MEASUREMENT ERROR MODELLING FOR INFRASTRUCTURE INSPECTION SYSTEMS

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1. INTRODUCTION

1.1 Definition of Measurement Errors

A general measurement process consists of the system under inspection which has objects with certain properties, and the measuring system which includes the components acquiring the measurement information, the components processing the information acquired to get a measured result, and the components interpreting the measured results. Measurement errors originate from the measured system, the measuring system, and the interface between them.

All measurements are subject to error because of imperfections in the measurement procedures and technologies used. Even if these imperfections were not present, there is inherent variability in the measured items and influences from uncontrolled sources that affect the result of measurement. This error can be defined as ε_t where:

$$\varepsilon_t = d - d^* \tag{1}$$

and d represents the measured value while d^* represents the true value. The components of the total error ε_t are discussed next.

1.2 Classification of Measurement Errors

The total error ε_t can have components that are intrinsic to the measured or measuring system, and can be observed in laboratory or experimental conditions when all known influencing factors are controlled. Alternatively, influence errors can arise due to factors that are not part of the measurement, such as physical variables in the measuring environment that were not controlled in an experimental setting during the design of a measurement system. For example, the presence of a film of water on a pavement surface when inspecting after a rainy period can change the reflectance property of the pavement surface. The system may not necessarily be designed to account for such a change.

Intrinsic and influence errors have both a systematic and a random component. Systematic measurement biases can be determined and eliminated if the measurement principles applied are known. Random measurement errors, on the other hand, cannot be predicted on an individual measurement basis, but can be statistically estimated from multiple measurements. They are sometimes referred to as uncertainty.

Consider the case of a pavement surface observed by human inspectors, who are making detailed maps of a sample of the pavement section (say a segment covering 20%) and measuring the distresses appearing on the surface to predict the extent of distress on the pavement section. If the pattern of distress occurrence varies from location to location in a non-systematic way, as on flexible pavements, then it would not be possible to predict

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the error due to sampling from a single inspection. Instead, the inspectors would have to perform multiple inspections on a random assignment of sample locations on the pavement segment to obtain a distribution of the measured values of distress. This distribution can then be used to estimate the sampling error.

Both systematic and random measurement errors can occur together, as demonstrated by the example above, and can arise from the same source. Most mathematical investigations carried out in the literature are aimed at estimating and controlling random measurement errors. Systematic errors require physical-technical investigations as well as statistical investigations to be detected. That is, one needs to know the physical process from which measurements result before being able to form a model that will allow the detection of systematic errors.

In addition, measurement errors can be classified as additive or multiplicative. Additive measurement errors are independent of the numerical value or size of the measured quantity. Multiplicative measurement errors vary with the size of the measured quantity. Resolution limitations are an example of a multiplicative error, as they only allow a technology to pick up distressed elements above a given size, thus missing a fraction of the true distress. This is demonstrated in more detail in the next section.

2. FORMULATION OF THE MEASUREMENT PROBLEM

In this section we develop measurement error models that account explicitly for the error types discussed in the past sections. Consider the measured extent of distresses d_{ijk} on section i, by technology j, measuring distress type k. The measurement depends on a number of parameters characterizing the technologies, distress types, and sections. The values of these parameters are unknown a priori, and the main objective is to determine the size, sign, and inter-relationships of these parameters.

Generally, this can be formulated as follows:

$$d_{ijk} = f(d^{*}_{ik}, \theta_i, \theta_j, \theta_k) \tag{2}$$

where:

$f(.)$ = function representing the relationship between the measured distress, the true value of distress, and factors affecting the measurement;

d^{*}_{ik} = the unobserved true value of distress on a section;

$\theta_i, \theta_j, \theta_k$ = vectors representing section, technological, and distress characteristics affecting measurement respectively.

The vectors θ_i, θ_j, and θ_k are termed "error-generating" factors, and qualitative descriptors are used to classify the effect of these factors on measurement. These descriptors are presented below and the influence of the characteristics they represent are derived.

(a) Section characteristics θ_i include the distresses that appear on a section and how they relate to the surrounding background, or the scene of measurement. They can be characterized by: (i) the density of distress occurrence (dense, moderate, and sparsely deteriorated pavements); (ii) the contrast between the distress and the background on which it occurs (high, moderate, or low contrast); (iii) the pattern of distress occurrence (systematic and even spacing or haphazard occurrence); and (iv) the location of the section with respect to the surrounding environment for effects such as shadows from trees, or oil spots.

