NOTES ON QUEUE DYNAMICS, DELAY AND KINETIC ENERGY LOSSES AT INTERSECTIONS

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INTRODUCTION

Two models of vehicle queue dynamics at intersections are compared relative to their use in computing delay and kinetic energy losses in traffic light control optimization software. It is shown that if delay is the only performance criterion to be used, then there is no need for a complex model of vehicle queuing behavior.

1. MACROSCOPIC REPRESENTATION OF TRAFFIC FLOW VARIABLES

Vehicular traffic is not represented from individual vehicles as in microscopic simulations, such as NetSim. Instead at any point of the roadway, moving traffic in is specified by any two of the following three quantities:

(i) traffic density (in number of vehicles per meter)
(ii) flow rate (in number of vehicles passing over the point per second)
(iii) vehicle speed (in meters per second)

The missing quantity is then derived from the evident relationship, [WARD52],

$$\text{Flow Rate} = \text{density} \times \text{speed} \quad (1)$$

Let's assume that the average link length occupied by one vehicle is α meters. An assumption that will be validated from its consequences in the following, is that the minimum space needed by a vehicle in motion at speed v can be expressed as $\alpha + \beta\ v$ where β is a constant. This can also be seen as the first two terms of the Taylor expansion of a function f : velocity -> space. (In earlier works, reported in [3], an equivalent function is expressed relating speed to density). Speed v is restricted to $[0,V]$ where V is the maximum legal speed. We shall say that the vehicle flow at one point of the roadway is not congested if the actual velocities of vehicles at that point is V; otherwise the flow will be said to be congested.

2. STREET LINK ACCUMULATION MODEL

2.1. Number of vehicles in the queue at time t

A model that uses actual queue lengths can be summarized as in Figure 1.

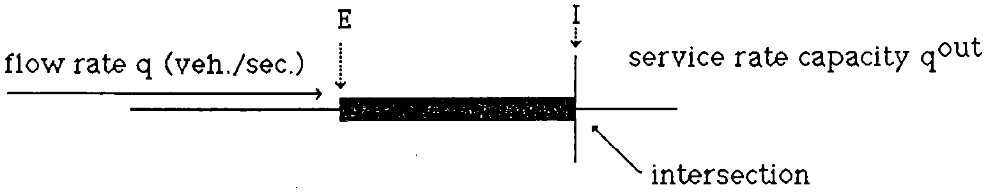

Fig. 1 : queue of vehicles at an intersection.

We shall suppose that at time t = 0, there is no vehicle in the queue. So, if x(t) denotes the number of vehicles in the queue at time t, we set x(0) = 0. For a queue to form we must have $q^{out} < q$, where q^{out} denotes the service rate out of the horizontal link upstream of the intersection, and q is the arrival rate (number of vehicles per second) into the link. To simplify the following exposition we shall assume that q^{out} is constant and that q is constant for a length of time and then equal to zero. This models the phenomenon of a platoon of vehicles moving along a street link.

In the phase of queue build up, let v_E denote the velocity of the end of the queue as it propagates upstream. We now determine v_E. (see [KALT81] for a more general derivation).

At time t, point E is located at $x(t) * (\alpha + \beta v)$ meters from the downstream intersection. Write that x(t), the number of vehicles in the queue at time t is equal to the number of vehicles that were already over segment EI (i.e. the not congested flow) plus the number of vehicles that entered EI during time interval [o,t], minus the number of vehicles that left EI during the same time interval:

$$x(t) = x(t)\,(\alpha + \beta v)\,\frac{q}{V} + q\,t - q^{out}\,t \qquad (2)$$

($x(t)\,(\alpha + \beta v)$ gives the length of segment EI; $\frac{q}{V}$ is the number of vehicles per meter for not congested flow). From this we get,

$$x(t) = \frac{V}{V - (\alpha + \beta\, v)\, q} \,(q - q^{out})\, t \qquad (3)$$

From (1) we have that $\frac{v}{\alpha + \beta\, v} = q^{out}$

so
$$v = \frac{\alpha\, q^{out}}{1 - \beta\, q^{out}} \qquad (4)$$

Then, from (3) and (4),

$$x(t) = \frac{V\,(1 - \beta\, q^{out})}{V\,(1 - \beta\, q^{out}) - \alpha\, q} \,(q - q^{out})\, t \qquad (5)$$

2.2. Velocity of end of the queue during build up.

now,

$$v_E = (\alpha + \beta\, v)\,\frac{dx(t)}{dt} = \frac{\alpha\, V\,(q - q^{out})}{(1 - \beta\, q^{out})\, V - \alpha\, q} \qquad (7)$$

2.3. Calculation of total delays to vehicles.

Suppose the arrival rate of q vehicles per second corresponds to a platoon length of L meters, as represented in Figure 2.

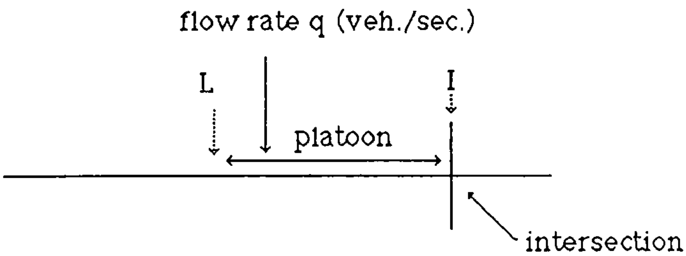

Fig. 2 : platoon of (uncongested) vehicle flow at time t = 0

The total delay to vehicles is defined as the sum of individual vehicle delays. The last vehicle of the platoon joins the queue at time

$$T = \frac{L}{V + v_E} \qquad (8)$$

The rate of vehicles joining the queue is :

$$\frac{dz}{dt} = \frac{dx(t)}{dt} + q^{out} = \frac{V(1-\beta\, q^{out}) - \alpha\, q^{out}}{V(1-\beta\, q^{out}) - \alpha\, q}\, q \qquad (9)$$

where v is given by (5).

