TESTING THE LINEAR INVERSE POWER TRANSFORMATION LOGIT MODE CHOICE MODEL

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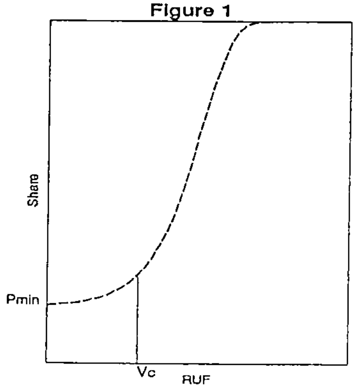
INTRODUCTION

This paper deals with three very practical problems of mode choice modelling: threshold effects, modal captivity and consistency with the independence from irrelevant alternatives axiom (IIA), which excludes the possibility of complementarity among alternatives. The first two are illustrated in Fig. 1, where the share of the mode is on the y-axis and the Representative Utility Function (RUF) corresponding to this mode is on the x-axis. Intuitively, a threshold effect occurs when the utility reaches a critical value (V_c), after which a further increment provokes very substantial growth of the share of the mode. In this paper, threshold effects result from an asymmetry of the reaction curve, a property not taken into account with well known Logit and Probit models.

Figure 1



Another practical issue is the possibility of having non zero observed shares for very unattractive modes. In Fig. 1 this means that the minimum share is P_{min} no matter how poor the level of service is. So far, the only model that handles captivity or loyalty towards a mode is the dogit model (Gaudry and Dagenais, 1979). The IPT model developed by Gaudry (1981) takes into account both captivity and threshold effects. In this paper, we study the empirical and numerical properties of one member of the IPT class, the Linear IPT model.

Because of the way in which the IPT model deals with threshold and captivity effects, it allows as a by-product the use of full or complete representative utility functions for <u>each</u> mode: not only can the characteristics of <u>all</u> modes can be entered in <u>each</u> utility function, but the socioeconomic variables as well. The enriched specifications of representative utility functions are made possible by either the asymmetric shape or the variable tail thickness of the reaction function. The enriched specification enables complementarity among alternatives, is unconstrained by the IIA assumption and, lastly, permits the estimation of a generic socioeconomic specification unaffected by the choice of a reference mode.

In this paper, a completely full form of the utility functions is specified and estimated, seemingly for the first time in mode choice analysis. Furthermore, the interactions among such enlarged specifications of utility functions, asymmetry of the reaction functions and modal captivity are analyzed. Statistical tests are carried out on a binomial case using urban data in order to examine both the theoretical properties of estimators and the empirical gains of the Linear IPT Logit model over its simpler linear Logit root. All three additional dimensions are found to be of practical use.

1. THE LIN-IPT-LOGIT MODEL

Formally, the Linear IPT-Logit model is written

$$P_i = \Psi_i \,/\, \left(\sum_{j=1}^{M} \Psi_j\right), \text{ where} \tag{1}$$

$$\Psi_i = (\lambda_i \exp(V_i) + 1)^{(1/\lambda_i)} + \mu_i, \qquad \lambda_i \neq 0 \tag{2-A}$$

$$= \exp(\exp(V_i)) + \mu_i, \qquad \lambda_i \to 0 \tag{2-B}$$

with $\lambda_i \geq 0$ and $\mu_i \geq -1$, (3)
where P_i is the share of the i^{th} of M alternatives and V_i is called the representative utility function (RUF) of alternative *i*. The Logit model is a nested special case and is obtained when $\lambda_i = 1, \mu_i = -1, \forall i$. It is useful to insure by (3) that computed shares be non-negative and smaller than one for all potential values of V_i's .