(b) Technological characteristics θ_j include components of the measurement technology such as instrumentation, data processing hardware and software, and data collection strategies. They can be characterized by the: (i) measurement principle (direct or indirect); (ii) inspection strategy (sampling or no sampling); (iii) data reduction format (individual recording, range estimate, or average across multiple distresses in a section); and (iv) objectivity of the data collection process.

(c) Distress characteristics θ_k include the attributes of distresses appearing on a pavement surface, that affect the result of measurement. They can be characterized by the dimensions of a distress (linear, areal, volumetric) and distress connectivity (high or low connectivity between distress elements).

The function $f(.)$ in equation (2) can be expressed in a linear form with respect to the true distress, which is simple to use and has an intuitive interpretation.

$$d_{ijk} = \alpha_{jk} + \beta_{jk} d^*_{ik} + \varepsilon_{ijk} \tag{3}$$

where:

α_{jk}, β_{jk} = systematic additive and multiplicative error respectively, of technology j measuring distress type k; and

ε_{ijk} = additive random error of technology j while measuring distress of type k in section i.

The linear form in (3) assumes that the effects of the section, technological, and distress characteristics can be included in the parameters α_{jk} and β_{jk} and the random error term ε_{ijk} as follows:

$$\alpha_{jk} = g_1(\theta_j, \theta_k);$$

$$\beta_{jk} = g_2(\theta_j, \theta_k);$$

$$\varepsilon_{ijk} = g_3(\theta_i, \theta_j, \theta_k); \text{ and}$$

$$E(\varepsilon_{ijk} \mid d^*_{ik}) = 0.$$

The components of $(\theta_i, \theta_j, \theta_k)$ that enter into the functions $g(.)$ are discussed next.

3. CHARACTERIZATION OF THE COMPONENTS OF ERROR

3.1 Basic Formulation

Consider a single pavement section and define the following:

- Each technology has the capability of viewing a fixed portion of a pavement section. The size of this area is denoted by A_j.
- Without loss of generality, let a cell size be uniquely defined as the smallest area viewed by any of the technologies evaluated. This will be referred to as the "reference cell size" **a**. This can be expressed as follows:

$$\mathbf{a} = \operatorname*{Min}_j A_j$$

Let a pavement section be divided into segments where each segment is being inspected by a technology. Dividing each segment into mutually exclusive and collectively exhaustive cells of size a, we obtain a collection of cells Z_i for every pavement segment. Consider the situation where a technology is measuring a single distress type on this section, and assume the pavement segment contains a distress whose total extent d_{iv}^* (e.g., sqft of cracking) is unknown, and each cell contains distresses whose extent d_{ivz}^* is also unknown. Let the total true extent of distress on the pavement section be d_i^*. Thus,

$$d_i^* = \sum_{v=1}^{V_i} \sum_{z=1}^{Z_i} d_{ivz}^* \tag{4}$$

Define a fraction p_{jz} of the extent of distress that a technology can detect in a cell. Similarly, define fractions p_{jv} and p_{ij}, for a segment and section respectively.

Let us now introduce two terms that will be referred to frequently in the following text. The measured value of distress on a pavement is jointly affected by the capability of a technology to measure distress, and the detectability of the distress itself. These terms are discussed in detail below.

(1) distress "detectability" is defined as the fraction of distresses falling in a particular cell z, which can be expressed as the fraction given below:

$$w_{ivz}^* = \frac{d_{ivz}^*}{d_i^*} = \frac{d_{ivz}^*}{\sum_{v=1}^{V_i} \sum_{z=1}^{Z_i} d_{ivz}^*} \tag{5}$$

which is the probability that a distress is present in cell z given that the section has a total extent of distress d_i^*. The relationship in equation (5) is a function of distress and section characteristics such as crack length, crack orientation, and location of the cell with respect to the segment such as shoulder, center-line, or wheel-track, and size of the cell. It is also dependent on the pattern of distress occurrence. If distresses are uniformly distributed across a section, then the probability of finding a distress in a cell z will be equal to that of finding a distress in cell $z+1$. If distresses are sparse in some sections and dense in others, the probability of finding a distress in a cell z will be different from that of finding a distress in a cell $z+1$. The variable w_{ivz}^*, therefore, captures the effects of the section characteristics such as density and pattern of distress occurrence. Since d_i^* is latent, the distress detectability is also latent (unobserved).

