CALIBRATING TRAVEL DEMAND MODELS FROM TRAFFIC COUNTS: STATISTICAL PERFORMANCES AND COMPUTATIONAL ASPECTS.

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INTRODUCTION

Demand models play a central role in the process of designing and simulating transport system.

Calibration and specification of demand models are usually based on expansive, time consuming ad hoc surveys. Calibrated models may also not satisfactorily reproduce observed flows.

On the other hand traffic counts on network links are an easily collectable and cheap information source on travel demand. In past years considerable attention has been paid to the use of traffic counts to estimate or update O/D matrices. Statistical framework and a review of different estimators can be found in Cascetta and Nguyen (1988). Numerical analysis of relative statistical performances in Di Gangi (1989).

The problem of integrating traffic counts in the process of calibration/specification of demand model, received comparatively less attention. (Cascetta 1986, Willumsen 1981, Hogberg 1976) Willumsen and Tamir, 1990). In this paper some GLS or Bayes estimators proposed by Cascetta (1986) are analysed. Their statistical formulations are briefly stated, computational aspects are described and statistical performances are analysed using numerical tests on a small network and real data on a medium size italian town.

1. NOTATION AND STATEMENT OF THE PROBLEM

Consider a trasport network, abstracted into a graph model, consisting of a set N of Nodes and a set R of directed links (arcs). A node at which trips originate and/or terminate is called a centroid. Let t_{rs} denote the average number of trips going from centroid r to centroid s, within a given time period. Let $\underline{t}$ be a vector with conponents t_{rs}, $\underline{t}$ is usually referred to as an origin-destination trip matrix.

Each origin-destination flow t_{rs} subdivides on the network into path flows F_{rsk}, $k \in I_{rs}$ where I_{rs} is a subset of all paths connecting the pair of centroids r and s.

For a given link $l \in R$, the sum of all path flows traversing this link is denoted by the link flow f_l:

$$f_l = \sum_{rs} \sum_{k \in I_{rs}} a_{lk} F_{rsk} \quad (1.1)$$

where a_{lk} equals 1 if path k traverses link l and 0 otherwise. In a more compact notation, eqn (1.1) may be rewritten as

$$\underline{f} = \underline{A}\ \underline{F} \quad (1.2)$$

where $\underline{A}$ is the usual link-path incidence matrix, and $\underline{f}$ and $\underline{F}$ are, respectively, vectors of link and path flows.

To every arc $l \in R$ is associated an average unit travel cost $c_l(\underline{f})$, which may be a fixed constant or a function of link flows $\underline{f}$.

Define p_{krs} as the proportion of trip demand t_{rs} traveling on path k connecting centroids r and s. The trip proportions p_{krs} are generally implicit functions of the travel cost vector $\underline{C}$, and satisfy

$$\sum_{k \in I_{rs}} p_{krs} = 1 \quad \forall rs$$
$$p_{krs} \geq 0 \quad \forall k \in I_{rs} \quad \forall rs \quad (1.3)$$

The aim of an assignment model is to produce an approximation of the trip proportions p_{krs} based on assumption concerning the trip makers'route choice behaviour. From now on, p_{krs} will denote the assignment model's predicted proportions. The flow on a path k can be expressed as:

$$F_k = t_{rs}\, p_{krs} \quad (1.4)$$

In a more compact notation eqn. (1.4) may be rewritten as

$$\underline{F} = \underline{P}\ \underline{t} \quad (1.5)$$

$\underline{P}$ denotes the matrix of predicted path-choice proportions, where each row corresponds to a specific path and each column to a pair of centroids. The equations 1.2 and 1.5 can be combined obtaining the predicted link flow f^*:

$$\underline{f}^* = \underline{A}\ \underline{P}\ \underline{t} \quad (1.6)$$

The product of the first and second term on the right-hand side of this equation is referred to as the assignment matrix and denoted by:

$$\underline{H} = \underline{A}\ \underline{P} \tag{1.7}$$

The above equations apply separately to networks of different modes (e.g cars, transit, pedestrians) as it has been shown by Cascetta and Nguyen (1988).