1.1. Full representative utility functions, IIA and complementarity

In general, the representative utility function (V_i) contains two categories of variables:
- service level or network variables (N_i) that differ across the M modes or alternatives;
- socioeconomic variables (S) that do not vary across alteratives.[1]

The Lin-IPT-L model makes it possible to enlarge the representative utility function of each mode with all the variables: level of services variables of all modes as well as socioeconomic variables. The reason for this is that a term common to all utility functions cannot be factored out any more: this is obvious in (2) when the λ_i differ from 1 but is also true if they are equal to one because of the remaining 1's and μ's . The application of the Lin-IPT-L form therefore solves the underidentification problem of the Logit.

We now define various specifications of the representative utility function which exploit this property, starting with the simplest Logit format. For simplicity, the number of modes is assumed equal to three.

1.1.1. Alternative-Specific Logit (A-S. L)

The alternative-specific Logit specification of the representative utility functions ($\lambda_i = 1, \mu_i = -1$) is given in Table 1. The RUF are clearly incomplete since network variables only appear once and measure the direct effect of network variables of a specific mode on the corresponding utility function. Moreover, socioeconomic variables only appear in the first two utility functions, because $\beta_1^1, \beta_2^2, \beta_3^3, \beta_4^1$, and β_4^2 are the only parameters that can be identified: here the third mode is assumed to be the reference mode.

1.1.2. Alternative-Specific Logit Extended (A-S, LE)

When asymmetry and/or captivity are present, the Lin-IPT-L model enables to "extend" the above specification by including socioeconomic variables in the utility functions of all the alternatives, as indicated in Table 2. We shall refer to this specification as the alternative-specific "Logit Extended" (LE) specification. The Logit specification is restrictive since it means that β_4^3 is zero or that only the difference of $(\beta_4^2 - \beta_4^3)$ and $(\beta_4^2 - \beta_4^3)$ can be estimated.

The estimation of the extended utility function is interesting since it renders possible to identify the effects of socioeconomic variables on each utility function. For instance, as income increases the utility of each mode may increase but, since the shares must add-up to one, at least one share must increase and another must decrease. Therefore, income effects on the shares may be misleading as one could conclude that modes are inferior. We argue that a more appropriate measure of the effect of one socioeconomic variable on one mode is through its effects on the RUF. This means that it could be possible for a transportation mode to be a normal or non-inferior good, when measured through its effect on the RUF, even though its share decreases as income increases.

1.1.3. Alternative-Specific Universal Logit (A-S, UL)

Again, if asymmetry and/or captivity are present, the Lin-IPT-L model enables to fill Table 1 with network variables in each utility function as shown in Table 3. We call this specification of the RUF the "Universal Logit" (UL) specification. The UL specification ensures that the Lin-IPT-L model is no longer consistent with the IIA assumption since relative shares now depend upon service levels of all modes.

As a consequence of the UL specification, complementarity among modal shares may arise, which means that improvement of the level of service of one mode may not necessarily reduce the share of the other modes, as occurs with the Logit model. To see this, let X_k be an attribute of mode i and be included not only in the vector of network variables N_i, but in the others as well $(N_j, j = 1, .., M, j \neq i)$; the elasticity of the mode i and j with respect to X_k are for own and cross elasticities, respectively:[2]

$$\eta_k^i \equiv \frac{\partial P_i}{\partial X_k}\frac{X_k}{P_i} = \left(\beta_k^i \Psi_i^* \left(\sum_{l=1}^{M} \Psi_l\right) - \Psi_i \left(\sum_{l=1}^{M} \Psi_l^* \beta_k^l\right)\right) X_k \Big/ \left(\left(\sum_{l=1}^{M} \Psi_l\right) \Psi_i\right) \qquad (4-A)$$

$$\eta_k^j \equiv \frac{\partial P_j}{\partial X_k}\frac{X_k}{P_j} = \left(\beta_k^j \Psi_j^* \left(\sum_{l=1}^{M} \Psi_l\right) - \Psi_j \left(\sum_{l=1}^{M} \Psi_l^* \beta_k^l\right)\right) X_k \Big/ \left(\left(\sum_{l=1}^{M} \Psi_l\right) \Psi_j\right) \qquad (4-B)$$