(2) technological "capability" is defined as the fraction of distresses on a pavement section detected by a technology, which can be expressed as follows:

Let m_{ivzj} define the theoretically measurable extent of distress in any cell by a technology. Let this measure incorporate the ability of a technology to measure distress given that the distress is present. It is a function of technological characteristics such as detection limitations (e.g., resolution) and classification or interpretation limitations (e.g., confounding effects like oil spots). This was defined earlier as p_{ij}, the fraction of distresses that a technology is capable of detecting in a section. The following relationship demonstrates these concepts.

$$p_{ij} = \frac{\sum_{v=1}^{V_i} \sum_{z=1}^{Z_i} m_{ivzj}}{\sum_{v=1}^{V_i} \sum_{z=1}^{Z_i} d^*_{ivz}} = \left(\frac{m_{ij}}{d^*_i} \right) \tag{5}$$

Assuming that a technology is capable of viewing all segments and cells in a section, the following situations can occur:

- $m_{ivzj} < d^*_{ivz}$ if a technology has detection limitations, such as resolution which result in a measured value less than the true value, and $p_{ij} < 1$; and
- $m_{ivzj} > d^*_{ivz}$ if a technology has classification or interpretation limitations, such as confounding effects that cause error, and $p_{ij} > 1$.

If we assume that detection limitations, such as resolution, are the same across all pavement sections of the same type, and that all other effects such as classification and interpretation limitations (e.g., confounding effects) occur randomly in a section, and therefore, do not generate a systematic component into p, then the expected value of distress that is theoretically measurable by a technology is:

$$m_{ij} = p_j d^*_i \tag{6}$$

This value can be defined from the measurement principle applied by a technology and the physical system representing the measurement situation. For example, for the case of measuring a crack on a pavement surface using optical measurement principles, it can be defined as the physical separation of the pavement surface (crack) which can be detected as a difference in intensity of light on a light-sensitive medium, such as film, and can be obtained in a laboratory environment.

3.2 Formulation to Include Coverage Limitations

So far the discussion has concentrated on the "technological capability" and "distress detectability". Typically, one can observe only a limited number of cells from a pavement segment say Z_{ij} cells, where $Z_{ij} \leq Z_i$. Likewise, one can observe only a limited number of segments $V_{ij} \leq V_i$.

Equation (6) represents the total true level of distress on a section when all cells are observed. When only a fraction of the cells are observed, the fractional true value of distress on the measured area can be expressed as :

$$d_{ivzj} = p_j d^*_{ivz} + e_{ivzj} = m_{ivzj} + e_{ivzj} \tag{7}$$

where e_{ivzj} is a random error that affects the theoretically measurable extent of distress. Define the following:

$Z_{ij} V_{ij}$ = total number of cells observed by a technology in a section; and

$Z_i V_i$ = total number of cells in a section.

Since each cell Z_i is of size a, the total area of the section is aZ_iV_i and the total area observed by a technology is $aZ_{ij}V_{ij}$. The percentage area observed by a technology is:

$$\frac{Z_{ij} V_{ij}}{Z_i V_i} \tag{8}$$

A natural estimator of the total distress on a section is obtained from equations (7) and (8) as follows:

$$\frac{Z_iV_i}{Z_{ij}V_{ij}}\sum_{z=1}^{Z_{ij}}\sum_{v=1}^{V_{ij}} d_{ivzj} = p_j\frac{Z_iV_i}{Z_{ij}V_{ij}}\sum_{z=1}^{Z_{ij}}\sum_{v=1}^{V_{ij}} d^{*}_{ivzj} + \frac{Z_iV_i}{Z_{ij}V_{ij}}\sum_{z=1}^{Z_{ij}}\sum_{v=1}^{V_{ij}} e_{ivzj}$$

$$Z_iV_i\overline{d_{ij}} = p_j\left(Z_iV_i\overline{d^{*}_i}\right) + Z_iV_i\overline{e_{ij}} \tag{9}$$

