TIME-SPACE MAPPING BASED ON TOPOLOGICAL TRANSFORMATION OF PHYSICAL MAP

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INTRODUCTION

It is considered desirable that the travel time-distances between any points can be represented by the physical distances measurable with a normal ruler. The focus of this paper is the development of such a map, which may be called a time-space map.

There is a complicated distortion of time-space from physical space due to transportation facilities, traffic congestion, man-made and physical barriers, etc. It is generally not possible to project time-space onto a two-dimensional plane as accurately as with physical space. If a time-space map is constructed with acceptable error, however, it will provide useful information for regional and urban planning. A time-space map shows visually an outline of the transportation level of service of a region. Two time-space maps before and after transportation improvements may visualize their effect. Furthermore, comparisons between time-space maps of week days and holidays, and of rush hours and day time may highlight some problems that face urban planners.

Isochrone maps are familiar as time-space maps. However, these are only based on the time-distances between one central point and many surrounding points, and hence cannot portray time-distances between the surrounding points (Angel et al., 1972; Ewing, 1974). This study concerns the construction of a time-space map on which distances between any points are in close agreement with the time-distances between them.

There have been some attempts to represent time-space in a two-dimensional Euclidean space mainly in the field of geography. The most successful have been the applications of multi-dimensional scaling (MDS) techniques (Marchand, 1973; Ewing, 1974; Forer, 1978; Sugiura, 1980; Yoshimoto,1981). Given a matrix of proximities for all pairs of some points, MDS gives the configuration of these points in a space of given dimensionality such that the distances between points are most consistent with given proximities. If time-distances are given as proximities between points and the configuration in a two-dimensional space is assumed, MDS gives locations of these points in the time-space map. Though MDS is seemingly relevant for a time-space mapping procedure, it has the following problems.

A time-space map should include other map features such as administrative boundaries, road networks, buildings, etc. In order to represent these features on the time-space map, they should be interpolated based on the points configurated by MDS. More important is that, even if map features are interpolated, this does not assure that topological properties of map features on the physical map are preserved on the time-space map (Ewing et al., 1977). This is because MDS gives only the projection of points, which are configurated in a space of three or more dimensions, onto a two-dimensional plane. This causes serious distortions such as two roads separated on the physical map may intersect on the time-space map, and a facility located in a certain district on the physical map may be included in another district in the time-space map.

This paper proposes a new time-space mapping procedure which integrates a topological transformation with the conventional MDS technique. It is assumed that the time-space map is reconstructed by a geometric transformation of the physical map. This transformation is defined by functions which preserve topological properties of map features. Given time-distances between some points on the physical map, transformation functions are calibrated such that the distances between these points on the time-space map are in close agreement with the given time-distances. The physical map is geometrically transformed by the calibrated function.

1. DEFINITIONS OF TIME-SPACE AND TIME-SPACE MAP

Before entering into a detailed discussion of time-space mapping, the definitions of time-space and time-space map in this study are clarified.

1.1 Definition of Time-space

1) Time-space is constituted by the same set (set of points) as physical space.

2) Time-space is metric space (Gatrell, 1983). Therefore, time-distance can be identified by measurement or cognition, and the time-distance between any two points p and q in time-space, t_{pq}, satisfies the following properties:

$$\text{a) if } t_{pq}=0, \text{ then } p=q; \quad \text{b) } t_{pq}=t_{qp}; \quad \text{c) } t_{pq} \le t_{pr}+t_{rq}, \text{ for any r in time-space.} \qquad (1)$$

If there are time-distances which do not satisfy the above properties, it is assumed to be caused by random errors.

3) Time-space is k-dimensional Euclidean space.

1.2 Definition of Time-space Map

1) The points which are to be represented on the time-space map are not necessarily all points of the time-space. The time-space map is constituted by a subset, V, of the time-space.

2) A map must be easy to understand. The dimensionality of the time-space map should be less than that of the time-space, that is k. The time-space map is a result of mapping a set V of time-space onto m-dimensional ($m<k$) Euclidean space.

