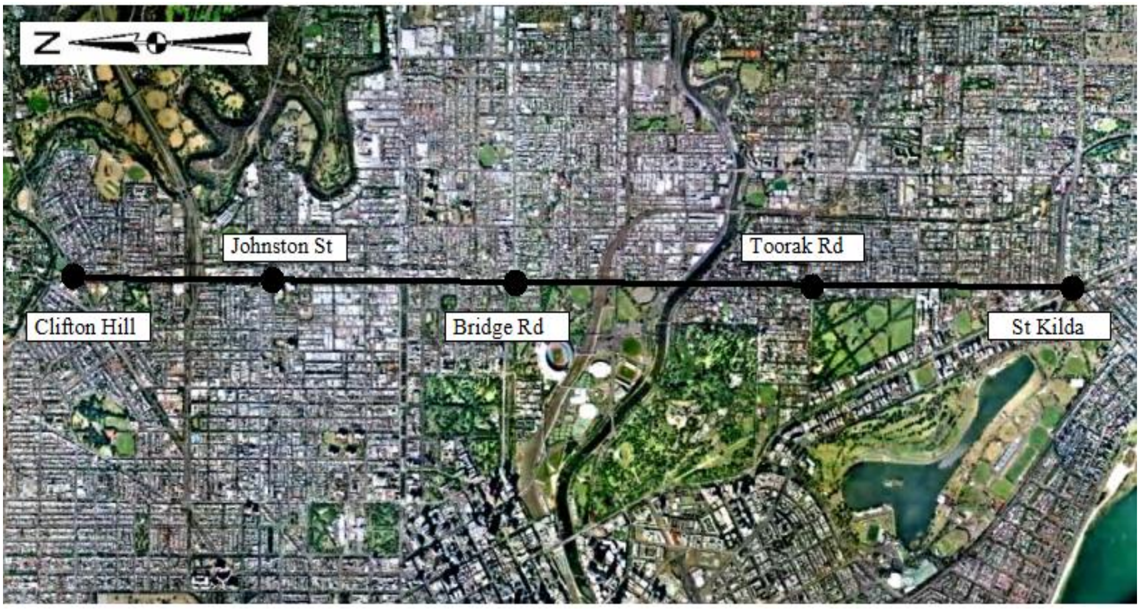

Figure 2: A schematic presentation of the study test bed.

A collection of the buses over this route were equipped with GPS devices recording bus arrival/departure times corresponding to timing point stops. A six month travel time dataset (starting from February 2007) was supplied for the research, which includes about 1,800 travel time observations corresponding to each section between two consecutive timing points. To show how travel times might vary over different days, Figure 3 presents the distribution of travel times across the day over one of the sections (Johnston St. to Bridge Rd). Accordingly, travel times vary considerably over different days. For instance, travel times at 2:00pm range from about 200 seconds to just over 500 seconds over different days. This might be attributable to many factors such as variations in passenger demand and traffic flow over different days as well as various signal delays experienced by different buses, and dissimilar driving behaviours. Figure 3 also suggests that travel times can be classified with respect to four time periods: AM peak (7am-10am), Inter peak (10am-4pm), PM peak (4pm-7pm) and Off peak (before 7am and after 7pm).

Signalized intersections over each section are equipped with the SCATS loop detectors collecting traffic counts and Degree of Saturation (DS) values. For each signalized intersection, traffic information over different signal cycles were collected by the detectors, and average traffic counts and DS values over different signal cycles are derived.

Alternative weather conditions have been reported to affect bus travel time by influencing drivers' performance and vehicle headways (Hofmann & O'Mahony, 2005). The amount of rain (in millimetres) that fell over the corresponding hour is also explored if it affects bus travel times.

Schedule adherence at each upstream timing point can affect bus travel time to the next timing point stop (Mazloumi et al., 2008). Early buses show higher travel times compared to those that are late relative to the scheduled travel time. This variable is also examined for inclusion in the model, and is quantified by subtracting observed arrival time from scheduled arrival time at each timing point stop.

