DYNAMICS OF URBAN SPATIAL STRUCTURE AND TRIP DISTRIBUTION MODEL CALIBRATION

Youzhen CHENG
Postgraduate Student
School of Civil Engineering
University of New South Wales
Sydney - Australia

John BLACK
Professor of Transport Engineering
School of Civil Engineering
University of New South Wales
Sydney - Australia

INTRODUCTION

In transport planning practice, the gravity model of trip distribution is calibrated against base-year survey data and applied to forecast the future inter-zonal traffic pattern. It is based on the assumption that the calibration parameters are stable over time. Such parameters relate to the transport impedance, or the deterrence function, associated with the trip-length frequency distribution. Irrespective of whether a mathematical function such as the power or exponential is specified, or whether a set of empirically-derived friction factors are used in the model temporal stability is assumed. This temporal stability of parameters has been questioned by a number of researchers and is not a new problem (for example, Ashford and Covault, 1969). Intuitively, as urban spatial structure (the relative spread and distribution of land-use activities) changes over time we would expect trip-lengths to change, given an overall stability in traveller preferences for commuter distances. Hutchinson and Smith (1978) show a statistical relationship between the calibration parameter for the journey-to-work and population size of the city, and the friction factors in the Quick-Response Urban Travel Estimation Techniques (Sosslau, *et al*, 1978) also vary by urban population.

This largely theoretical paper examines the relationship between urban spatial structure (the spatial distribution of homes and workplaces) and the mathematical form of the gravity model impedance function and its calibration parameters. By analogy with moments of inertia in mechanics, a spatial index is defined that summarises different distributions of residences and workplaces around a central point. As the gravity model has one mathematical function that attempts to describe trip length behaviour, we have similarly proposed one function that summarises urban spatial structure. This spatial index is responsive to dynamic changes in land-use distribution. We hypothesis a direct relationship between this index (moment of inertia) and trip distribution model calibration.

An outline of the theoretical concepts of moments of inertia, using no more than advanced level higher school mathematics, is presented in Section 2. By analogy, an index is derived to describe quantitatively urban spatial structure. The example cited is the distribution of homes and workplaces. In Section 3, the dynamic behaviour of this index is investigated with changing distributions of homes and workplaces using hypothetical urban forms, as used, for example, by Allen (*et al*, 1982) or by Black and Katakos (1987). In Section 4, the link between the dynamics of urban spatial structure and the trip distribution model calibration parameter is tested using Australian Bureau of Statistics (ABS) Census journey-to-work data for Sydney, Australia, for 1961, 1971, 1976 and 1981 . During this period of total number of metropolitan jobs increased from 0.86 million to 1.2 million. (Journey-to-work data for 1986 contained too many errors for ABS to release the information, and the 1991 data is not due for release before 1993). Section 5 discusses the implications of this research for transport modelling practice. The conclusion suggests areas for further work.

1. PROBLEMS WITH GRAVITY MODEL FORECASTS

The standard gravity model, as used in transport practice, is specified in Section 1.1. Its use as a forecasting tool is noted together with the assumptions about changes in the values of zonal variables and parameters over time. The temporal stability of model parameters was tested using gravity models (with three different transport impedance functions) on data for Sydney from 1971 to 1981 (Section 1.2). The lack of parameter stability and its practical implications are also assessed.

1.1 Temporal Stability of Parameters

The standard method in urban transport modelling practice is to calibrate a spatial interaction model to estimate traffic say for the, journey-to-work, from zone i to zone j, $\hat{Q}_{ij}$, for the base year (denoted by subscript t). The fully-constrained gravity model to be calibrated is:

$$\hat{Q}_{ij(t)} = X_{i(t)}\, Y_{j(t)}\, R_{i(t)}\, E_{j(t)}\, f\left(T_{ij(t)}\right)$$

where,

$X_{i(t)}$ = balancing factor for zone i at time t;
$Y_{j(t)}$ = balancing factor for zone j at time t;
$R_{i(t)}$ = total number of workers in zone i at time t;
$E_{j(t)}$ = total number of jobs in zone j at time t;
$f(T_{ij(t)})$ = some function of transport impedance (distance, travel time , or generalized cost) from zone i to zone j at time t.