3) The mapping must be identified by given time-distances between points. Let W be a subset of V and W be constituted only by points between which time-distances for all pairs are given. Time-space mapping is identified based on time-distances between the points of subset W, and then the remaining points of set V are transformed by the derived functions onto the time-space map.

1.3 Study Assumptions

Furthermore, this study assumes the following from the practical viewpoint of time-space mapping.

1) There exists a physical map representing set V. That is to say that the time-space map is a result of mapping the physical map.

2) Set V is approximately represented as graphs (points and their connection relations) within an acceptable accuracy on a physical map. (This is a usual assumption when drawing a map by a plane table surveying or constructing a digital map data-base.) With this, a time-space map can be constructed by mapping a practical finite number of points.

3) The time-space map is represented in m-dimensional space($m<k$). For ease of purpose, however, a plane map is the most familiar and practical. Therefore, the time-space map is assumed to be two-dimensional for convenience in later discussion. However, the proposed mapping procedure does theoretically hold in more than two-dimensional cases.

The time-space mapping is summarized as follows;

$$u = f(x,y); \quad v = g(x,y) \tag{2}$$

where (x,y) is the coordinate of a point belonging to set V on the physical map and (u,v) is that on the time-space map, and f and g are the transformation functions from the physical map to the time-space map. These functions are identified based on the time-distances between the points i and j, t_{ij}, which belong to subset W. Suppose that (x_i,y_i) and (x_j,y_j) are the coordinates of the points i and j on the physical map, then the coordinates of the points i and j on the time-space map, (u_i,v_i) and (u_j,v_j), are given as follows;

$$u_i = f(x_i,y_i); \quad v_i = g(x_i,y_i) \tag{3}$$

$$u_j = f(x_j,y_j); \quad v_j = g(x_j,y_j) \tag{4}$$

The functions are calibrated such that the distance between the points i and j on the time-space map, d_{ij}, that is,

$$d_{ij}=\sqrt{(u_i-u_j)^2+(v_i-v_j)^2} \tag{5}$$

is most consistent with the given time-distance, t_{ij}.

Figure 1 shows the concept of the above mentioned time-space mapping procedure.

2. APPLICABILITY OF MDS TO TIME-SPACE MAPPING

This chapter describes the basic formulation of the multi-dimensional scaling (MDS) technique and discusses the applicability of MDS to the time-space mapping.

2.1 Formulation of MDS

MDS seeks a configuration of the points in a given dimensional space such that the solution distance between points i and j, d_{ij}, is most consistent with the given distance, t_{ij}. MDS techniques can be classified into metric MDS and non-metric MDS according to whether t_{ij} is regarded as an interval scale or an ordinal scale respectively (Torgerson, 1952; Shepard, 1962; Kruskal, 1964; Shepard, 1966). A variety of formulations and algorithms for the solution have been provided. The basic formulations of the metric MDS and the non-metric MDS are given as follows.

1) Metric MDS

$$\min. \quad \sum_{i<j} (t_{ij} - d_{ij})^2 \qquad (6)$$

2) Non-metric MDS

$$\min. \quad \sum_{i<j} (e_{ij} - d_{ij})^2 \qquad (7)$$

where eij is an interval scale measure satisfying the following;

$$\text{if } t_{ij}<t_{kl}, \text{ then } e_{ij}\leq e_{kl}; \quad \text{if } t_{ij}=t_{kl}, \text{ then } e_{ij}=e_{kl} \qquad (8)$$