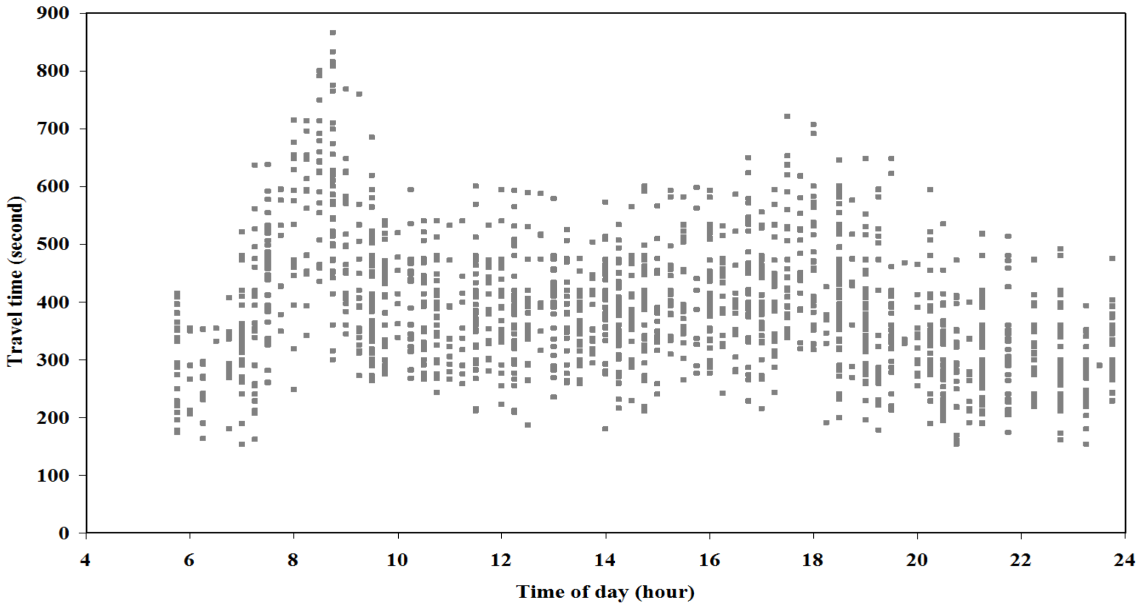

Figure 3: Travel time observations of the section Johnston St. to Bridge Rd.

Input variable selection

To select input variables for prediction, the ANalysis Of VAriance (ANOVA) technique is used, which identifies the variables that best explain the variability in a dependent variable. In this study, the dependent variable is bus travel time for each section between two consecutive timing point stops, and variables whose effects on bus travel times are explored include the SCATS traffic flow data, weather conditions, and schedule adherence measured at the upstream end of each route section.

The SCATS data for each signalized intersection include traffic counts and DS values averaged over a predefined aggregation period (k) prior to the departure of each bus from the upstream timing point. To examine the effect of different aggregation period lengths on the level of variability explained, alternative periods of 2, 15, 30 and 60 minutes are considered. When k equals to 15 minutes, the average of traffic counts and DS values over the signal cycles of the previous 15 minutes are taken into account for the analysis. Table 2 reports the portion of the variability explained by each variable in terms of the adjusted R^2 .

As the results suggest, although both traffic counts and DS values affect travel times, use of DS values acts to provide a more significant explanation of the variability of travel time. This could be because traffic counts describe demand fluctuations, whereas DS values capture variations caused by changes in supply (signal cycle/green time) as well as those in demand. In addition, when DS values are combined with traffic counts, minor improvements can be seen in the corresponding adjusted R^2 . As a result, only DS values are adopted to account for dynamic changes in traffic flow.

In each route section, when the weather variable (rainfall) is the only variable examined, the adjusted R^2 becomes zero implying that rainfall does not contribute to the variability of the travel time values. This conclusion might be related to the low number of rainfall observations made during the analysis period. Schedule adherence is associated with a positive adjusted R^2 suggesting that this variable should be adopted for predicting travel time.

Table 2: Proportion of travel time variation explained by different variables in terms of adjusted R^2.