In the following it will be assumed that the vector $\underline{f}$ may include flows on links belonging to different modal networks and that the incidence matrix, path choice fractions and modal O/D flows are consistently defined.

Furthermore, let us assume that a demand function relating the multimodal O-D matrix to exogenous variables via a K-dimensional parameters vector $\underline{\beta}$ is correctly specified.

The assumed demand model can be seen as a vectorial function $\underline{t}$ ($\underline{\beta}$) transforming the parameter space S^K into the O-D pairs space S^n.

In general the actual O-D matrix do not conform with that predicted by the model, even if we use the "true" vector $\underline{\beta}$. This is because the model is only an approximation to real phenomena underlying mobility; furthermore mobility, strictly speaking, should be seen as a stochastic process over homogeneous time periods with a mean that is usually assumed to be given by the model.

The above discussion can be summarized in the following:

$$\underline{t} = \underline{t}(\underline{\beta}) + \underline{\tau} \qquad E(\underline{\tau}) = \underline{0} \tag{1.8}$$

Assume also that traffic flows on a subset M of R have been observed. Let $\underline{f}$ denote the vector of these counts. Due to measurement errors, random variation in trip demand and trip maker route selections over time, and inherent errors in the network and assignment models, it is natural to assume that the traffic counts f_l , $l \in M$, are observations of random variables. It is further assumed that the model's predicted value f_l represents f_l 's mean value. This can be expressed as:

$$\hat{\underline{f}} = \underline{Ht} + \underline{\epsilon}, \qquad E(\underline{\epsilon}) = \underline{0} \qquad E(\underline{\epsilon},\underline{\epsilon}') = \underline{W} \tag{1.9}$$

Finally, assume that additional information on parameters of demand models $\underline{\beta}$ -such as sample survey estimates, outdated estimates or values calibrated in similar areas-is also available. The general problem addressed here can be stated broadly as:

Determine an estimate of the parameters of demand models $\underline{\beta}$ by efficiently combining traffic counts based data and all other available information.

Depending on the nature of the available information, the above problem may be formulated differently. For instance, if sample based estimates are given, the estimation problem can be seen as that of attempting to combine two distinct sets of experimental data (survey estimates and traffic counts), related to the unknown parameters demand models $\underline{\beta}$.

On the other hand, if an a priori probability distribution of $\underline{\beta}$-formalizing analyst's subjective knowledge about the parameters β_i -is given, then the estimation problem will reduce to that of attempting to combine a priori information and experimental data.

2. FORMULATIONS OF THE ESTIMATION PROBLEM

This section formulates the parameters estimation or updating as optimization problems.
The objective function of the latter depends on the statistical inference techniques adopted.
Within the classical inference approach, the generalized least-squares (GLS) methods for deriving objective functions will be examined and contrasted to the Bayesianmethod.

2.1. GLS estimator

In this approach no distributional assumptions on the sets of data are needed. Let $\underline{\beta}$ denote the survey estimates of $\underline{\beta}$ obtained through traditional estimators, typically Maximum Likelihood ones. Considered the following stochastic system of equations in $\underline{\beta}$:

$$\hat{\underline{\beta}} = \underline{\beta} + \underline{\delta}$$

$$\hat{\underline{f}} = \underline{Ht}\ (\underline{\beta}) + \underline{\epsilon} \qquad (2.1)$$

in which $\underline{\delta}$ is the sampling error with a variance-covariance matrix $\underline{D}$, and $\underline{\epsilon}$ is the traffic count error with dispersion matrix $\underline{W}$ introduced earlier. The nonlinear Generalized Least Square (GLS) estimator $\underline{\beta}^{GLS}$ of $\underline{\beta}$ may then be obtained by solving:

$$\underline{\beta}^{GLS} = \arg\min_{\underline{\beta}\in\underline{S}} (\hat{\underline{\beta}} - \underline{\beta})\underline{D}^{-1}\ (\hat{\underline{\beta}} - \underline{\beta}) + (\hat{\underline{f}} - \underline{Ht}(\underline{\beta}))\underline{W}^{-1}(\hat{\underline{f}} - \underline{Ht}(\underline{\beta})) \qquad (2.2)$$

S is the feasibility set of $\underline{\beta}$ values e.g. non positive value for "cost" parameters in distribution or modal choice models.