The mathematical function adopted for the transport impedance term, is usually a power function with a calibration parameter, α, an exponential function, with a parameter, β, or Tanner's function, with two parameters, α and β. Correctly, the most appropriate function and its parameter(s) should be determined as part of model calibration, but, in practice, impedance are sometimes selected without any evaluation. Planning practice in the USA is to use a set of empirically determined "friction factors" instead of a mathematical function.

When applied to forecast journey-to-work travel at some future time, t + n, where n may be 20 or 30 years, the model becomes:

$$\hat{Q}_{ij(t+n)} = X_{i(t+n)}\;\; Y_{j(t+n)}\;\; \hat{R}_{i(t+n)}\;\; \hat{E}_{j(t+n)}\;\; f\left(\hat{T}_{ij(t+n)}\right).$$

This model is essentially static and it is important to consider the following when analysing change over time:

(a) $\hat{R}_{i(t+n)}$ is an <u>exogeneously</u> forecast value, which may take on a value less than $R_{i(t)}$, implying resident workers loss, a value equal to $R_{i(t)}$, or, a value greater than $R_{i(t)}$, implying resident workers gain;

(b) $\hat{E}_{j(t+n))}$ is an exogeneously forecast value, which may be taken on a value less than $E_{j(t)}$, implying job losses, a value equal to $E_{j(t)}$, or, a value greater than $E_{j(t)}$, implying employment gain;

(c) $\hat{T}_{ij(t+n)}$ represents the future transport system and its forecast level of service to users, based on alternative plans under consideration;

(d) $f\left(\hat{T}_{ij(t+n)}\right)$ the parameters in this function, α, or β, or α and β, are assumed to remain the same for t + n as for the calibration in the base year, t;

(e) $Y_{j(t+n)}$, $X_{i(t+n)}$ these are mathematical balancing factors will differ from $X_{i(t)}$ and $Y_{j(t)}$ depending on (a), (b) and (c) above.

Model parameter stability and forecasting accuracy (including for the variants of the fully-constrained model, the unconstrained and production-constrained models) is assessed by checking on base-year and forecast traffic flows, changes to zonal workers living in zone i, changes to zonal jobs in zone j, and changes to inter-zonal transport impedance. Such comprehensive assessments are rarely conducted. The following case study is based on known values of the above considerations at two points and it shows that the calibration parameters are not constant with time.

1.2 Case Study of Sydney

The temporal stability of different transport impedance functions had been tested by Palgunadi (1986) using Census of Population journey-to-work data (O-D traffic, zonal workers, and zonal jobs) for metropolitan Sydney for 1971 and 1981, spatially stratified into 44 zones (local government areas). Models were calibrated on Census data for both years. We would expect the 1971 transport impedance function with the same parameters to give the best fit model in 1981. As inter-zonal travel time and cost data for the two time periods were unreliable, over-the-road distances were used. The parameter values for the production-constrained and the fully-constrained gravity models for both years were found not to be stable. The transport impedance functions were ranked in order of relative accuracy (using chi-squared, correlation coefficient, and root-mean squared tests) in reproducing the survey origin-destination pattern of traffic. This order was found to remain the same for 1971 and 1981 - namely, Tanner's, power and exponential. The percentage change in parameter values was from 3 to 14 per cent over 10 years.