Section	Aggregation period k (minute)	Selected input variable*					
		Traffic Count (TC)	Degree of Saturation (DS)	TC & DS	Weather (W)	Schedule Adherence (SA)	DS, W, & SA
Clifton Hill to Johnston	2	0.49	0.50	0.50	0.00	0.02	0.51
	15	0.51	0.52	0.52			0.53
	30	0.44	0.45	0.46			0.47
	60	0.35	0.35	0.34			0.35
Johnston to Bridge	2	0.26	0.38	0.38	0.00	0.02	0.40
	15	0.28	0.39	0.40			0.42
	30	0.23	0.32	0.32			0.34
	60	0.19	0.28	0.29			0.31
Bridge to Toorak	2	0.47	0.49	0.50	0.00	0.03	0.51
	15	0.54	0.55	0.55			0.57
	30	0.50	0.51	0.52			0.54
	60	0.45	0.45	0.46			0.49
Toorak to St Kilda	2	0.21	0.21	0.22	0.00	0.02	0.23
	15	0.22	0.22	0.24			0.25
	30	0.18	0.18	0.18			0.20
	60	0.17	0.17	0.17			0.18

* All two and three way interaction terms are considered

Different aggregation period lengths k for collecting the SCATS data show differing influences on the level of variability explained. The results suggest that the optimal k equals to 15 minutes. In larger intervals, travel time values are less related to the selected variables. In shorter intervals, say 2 minutes, sometimes two signal cycles fall into the interval, while in some other occasions only one signal cycle corresponds to the interval. This causes non-smooth average traffic counts/DS values in successive intervals.

Based on the preceding results, the explanatory variables adopted include the average of DS values (in the last 15 minute interval prior to the departure of the bus from the upstream timing point stop) along with schedule adherence.

Model development

To train the ANNs, the travel time dataset of each section randomly split up into a training dataset (80% of data) and a testing dataset (20% of data). To develop an ANN, the number of its hidden layer neurons has to be determined. This task is a trade-off between model complexity and its generalizing ability. More hidden neurons may empower the model to better describe the relationship between input and output values, which may make the model more prone to overfit the data and hence have poor generalization. On the other hand, a simple model may not be able to adequately describe the complexity of a problem.

The study adopts a conventional trial and error procedure using different numbers of hidden neurons for each neural network. Several network structures with differing hidden layer neuron numbers are tested, and those with the least error are selected. For each model, the Route Mean Squared Error (RMSE) is reported, which is calculated using Equation (4):

$$RMSE = \sqrt{\frac{\sum_{n=1}^{M}(Y_n(o) - \overline{Y}_n(o))^2}{M}} \qquad (4)$$

where *M* is the total number of input-output pairs in the testing dataset, $Y_n(o)$ is the n^{th} observed travel time value, and $\overline{Y}_n(o)$ is the corresponding predicted travel time. The results with respect to different route sections are reported in Table 3. This Table also shows the number of input variables used to predict each section travel time. The number of input variables minus one reflects the number of signalized intersections within each route section.

Table 3: The summary of training different the ANN_{avg} and ANN_{var} models.

Dependent variable	Section	Number of input variables	Number of training examples	Number of hidden layer neurons	RMSE*
Average travel time (ANN_{avg})	Clifton Hill to Johnston	5	1390	3	82
	Johnston to Bridge	6	1400	2	88
	Bridge to Toorak	6	1400	2	73
	Toorak to St Kilda	6	1300	3	94
Variance of travel times (ANN_{var})	Clifton Hill to Johnston	5	1390	2	14062
	Johnston to Bridge	6	1400	2	11790
	Bridge to Toorak	6	1400	2	7436
	Toorak to St Kilda	6	1300	3	19508

* The unit of RMSE for ANN_{avg} models is seconds and for ANN_{var} models is seconds^2

Model evaluation

As noted, for each route section, given a certain set of values for input variables (DS values and schedule adherence), travel times (which may occur) will form a probability distribution and each observed travel time is a randomly drawn value from the distribution. The ANN_{avg} predicts the average, whereas the ANN_{var} predicts the variance of this distribution. These two give rise to a prediction interval, say 95% prediction interval, corresponding to each input value set. This interval reflects a range where individual travel times may occur with 95% probability, and might have resulted from different buses experiencing different dwell times and signal delays.