A vehicle that joins the queue at time t, must travel v_E t meters at speed v. The delay it incurs from joining the congested flow is then,

$$D(t) = \frac{v_E\, t}{v} - \frac{v_E\, t}{V}$$

$$= \frac{(1-\beta\, q^{out})\, V - \alpha\, q^{out}}{(1-\beta\, q^{out})\, V - \alpha\, q} \; \frac{q - q^{out}}{q^{out}} \qquad (10)$$

Making use of (8) and of the fact that

$$V + v_E = \frac{V(1-\beta\, q^{out}) - \alpha\, q^{out}}{V(1-\beta\, q^{out}) - \alpha\, q}\; V$$

The total delay is then,

$$TD_L = \int_0^T D(t)\, \frac{dz}{dt}\, dt = \frac{1}{2}\, \frac{L^2}{V^2}\, \frac{q}{q^{out}}\, (q - q^{out}) \qquad (11)$$

2.4. Calculation of total kinetic energy losses.

Recall that \f(dz,dt) is the rate at which vehicles join the queue; this over time interval $[0, \frac{L}{V+v_E}]$.
then,

$$TKEL_L = \int_0^{\frac{L}{V+v_E}} \frac{1}{2} (V^2 - v^2) \frac{dz}{dt} dt = \frac{1}{2} \frac{q\,L}{V} [\,V^2 - (\frac{\alpha\, q^{out}}{1 - \beta\, q^{out}})^2\,] \qquad (12)$$

3. NODE ACCUMULATION MODEL

In this model the vehicles accumulate only at the intersection according to the input/output balance equation:

Queue = arrivals - departures.

This is as if the queue was not covering any length of roadway upstream of the intersection and the vehicles were piled up vertically at the intersection (or node in the street network).

3.1. Computation of Total Delay

In that case, the queue increases linearly at the node until time $t = \frac{L}{V}$ and then decreases linearly until time $T = \frac{L\, q}{V\, q^{out}}$ when it is empty.

then,

$$x(t) = (q - q^{out})\, t \quad \text{for} \quad 0 \leq t \leq \frac{L}{V} \qquad (13)$$

$$= \frac{q\, L}{V} - q^{out}\, t \quad \text{for} \quad \frac{L}{V} \leq t \leq \frac{L\, q}{V\, q^{out}} \qquad (14)$$

$$= 0 \quad \text{otherwise}$$

and the Total Delay is,

$$TD_N = \int_0^T x(t)\, dt = \frac{1}{2} \frac{L^2}{V^2} \frac{q}{q^{out}} (q - q^{out}) \qquad (15)$$

Notice that this expression is identical to that in (11), which was to be expected since the two models differ only as to when the delays occur.

3.2. Determination of Total Kinetic Energy Loss.

$$\frac{dx}{dt} = q - q^{out} \qquad \text{for} \quad 0 \le t \le \frac{L}{V} \qquad (16)$$

$$x(0) = 0$$

The kinetic energy loss for one vehicle joining the vertical queue is that of a full stop, so it is $\frac{1}{2} V^2$. Then,

$$TKEL_N = \int_0^{\frac{L}{V}} \frac{1}{2} V^2 \frac{dx}{dt} dt = \frac{1}{2} V L (q - q^{out}) \qquad (17)$$

Observation : The node accumulation model underestimates kinetic energy losses.

We show that $TKEL_N < TKEL_L$

Let $DKL = TKEL_N - TKEL_L = \frac{1}{2\,V} \; (v^2 q \; - \; V^2 q^{out}) \qquad (18)$

and $\quad q_0 = \frac{V^2}{v^2} q^{out}$; then $DKL > 0$ implies $q > q_0$

Now $v \le V$ implies $q_o \ge q^{out}$; so two cases are to be considered,

(i) $q^{out} \le q \le q_0$.

or

(ii) $q > q_0$

Let q^{max} be the maximum flow rate attainable;

$$q_{max} = \frac{V}{\alpha + \beta \; v} \qquad (19)$$

We show that $q0 \ge q^{max}$

so $q_0 = q^{max}$ and case (i) holds.

Define $f(q^{out}) = q_0 - q^{max}$; where $q_0 = \dfrac{(1 - \beta\, q^{out})^2}{(\alpha\, q^{out})^2}\, V^2\, q^{out}$

A simple derivation shows that the sign of $f'(q^{out})$ is that of $\beta\, q^{out} - 1$. So $f'(q^{out}) < 0$ for

$$0 \le q^{out} \le q^{max} = \frac{V}{\alpha + \beta\ v} < \frac{1}{\beta}\ ; \qquad\qquad (\beta > 0)$$

So $f(qout) = q0 - q^{max} \ge 0$ and case (i) holds and $TKEL_N < TKEL_L$

This result is not obvious since one could have expected the reverse, arguing that vehicles are experiencing less velocity reduction when joining the "realistic" queue. (However this argument would forget to compare the number of vehicles joining the respective queues).

CONCLUSION

Two models of queue accumulation for use in traffic control optimization models have been presented. It has been shown that if total delay to vehicles is the only (or main) criterion of optimization, then there is no need to perform the extra calculations required by the more realistic street link accumulation model. If kinetic energy reductions is considered a major objective, then it may be difficult to avoid using the more complex model.

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