There are considerable practical implications for transport planners of this failure to account for such parameter changes. By using the 1971 calibrated models to estimate 1981 traffic patterns there is an under-estimation of the true value of the 1981 mean trip length (15.97 km) by 3 per cent with Tanner's function, 1.9 per cent by the power function, and 5.4 per cent by the exponential function. More significantly, a comparison of the differences - both overpredictions and underpredictions - between the major desire lines for 1971 model estimates for 1981 and the 1981 survey desire lines show systematic residuals, especially with destinations focussing on the central business district. In general, the model over-estimates radial trips from origin zones within 9 km of the city centre and underpredicts trips from an arc of middle-distance suburbs from the north west and south west of the city centre (Palgunadi, 1986, Figure 4.8, p. 93).

2. SPATIAL INDEX CONCEPT

In mechanics, a geometric quantity which is of great importance in discussing the motion of rigid bodies is called the moment of inertia. The moment of inertia (I) of a particle of Mass, m about an axis is defined as:

$$I = m\rho^2$$

where,

ρ = distance from the particle to the axis.

The moment of inertia of a system of particles ($v = 1$ to n) with masses, $m_1, m_2 \ldots m_n$, about an axis is defined as:

$$I = \sum_{v=1}^{n} m_v \rho_v^2$$

Similarly, the moment of inertia of a continuous distribution of mass, such as a solid rigid body, is obtained by an integral function. Standard engineering mathematics handbooks, such as Tuma (1970), on integral calculus, provide the properties of regular plane areas and solids and their moments of inertia, where the axes are defined in the x-, y- and z- planes (the perpendicular axis).

The spatial index, SI, for any zone i, for the moments of inertia of a population distribution (and resident workers) or of an employment distribution, can be defined by analogy as

$$SI_i = \sum_{j=1}^{n} L_j t_{ij}^2$$

where,

L_j - land-use activity level in zone j; and

t_{ij} - travel-time, cost of distance from zone j to zone i.

The total value of this spatial index for the whole of the area may be obtained by summing the zonal values, as illustrated in Section 3.

The physical concept of moments of inertia relates to the kinetic energy of a particle rotating at a constant angular velocity at a fixed distance from a point, or of a solid body rotating about an axis through it with constant angular velocity. By considering the distribution of either homes or workplaces as the "particles", with their mass corresponding to levels of land-use activity, and the centroid of the whole urban area as the point through which the perpendicular axis passes, the moment of inertia can be easily calculated by summation or integration of the distribution. Dynamic changes of spatial structure (the distribution of home and workplaces) will be shown by a corresponding change in the value for the moment of inertia, as explained in the following section. Furthermore, there is a systematic relationship for the spatial index between centralised and dispersed land-uses.

3. DYNAMICS OF SPATIAL STRUCTURE AND ITS INDICES

To illustrate the concept of a spatial index, and its relationship between the dynamics of urban spatial structure and the resultant moments of inertia, calculations are made for hypothetical urban forms. The building blocks of zones, land-use activity levels, and inter-zonal transport are described in Section 3.1. The results for 12 different square urban forms - varying in geographical area and employment size - are summarised in Section 3.2, where the spatial index distinguishes amongst centralised, multi-nodal and dispersed land-uses.

3.1 Theoretical City Structure

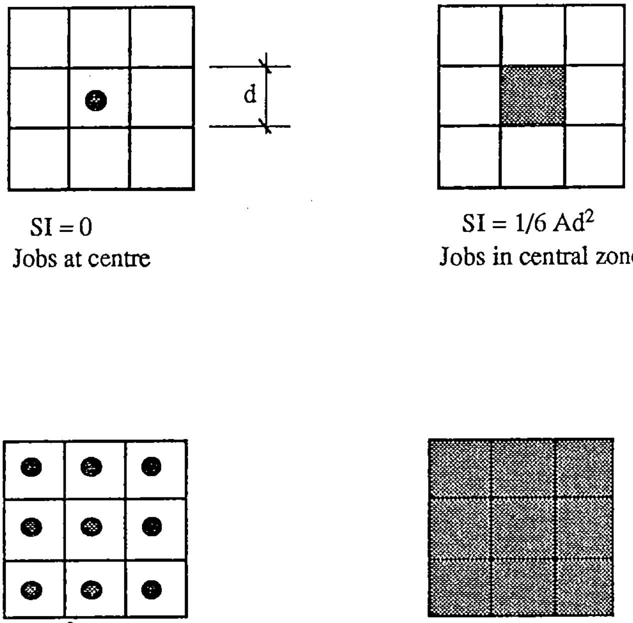
The 'building block' for theoretical city structures was a square zone of length 5 km where all land-use (restricted to homes and workplaces) is assumed either to be at the geographical centre of the zone (zone centroid) or spread evenly throughout the zone. These building blocks were juxtaposed in increasing numbers - 3 x 3, 5 x 5 and 7 x 7 - to reflect bigger cities that they might correspond to the distances and dimensions encountered in some of the largest cities of the world.