Or:

$$d_{ij} = p_jd^{*}_i + e_{ij} \tag{10}$$

This estimator is unbiased if the cells and segments were randomly selected, or if distress is uniformly distributed across a section, such that detectability is:

$$w^{*}_{ivz} = \frac{d^{*}_{ivz}}{d^{*}_i} = \frac{1}{Z_i}$$

However, most automated technologies view a fixed portion of the pavement area, in which case the cells inspected are not randomly selected. This introduces a bias in the estimate of true distress expressed by equation (9). The nature of this bias can be expressed as:

$$d^{*}_i = a_{ij} + b_j\left(Z_iV_i\overline{d^{*}_i}\right) \tag{11}$$

where a_{ij} captures the effect of the distribution of distress on a section on coverage error and b_j captures the coverage limitation of a technology. These are the biases induced in the estimate of true distress due to the systematic sample inspected by a technology or the non-uniformity of distress distribution on a section. Substituting (11) in (10) gives:

$$d_{ij} = p_j\left[a_{ij} + b_j\left(Z_iV_i\overline{d^{*}_i}\right)\right] + e_{ij}$$

$$d_{ij} = (p_ja_{ij}) + (p_jb_j)d^{*}_i + e_{ij}$$

Let

$$p_ja_{ij} = \alpha_{ij}$$

$$p_jb_j = \beta_j$$

If we assume that for a particular group of sections, say flexible pavements, the pattern of distress distribution is the same, then $\alpha_{ij} = \alpha_{i'j}\, for\ i \neq i'$, and:

$$d_{ij} = \alpha_j + \beta_jd^{*}_i + e_{ij} \tag{12}$$

where α_j and β_j capture the effects of technological characteristics and coverage limitations. Spatial models are necessary to identify the nature of α_{ij} for cases where the pattern of distress distribution is varying.

3.3 Formulation to Include Measurement Principle

The derivations presented so far in equations (4) through (12) represent the situation where a technology is performing direct measurement. When technologies are employing indirect measurement principles, and one wants to include the effect of measurement principle on the measurement errors, an alternate formulation which incorporates data processing limitations is required. Consider the following measurement situations:

1. The technology is measuring distress directly but measures it with error. For example, a visual inspection of pavement cracking where the areas cracked are measured by a yardstick.
2. The technology is measuring a substitute or proxy for the quantity of distress, and

an estimate of quantity of distress is made through various data processing stages. For example, a video technology is used to measure areas cracked, where it measures the intensity values of cracking on a piece of film and uses them to estimate the areas covered by cracking. Such proxies will be referred to as indicators of cracking.

The situations mentioned above are referred to as direct and indirect measurement respectively, in the remainder of this paper.

Direct Measurement:

Recall the general model in equation (3). This model is for the case where a technology is performing direct measurement. Here d_{ij} would be the output of the measurement process in sqft, d_i^* would be the 'true' area of alligator cracking on the pavement surface. The parameters α_j and β_j would be the additive and multiplicative systematic biases of inspector j respectively, and ε_{ij} would be the random error of the measurement process. If we know α_j and β_j our best estimate of d_i^* from a single measurement, assuming $E(\varepsilon_{ij}) = 0$ would be:

$$\hat{d}_i^* = \frac{d_{ij} - \alpha_j}{\beta_j} \tag{13}$$

Indirect Measurement:

So far we have included the effect of influence errors into the formulation by introducing the multiplicative and additive error parameters α_j, β_j, that depend on the technological characteristics and the inspection strategy (in terms of coverage) used. Now let us consider the situation where the measurement process is indirect. The error due to measurement principle has two components: the data acquisition error and the data processing error. These can be introduced as follows:

$$\delta_{ij} = \tilde{a}_j + \tilde{b}_j d_i^* + \tilde{\varepsilon}_{ij} \tag{14}$$

where:

δ_{ij} = measured level of an indicator of distress on an "inspected" pavement section by a technology;

d_i^* = 'true' level of distress on an "inspected" pavement section; and

$\tilde{a}_j, \tilde{b}_j$ and $\tilde{\varepsilon}_{ij}$ = parameters and random error of indirect measurement describing data acquisition

The parameters $\tilde{a}_j, \tilde{b}_j$ and $\tilde{\varepsilon}_{ij}$ in equation (14) are not the same as those in equation (3). In (3) the parameters reflected only the mapping from d^* to δ. In (14) the parameters also incorporate the systematic measurement errors in the process of measuring δ.

The technological relationship between d_i^* and δ_i is represented in the form of a processing algorithm that maps the indirect measure δ_i to the true level of distress d_i^*. This is derived below from a linear processing function, where γ_j and λ_j are scale parameters that are supposed to calibrate the proxy of distress δ_i^* to the true level of distress d_i^*, and is realized as the measured distress d_{ij}. That is, the mapping $\delta \rightarrow d_{ij}$ can be expressed as:

$$d_{ij} = \lambda_j + \gamma_j \delta_i + \nu_i \tag{15}$$

where ν_i is the random error of processing. Note that in the absence of systematic measurement errors $\lambda_j = \frac{-a_j}{b_j}$ and $\gamma_j = \frac{1}{b_j}$.