Statistical properties of nonlinear GLS estimators are only asymptotic, it can be shown that they are consistent and asymptotically normally distributed. Furthermore an approximate expression for the dispersion matrix $\underline{B}^{GLS}$ of $\underline{\beta}^{GLS}$ (GLS estimator) can be obtained, (Cascetta, 1986), and show that the use of informations contained in traffic flows reduces the variances of model parameter estimates.

It is well known that under a normality assumption for both $\underline{\beta}$ and $\underline{f}$, nonlinear GLS and Maximum Likelihood estimators coincide so that in this case also the ML estimators asymptotic efficiency can be credited to $\underline{\beta}^{GLS}$.

2.2. The Bayesian approach

In the Bayesian inference framework, a priori information on the parameters of demand models $\underline{\beta}$ is expressed as an a priori probability function $g(\underline{\beta})$, the parameters of which are a function of the a priori estimates β_i of the unknown β_i. The β_i are either rough or obsolete estimates, or estimates from other areas.

On the other hand, the traffic counts represent an additional source of information about $\underline{\beta}$ with given probability $L(\underline{f}/\underline{\beta})$; Bayes theorem allows these two source of information to be combined to provided the posterior probability function $m(\underline{\beta}/\underline{f})$ (i.e., the probability of observing $\underline{\beta}$ conditional to the traffic count $\underline{f}$):

$$m(\underline{\beta}/\hat{\underline{f}}) \propto L(\hat{\underline{f}}/\underline{\beta})\, g(\underline{\beta}) \qquad (2.3)$$

In principle, the above posterior probability function allows a confidence region to be generated for $\underline{\beta}$. In practice, due to computational complications, only point estimators can be obtained as the maximum value of (the logarithm of) the posterior distribution (the mode of the posterior distribution):

$$\underline{\beta}^{B} = \arg\max_{\underline{\beta}\in\underline{S}} \ln m(\underline{\beta}/\hat{\underline{f}}). \qquad (2.4)$$

The maximum posterior distribution estimator is a substitution for the expected value, when this latter requires complex numerical integration.

If multivariate normal distibutions are assumed for the traffic counts, then the likelihood function $L(f/\beta)$ is given by (Cascetta 1984, 1986)

$$\ln L(\hat{\underline{f}}/\underline{\beta}) = -\tfrac{1}{2}(\hat{\underline{f}}/\underline{Ht}(\underline{\beta}))\, \underline{W}_{\beta}^{-1}(\hat{\underline{f}} - \underline{Ht}(\underline{\beta})) + \text{constants} \qquad (2.5)$$

On the other hand if is used a multivariate normal prior distribution, with mean $\underline{\beta}$ and dispersion matrix $\underline{D}_B$ (Cascetta, 1986):

$$g(\underline{\beta}) \propto \exp\left(-\tfrac{1}{2} (\underline{\beta}-\hat{\underline{\beta}})' \underline{D}_B^{-1} (\underline{\beta}-\hat{\underline{\beta}}\right)$$

yielding:

$$\text{Ln } g(\underline{\beta}) = -\tfrac{1}{2} (\underline{\beta}-\hat{\underline{\beta}})' \underline{D}_B^{-1} (\underline{\beta}-\hat{\underline{\beta}}) + \text{constant} \qquad (2.6)$$

This formulation can be seen as a particular case of problem (2.4), when the objective is the sum of (2.5) and (2.6).

Note that the nonlinear GLS and Bayes problems under the assumption made are formally similar even though their statistical interpretation are quite different.