The population distribution within each city structure was assumed to be an even one with a population of 250 000 per zone (corresponding to the approximate gross population density of fifteen inner London boroughs). The work-force participation rate was assumed to be 0.4 (that is 40% of the population employed) with the same participation rate for each zone. Employment distributions tested were centralised at one zone centroid, centralised but distributed across that zone, decentralised with an equal number of jobs at each zone centroid (multi-nodal), and decentralised with jobs spread evenly over the whole area.

Consistent with the aim of keeping the analysis simple, airline distance from zone centroid to zone centroid was used as the measure of inter-zonal transport system impedance. Intra-zonal distance was assumed to be 2 km. These simplifications avoided the need to define transport network configurations superimposed over the land-use zones. (If more realistic network distances are required then the route-factor concept - the ratio of the *actual* distance travelled from an origin to a destination over the transport network to the direct (airline) distance for that same origin and destination pair- can be exploited.)

These building blocks allowed twelve different urban structures to be analysed: by three different population sizes, each with increasing areas, containing from 900,000 jobs to 4.9 million jobs; and by four different, and extreme, employment distributions - completely centralised (at one zone centroid), evenly distributed over the whole area, evenly distributed in the central zone, and clustered evenly at each zone centroid. Figure 1 illustrates these four employment distributions for an urban structure comprised of 3 x 3 zones, where the shaded circles indicate point locations at zone centroids and the shaded squares uniformly distributed locations. Similar employment distributions are produced for larger cities - comprised of 5 x 5 and 7 x 7 zones.

Figure 1
Theoretical Urban Structures and the Spatial Distribution of Employment



SI = 0
Jobs at centre

SI = 1/6 Ad^2
Jobs in central zone

$SI = \sum_{j=1}^{9} A_j d_{ij}^2$
Multi-nodal

SI = 1/6 $A(3d)^2$
Dispersed jobs

3.2 Spatial Indices

Figure 1 gives the formulae used in each of the four urban structures to calculate the spatial index SI. The full results of calculating the moments of inertia for twelve theoretical urban structures differentiated by their size (number of zones and total employment) and employment distribution are given in Table 1.. Employment distributions are ordered from the theoretical bounds of highly centrelised in the left-hand column to completely dispersed in the right-hand column. Employment distributed uniformly in the central zone and employment distributed evenly amongst every zone centroid (multi-nodal) represent intermediate distributions.

From Table 1, the behaviour of the spatial index can be seen to conform to some general notions of the dynamics of spatial structure. The index increases from zero for a completely centralised employment distribution to a maximum value defined by the geographical size of the area and its total employment base. In the case of a city with about 0.9 million jobs in an area of 25 km^2 this range is from 0 to 3.4 x 10^7. Similarly, as the city both increases in area, and increases in the number of jobs contained in that area - but holding the employment distribution the same - then the index also increases. For example, in a multi-nodal structure, this is from 3 x 10^7 for 0.9 million jobs to 98 x 10^7 for 4.9 million jobs, and from 3.4 x 10^7 for 0.9 million jobs to 100 x 10^7 for 4.9 million jobs in a dispersed city.