For illustration purposes, an arbitrary selected trip is used as an example to demonstrate the outcomes of the proposed prediction model. Figure 4 shows information about this bus trip over alternative route sections, including the observed and predicted (average) travel times together with the lower and upper bounds of the 95% prediction interval. Each observed travel time could have been randomly drawn from a distribution of travel times that are likely to occur based on a certain set of input values. Hypothesized distributions are also schematically drawn in Figure 4. The ANN_{avg} model predicts the average in this distribution, which does not necessarily always correspond to the observed travel time (for instance, Bridge to Clifton section). The ANN_{var} model determines the spread of the distribution (e.g. the distance between the lower and upper bounds in each prediction interval) by predicting the distribution variance.

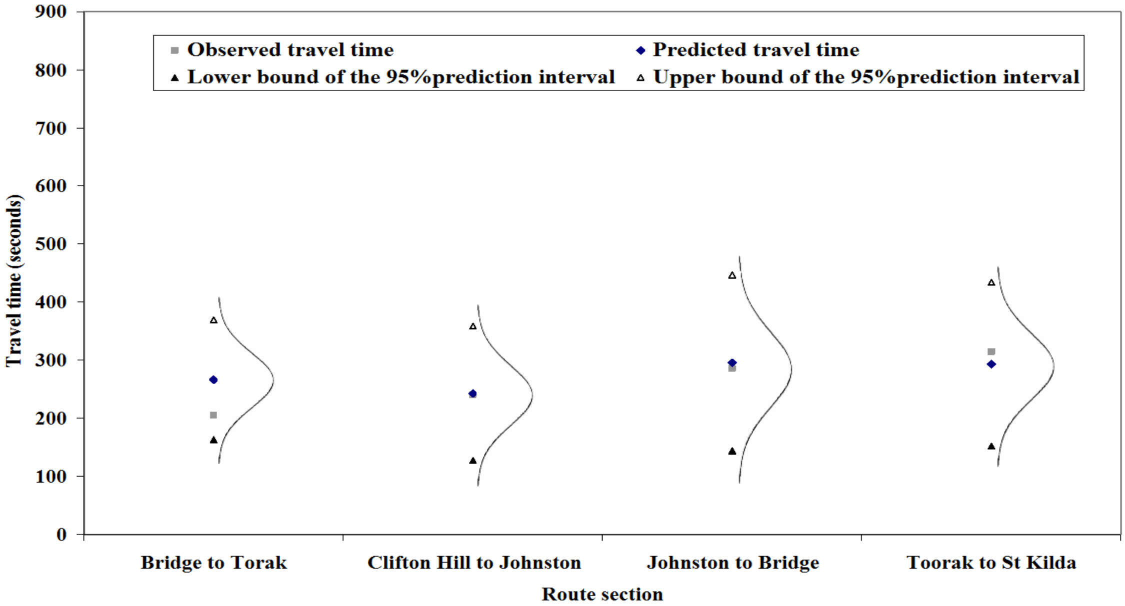

Figure 4: A sample of travel time observations, predicted travel times and prediction intervals.

For better presentation of the results, Figure 5 shows the values of predicted average travel time, the lower bound and the upper bound of each prediction interval aggregated in 15 minute intervals across the day (for the section Johnston St. to Bridge Rd.). It can be seen that unlike the average travel time, which is not able to explain the whole variability in travel times, the 95% prediction interval length provide a more reliable outcome by fairly representing the range of possible travel time values. This range can benefit planners in offline designs such as defining slack times in timetables as well as in real time strategies such as holding and express services. The provision of this range to passengers can also act to ease stress and anxiety associated with variability in travel time.

Further insights into model performance can be obtained by calculating the Prediction Interval Coverage Probability (PICP) and comparing it to the expected coverage probability. It is expected that the 95% prediction intervals encompass the observed travel times at 95% of the occasions. Table 4 reports the PICP of each route section by time period. As seen, except for minor deteriorations in the AM peaks, in other periods, the probability range covers the observed travel times well. In the AM peaks, the coverage probability values are less than 95 suggesting that the prediction intervals underestimate the actual/observed 95% intervals. Further variables (such as passenger demand) impacting travel times may exist in the AM peaks, whose unavailability makes the predictive model to underestimate the actual observed intervals.