where, $\Psi_i^* = (\Psi_i - \mu_i)^{(1-\lambda_i)} \exp(V_i)$. $\qquad (5)$

First, notice how, if $\lambda_i = 1$, and $\mu_i = -1$ for $\forall i$, $\Psi_i^* = \Psi_i = \exp(V_i)$, and $\beta_k^j = 0$, for $i \neq j$, then η_k^i and η_k^j reduce to the well-known formulas for the Logit model:

$$\eta_k^i = \beta_k^i (1-P_i) X_k \quad \text{for the own elasticities and} \qquad (6-A)$$

$$\eta_k^j = -\beta_k^i P_i X_k \quad \text{for the cross elasticities} \qquad (6-B)$$

From (6), modes i and j are necessarily substitutes in the Logit model given the sign of β's : (6-B) is necessarily of a sign opposite to that of (6-A). Moreover the cross-elasticity is the same for all j modes. Now two properties are of special interest.

Complementarity. In the real world, cross elasticities may be positive or negative, may differ among mode pairs considered. Two modes may even be substitute or complements depending on the level of the variables. For instance bus and car may be substitutes, if travel time by car is relatively low but may be complements as travel time by car becomes relatively high. Depending upon the conditions in a specific market, complementarity or substitution may arise.

All the preceding cases are possible with the Lin-IPT-L model. From (4-B), if the number of modes is greater than two, η_k^j may be positive or negative and may vary with i or j. Moreover, η_k^j may be negative (positive) with low value of X_k but positive (negative) with high value of X_k since Ψ and Ψ^* functions are not, in general, independent of X_k .

Rejection of the IIA property. One important limitation of the Logit model is its inability to yield a differentiated reaction from existing modes following the introduction of a new mode. A new mode getting 5% of the market means that the shares of the existing modes have all been reduced by 5%. As shown by equation (7)[3] if RUF does not have a UL specification, then the RUF will be unaffected by the introduction of the new mode M+1, and the result will follow. But with service variables of all modes in each RUF then the RUF are modified by the introduction of a new mode and a differentiated reaction of modes is now possible, as in equation (8). This is not surprising in view of the fact that cross elasticity patterns are different for each mode pair considered.

$$\frac{(P_i^B - P_i^A)}{P_i^B} = 1 - \frac{P_i^A}{P_i^B} = \Psi_{M+1} \;/\; \left(\sum_{j=1}^{M+1} \Psi_j\right) = P_{M+1} \quad ,\text{if } \Psi_i^A = \Psi_i^B, \quad \forall i \qquad (7)$$

$$= 1 - \left(\Psi_i^A \;/\; \left(\sum_{j=1}^{M+1} \Psi_j^A\right)\right)\left(\sum_{j=1}^{M} \Psi_j^B \;/\; \Psi_i^B\right) \quad ,\text{if } \Psi_i^A \neq \Psi_i^B, \quad \forall i \qquad (8)$$

1.1.4. Alternative-Specific Universal Logit Extended (A-S, ULE)

Combining the two preceding generalizations, one obtains the ULE specification, which admits of complements, is generally not consistent with IIA, and clearly does not exhibit underidentification of parameters associated with socioeconomic variables or mode-specific constants.

1.1.5. Abstractness and specificity of the utility function

So far we have defined the utility function by assuming that explanatory variables N_i and S have alternative-specific influences on the RUF. If one is interested in the potential effect of a new mode, an abstract or generic specification may be required. Table 5 describes the general form of a generic spefication for the Logit model. It is readily observable that socioeconomic variables are not "really" generic. One well-known problem

Table 1. Alternative-Specific Logit (A-S, L)	Utility function	Network variables		Socioeconomic variables
		Direct	Cross	
	$V_1 =$	$\beta_1^1 N_1$ +		$\beta_4^1 S$
	$V_2 =$	$\beta_2^2 N_2$ +		$\beta_4^2 S$
	$V_3 =$	$\beta_3^3 N_3$		