3. A SOLUTION ALGORITHM

3.1. The projected gradient algorithm

If in the objective function (2.2) or (2.4) we ignore the covariance terms in the matrix $\underline{D}$ and $\underline{W}$, the optimizazion problem can be reformulated as follows:

$$\underline{\beta}^* = \arg \min_{\underline{\beta} \in \underline{S}} \left[Z(\underline{\beta}) = \Sigma_i \frac{(\beta_i - \hat{\beta}_i)^2}{\text{Var } (\delta_i)} + \Sigma_l \frac{[f_l - \Sigma_{rs} h_{lrs} t_{rs}(\underline{\beta})]^2}{\text{Var } (\epsilon_l)} \right] \qquad (3.1)$$

Different computation methods exist that allow solving the optimization problem (3.1); all different techniques provide, for every k-iteration, three principal steps:

a) research of feasible discent direction $\underline{h}_k$
b) performe a line search to obtain the value μ, minimizing the objective function along this direction;
c) compute a new point (if a stop test is not met)

$$\underline{\beta}_{k+1} = \underline{\beta}_k + \mu_k \underline{h}_k$$

The gradient projection algorithm of Rosen with linear constraints was adopted (Shetty and Bazaraa, 1980). This method projects the gradient of the binding constraints in such a way that improves the objective function maintaining feasibility.
In this application a modified version of the algorithm

resulted more efficient. After a number of "classical" iterations, these were alternated with iterations maximizing the objective function with respect to a sub-set of parameters (mode choice parameters) influencing the objective function less markedly than the others.

3.2. Gradient of the objective function

The calculation of partial derivatives of the objective function $Z(\underline{\beta})$ in eqn. (3.1), is developed analytically, except for the part inherent the demand vector $\underline{t}(\underline{\beta})$, whose gradient is calculated numerically by finite differences method. To reduce computational times numerical derivatives were calculated using the forward difference with increment 0.01. Finally the gradient components $\nabla \underline{z}$ is obtained with the expression:

$$\frac{\partial z}{\partial \beta_i} = 2\frac{(\beta_i - \hat{\beta}_i)}{Var(\delta_i)} + 2\Sigma_l \frac{(\Sigma_{rs}\, h_{lrs}\, t_{rs}\, (\underline{\beta}) - \hat{f}_l)}{Var(\epsilon_l)}\, \Sigma_{rs} h_{lrs} \frac{\partial t_{rs}(\beta)}{\partial \beta_i}$$

4. STATISTICAL PERFORMANCES OF THE ESTIMATOR

Statistical performances of the estimators (3.1) were evaluated numerically applying it to the estimation of the parameters for a classic "four stages" demand model on a test network.
The estimator was also applied to a real urban network.

4.1. Demand model and test network

In this application a classic "four stages" generation/ distribution/mode choice/assignment demand model was used with elastic demand on mode choice.
To simulate demand in the a.m. peak period two trip purpose were considered: home-to-work and home-to-school. The specification and the parameters of different sub-models resulted from a multi year research project carried out in the context of PFT-CNR (Cascetta and Nuzzolo, 1992).

The matematical formulation of the home -to-work model is the following:

$$t_{rsm} = \beta_1\, Act_r \frac{Emp_s^{\beta 2}\, D_{rs}^{\beta 3}}{\Sigma_{s'} Emp_{s'}^{\beta 2}\, D_{rs'}^{\beta 3}} * \frac{\exp\, (V_m)}{\Sigma_{m'} (\exp V_{m'})}$$

where:
Act_r is the number of employed people living in zone r;
Emp_s " " " working in zone s;

D_{rs} is the network distance between zones r and s(in meters * 100).
The systematic utilities V_m used for the three modes mode choice model are the following:

$$V_{WALK}=\beta_4 T_w; V_{CAR}= \beta_5 T_C + \beta_6 C_C + \beta_7; V_{BUS} = \beta_5 T_B + \beta_6 C_B + \beta_8$$

Where $T_W/T_C/C_C/T_B/C_B$ are respectively the walking time, the car time, the car cost, the bus time and the bus cost relative to travel between zones r and s.

The home-to-school model keeps the some structure with nr. of students living in zone r, nr. of school places offerend in zone s and a two mode (walk/bus) modal choice model.

The assignment models for the calculation of the $\underline{H}$ matrix, have been obtained with an All or Nothing (A o N) models for pedestrian and transit networks, assuming both networks non congested, and with a Stochastic User Equilibrium that (SUE) model for the road network.

The values of parameters resulting from the quoted research are assumed as the "true" ones and reported in table 1.
The performances of the estimators were evaluated on a test network with 5 zones, 11 nodes road and pedestrian network, 18 nodes transit network (see fig. 1).
In the evaluation it was assumed that counts were available for all "indipendent" links.

