OPTIMIZING INFRASTRUCTURE MANAGEMENT DECISIONS UNDER MEASUREMENT AND FORECASTING UNCERTAINTY

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1. INTRODUCTION

Infrastructure management is the process by which agencies monitor and maintain built systems of facilities, with the objective of providing the best possible service to the users, within the constraints of available resources. More specifically, the management process refers to the set of decisions made by an infrastructure agency concerning the allocation of funds among a network of facilities and over time. The basic maintenance and rehabilitation (M&R) decision that an agency has to make is: "in every time period, what M&R activity should be performed on each facility in the network ?" This decision is made more difficult due to the uncertainty in forecasting infrastructure condition.

State-of-the-art infrastructure management systems utilize Markov Decision Processes as a methodology for M&R decision-making (Golabi et al 1982, Carnahan et al 1987, Feighan et al 1988). In this methodology, the facility condition at any time is measured by a discrete state and the deterioration process is represented by discrete transition probabilities of the form:

$p(x_{t+1}=j \mid x_t=i, a_t); \quad 1 \le i, j \le n; \quad t=0,1,\ldots T-1$

where:

x_t = condition state of the facility at the beginning of year t,

i, j = indices of elements in the set of discrete conditions,

a_t = M&R activity performed during year t,

n = number of possible states the facility can be in,

T = number of years in the planning horizon.

Because the probabilistic distributions of the future condition states can be obtained by using the transition probabilities, the decision-maker can, at the beginning of the planning horizon, evaluate alternative M&R policies for the entire horizon. The underlying assumption in this methodology is that an inspection is performed at the beginning of every year, and that inspections <u>reveal the true condition state of the facility, with no error</u>. As a result, after an inspection, the decision maker can apply the activity prescribed by the optimal policy for that condition state of the facility.

There are two major limitations in this approach. First, it assumes that inspections are error-free. Second, it depends on a fixed, not necessarily optimal inspection schedule (an inspection has to be performed at the beginning of every time period).

The first assumption has been demonstrated to be incorrect in several empirical studies (for example, Humplick 1991): there is substantial measurement uncertainty in infrastructure inspection. This uncertainty affects M&R decisions because a measurement error will lead to the selection of a "wrong" activity if the prescribed M&R activity for the true condition and that prescribed for the measured condition are different. This "wrong" selection can translate into an increase in the total lifecycle costs of an infrastructure facility, if the "correct" M&R activity is the one that achieves minimum cost.

The second assumption reflects the absence of a systematic methodology for making inspection decisions in the field of infrastructure. Although it is possible to formulate a MDP where the times of inspections are not constrained (Klein 1962, Rosenfield 1976), such an approach has never been implemented in the field of infrastructure.

In this paper, a methodology for M&R activity selection, which accounts for the presence of both forecasting and measurement uncertainty, is presented. This methodology is the Latent Markov Decision Process (Madanat 1991, Ben-Akiva et al 1992). The Latent Markov Decision Process (LMDP) is an extension of the traditional MDP methodology, but differs from it in one major aspect: it does not assume the measurement of facility condition to be necessarily error-free. Instead, it assumes that the decision-maker observes "outputs" from the measurement which are only probabilistically related to the true condition of the facility (Eckles 1968, Smallwood and Sondik 1973).

We then extend this methodology to handle inspection decision-making. This is done by recognizing the trade-offs between inspection costs and M&R costs. To visualize this trade-off, note that increasing the frequency of inspections increases inspection costs but enhances the quality of information available to the decision-maker. This leads to more cost-effective M&R decisions, hence reducing lifecycle M&R costs.

2. THE LATENT MARKOV DECISION PROCESS WITH FIXED INSPECTION FREQUENCY

The difficulty with introducing measurement uncertainty into the MDP problem is that it violates the central assumption of knowledge of the condition state after an inspection. What the decision maker observes at the beginning of t is now a measured state, which is only probabilistically related to the true state of the system. This relation can be mathematically stated as:

$$q(\hat{x}_t = k \mid x_t = j); \quad 1 \le j,k \le n \quad t = 0,1,\dots,T-1 \tag{1}$$

where:

$\hat{x}_t$ = measured condition state of the facility at start of t,

x_t = true condition state of the facility at start of t,

j, k = indices of elements in the set of discrete condition states,

q = a known probability mass function.