Table 2. Alternative-Specific Logit Extended (A-S, LE)	Utility function	Network variables		Socioeconomic variables
		Direct	Cross	
	$V_1 =$	$\beta_1^1 N_1$ +		$\beta_4^1 S$
	$V_2 =$	$\beta_2^2 N_2$ +		$\beta_4^2 S$
	$V_3 =$	$\beta_3^3 N_3$ +		$\beta_4^3 S$

Table 3. Alternative-Specific Universal Logit (A-S, UL)	Utility function	Network variables		Socioeconomic variables
		Direct	Cross	
	$V_1 =$	$\beta_1^1 N_1$ +	$\beta_2^1 N_2 + \beta_3^1 N_3 +$	$\beta_4^1 S$
	$V_2 =$	$\beta_2^2 N_2$ +	$\beta_1^2 N_1 + \beta_3^2 N_3 +$	$\beta_4^2 S$
	$V_3 =$	$\beta_3^3 N_3$ +	$\beta_1^3 N_1 + \beta_2^3 N_2 +$	

Table 4. Alternative-Specific Universal Logit Extended (A-S, ULE)	Utility function	Network variables		Socioeconomic variables
		Direct	Cross	
	$V_1 =$	$\beta_1^1 N_1$ +	$\beta_2^1 N_2 + \beta_3^1 N_3 +$	$\beta_4^1 S$
	$V_2 =$	$\beta_2^2 N_2$ +	$\beta_1^2 N_1 + \beta_3^2 N_3 +$	$\beta_4^2 S$
	$V_3 =$	$\beta_3^3 N_3$ +	$\beta_1^3 N_1 + \beta_2^3 N_2 +$	$\beta_4^3 S$

Table 5. Generic Logit (G, L)	Utility function	Network variables		Socioeconomic variables
		Direct	Cross	
	$V_1 =$	$\gamma_1 N_1$ +		$\gamma_3 S$
	$V_2 =$	$\gamma_1 N_2$ +		$\gamma_3 S$
	$V_3 =$	$\gamma_1 N_3$		

Table 6. Generic Universal Logit Extended (G, ULE)	Utility function	Network variables		Socioeconomic variables
		Direct	Cross	
	$V_1 =$	$\gamma_1 N_1$ +	$\gamma_2 (N_2 + N_3) +$	$\gamma_3 S$
	$V_2 =$	$\gamma_1 N_2$ +	$\gamma_2 (N_1 + N_3) +$	$\gamma_3 S$
	$V_3 =$	$\gamma_1 N_3$ +	$\gamma_2 (N_1 + N_2) +$	$\gamma_3 S$

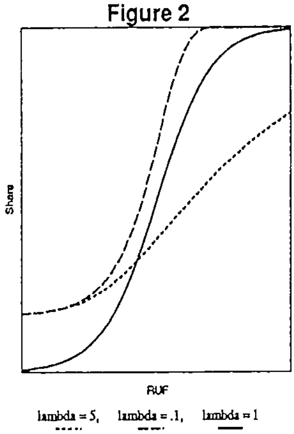
related with this specification is that estimated γ coefficients are not invariant with respect to the mode of reference, in this example the third mode. This problem did not occur in alternative-specific logit formats. Here, the underidentification of parameters associated with variables common to all alternatives, combined with the further requirement that the β_4^1 and β_4^2 of Table 1 be equal, yields γ parameters that depend on the mode of reference (except in the two-alternative case).

Again, if asymmetry and/or captivity are present, the Lin-IPT-L model enables to define an abstract or generic version of the Universal Extended Logit of the RUF as shown in Table 6.[4] The generic version of the ULE is completely free from an *ad hoc* choice for the reference mode.