4.2. The evaluation method

The evaluation of the statistical performances was carried out using the test network. Flows, calculated assigning the demand obtained from the described system system of models with "true" parameters, are considered as true ones in the procedure implicitly assuming no demand model and assignment errors.

Initial estimates β_i of the parameters were generated through a MonteCarlo procedure sampling from a normal variate with mean equal to the "true" value β_{true} and with variance $(CVB\ \beta_{true})^2$ where CVB is the variation coefficient assumed for initial estimates errors.

Althoug the convexity of objective function has not been prooved analitically, a numerical analysis showed that for the model structure assumed the convexity assumption is fully accetable, (Russo and Iannò, 1991).

The "corrected" estimates β^{op} were obtained by imposing only sign constraints on the parameter (e.g. negative costs).

The statistic used as an indicator for the extimator's efficiency is the Mean Square Error (MSE), given by (Judge et alii, 1988):

$$MSE(\hat{\beta}_i) = \Sigma \{ var(\hat{\beta}_i) + [bias(\hat{\beta}_i)]^2 \}$$

Two types of statistics are reported. The first is relative to a single trial while the second refers to the average over 20 trials.

The following values were computed:

N = number total trials

$\underline{\beta}^t = \beta_{true}$ = vector of true parameters (tab. 1)

$\hat{\underline{\beta}}^{op}_n$ = optimal vector of parameters at trial n

$\hat{\underline{\beta}}^{in}_n$ = initial vector of parameters at trial n

$\hat{\underline{\beta}}^{op}_m : \hat{\beta}^{op}_{m,k} = \frac{1}{N} \sum_{n=1}^{N} \hat{\beta}^{op}_{n,k}$ mean vector of the optimal parameters after N trials

$\hat{\underline{\beta}}^{op}_m : \hat{\beta}^{in}_{m,k} = \frac{1}{N} \sum_{n=1}^{N} \hat{\beta}^{op}_{n,k}$ mean vector of initial parameters after N trials

$bs(\hat{\underline{\beta}}^{op}) = (\hat{\beta}_m - \beta^t)$ estimates of the bias of the vector $(\hat{\underline{\beta}}^{op})$

$MSE(\hat{\beta}^{op}_k) = \frac{1}{N} \sum_{n=1}^{N} (\hat{\beta}^{op}_{n,k} - \beta^t_k)^2$ final mean square error of the parameter β_k (tab 1)

$MSE(\hat{\beta}^{in}_k) = \frac{1}{N} \sum_{n=1}^{N} (\hat{\beta}^{in}_{n,k} - \beta^t_k)^2$ initial mean square error of the parameter β_k (tab.1)

$MSE\%(\beta_k) = [MSE(\hat{\beta}^{in}_k) - MSE(\hat{\beta}^{op}_k)]/MSE(\hat{\beta}^{in}_k)$ proportional reduction of the error in the parameter k(tab. 1)

In addition to the measure of the deviation of the parameters from the true ones, deviations of the demand vector estimated with $\underline{\beta}^{op}$, $t(\underline{\beta}^{op})$ and of the resulting flows vector $\underline{f}=Ht(\underline{\beta}^{op})$ from true values were computed by using, efficiency indicators similar to those adopted for the parameters.

4.3. Analysis of results.

In numerical tests it was assumed that the variance of flow measurement errors (ϵ_l) is proportional to the flows f_l: $Var(\epsilon_l) = (CVF\ f_l)^2$.
Measured flows were virtually assumed as the true ones putting CVF = 0,01.

On the other hand the coefficient of variation of initial estimates errors, CVB, was assumed equal to 0.40. Results for two single trials and average values over 20 trials are reported in table 1.
On the basis of the obtained results and of the calculated indicators various considerations can be made.

The "corrected" estimates of parameters are generally very satisfactory. Looking at numerical detail it is possible to observe that parameters of generation and distribution models are always unbiased, in fact table 1 shows as these parameters have a zero MSE between the true values and those obtained after the correction.