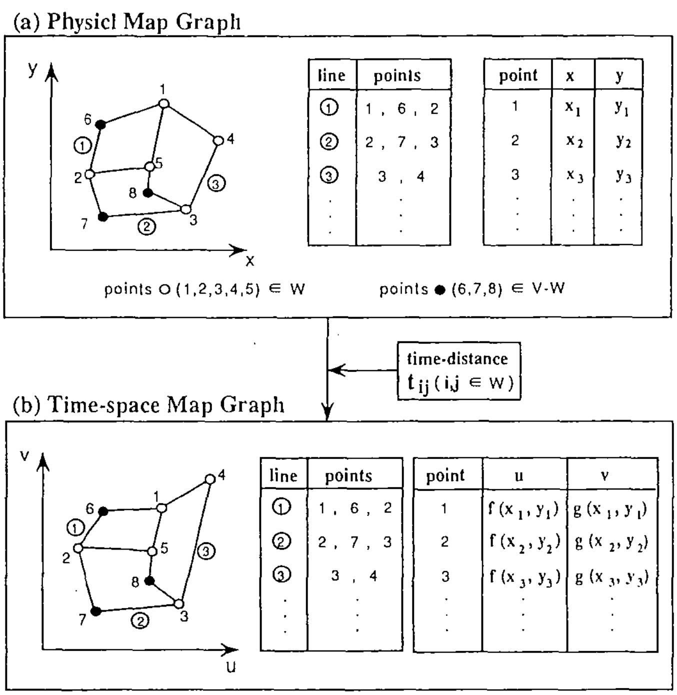


Fig.1 Concept of Time-space Mapping

2.2 Applicability of MDS to Time-space Mapping

Time-space mapping appears to be a typical application problem of MDS in the case that t_{ij} is given as a time measure and the space in which the points are configurated is given as a plane. There have been many researches in which the configuration of the points by MDS is regarded as the time-space map (Ewing, 1974). The conventional MDS technique, however, is insufficient for the above defined time-space mapping procedure for the following reasons.

1) MDS configurates the points only based on distances between them, and does not consider the topological properties of the map graphs. That is to say that MDS is not topological transformation (homeomorphic mapping). Therefore, it is possible that MDS causes the destruction of topology of map graphs. This shows the possibility that two roads separated on the physical map may intersect on the time-space map. The destruction of topology is sometimes effective in showing how large the time-space is distorted from the physical space. However, in the case of a time-space map utilized for the presentation of transportation level of service and its change due to transportation improvements, there is a risk that the destruction of topology creates confusion for map users.

2) The mapping from the physical map onto the time-space map is applied only to the points between which time-distances are given, that is, subset W. It does not determine the mapping of the other points, that is, subset V-W. It must be noted again that the time-space map constructed by MDS is a configuration of points where the time-distances between them are given. In order to represent the other map features (V-W) on the time-space map, it is required to determine the mapping for them (Tobler, 1978).

3. FORMULATION OF MDS CONSTRAINED BY TOPOLOGICAL TRANSFORMATION

This chapter proposes a new time-space mapping procedure which solves the problems of the MDS technique. The characteristics of the proposed procedure are as follows; 1) The mapping of the physical map onto the time-space map is based on the topological transformation which preserves the topological properties of the physical map graph, and 2) This topological transformation is identified by the same optimal criteria as those of the conventional MDS shown in Equations (6) and (7).

3.1 Conditions for Topological Transformation

The condition that a certain mapping is a topological transformation, that is homeomorphic transformation, is that the mapping is a 1-1 mapping and continuous, and its inverse mapping is also continuous. The typical topological transformations are an affine transformation and a projective transformation as given below;

1) Affine transformation

$$u = ax+by+c; \quad v = dx+ey+f; \qquad \begin{vmatrix} a & b \\ d & e \end{vmatrix} \neq 0 \tag{9}$$

where a,b,...,f are unknown parameters.

2) Projective transformation

$$u = \frac{ax+by+c}{px+qy+r}, \quad v = \frac{dx+ey+f}{px+qy+r}, \qquad \begin{vmatrix} a & b & c \\ d & e & f \\ p & q & r \end{vmatrix} \neq 0 \tag{10}$$

where a,b,...,f and p,q,r are unknown parameters.