To address the problem created by the violation of the assumption of perfect inspection, the method of state augmentation (Bertsekas 1987) is utilized. State augmentation consists of redefining the state of the system at any point in time to represent all the information that is available to the decision-maker and that is relevant to future decisions.

When the condition state of the facility is measured with uncertainty, the information available to the decision maker at the beginning of t includes the entire history of measured states up to t and the decisions made up to t-1. Moreover, since the measured state at t is only probabilistically related to the true state at t, knowledge of the measured state is not sufficient for decision making. As a result, all the previous measured states and decisions in the history can be relevant to future decisions, and have to be included in the augmented state. Denoting the new state by I_t, we have:

$$I_t = \{I_0, a_0, \hat{x}_1, a_1, \dots, \hat{x}_{t-1}, a_{t-1}, \hat{x}_t\}; \qquad t = 1,2,\dots,T$$

$$I_0 = \{\hat{x}_{-\tau}, a_{-\tau}, \dots, \hat{x}_{-1}, a_{-1}, \hat{x}_0\} \tag{2}$$

where:

τ = number of years between first inspection of facility and start of planning horizon.

It follows that:

$$I_t = \{I_{t-1}, a_{t-1}, \hat{x}_t\} \qquad t = 1,\dots,T \tag{3}$$

From which we can write the following transition probability:

$$P(I_t \mid I_0, a_0, \hat{x}_1, I_1, \ldots, I_{t-2}, a_{t-2}, \hat{x}_{t-1}, I_{t-1}, a_{t-1}) = P(I_t \mid I_{t-1}, a_{t-1}) \quad (4)$$

This equality follows from (3). Assuming I_0 to be known, the transition probabilities $P(I_t \mid I_{t-1}, a_{t-1})$ define the evolution of the state of information, and this evolution is Markovian, by virtue of equation (4).

We can thus write a Dynamic Programming formulation over the space of the information states (for notational simplicity, this will be done using the generic cost function $g(x_t, a_t)$). To do that, the cost function has to be rewritten in terms of the new variables. The cost per stage as a function of the new state, I_t, and of the activity a_t, is:

$$\tilde{g}(I_t, a_t) = \underset{x_t}{E}\{g(x_t, a_t) \mid I_t\} \quad (5)$$

where:

$\underset{x_t}{E}\{g \mid I_t\}$ is the conditional expectation of g over x_t conditional on I_t.

The Dynamic Programming formulation is given by:

$$J_T(I_T) = \underset{x_T}{E}\{g(x_T) \mid I_T\}$$

and

$$J_t(I_t) = \min_{a_t}\left(\underset{x_t}{E}\{g(x_t, a_t) \mid I_t\} + \alpha \sum_{\forall L_{t+1}} P(I_{t+1} = L_{t+1} \mid I_t, a_t) J_{t+1}(L_{t+1})\right) \quad t = 0, \ldots, T-1 \quad (6)$$

where:

α = discount amount factor,

$P(I_{t+1} = L_{t+1} \mid I_t, a_t)$ = transition probabilities for the state of information, and

$L_{t+1} = \{I_t, a_t, \hat{x}_{t+1}\}$.

In order to solve program (6), it is necessary to evaluate expression (5). For this, we need to know the distribution of the true condition state at t conditional on I_t , that is:

$$p_t(x_t \mid I_t), \quad \forall x_t, \quad \forall I_t, \quad \forall t \quad (7)$$

or, in vector form,

$$P_t \mid I_t, \quad \forall I_t, \quad \forall t \quad (7a)$$

where:

$P_t \mid I_t$ = n-dimensional vector (the information vector) with elements $p_t(x_t \mid I_t)$.

If we assume $P_0|I_0$ to be known, then $P_t|I_t$ can be calculated recursively for all t, starting from t=1 to t=T, using Bayes' law, the known measurement probabilities of (1) and the known transition probabilities. Given $I_t=\{I_{t-1},a_{t-1},\hat{x}_t\}$, each element of $P_t|I_t$ can be calculated as follows:

$$p_t(x_t=j\,|\,I_t)=\frac{prob(x_t=j,I_t)}{prob(I_t)}$$

$$=\frac{q(\hat{x}_t\,|\,x_t=j)\sum_i p(x_t=j\,|\,x_{t-1}=i,a_{t-1})\,p_{t-1}(x_{t-1}=i\,|\,I_{t-1})}{\sum_j q(\hat{x}_t\,|\,x_t=j)\sum_i p(x_t=j\,|\,x_{t-1}=i,a_{t-1})\,p_{t-1}(x_{t-1}=i\,|\,I_{t-1})}\quad j=1,..,n \qquad (8)$$

