OPTIMAL LOCATION PATTERNS IN A GENERAL SPATIAL ECONOMIC EQUILIBRIUM MODEL

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1. Introduction

The fundamental concept, the distinguishing element, of location theory as against other areas of economics, is distance (Beckmann, 1982). Distance makes its appearance in two ways : in transportation cost and in neighborhood effects. While location theory has made the most of transportation cost, it has hardly scratched the vast complexities of neighborhood effects, those disturbing forces that generate external economies and diseconomies through the propinquity of locations.

From a purely formal point of view distance is any nonnegative number **d** assigned to a pair of locations $\mathbf{x}_1$, $\mathbf{x}_2$ that satisfies the following requirements,

$d(x_1 , x_2) > 0$

$d(x_1 , x_2) = 0$ if and only if $x_1=x_2$

$d(x_1 , x_2) + d (x_2 , x_3) \geq d(x_1 , x_3)$; this is the triangle inequality.

Symmetry is not required in location theory ($d(x_1 , x_2) = d(x_2 , x_1)$).

How distance should be measured is an empirical question. In order to measure distances, in general, locations of activities have to be known. Finding optimal locations for both producers and consumers, which lead to a spatial economic equilibrium is the most important problem that has to be solved here.

2. Spatial economic equilibria; Tinbergen Bos Systems.

Tinbergen and Bos define production centres and systems of centres. A production centre is a spatial point where one or more industrial sectors are produced; a system of centres is a set of centres where all defined industrial sectors are located (Paelinck, 1985).

In the model two types of activities are distinguished, viz. agricultural and industrial activities, within a closed area; agricultural production is distributed homogeneously over the area, industrial production is located in production centres and so is population.

An important hypothesis of these models concerns the size of production plants. The optimal plant size of sector i is denoted by $\bar{Y}_i$, the total number of firms (or elementary production units) required to produce Y_i (this is an exogenous variable in the model) is calculated as :

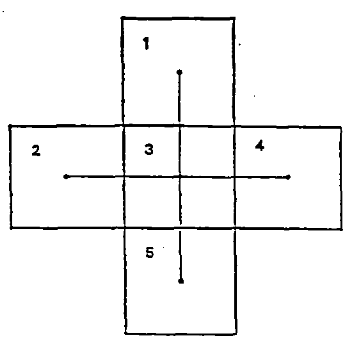
$$n_i = \frac{Y_i}{\bar{Y}_i} ;$$

Each production centre contains only one production unit (or firm) of the highest ranking sector.
It is recommended that $n_1 > n_2 > \ldots > n_I$.
Sector I showing the highest rank; production centres are characterised by the sector of highest rank located in the centre. Trade between centres of the same rank is excluded.

3. Optimal location patterns.

An important step to a more general spatial structure (in Kuiper, Paelinck, Rosing, 1990) was the introduction of "Manhattan circles" where all sectors are produced. The analysis will be based upon the following assumptions :

1. The area to be studied is organised around a rectangular network with unit mazes.
2. The area limits are located around a "Manhattan circle" with integer "radius"; a Minkowski-metric p=1 is used;
3. The area is structured by unit Manhattan-circles (squares) centred on the network nodes; this implies that the area's border is located outside the Manhattan-circle stated in 2.
4. The spatial economy functions according to Tinbergen-Bos assumptions.



(fig 3.1 A Manhattan circle)

In order to compute optimal location patterns, transportation cost have to be minimized; next variables are used:

x_{lk}^{ij} - transported quantity of sector i in order to produce sector j, transported from location l to k.

y_l^i - number of firms of sector i located in l.

Y - total income of the system (exogenous).
n_i - number of firms of sector i to be located in the circle (exogenous).
n - number of locations available in the circle (exogenous).
a_i - propensity to consume i (exogenous).
d_{lk} - Manhattan distance between l and k (exogenous).
t_i - transportation tariff of i (exogenous).