3.2 Formulation of MDS Constrained by Topological Transformation

The integration of the metric MDS and the topological transformation is realized by uitilizing Equations (3),(5) and (6).

$$\min. \quad \sum_{i<j} (t_{ij}' - d_{ij})^2 \tag{11}$$

$$d_{ij} = \sqrt{(u_i - u_j)^2 + (v_i - v_j)^2} \tag{12}$$

$$u_i = f(x_i, y_i); \quad v_i = g(x_i, y_i) \tag{13}$$

Equations (11) and (12) are those of the conventional metric MDS. The difference is that the coordinates of the time-space map, (u_i, v_i), are constrained by the topological transformations, $f(x_i, y_i)$ and $g(x_i, y_i)$. While the metric MDS gives directly the coordinates, (u_i, v_i), the proposed procedure calibrates the parameters of the transformation at the first step, and then determines the coordinates, (u_i, v_i). Given the appropriate initial values of the unknown parameters, optimization can be realized by the quasi-newton method.

Also in the case of non-metric MDS, the estimation of the coordinates, (u_i, v_i), is not different from that in the metric MDS. Therefore, the proposed procedure can be also integrated with the non-metric MDS.

4. EXPERIMENTAL STUDY

A simple experiment was carried out in order to evaluate the performance of the proposed time-space mapping procedure. Figure 2 shows the physical map graph and time-distances which were given for the experiment. Time distances for all pairs of the points constituting the map graph are given in this case. Therefore, the time-space map can be constructed even by the conventional MDS. The metric MDS was applied to the time-space mapping.

For comparison, the metric MDS techniques constrained by the transformation functions shown below were applied.

1) Affine transformation expressed as in Equation (9).

2) Quadratic affine transformation, which is in this study an expanded form of the affine transformation to a quadratic function. This is not theoretically a topological transformation.

$$u=a(x-b)^2+c(y-d)^2; \quad v=e(x-f)^2+g(y-h)^2 \tag{14}$$

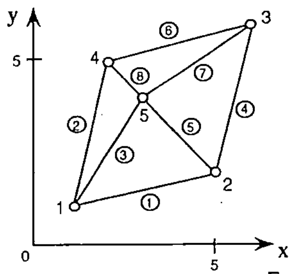
3) Cubic affine transformation, which is in this study an expanded form of the affine transformation to a cubic function. This is theoretically a topological transformation.

$$u=a(x-b)^3+c(y-d)^3; \quad v=e(x-f)^3+g(y-h)^3 \tag{15}$$

4) Projective transformation expressed as in Equation (10).

5) Polynomial transformation, which is not theoretically a topological transformation.

$$u=ax+by+cxy+dx^2y+exy^2; \quad v=fx+gy+hxy+ix^2y+jxy^2 \tag{16}$$

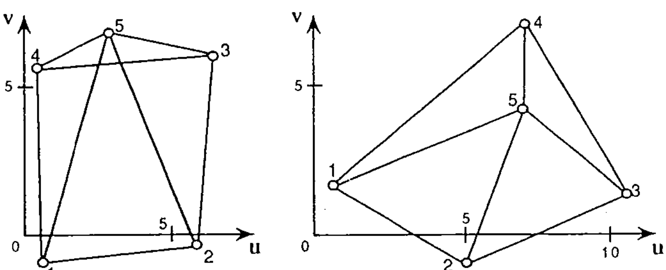


line	points
①	1 , 2
②	1 , 4
③	1 , 5
④	2 , 3
⑤	2 , 5
⑥	3 , 4
⑦	3 , 5
⑧	4 , 5

point	x	y
1	1	1
2	5	2
3	6	6
4	2	5
5	3	4

$$t_{ij} = \begin{bmatrix} 0 & 5 & 11 & 7 & 10 \\ & 0 & 6 & 12 & 9 \\ & & 0 & 6 & 5 \\ & \text{sym.} & & 0 & 3 \\ & & & & 0 \end{bmatrix}$$

Fig.2 Physical Map Graph and Time-distance



(a) Metric-MDS

(b) Metric-MDS Constrained by Cubic Affine Transformation

Fig.3 Time-space Map Graph

Table 1 shows the accuracy of the time-space mapping indicated by the correlation coefficient between time-distance (t_{ij}) and distance on the time-space map (d_{ij}). Table 1 also shows the preservation or distortion of the topological properties of the physical map on the time -space map. Figure 3 shows the configurations of the time-space maps constructed by the conventional metric MDS and by the metric MDS constrained by the cubic affine transformation. The following implications are obtained from the experiment.