Using the elements $p_t(x_t=i\,|\,I_t)$ calculated above, (6) can be rewritten as:

$$J_T(I_T)=\sum_{i=1}^{n}p_T(x_T=i\,|\,I_T)g(x_T);\quad \forall I_T$$

and

$$J_t(I_t)=\min_{a_t}\Big(\sum_{i=1}^{n}p_t(x_t=i\,|\,I_t)g(x_t,a_t)+$$

$$\alpha\sum_{k=1}^{n}P(\hat{x}_{t+1}=k\,|\,I_t,a_t)J_{t+1}(I_t,a_t,\hat{x}_{t+1}=k)\Big)\quad \forall I_t\quad t=0,\ldots,T-1 \qquad (9)$$

The expression $P(\hat{x}_{t+1}=k\,|\,I_t,a_t)$ can be decomposed into known quantities:

$$P(\hat{x}_{t+1}=k\,|\,I_t,a_t)=\sum_{j=1}^{n}q(\hat{x}_{t+1}=k\,|\,x_{t+1}=j)\sum_{i=1}^{n}p(x_{t+1}=j\,|\,x_t=i,a_t)\,p_t(x_t=i\,|\,I_t)$$

Substituting into (9), we obtain:

$$J_T(I_T)=\sum_{i=1}^{n}p_T(x_T=i\,|\,I_T)g(x_T)\quad \forall I_T$$

and

$$J_t(I_t)=\min_{a_t}\Big(\sum_{i=1}^{n}p_t(x_t=i\,|\,I_t)g(x_t,a_t)+$$

$$\alpha\sum_{i=1}^{n}p_t(x_t=i\,|\,I_t)\sum_{j=1}^{n}p(x_{t+1}=j\,|\,x_t=i,a_t)\sum_{k=1}^{n}q(\hat{x}_{t+1}=k\,|\,x_{t+1}=j)J_{t+1}(I_t,a_t,\hat{x}_{t+1}=k)\Big)$$

$$\forall I_t,\quad t=0,\ldots,T-1, \qquad (10)$$

which consists entirely of known quantities.

The model defined by this formulation will be referred to as the Latent Markov Decision Process with annual inspections, because it assumes that the state of the facility is latent, and because it assumes that a measurement of facility condition $\hat{x}_t$ is available at the start of every year, or every time period, t.

3. DYNAMIC PROGRAMMING SOLUTION

Once the true condition state probabilities are calculated from t=1 to t=T, using (8), we could solve program (10) recursively from t=T to t=0, and for all states of the information I_t at each year t, to obtain the minimum expected costs $J_0(I_0)$ and the optimum policy $\pi^* = \{\mu_0^*(I_0), \mu_1^*(I_1), \ldots, \mu_{T-1}^*(I_{T-1})\}$. This, however, would be computationally very expensive. The reason for this is that the number of states I_t for which equation (10) has to be solved at each year grows exponentially with t. The problem is that even when two states I_t have similar "information contents" as far as future decisions are concerned (that is, they produce similar distributions of the true state $p_t(x_t = i \mid I_t)$, $i = 1, \ldots, n$), they still are considered as separate states by the Dynamic Programming algorithm.

It is thus of interest to replace I_t with a quantity of smaller dimension, but which would have the same information content. Such a quantity is referred to as a "sufficient statistic" for I_t. Since, as observed earlier, I_t affects decisions only through $p_t(x_t = i \mid I_t)$, $i = 1, \ldots, n$, an ideal sufficient statistic is the vector $P_t|I_t$.

The advantage of this statistic is that it allows for direct comparison among states at a given t. Two states of the information which consist of very different histories I_t may have similar $P_t|I_t$ vectors. It is possible to compare the information vectors $P_t|I_t$, by pairwise comparison of corresponding elements. When two states are found to have equal, or almost equal, values of $P_t|I_t$, they can be combined into a single state, which reduces the number of times equation (10) has to be applied.