A relationship between the measured distress level and the true distress level can be derived from:

$$d_{ij} = \lambda_j + \gamma_j(\bar{a}_j + \bar{b}_j d_i^* + \bar{\varepsilon}_{ij}) + \nu_i$$

$$d_{ij} = (\lambda_j + \gamma_j \bar{a}_j) + (\gamma_j \bar{b}_j) d_i^* + (\gamma_j \bar{\varepsilon}_{ij} + \nu_i) \tag{16}$$

which gives us the familiar equation:

$$d_{ij} = \alpha_j + \beta_j d_i^* + \varepsilon_{ij}$$

where α_j and β_j now include the effects of the data acquisition $(\bar{a}_j, \bar{b}_j)$ and processing λ_j, γ_j errors.

An example of such a situation would be measuring alligator cracking on a pavement surface using optical techniques. The cracking would be observed as an intensity level δ_i which will be recorded as a density on a piece of film, which would be processed to get an estimate of the sqft of alligator cracking d_{ij} from equation (16). The next section describes a case study demonstrating the applicability of such a representation for testing the impacts of various factors on measurement accuracy.

4. ACCURACY IN VARYING MEASUREMENT SCENES

The parameters α, β and ε in the past sections included the combined effect of technological and distress characteristics. To investigate the independent effects directly from the derivations so far (equations (4) through (16)) requires a large data set, with all effects of interest appearing in a great enough frequency to allow estimation of the parameters of error. Such a data set could not be obtained. Instead an existing data set (see Hudson et al, 1987) was used to demonstrate the difference in error parameters in the generalized measurement equation (3) for technologies with varying characteristics when measuring linear, areal, and volumetric distresses. The technologies which differed in their measurement principle, data reduction strategies, and measurement resolution are described in Table 1.

Table 1: Technological Factors Affecting Measurement

Technology	Measurement Principle	Data Reduction	Resolution
Logging	Direct	Yes	High (Human eye)
Photol	Indirect	No	High
Video	Indirect	Yes	Low

The measured results from these technologies were used to estimate the errors in equation (3). The estimation was done using latent variable modeling techniques as described in Ben-Akiva and Humplick (1991). To demonstrate the effect of technological and distress factors, the multiplicative biases were compared as shown in Figure 1. This figure plots the squared multiplicative error $(1 - \beta_j)^2$ for each technology when measuring linear (longitudinal and transverse cracks), areal (alligator and block cracks), and volumetric (potholes and rutting) distresses.

As indicated by Figure 1, the technology with the least multiplicative bias for linear cracking is Video, an indirect measurement, low resolution technology using data reduction techniques. Thus, the impact of these technological factors on the accuracy of linear measurements is minimal. The technology with the least multiplicative bias for areal cracking is Photo1, a high resolution photographic technique which measures each individual distress. The impact of low resolution and data reduction, such as with the Video technology, showed up as a definite higher bias in Figure 1 for areal cracking. This result supports the added benefit of high resolution when measuring areal distresses. The superiority of direct measurement shows up when the measurement scene becomes more complex, such as when measuring volumetric distresses. This is demonstrated by the Logging technology employing human inspectors, which has the lowest multiplicative bias when measuring volumetric distresses. Such an effect was theoretically derived in equations (5) and (6) where a human inspector would be capable of noticing the density and pattern of volumetric distresses as opposed to the photographic techniques which would be confounded by such factors. For a full blown hypothesis testing of the impact of error-generating factors on the results of measurement see Humplick (1992).

5. REFERENCES

Ben-Akiva, M., and Humplick, F.. "A methodology for estimating the accuracy of inspection systems". working paper submitted to Transportation Science. 1991.

Hudson, W.R., Elkins, G.E., Uddin, W., and Reilley, K.T.. Improved methods and equipment to conduct pavement distress surveys. Final Report # FHWA-TS-87-213. 1987

Humplick, F.. "Identifying error-generating factors in infrastructure condition evaluations". Transportation Research Board # 9069. Washington D.C., January 12-16, 1992.

Figure 1: Factors Affecting Measurement Error