1.2. Asymmetry and modal captivity

The role of the λ and μ parameters can be analyzed with the reaction curve. The parameters λ and μ determine the asymmetry of the reaction function and the captivity level, respectively. In Figure 2, three different reaction functions are drawn. The continuous line denotes the reaction curve of the Logit model: its sigmoid shape has an inflection point where modal shares are equal. If the λ parameter is smaller than one, the reaction curve becomes steeper or introduces the threshold effect mentioned above. The share is less sensitive to the RUF if λ is greater than one. The minimum share of a mode is different from zero, and a captivity effect is present, if μ is greater than -1. We now present a way to measure both the asymmetry of the reaction curve and the level of captivity.

Figure 2



1.2.1. Modal captivity

The modal captivity of mode i is defined as the modal share (P_{min}) with a very poor level of service ($V_i \to -\infty$) or, more formally, as a non-zero limit of the share:

$$P_{\min}(i) \quad = \quad (1+\mu_i) \quad / \quad \left(1+\mu_i+\sum_{\substack{j=1 \\ j\neq i}}^{M}\Psi_j\right) \quad . \tag{9}$$

It is clear that, in practice, captivity is a limit concept and that its presence can be explained by the absence of various explanatory variables in the V_i functions.[5]

1.2.2. Asymmetry measure

Although the presence of asymmetry is quite intuitive, a more formal definition is useful. First note that, on the x-axis in Fig.2, we show the RUF and are therefore "in utility space". We might as well have shown any variable contained in RUF, for instance a service level, and presented the curve "in characteristics space". Had we shown the service level, however, it would not have been true that asymmetry is not possible with a Logit or Probit model: indeed, non linear transformations, such as the natural logarithm of a variable present in a linear-in-parameters RUF, imply an asymmetric response for these models in characteristics space -but not in utility space.

Let (RUF_f) be the value of the RUF associated with the inflexion point of the reaction curve, i.e. the point at wich its curvature changes from concave to convex. This point can be found by equating to zero the second derivative of the Lin-IPT-L mode share,

$$\partial(P_i(V_i=RUF_f))^2 \;/\; \partial^2 V_i \;=\; 0 \;. \tag{10}$$

A reaction curve can be said to be symmetric if any points equidistant from RUF_f yield the same variation from the share evaluated at the inflexion point,

$$P_i(RUF_f)-P_i(RUF_f-\Delta) \;=\; P_i(RUF_f+\Delta)-P_i(RUF_f) \;,\; \forall\Delta \;. \tag{11}$$

This suggests a definition of asymmetry, or υ_i , of a curve in terms of the partial correlation of two series of numbers, $P_i(RUF_1)$, and $1-P_i(RUF_2)$:

$$\upsilon_i \;\equiv\; \rho[\,P_i(RUF_1), 1-P_i(RUF_2)\,]$$

with $\quad RUF_1(t) \;=\; RUF_f \;+\; \Delta_t$, and

$$RUF_2(t) \;=\; RUF_f \;-\; \Delta_t\,,\quad t=1,..,T \;. \tag{12}$$

The υ_i measure has the property that it is equal to one if the reaction curve is symmetric ($\lambda_i = 1$) and is between 0 and 1 if the curve is asymmetric ($\lambda_i \neq 1$) . Note that, with two modes, an asymmetric reaction curve for one mode does not imply a complementary asymmetric reaction curve for the other mode. This is possible since the reaction curve is defined with the utility function and the share of the same mode. In consequence, the complementary share of a mode is not the reaction curve of that other mode.

2. NUMERICAL AND STATISTICAL ISSUES

2.1. Data base

The empirical model to be used is a variant of Cléroux *et al.* (1981), the ULE version of this model is:

$$V_{bus} = \beta_1^1 TZIMP1 + \beta_2^1 TFBUS + \beta_3^1 AUTIM1 + \beta_4^1 TAUT + \beta_5^1 OINC + \beta_6^1 OCAR + \beta_7^1 MEN1 + \beta_8^1$$

$$V_{car} = \beta_1^2 AUTIM1 + \beta_2^2 TAUT + \beta_3^2 TZIMP1 + \beta_4^2 TFBUS + \beta_5^2 OINC + \beta_6^2 OCAR + \beta_7^2 MEN1 + \beta_8^2 \tag{13}$$