So far , the cost per stage of the problem has been assumed to be captured by the function $g(x_t, a_t)$. It is necessary, in order to formulate the Dynamic Programming for the M&R decision problem, to specify the components of this cost function. These components are:

· The expected cost of performing the M&R activity. The cost of an activity depends of the type of activity and on the extent of this activity. The extent of an activity is a function of the true state of the facility, x_t. Since x_t is not known, we use the expected cost of an activity, taken over the distribution of x_t, denoted by:

$$\sum_{i=1}^{n} p_t(x_t = i \mid I_t) * ca(a_t \mid x_t = i) \qquad (11)$$

where:
$ca(a_t \mid x_t = i)$ = cost of performing activity a_t, when the true condition state of the facility is i.

· The inspection cost, which is incurred at the beginning of every time period t, and is constant:

$$cm \qquad (12)$$

· Expected user costs. User costs are a function of the condition state of the facility; since the condition state is not known, its distribution is used. Because user costs models for infrastructure facilities are not easiliy defined, they can be approximated by using a minimum allowable condition state, which acts as a constraint in the cost-minimization algorithm. This constraint is introduced by specifying a penalty to take a value of zero if the condition state is above the minimum allowable state and a value of infinity if it falls below that minimum. The mathematical expression for the expected penalty is:

$$\alpha \sum_{i=1}^{n} p_t(x_t = i \mid I_t) \sum_{j=1}^{n} p(x_{t+1} = j \mid x_t = i, a_t) cu(x_{t+1} = j) \qquad (13)$$

where:
$cu(x_{t+1} = j)$ = penalty incurred for year t if the facility is in state j at the end of the year.

We can now write the Dynamic Programming formulation of the M&R decision problem, over the state space of the information vectors $P_t \mid I_t$. It is presented below:

$$J_T(P_T \mid I_T) = 0, \quad \forall P_T \mid I_T$$

and

$$J_t(P_t \mid I_t) = \min_{a_t} \{ \sum_{i=1}^{n} p_t(x_t = i \mid I_t)[ca(a_t \mid x_t = i) + \alpha \sum_{j=1}^{n} p(x_{t+1} = j \mid x_t = i, a_t) cu(x_{t+1} = j) +$$

$$\alpha cm + \alpha \sum_{j=1}^{n} p(x_{t+1} = j \mid x_t = i, a_t) \sum_{k=1}^{n} q(\hat{x}_{t+1} = k \mid x_{t+1} = j) J_{t+1}(P_{t+1} \mid I_{t+1})]\}$$

$$\forall P_t \mid I_t, \quad t = 0, 1, \ldots, T-1 \qquad (14)$$

where:

$$I_{t+1} = \{I_t, a_t, \hat{x}_{t+1} = k\}$$

We assume that $P_0 \mid I_0$ is given, in order to be able to calculate $P_t \mid I_t$, for all t, using (8). This initial vector, the prior distribution of the true states, represents the prior belief of the decision-maker regarding the state of the facility.

Solving program (14) for a specified planning horizon T will yield the minimum expected cost $J_0(P_0 \mid I_0)$ and the optimal sequence of policies
$\pi^* = \{\mu_0^*(P_0 \mid I_0), \mu_1^*(P_1 \mid I_1), \ldots, \mu_{T-1}^*(P_{T-1} \mid I_{T-1})\}$, where $\mu_t^*(P_t \mid I_t)$,

the optimal policy for period t, specifies the optimal M&R activity for each possible information vector at time t, $P_t | I_t$.

4. THE LATENT MARKOV DECISION PROCESS WITH UNCONSTRAINED INSPECTION FREQUENCY

So far, the assumption has been that at the beginning of each year t, the decision-maker performs an inspection which provides him with a measurement of the condition state of the facility $\hat{x}_t$. Hence, the approach so far does not address the decision of whether to inspect in a given year or not. However, including the inspection decision within the existing formulation is a straightforward extension of the ideas presented in the previous sections.

In program (14), we can represent the precision of the measurement technology used, denoted by r_t, by re-writing the measurement probabilities as $q(\hat{x}_t = k \mid x_t = j, r_t)$. The better the precision of r_t, the higher $q(\hat{x}_t = j \mid x_t = j, r_t)$ and the lower $q(\hat{x}_t = k \mid x_t = j, r_t)$, where $k \neq j$. For a perfectly precise technology r_t' (one having no measurement errors), we have:

$$q(\hat{x}_t = k \mid x_t = j, r'_t) = 1 \quad if \quad k = j, \quad 0 \quad otherwise; \quad k, j = 1, \ldots, n. \tag{15}$$

At the other extreme, for a totally imprecise technology $\tilde{r}_t$, we have:

$$q(\hat{x}_t = k \mid x_t = j, \tilde{r}_t) = \frac{1}{n}; \quad k, j = 1, \ldots, n. \tag{16}$$

Expression (16) represents an inspection technology which provides no new information to the decision-maker, or that does not change any previous information.