The model minimizing transportation cost T :

$$\min T = \sum_{ij} \sum_{lk} x_{lk}^{ij} d_{lk} t_i \qquad i \neq j$$

$$i=0,\ldots,n_i \quad j=0,\ldots,n_j \quad l=1,\ldots,n \quad k=1,\ldots,n$$

$$s.t. \sum_{l} x_{lk}^{ij} = \frac{a_i a_j y_k^l Y}{n_j} \qquad \forall\ i,j,k \quad l \neq j$$

$$y_k^l = 0 \ \vee \ y_k^l \geq 1 \qquad \forall\ i,k$$

$$\sum_k y_k^l = n_i \qquad \forall\ i \qquad i \geq 1$$

$$x_{ij}^{lk} \geq 0$$

3.1 Optimal location patterns.

If only one industrial sector is considered, the location of this sector is determined completely by the value of the distances to the agricultural sector. Agriculture is distributed homogeneously in the circle so the industrial sector also will distribute homogeneously over the area if the number of firms grows. Adding a second industrial sector will change this picture. Depending on their propensity to consume values, these industries will attract each other.

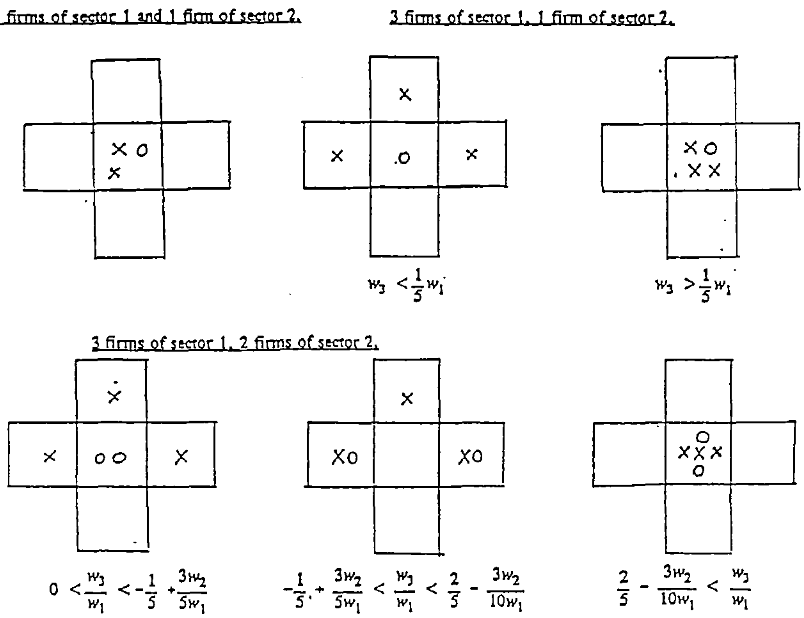
Optimal patterns can be computed for different values of the propensity to consume.

Suppose : $w_1 = a_1\, a_0\, t_1$,

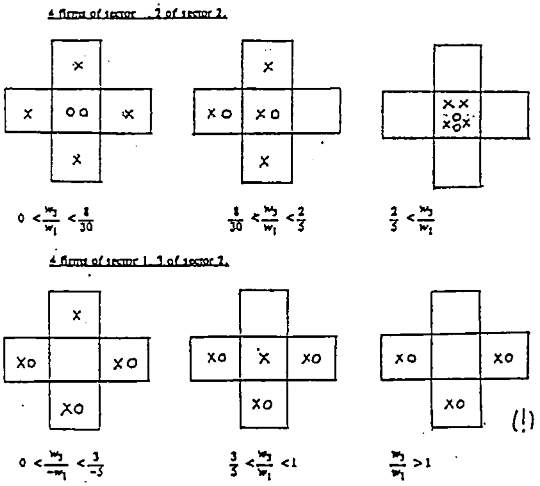
$w_2 = a_2\, a_0\, t_2$ and

$w_3 = a_2\, a_1\, t_2$.