Modal choice parameters perform less satisfactorily. In particular some parameters for home-to-school trips remain with significant MSE values, although reduced with respect to initial estimates. Such a result may be explained considering that the choice represented by this model is almost constrained because the students have only the alternative walk-bus and given the average distance of trips even values of the parameters slightly different from the trues ones produce the true values of the flows.

A similar differences, in terms of the MSE occur in the parameters of the generation and distribution, finding a better correction for the home-work with respect to home-school.

Finally two trials starting from completely unrealistic initial vectors β^{in} were performed. The results are yet congruent with what seen so far. In the first trial we have considered a initial vector with all the entries equal to 1 while in the second all entries are equal to 0.5. Again values pratically equal to the true ones for the parameters of generation and distribution models, but less similar values were obtained for modal choice.

In addition to the indicator regarding the parameter vector, some indicators for the demand vector obtained with the optimal parameters relative to each trial were computed.
Such indicators give a more direct measure of the demand reproduction capability whic is one the mair targets of any demand modelling exercise. The results are very good, in view of the fact that in all trials at least 99% error reduction has been reached.

The indicators concerning the flows, confirm the goodness of the estimators, as we except the flows being directly derived from the demand vector by mean of an assignment matrix. For all links in all trials a reduction of the initial error larger than 99% were obtained. Finally a number of trials, modifying the CVF and maintaining

unchanged the CVB, were performed keeping the same initial vector.
It resulted that by increasing the CVF to 0,1 the average error reduction on the parameters is slightly inferior (95%) but the computation time is reduced of the 80%.

4.4. Application to a real medium-size town

The estimator defined in section 2 with the procedure described in section 3 was applied to multimodal network of the city of Reggio Calabria (about 180000 inhabitans). It was used a zoning with 20 internal zones, and with 6 external centroids. The tree modes present in the city have been considered: walk, car and bus; the same models used in the test network, refering to the period 7,30 a.m. - 8,30 a.m. were applied. The links flows were actually measured on 62 links: 30 road, 26 pedestrian, 6 public.

Two calibrations have been performed, the first with the initial vector calculated in the city of Parma, and the second with a random vector generated as in the test network.

The initial values and the obtained results are reported in table 2. The optimal parameters of both trials are very similar, as are the values of objective function.

A specific analysis of the corrected values shows some differences from those calculated for other towns. Such differences can be explained by the use of a single home-to-work purpose with parameters obtained specifically for commuters mobility. This would lead to an underestimations of trips for business and other purposes. Since the generation parameter for school trips is quite similar to that of Parma, extra mobility is explained by expanding the home-to-work generation coefficient. Such an interpretation is strengthened by the values of the distribution parameters.
In fact while the parameter of the attraction variable is slightly reduced the distance parameter is reduced to 1/3.

The parameters of the home-to-school models turn out to be basically confirmed both for generation and distribution models.

The parameters of modal choice are yet quite near to those of Parma, with a larger influence of the walking time.

5. CONCLUSION

Estimators of demand model parameters were proposed and their statistical performances were evaluated both on a test network and on a real urban network.

The .results are in general satisfactory showing the capability of the proposed estimator to reduce significantly the errors in initial estimates. This is particularly true for generation and distribution models, while the objective function is more stable with respect to changes in mode choice parameters. In any case modal demand and link flows obtained using corrected parameters are virtually coincident with the "true" ones. The application of the estimator to a real network showed its capability to "adapt" to a model mis-specification correcting generation and distribution parameters to take into account business and other purposes trips initially non included in the model but significant in the real case.