The transit impedance time (*TZIMP1*) variable is defined as the sum of the travel time plus three times the non-travel time (access, egress and waiting time). The other variables are trip transit price (*TFBUS*), travel time by car (*AUTIM1*), trip car price (*TAUT*), average income per household in the origin zone (*OINC*), average car ownership per household in the origin zone (*OCAR*) and proportion of men travelling at the peak hour (*MEN1*). The data base comes from a 1976 origin-destination survey (20% of households) in Winnipeg. The calibration is carried out using work trips during the 7:30-8:30 AM peak period. The origin-destination pairs were chosen according to two criteria: that there should be trips by car and by bus, and that the total numbers of trips by the two modes be greater than 60. There are 211 pairs which satisfy those two criteria.

2.2. Estimation procedure

2.2.1. Log-Likelihood function

In order to define the log-likelihood function, we randomize the Lin-IPT-L model by associating to each Ψ_{it} term a random term ε_{it} pertaining to the share i for the O-D pair t. The statistical Lin-IPT-L model is then defined as

$$P_{it} = \Psi_{it} \exp(\varepsilon_{it}) \; / \; \left(\sum_{j=1}^{2} \Psi_{jt} \exp(\varepsilon_{jt}) \right) \text{, where} \tag{14}$$

$$\Psi_{it} = (\lambda_i \exp(V_{it}) + 1)^{(1/\lambda_i)} + \mu_i, \qquad \lambda_i \neq 0$$

$$= \exp(\exp(V_{it})) + \mu_i, \qquad \lambda_i \to 0 \tag{15}$$

with $\lambda_i \geq 0$ and $\mu_i \geq -1$, (16)

Assuming that each random term is normally distributed with constant variance, the log-likelihood function ($\mathcal{L}$) corresponding to (14) can be written for our bimodal case as:

$$\mathcal{L} = - \frac{1}{2} \sum_{t=1}^{T} \left(\ln\left(\frac{P_{1t}}{P_{2t}} \right) - \Psi_{1t} + \Psi_{2t} \right)^2 - \frac{T}{2} \ln(2\pi) \tag{17}$$

The maximization of (17) was done with the SHARE program (Liem and Gaudry, 1989) which uses the Davidon-Fletcher-Powell algorithm (Fletcher and Powell 1966).

2.2.2. Numerical constraints

In order to insure that no numerical problems arise during the estimation of the Lin-IPT-L model, eq.(14)-(16), several numerical constraints are needed.

Constraints against underflow. Very small V_{it} may cause problems related to the computation of the exponential function and the computation of the inverse Box-Cox transformation. We need,

- $V_{it} > B_I$, (18)
- $\Psi_{it} > \mu_i + 1$. (19)

Constraint (18) is meant to garantee that the computation of the exponential function is above the inferior limit (B_I) of the computer. Constraint (19) garantees that the inverse Box-Cox transformation is numerically meaningful.

Constraints against overflow. Reciprocally, very high V_{it} may cause problems when computing the exponential function or the inverse Box-Cox transformation. Indeed the argument of the exponential function should be smaller than the superior limit (B_S) of the computer,

$$\bullet\ V_{it} \ < \ B_S \ . \tag{20}$$

The inverse Box-Cox transformation is numerically meaningful if the addition of the number one to the base is numerically significant, namely if

$$\bullet\ (\lambda_i \exp(V_{it}))^{(1/\lambda_i)} \ < \ (\lambda_i \exp(V_{it})+1)^{(1/\lambda_i)} \tag{21}$$

Note that eq. (21) must be tested only with positive values of V_{it}, since it will always hold when $V_{it} < 0$ and its computation with negative values may lead to underflow problems.