Table 1
Spatial Index Values for Different Employment Distributions

Urban Structure		Index (x 10^7) by Employment Distribution			
Zones	Total jobs	At centre	Central zone	Multi-nodal	Dispersed
9	900,000	0	0.38	3.00	3.38
25	2,500,000	0	1.04	25.00	26.04
49	4,900,000	0	2.04	98.00	100.04

Spatial indices could have been calculated for the distribution of population (resident workers). Because the assumption is for an uniformly dispersed pattern of resident workers then the spatial index would be identical to the sixth column of Table 1 headed "dispersed". The spatial index for residential workers can be calculated for an exponentially distributed population around a city centre, for example, or for any other empirical distribution. Section 4 gives results for a case study that is based on actual distributions of resident workers and employment.

4. CASE STUDY, SYDNEY 1961 TO 1981

This section presents the results of analysing ABS Census data for the journey-to-work in Sydney from 1961 to 1981. (The 1986 data were not released because of errors). These data are described in Section 4.1. Sources of these data allow: the parameters of a fully-constrained gravity model to be calibrated, as in Section 1; the spatial index (moments of inertia) for both jobs and resident workers to be calculated, as in Sections 2 and 3; and the relationship between the indices and gravity model parameters to be explored, as in Section 4.2 below.

4.1 Data Sources

The Census of Population and Housing, journey-to-work tabulations provide the basic data for the Sydney metropolitan region from 1961 to 1981. The questions asked in the 1961 Census by the Commonwealth Statistician included the name and address of employers, but the answers were not published. From this, a 10 per cent sample of the work force at June, 1961, was selected by the Commonwealth Statistician. Subsequent ABS Census journey-to-work data (for 1971, 1976 and 1981) were a full census of journeys-to-work, but excluded those with no fixed place of employment.

These journey origins and destinations were aggregated by local government area (LGA) into 40 zones for metropolitan Sydney plus four peripheral zones of Wollondilly, Blue Mountains Gosford and Wyong. Although there have been boundary changes to some local government areas, especially those around the outer fringe of the metropolis, this analysis has minimised and effect on boundary changes.

The distances travelled by people to work in the period 1961 to 1981 are measured in kilometres and tenths of kilometres over the road by the shortest path from the origin local government area zone centroid to the destination local government area zone centroid. The State Transport Study Group of New South Wales provided the matrix of inter-zonal distances based on the location of centroids and a computer algorithm (skin trees over the highway network). Intra-zonal distance for each local government area reflects an average distance to travel within the zones and is a function of the size and shape of each local government area. Travel times by private and public transport were not available for 1961 as they were in 1971 and 1981 to allow a comparative study between 1961 and 1981.

4.2 Results

For the observed origin-destination traffic flows and the spatial distribution of employment and resident workers from 1961 to 1981, the spatial indices, the journey-to-work mean trip lengths, and the calibrated parameters for Tanner's function were estimated. These results are given in Table 2. The criterion of the correlation coefficient between survey and model O-D matrices was used to establish the best transport impedance function - in this case, Tanner's function. The spatial indices increase over twenty years reflecting growth, suburban expansion, and the relative suburbanisation of homes and workplaces. The spatial index for employment has increased three-fold, and the spatial for residences has also increased by almost that amount but from a higher value in 1961. The mean journey-to-work trip length has also increased from 12.8 km to 16 km.

Table 2
Spatial Indices and Trip Distribution Model Parameters, Sydney 1961-1981

Year	Spatial Index ($x10^7$)		Mean Trip Length	Parameters for Tanner's Function	
	Employment	Residences	(km)	α	β
1961	25.130*	36.131*	12.78	2.234	0.029
1971	49.867	68.133	14.27	1.973	0.044
1976	61.484	81.256	14.85	1.879	0.046
1981	78.679	103.958	15.97	1.724	0.051

(* - based on 10 per cent sample)

Despite there being only four data points, the relationship between the gravity model transport impedance function parameters, α and β, were plotted against the spatial index for employment and residences (resident workers) in turn. All graphs showed a consistent trend. The results for both the spatial index for employment and for the spatial index of resident workers as explanatory variables were very similar. For this reason we have combined the indices into an average value for Sydney in 1961, 1971, 1976 and 1981.