1) Under the transformation constraints, the accuracy of the time-distances becomes low in comparison with the conventional MDS. In particular, high accuracy cannot be obtained by the affine transformation. The expansion of the affine transformation to a polynomial function may be required for the application in time-space mapping.

2) The conventional MDS cannot preserve the topological properties, as shown in Fig.3. For example, lines 5 and 6 which are separated on the physical map intersect on the time-space map, and point 5 within the quadrilateral 1-2-3-4 on the physical map is located without it on the time-space map. Topological properties were not preserved by the polynomial transformation, either. On the other hand, the quadratic affine transformation, which is not theoretically the topological transformation, preserved the topological properties of the physical map. This shows that the various transformations, without regard to whether they are theoretically topological transformations or not, can be alternatives in practice.

Table 1 Accuracy of Time-space Mapping

procedure	correlation coefficient	topology
metric MDS	0.942	not preserved
affine transformation	0.771	preserved
quadratic affine transformation	0.873	preserved
cubic affine transformation	0.866	preserved
projective transformation	0.806	preserved
polynomial transformation	0.941	not preserved

5. APPLICATIONS

The proposed procedure was applied to a time-space mapping of Japan based on travel time-distances by Japan Railways (JR) group. Twenty-nine cities were selected considering the regional balance. Figure 4 shows the physical map of Japan to be represented on the time-space map in this study. The map features are the coast lines of the four main islands (Honshu, Hokkaido, Kyushu and Shikoku) and the locations of selected cities. The time-space maps were constructed at three points of time; 1962 (just before the operation of Shinkansen ("Bullet train")), 1992, and after the completion and operation of all currently planned Shinkansen lines. The trunk railway network employed in this study is shown in Fig.5. Figure 6 shows the Shinkansen projects including the existing and currently planned lines. Travel time-distances were calculated for all pairs of these cities based on the Dijkstra's method for the shortest path search after giving the travel times at three points of time to the links of the railway network.

Figures 7 and 8 show the time-space maps in 1962 and 1992 respectively. Both maps were constructed by the metric MDS constrained by the cubic affine transformation. The correlation coefficients between the distances on the time-space map and the given time-distances were 0.971 and 0.956 respectively, which were considered to be satisfactory in application. Figure 8 well presents the outline of the current railway level of service of Japan. It is clearly shown that the railway level of service of Honshu, in particular, in the north-south direction, is comparatively high. The comparison between the two time-space maps in 1962 and 1992 shows the railway service improvements in Japan, mainly by the Shinkansen. It is visualized that the railway service improvements in Honshu have made progress in the last thirty years. Such improvements, on the other hand, have not been provided to Hokkaido, Kyushu and Shikoku while in this period.

In order to examine the effects of the Shinkansen projects which are currently planned, an additional time-space mapping was attempted. Figure 9 shows the time-space map of Japan at the time when these projects will be completed and the Shinkansen lines operating. The result visually describes that the Shinkansen projects will contribute to the formation of a regional well-balanced time-space of Japan, as shown in the scale-down of the areas of Hokkaido and Kyushu and in the reduction of distance between the east and west sides of Honshu.

6. CONCLUSION AND FUTURE PROSPECTS

While the isochrone map, which is a conventionally typical time-space map, represents only time-distances from a central point to the surrounding points, the time-space map constructed by this study represents the time-distances between any points as accurately as possible. This time-space mapping procedure provides the possibility to effectively present the outline of the transportation level of service and its change caused by transportation improvements and traffic congestion. It can also be applied to the analyses on the cognitive distance or space of citizens.