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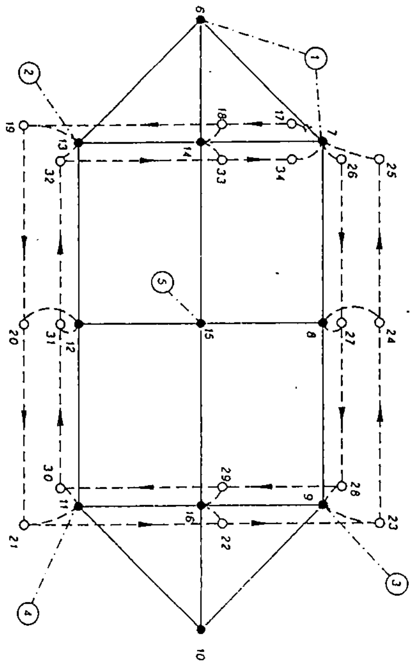
Results obtained and statistical performances in the test network										tab. 1
Model	Purpose	Attribute	β^{true}	Trial 5 β^{in}	Trial 5 β^{op}	Trial 10 β^{in}	Trial 10 β^{op}	$MSE\hat{\beta}^{in}$	$MSE\hat{\beta}^{op}$	MSE%β
Gener.	H-W	Actives	0,46	0,585	0,459	0,057	0,455	0,0406	$1*10^{-5}$	100,0
Gener.	H-SC	Students	0,86	0,275	0,870	0,626	0,864	0,0793	$4*10^{-5}$	99,9
Distr.	H-W	Distances	0,70	1.084	0,648	0,801	0,707	0,0885	$8*10^{-4}$	99,1
Distr.	H-W	Employees	1,02	1,338	1,033	0,571	1,032	0,2417	$1*10^{-4}$	99,9
Distr.	H-SC	Distances	0,93	0,774	1,088	0,640	0,849	0,1519	$4*10^{-3}$	97,1
Distr.	H-SC	School pl.	0,35	0,722	0,354	0,421	0,358	0,0249	$3*10^{-4}$	99,9
Mod.ch.	H-W	Walking T.	1,19	1,194	2,069	1,103	1,263	0,1493	0,0186	87,5
Mod.ch.	H-W	Car/Bus T.	0,54	0,894	1,074	0,283	0,411	0,0512	0,0448	12,4
Mod.ch.	H-W	Car/Bus C.	1,80	0,763	0,958	1,703	1,891	0,4396	0,3168	27,9
Mod.ch.	H-W	Car m.s.	2,56	1,504	1,274	4,765	3,064	1,0377	0,3439	66,9
Mod.ch.	H-W	Bus m.s.	2,29	0,284	0,735	1,736	2,935	0,6203	0,4465	28,0
Mod.ch.	H-SC	Walking T.	2,18	1,561	1,167	3,763	2,546	0,6057	0,0610	89,9
Mod.ch.	H-SC	Bus Time	0,39	0,641	0,415	0,252	0,018	0,0300	0,0388	29,6
Mod.ch.	H-SC	Bus Cost	1,58	1,562	0,431	2,340	2,344	0,3570	0,1442	59,6
Mod.ch.	H-SC	Bus m.s.	1,53	1,613	1,586	0,955	0,958	0,4253	0,3939	7,4



The results obtained in the city of Reggio C.			tab 2			
Model	Purpose	Attribute	Trial 1 $\hat{\beta}_{in}$	Trial 1 $\hat{\beta}_{op}$	Trial 2 $\hat{\beta}_{in}$	Trial 2 $\hat{\beta}_{op}$
Gener.	H-W	Actives	0,46	0,604	0,230	0,602
Gener.	H-SC	Students	0,86	0,902	1,015	0,900
Distr.	H-W	Distances	1.02	0,346	1,103	0,347
Distr.	H-W	Employees	0,70	0,570	1,008	0,550
Distr.	H-SC	Distances	0,93	0,900	0,335	0,908
Distr.	H-SC	School pl.	0,35	0,272	0,346	0,269
Mod.ch.	H-W	Walking T.	1,19	1,424	1,848	1,649
Mod.ch.	H-W	Car/Bus T.	0,54	0,628	0,466	0,559
Mod.ch.	H-W	Car/Bus C.	1,80	0,100	1,541	0,100
Mod.ch.	H-W	Car m.s.	2,54	2,543	3,536	3,352
Mod.ch.	H-W	Bus m.s.	2,29	2,330	2.116	3,179
Mod.ch.	H-SC	Walking T.	2,18	2,207	3,436	2,737
Mod.ch.	H-SC	Bus Time	0,39	0,506	0,349	0,642
Mod.ch.	H-SC	Bus Cost	1,58	1,713	1,315	1,980
Mod.ch.	H-SC	Bus m.s.	1,53	1,544	0,796	2,632