Finally, the last constraint prevents the result of the inverse Box-Cox transformation from going beyond the numerical capacity of the computer:

$$\bullet (\lambda_i \exp(V_{it})+1)^{(1/\lambda_i)} \ < \ \exp(B_S) \ , \tag{22-A}$$

which may also be applied after a logarithmic transformation

$$\bullet \ln(\lambda_i \exp(V_{it})+1)/\lambda_{it} \ < \ B_S \ . \tag{22-B}$$

We did not meet any particular problems of maximization, which leads us to believe that the log likelihood is well behaved. However, the presence of constraints limit the usefulness of any plot of the function since the constraints produce local maxima. To clarify this point, we now proceed to a study of the distribution of the parameter estimates, thus focussing the analysis on the purpose of estimation-unbiased parameter estimates.

2.3. A Monte Carlo study

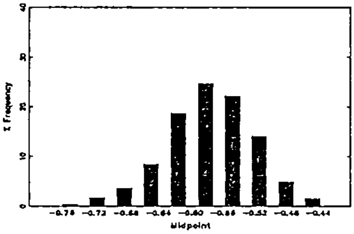
As the computational burden for the estimation of the Lin-IPT-L model is relatively high, a better understanding of the properties of the estimates is in order. For this reason, we decided to perform a Monte Carlo study. Instead of using synthetic data, the true model is based on a subset of variables contained in the Winnipeg data set: the variables *TAUT*, *TFBUS*, and *OINC* were left out of the Monte Carlo study. The use of real data is intended to reduce the inherent arbitrariness of a Monte Carlo study and at the same time provide a better understanding of the empirical results.

Based on 1000 replications, the β, λ and μ estimates are biased. But despite these biases, the predictive capability of the Lin-IPT-L model is very reasonable as the average forecast error[6] is only 0.0014 . This suggest that some compensations take place among parameters, notably across shares; a high value of λ_1 may be offset by a correspondingly high value of λ_2 and/or high values for the β parameters.

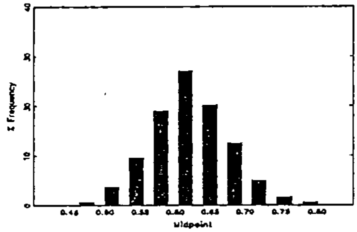
As equation (4) indicates, the computation of the elasticities involves all the parameters of the model. Clearly, if the parameters are offsetting one another, this should be revealed by the elasticities. Except for the constant terms, the estimated elasticities are unbiased as shown in Table 7. Moreover, the distributions of the estimated elasticities are relatively small: more than 10 times smaller than the true parameters. Generally speaking, the standard deviations of all the estimated elasticities are about 5 times smaller than that of the residuals (0.2). Despite the appearances, the distributions of the elasticities do not follow a normal distribution according to the Kilmogoroff-Smirnoff test. After examination of (4), the opposite would have been surprising.

Table 7. Empirical Distribution of the Elasticities

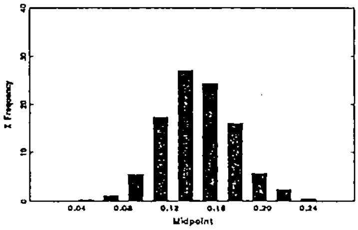
distribution of $\hat{\eta}_1^1$
($\eta_1^1 = -0.577$)



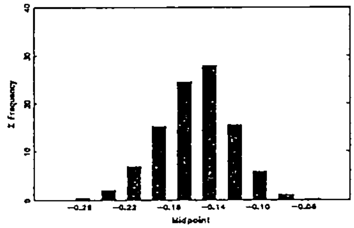
distribution of $\hat{\eta}_3^2$
($\eta_3^2 = 0.614$)



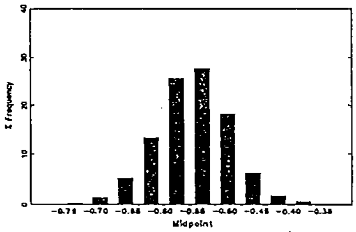
distribution of $\hat{\eta}_3^1$
($\eta_3^1 = 0.144$)