The relationship between the gravity model parameters and the mean of the spatial index over a twenty-year period is shown in Figure 2 for the α-parameter and in Figure 3 for the β-parameter of Tanner's function. Linear regression analysis lead to the following models:

$$\alpha = 2.483 - 8.414 \times 10^{-10} \text{ SI}$$

and

$$\beta = 0.0195 + 3.6591 \times 10^{-11} \text{ SI}$$

By making exogeneous forecasts for the future zonal distribution of resident workers and employment (see Section 1.1), or by postulating alternative strategic land-use plans, the future values of the spatial index may be easily calculated (Section 2). Substitution into the above equations will provide a better estimate of the future transport impedance parameters than is currently the case in transport planning practice. The implications for modelling practice are explored in the next section.

Figure 2
Relationship Between Mean Spatial Index and Model Parameter, in Alpha, Fully-Constrained Gravity Model, Sydney, 1961 to 1981

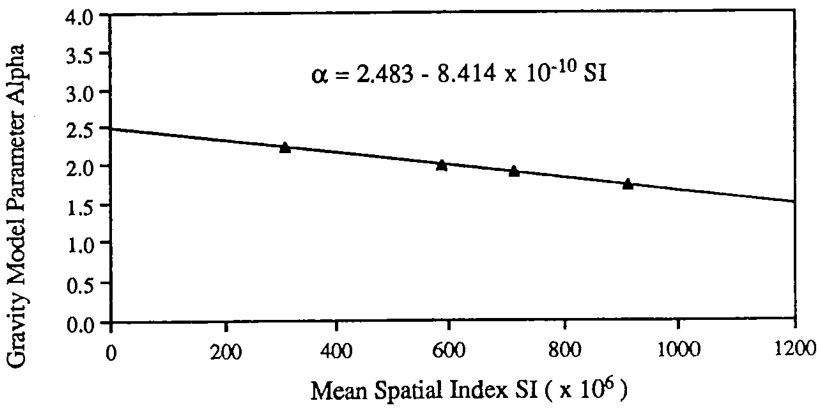
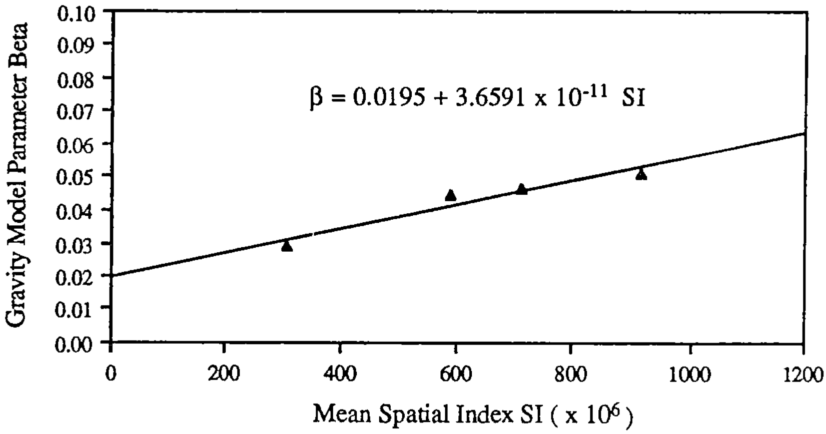


Figure 3
Relationship Between Mean Spatial Index and Model Parameter, Beta, in Fully-Constrained Gravity Model, Sydney, 1961 to 1981



5. IMPLICATIONS FOR TRANSPORT MODELLING PRACTICE

In those cities with rapid growth and extensions of their built-up areas, or with internal restructuring of land-use activities, it is unlikely that the traditional approach to trip distribution model calibration will give results of sufficient accuracy for strategic planning purposes. This research has suggested that dynamic urban spatial structures lead to changes in trip-length frequency distributions that are not modelled correctly by assuming stable calibration parameters in the gravity model transport impedance function. A simple method for adjusting the calibration method has been proposed in Section 4.