Since the effect of not inspecting the facility at the beginning of period t is to leave the previous information unchanged, we can model the no-inspection case as an inspection with a totally imprecise technology, such as $\tilde{r}_t$ of (16). The Dynamic Programming formulation of the joint inspection and M&R problem can hence be written, by extending (14) to include two inspection alternatives: inspect, using the available technology, or do not inspect (which is equivalent to inspecting with $\tilde{r}_t$).

The Dynamic Programming formulation is:

$J_T(P_T | I_T) = 0, \quad \forall P_T | I_T$

and

$$J_t(P_t | I_t) = \min_{(a_t, r_{t+1})} \{ \sum_{i=1}^{n} p_t(x_t = i | I_t)[ca(a_t | x_t = i) + \alpha \sum_{j=1}^{n} p(x_{t+1} = j | x_t = i, a_t) cu(x_t = j) +$$

$$\alpha cm(r_{t+1}) + \alpha \sum_{j=1}^{n} p(x_{t+1} = j | x_t = i, a_t) \sum_{k=1}^{n} q(\hat{x}_{t+1} = k | x_{t+1} = j, r_{t+1}) J_{t+1}(P_{t+1} | I_{t+1})]\}$$

$$\forall P_t | I_t, \quad t = 0, 1, \ldots, T-1 \tag{17}$$

where:

$I_{t+1} = \{I_t, a_t, r_{t+1}, \hat{x}_{t+1} = k\}$

Solving program (17) for a specified planning horizon T will yield the minimum expected cost $J_0(P_0 | I_0)$ and the optimal sequence of policies $\pi^* = \{\mu_0^*(P_0 | I_0), \mu_1^*(P_1 | I_1), \ldots, \mu_{T-1}^*(P_{T-1} | I_{T-1})\}$, where $\mu_t^*(P_t | I_t)$, the optimal policy for period t, specifies the optimal **inspection and M&R** activities for each possible information vector at time t, $P_t | I_t$.

5. PARAMETRIC STUDY AND CONCLUSIONS

In this section, a parametric study is performed to investigate the effect of measurement uncertainty on the minimum expected cost obtained by the two versions of the LMDP (LMDP with annual inspections and unconstrained LMDP). This study also demonstrates a useful capability of the Latent Markov Decision Process: the quantification of the benefits of using more precise measurement technologies.

In this empirical study, the condition scale of the facility was described by the PCI (Pavement Condition Index, Shahin and Kohn 1981) which has values in the range 0 to 100. This range was divided into 8 states, that is: i=0, 1,..., 7 (each state represents an average of 12.5 units on the PCI scale), as in Carnahan et al (1987). The transition probabilities and associated unit costs were adapted, with few modifications, from Carnahan et al (1987), for the following three activities: routine maintenance, two-inch overlay, and reconstruction. The minimum allowable state for the facility was set to i=3 (i.e., the penalty has a value of infinity for states 0, 1 and 2 and a value of 0 elsewhere).

The planning horizon (T) for the study was set to 10 years and the interest rate to 5 %, which corresponds to a discount factor (α) of 0.9524. The initial information vector, $P_0 | I_0$, was set to:

$p_0(x_0 = 7 | I_0) = 1.0, \quad p_0(x_0 = i | I_0) = 0.0, \quad i = 0, \ldots, 6.$

Five hypothetical cases of measurement precision were analyzed in the parametric study. The measurement errors were assumed to be normally distributed, with zero mean and standard deviations of 0.0, 2.5, 5.0, 7.5, and 10.0 PCI units respectively. These distributions were then transformed into discrete measurement probabilities by using basic theorems of probability.

The five hypothetical measurement technologies analyzed in the study were assumed to have equal unit costs. A comparison of existing technologies which was conducted for the FHWA (Hudson et al 1987) provided unit costs for different technologies. The average of these unit costs was used as the unit cost of measurement in this study.