The MDS, which has been most familiar as a technique for the analysis of a time-space, has two limitations in its application to time-space mapping. First, it cannot represent the map features except the points between which the time-distances are given. Therefore, remaining points must be interpolated based on the located points. Secondly, though a variety of geographical interpolation techniques can be applied, they cannot necessarily preserve the topological properties of the interpolated features. This study proposed a new MDS technique which integrates the conventional MDS and the topological transformation. With this, all features on the physical map can be transformed onto the time-space map with a perfect preservation of their topological properties.

Geographical information systems (GIS), which enable the efficient and effective management and application of maps by means of computers, have recently made remarkable progress and gained popularity (Burrough,1986; Nakamura et al.,1990). The GIS represents map features as coordinates and topological relations independently in a data-base. The proposed procedure can construct time-space maps, in which the topological properties are perfectly preserved, only by transformation of coordinates. If the proposed procedure is integrated into GIS, time-space maps can be automatically displayed by making only a change of coordinates in the data-base by the calibrated transformation functions. There is a good possibility that the proposed time-space mapping procedure will be utilized in GIS as both an easy and effective tool for regional and transportation analyses and the presentation of their results.

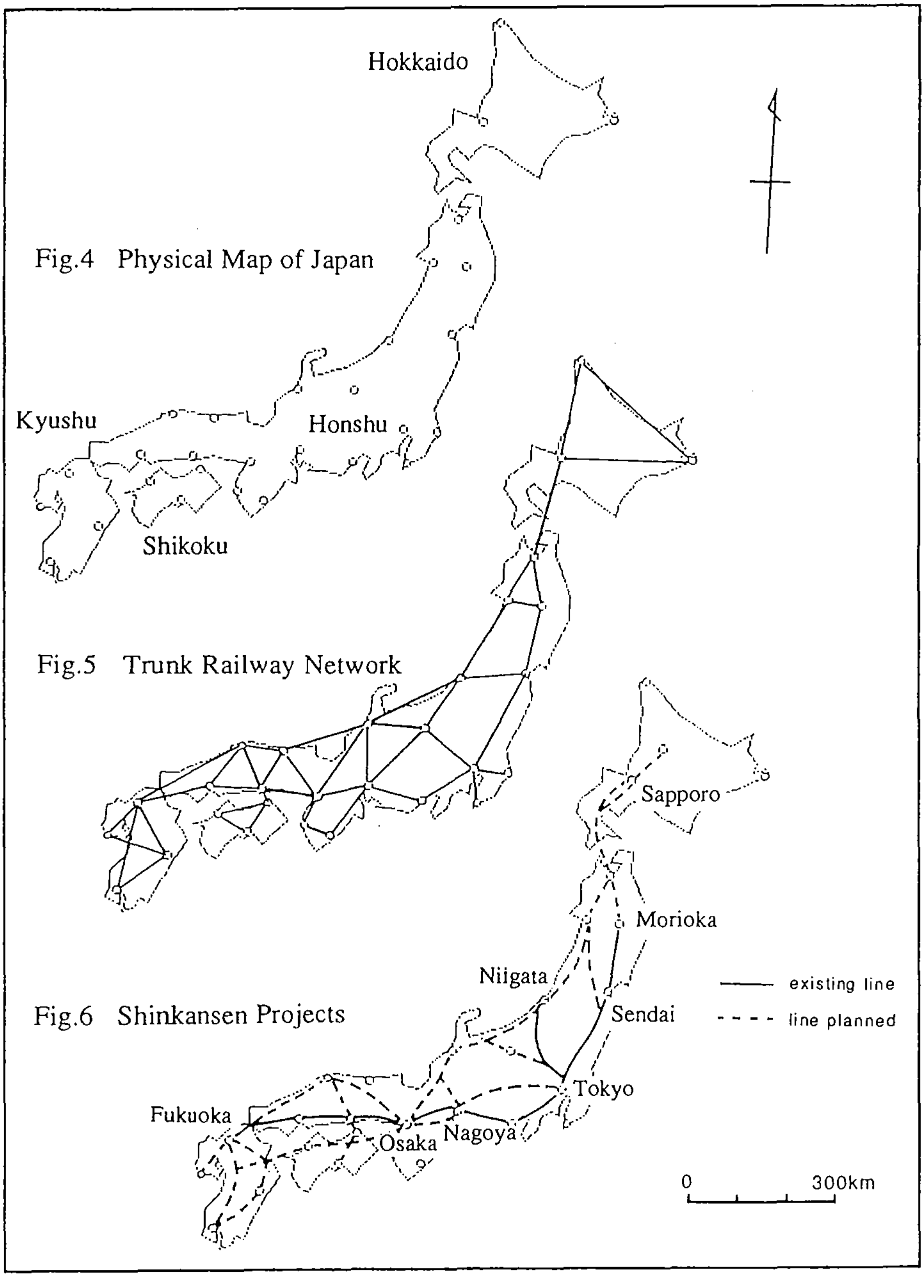


Fig.4 Physical Map of Japan

Fig.5 Trunk Railway Network

Fig.6 Shinkansen Projects

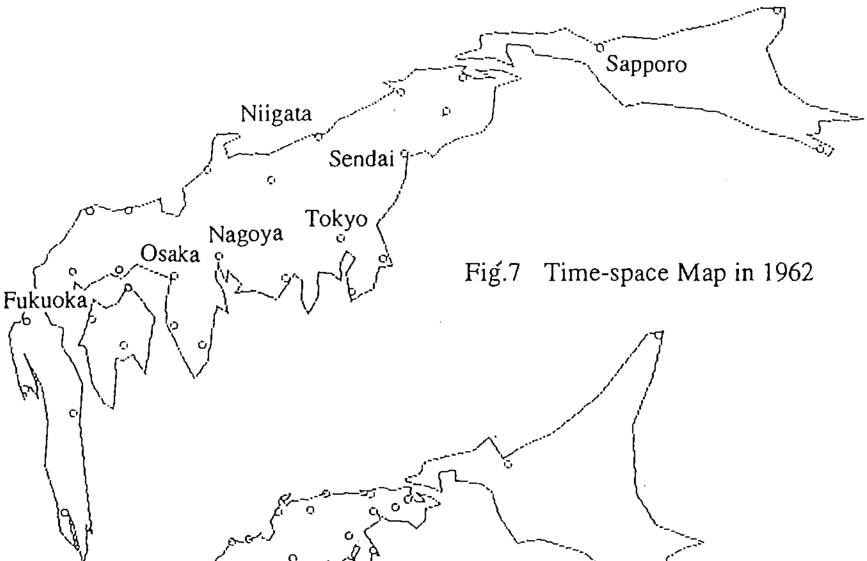


Fig.7 Time-space Map in 1962

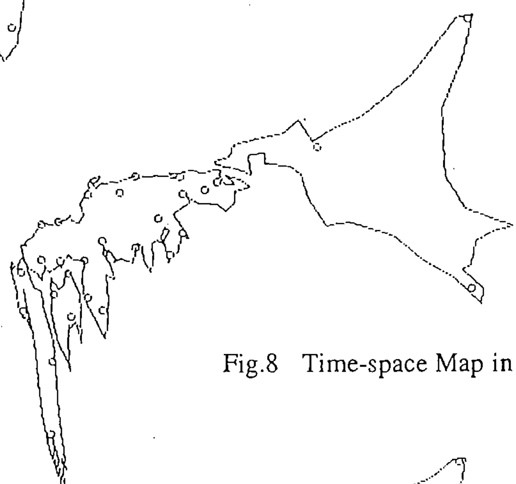
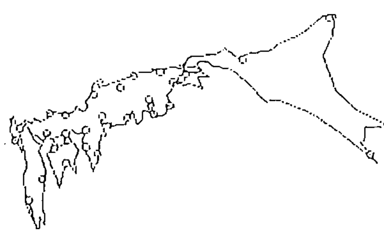


Fig.8 Time-space Map in 1992



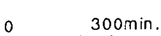


Fig.9 Time-space Map after the Operation of Currently Planned Shinkansen Lines

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