Next optimal locations are computed (Kuiper, Paelinck, 1991) :

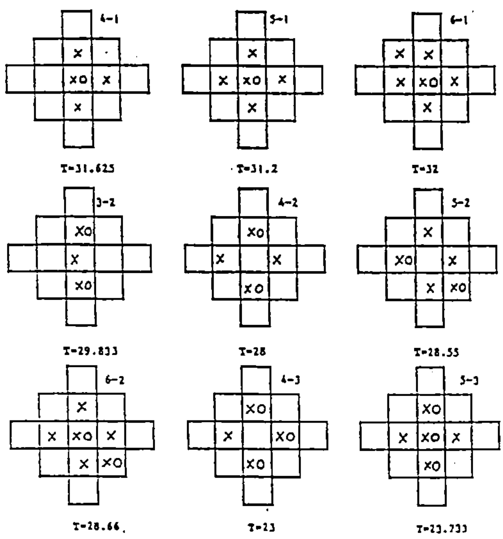


(fig 3.2 Optimal Location patterns, $a_i = a_j$, R=1)



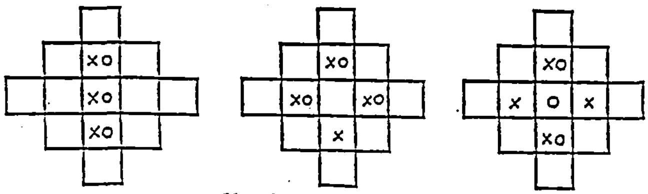
(fig 3.3 **Optimal Location patterns,** $a_i \neq a_j$ **, R=1)**

Next some optimal location patterns for R=2 are presented; first considering no differences between the values of the propensity to consume.



(fig. 3.4 **Optimal location patterns,** $a_i = a_j$, R=2)

Finally optimal location patterns are computed, for R=2, where 4 firms of sector 1 and 3 of sector 2 have to be located optimally, and different values for the propensity to consume are supposed :



$a_0 \le \frac{39}{2}\frac{a_2 a_1}{(11a_1 - a_2)}$ $\quad$ $\frac{39}{2}\frac{a_2 a_1}{(11a_1 - a_2)} \le a_0 \le \frac{26}{11}a_2$ $\quad$ $a_0 \ge \frac{26}{11}a_2$

(fig. 3.5. **Optimal Location patterns, 2 industrial sectors,** $a_i \neq a_j$ **, R=2)**

If the propensity to consume products of the agricultural sector are relatively high, the optimal location shows a deconcentrated picture; small values lead to a concentrated pattern. in the latter case, the capacity of firms of sector 1 is enlarged; this sector is produced in only 3 firms (instead of 4).

4. Conclusions.

In this paper a simplified mathematical program is introduced, that is solved for a number of cases where not more than 2 industrial sectors and a radius of 1 and 2 are used.

One of the problems in the mathematical program is the restriction concerning the number of firms of sector i located at k, y_k^i :

$$y_k^i = 0 \quad \vee \quad y_k^i \geq 1.$$

Without this restriction a simple lineair program remains, which can be solved without many problems; now very soon computations become time-consuming.

Optimal location patterns are determined by a number of variables; the propensity to consume (a_i) plays an important part in this respect. For a number of examples optimal location patterns are computed without taking into account different values of the propensity to consume. Optimal location patterns show a tendency to a homogeneous distribution of firms over the area as the number of firms increases; the industrial sectors are following the agricultural sector. If the number of firms in a sector increases there is a tendency towards deconcentration; the speed of this process is influenced by the transportation costs between this sector and the agricultural sector. If these costs are relatively high compared to the intersectoral transportation costs, deconcentration will mostly be optimal.

High transportation cost between 2 sectors leads to a more concentrated pattern. If there are many separate firms of equal size, it sometimes appears to be profitable to produce (the same total quantity) in larger firms. This result was found with 4 firms of sector 1 and 3 of sector 2; when propensity to consume 2 is relatively high, it is profitable to produce sector 1 in 3 firms instead of 4. So the capacity of these firms is enlarged !

Although just a few examples are computed here, the possibilities of the model are promising. Enlarging the number of producers, the number of different sectors and the radius of the circle is only a technical problem to be solved soon.

The model can be enriched introducing explicitly for example

- optimal locations for consumers;
- different properties of regions with respect to sectors;
- the cost of congestion.

Space can be used in an infinite number of ways; the number of agents determining the locations and the interactions is very large and so is the number of different tastes.
Spatial economics can contribute, in a modest way, in advising finding the optimal spatial distribution; it is one of the many different disciplines that are important in this respect, but since the problem is enormous, each serious contribution is important.

5. References.

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