The results of the study are summarized in Figure 1. It shows the variation in minimum expected costs, $J_0(P_0 | I_0)$ as a function of the standard deviation of measurement, for the two LMDP algorithms: annual inspections (upper curve) and unconstrained inspection frequency (lower curve).

The main observation that can be made regarding both curves is that the minimum for $J_0(P_0 | I_0)$ lies at the extreme left, that is, when the standard deviation of the measurement technology is the smallest. This trend shows that minimum expected lifecycle cost increase with an increase in measurement uncertainty, which is an intuitively appealing result. The difference in lifecycle costs between different levels of precision is the benefit of more precise measurements.

The difference between the first and the second curves shows the effect of forcing inspections to take place at every year in the LMDP model with annual inspections. This difference is equal to the savings achieved by the unconstrained LMDP by selecting to inspect only when it is cost-effective to do so.

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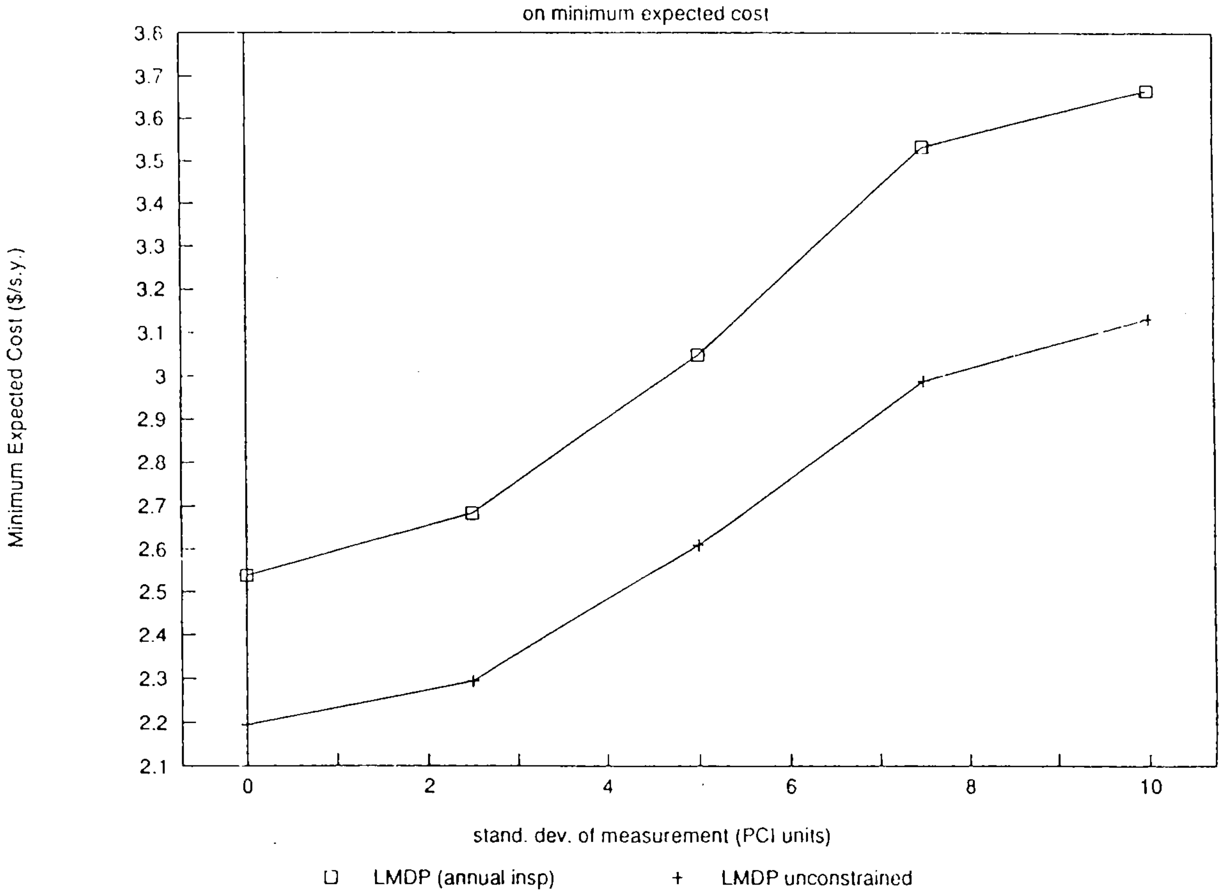
Fig 1: Effect of Measurement Uncertainty



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