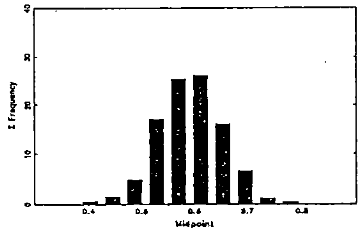
distribution of $\hat{\eta}_1^2$
($\eta_1^2 = -0.153$)



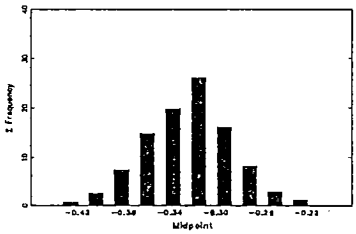
distribution of $\hat{\eta}_6^1$
($\eta_6^1 = -0.543$)



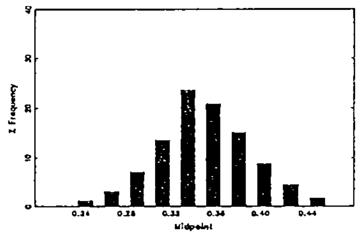
distribution of $\hat{\eta}_6^2$
($\eta_6^2 = 0.578$)



distribution of $\hat{\eta}_7^1$
($\eta_7^1 = -0.325$)



distribution of $\hat{\eta}_7^2$
($\eta_7^2 = 0.346$)



In essence, the Lin-IPT-L model envelops the exponential functions of the Logit model in an inverse power transformation, more specifically the inverse Box-Tukey transformation, which differs from the inverse Box-Cox transformation by the addition of the μ parameters. We have shown elsewhere that the direct Box-Cox transformation (Gaudry and Laferrière, 1987) is invariant to power transformation of the variable or of function to which it is applied. There is therefore no doubt that the inverse Box-Tukey used here implies unique values of the λ and μ parameters. However, the share format makes it easy for some parameters to behave as close substitutes to others -to "offset" them-, even if all parameters are strictly identified. This shifts interest away from the values of individual parameters to that of elasticities.

We argue on the basis of unbiased elasticities results that the estimation of the Lin-IPT-L model is certainly relevant and appropriate. With a very nonlinear model, the moments of the distributions for the estimated parameters (β'*s* , λ'*s* and μ'*s*) are far from being obvious and to some extent not so relevant. In our mind, it seems more interesting to have good distributions for the elasticities, which are, after all, the most meaningful model coefficients.

2.4. Bimodal urban application

This last section presents an empirical analysis of the Lin-IPT-L model using specification (13). The statistical test on which all the tests in the present subsection are based on is the likelihood ratio test. More details are supplied in the full paper, available from the authors.

Testing asymmetry. We have tested asymmetry by specifying the RUF as L or ULE and allowing one generic λ or two λ's. In all those cases, asymmetry of the reaction functions could not be rejected.

Testing captivity. With the same two specifications of the RUF, modal captivity cannot be rejected, wether one generic μ , or one μ specific to each mode, are used.

Testing the IIA assumption. Even if the number of modes availables is only two, it is possible to test the IIA asumption with the approach suggested in 1.1.3. This is done by testing wether or not the coefficients $\beta_3^1, \beta_4^1, \beta_3^2, \beta_4^2$ in (13) are simultaneously significantly different from zero. The IIA assumption is rejected under both UL and ULE specifications.

Specificity and interaction effects. In the Logit model, modal specificity is defined solely in terms of constraints on the β'*s*. In our case, it should be recognized at first that specificity with the Lin-IPT-L model may not necessarily be caused by β parameters but could be due to λ or μ parameters as well. Therefore, it might be interesting to know if specificity is brought about by an asymmetric reaction functions and/or by modal captivity and/or by the RUF. The answer to this question is that each dimension has a significant contribution to the modal specificity. Furthermore, it appears that positive interactions take place among the three dimensions.