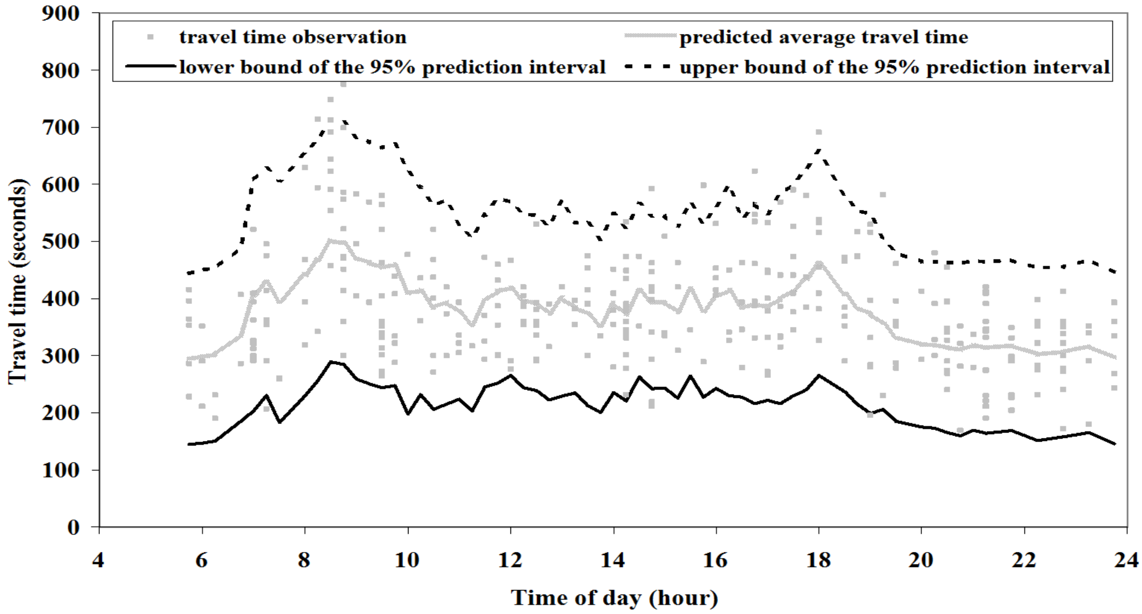

Figure 5: Travel time observations of the four sections along with predictions across the day

Table 4: The obtained coverage probability of the 95% prediction interval (percent).

Section	AM peak 7am - 10am	Inter peak 10am - 4pm	PM peak 4pm -7pm	Off peak 7pm -7am
Clifton Hill to Johnston	74	95	89	96
Johnston to Bridge	93	96	94	97
Bridge to Toorak	87	96	92	92
Toorak to St Kilda	87	90	98	95

Note: the expected coverage probability is 95%,

SUMMARY AND CONCLUSION

Despite the importance of traffic flow in bus travel time prediction, existing studies have not considered real world traffic flow data in predictions. Therefore, their models may not be able to effectively consider the dynamic changes of traffic flow in predictions. In addition, existing models mainly predict average travel time based on a given set of input values, and no research has been conducted on predicting the variance of travel times (travel time variability) and on providing a prediction interval rather than a prediction point.

This paper proposed an integrated framework consisting of two ANN models to predict both the average and variance of travel times. A part of a bus route in Melbourne, Australia, was the test bed of this study. Using the ANOVA technique, DS values collected by SCATS in the last 15 minutes prior to the departure of a bus from the upstream timing point and schedule adherence were found to have the greatest impact on the variation of travel time values, and hence were selected to predict travel times to the next timing point stops. It was found that the amount of rainfall did not contribute in the variation of travel times, and therefore was not selected for prediction purposes. The use of a traffic measure ensures that the model effectively responds to the dynamic changes in traffic flow when making predictions. The results suggest that unlike the predicted average travel times which are not able to explain the whole variability in travel times, prediction intervals could fairly construct the ranges where the travel times were most likely to occur.

The proposed modified ANN can be used to predict the variability of any variable with stochastic determinants. The robust ability of the proposed structure in quantifying the uncertainty in predictions suggests a wide range of applications in transportation engineering. In the context of public transport, the proposed framework can be a promising means to help operators in developing timetables and defining slack times to maximize on time performance. The outcomes of the proposed framework can be also disseminated to passengers acting to reduce their anxiety caused by uncertainty in travel decision making.

This research has explored the effect of traffic flow, schedule adherence and weather conditions on bus travel time. Expanding the analysis to include data on other important variables such as passenger demand could be a promising direction for future research.

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