The broader implications for transport modelling practice have been investigated using Canberra, Australia's National Capital as an example. The 1963 *Canberra Area Transportation Study* collected person trip information from road side interviews and postcard mail back surveys of car and bus travellers in order to calibrate traffic production, distribution and assignment models (Black, 1981, pp. 135-143). A production constrained gravity model with a power function was calibrated for journey-to-work trips and found to have a value of $\alpha = 0.53$ when Canberra was a compact settlement with about 40,000 people. In 1967, recognising that rapid population growth required a new study, *The Canberra Land Use Transportation Study*, examined six radically different development options to accommodate a future population of one million (Black, 1981, pp 154-165). This study transferred friction factors for the gravity model from those calibrated for Washington, D.C. and then adjusted them manually to reflect increasing trip lengths over time.

Whilst this study clearly recognised the connection between urban structure and gravity model calibration, the procedures were *ad hoc*. Furthermore, the six radical different options were assumed to be modelling with the same transport impedance function (friction-factors). This resulted in future trip lengths being similar in each case - a difference of only 2 per cent between a centralised and a decentralised form. Black and Katakos (1981) have shown that with different assumptions the linear growth pattern (Plan C) could entail 34 per cent more travel in the journey-to-work than a compact development (Plan A).

The data and transport models developed by the consultants in the 1967 Canberra Land Use Transportation Study were used by the authors to design a knowledge-based, computer system of land-use and transport interaction. A prototype has been successfully completed and demonstrated to the National Capital Planning Authority in Canberra. This program is currently being applied to further test some of the ideas contained in this paper, but with direct relevance to Canberra as a case study.

CONCLUSIONS

A case study of modelling journey-to-work census data in Sydney from 1971 to 1981 showed that the calibration parameters gravity model were not stable over time. This resulted in systematic errors in the forecast origin-destination matrix for the 1971 calibrated model when compared with 1981 data. We hypothesised that the transport impedance function in the gravity model was itself a function of evolving urban spatial structure - dynamic changes in relative home and workplace locations - and looked for a simple relationship between the two.

Urban spatial structure can be described quantitatively with an index analoguous with moments of inertia from mechanics. The properties of this index were tested using hypothetical urban forms varying in their population size (0.9 million jobs to 4.9 million jobs) and their zonal distribution of employment (centralised, spread across one central zone, multi-nodal, and evenly spread). The index increases in size from 0 (centralised employment) as the employment distribution decentralises, reaching a maximum for an evenly distributed employment pattern. The index also increases with the amount of land-use activity (population or employment).

The relationship between this index and the gravity model calibration parameter was examined with a case study of journey-to-work census data for Sydney for four time periods from 1961 to 1981. The index was calculated separately for the changing locations of resident workers, jobs, and for an average of resident workers and jobs. All were found to have a strong linear relationship with the parameters of different gravity model transport impedance functions (Tanner's, power, exponential). Tanner's function was found to be the best for Sydney.

Linear regression models with the calibration parameters, α and β, as the dependant variables and the spatial index as the explanatory variable were produced that allow future calibration parameters to be estimated from any dynamic change to the distribution of homes and work places. Whilst these findings that show a more dynamic spatial structure and its relationship with gravity model calibration parameters are no more than a provisional hypothesis - requiring further confirmation from data for Sydney for 1991, or from other cities with time series data - their practical application would lead to better gravity model forecasts. A knowledge-based software system is being written to assist model builders to estimate with more confidence future parameters values for radically different land-use plans in their study areas. Furthermore, the concept of transferable model parameters from one city to another is an appealing one to reduce the costs of transport model building.

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