TRAFFIC DIVERSION MODEL ON URBAN EXPRESSWAY BY FUZZY REASONING

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INTRODUCTION

The accurate forecast of traffic volume on urban transport networks are important. As the transport networks are generally consist of expressways and streets, the suitable traffic balance between them, in particular, should be investigated as an urban transport planning. The functional diversion model is commonly used to solve this problem. In this study, fuzzy reasoning is proposed to apply as an useful technique for describing the travel behavior alternatively.

First, important factors of route choice are displayed by fundamental analysis of travel behavior on urban expressway. Prototype fuzzy reasoning model is built for description of the diversion travel pattern with OD survey results. Second, implication forms based on T-norms are discussed to educe the effect on estimation with change of formulation. Lastly, two advanced reasoning techniques such as min-sum-gravity and product-sum-gravity are introduced to find out the better way to construct the fuzzy reasoning system to forecast the diversion rate.

1. BACKGROUND OF FUZZY DIVERSION MODEL

Traffic assignment model is generally used for network traffic demand forecasting. In case the urban transport network consists of expressways and streets, the share rate between them should be obtained at this forecasting stage. For the technical reason of traffic assignment on the practical scale network, traffic volume on expressway will be determined after each O-D traffic volume is obtained.

Diversion model have been proposed for forecasting the share rate of expressway. Many researches focus to estimate the diversion curve in realistic data. The relative researches with route choice behavior gave useful results to construct the forecasting model:(a) Travel time and travel cost (toll on expressway) are recognized to be important factors. (b) Different travel pattern occurs when the trip length of OD becomes short.

In addition, it is presumed that the route choice behavior should be described with linguistic variables, which represents human perception. Therefore, in this study, fuzzy reasoning model is employed to describe behavior of the motorists.

1.1. Fuzzy Reasoning Approach

The estimation system of fuzzy reasoning resembles to the production system which is a most popular technique to realize expert inference on the computer. The general procedure of fuzzy inference is illustrated in Figure 1. The basic concept of fuzzy reasoning (particularly used in fuzzy control field) can be introduced with referring to the figure[1],[2].

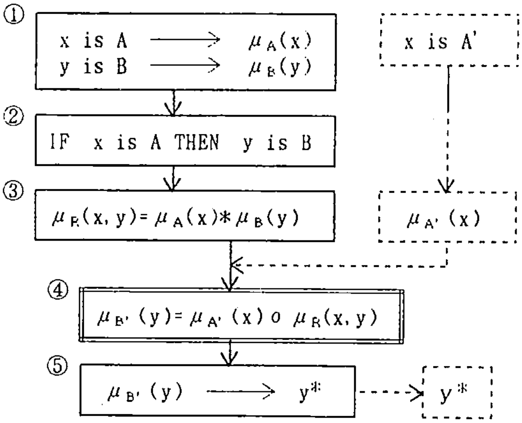


Figure 1 The Abstracts of Fuzzy Reasoning

(1)Propositions such as "x is A" and "y is B" are to be determined with fuzzy numbers. In other words, every membership function should be defined respectively at first stage of modeling.

(2)It should be determined how many IF/THEN rules are prepared. It means that the number of inference rules and their structure should be determined.

(3)Implication rule, which is denoted as (*) in formulation, is defined to indicate the relation between x and y. This relationship R is preserved as membership function of x and y. Most popular implication form of (*) is a minimum function of x and y. It is called as implication of Mamdani.

(4)Once the fuzzy relation is preserved in the system, the fact can be deduced from the observed information "x is A'" . The observed fact, A' is also a fuzzy number. The deduced result will be given from the composition, B'=A'oR, according to the formulation in the figure. The composition, o, is usually defined as max-min.

(5)Each deduced result of an inference rule have a membership distribution. If plural rules simultaneously fires, the union of each fuzzy number becomes a final result. The union is usually defined as largest value of membership function in meaning of the union of fuzzy sets. Finally, the estimated value (non-fuzzy number) to the problem is determined by means of a center of gravity under the membership grade curve on a previous union set.

The prototype of fuzzy reasoning model consists of "min-max-gravity" reasoning, because the first term, 'min', corresponds to the implication (step 3), the second term, 'max', represents how to combine fuzzy result (step 5) and the last one shows the deffuzification method (step 5).

The advantages of the fuzzy reasoning model are summarized from the results of recent studies: First, it is helpful to describe nonlinear relationship among variables. This property is quite different from the results of techniques based on function. Second, since a large number of rules should be defined in case of crisp rules to cover every conditions. On the contrary, fuzzy rules will contribute to saving rules.

1.2. The diversion model

The objectives of the section is to construct the diversion model in order to estimate the OD traffic volume on the expressway. The prototype model is formulated by using typical fuzzy reasoning (max-min-gravity method) as mentioned previously.

1.2.1. Objective Area

In order to obtain the actual traffic flow patterns as examples, OD traffic on eleven zones each other were counted. This objective transport network consists of Hanshin Expressway and the street around Osaka city area, which is commonly used for traffic assignment. A hundred samples of OD traffic are chosen at random to be typical example cases.

1.2.2. Factors and their description

Three explanation factors are prepared to form inference rules. The linguistic labels are also defined on the factors to realize fuzzy reasoning[2].

a) Difference of travel time

This factor is defined on each OD pair as follows:

$t_1=t_s-t_h$,

where t_s:travel time on the street

t_h:travel time on the expressway

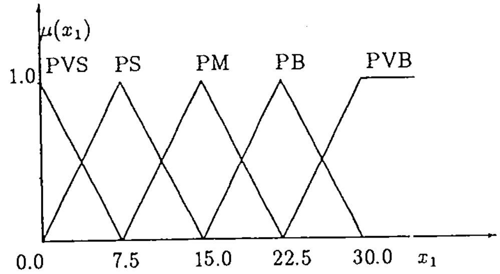


Figure 2 The Membership Functions for Time Difference

The drivers make decision of route choice mainly through the conditions of the factor. It is, however, difficult to give an answer to how precisely the driver perceive to the travel time. Even accurate information is provided to the drivers, their perceived times are not observed with certainty.

Therefore, five linguistic variables are prepared to represent the perceived degree of difference between street travel time and expressway travel time. Triangular fuzzy numbers (T.F.N.) in Figure 2 are employed to describe the linguistic labels.

b) The cost for unit travel

The perception of the drivers to the entrance fee is measured as a cost of unit distance of travel on the expressway. Uniform fare is charged on urban expressways in Japan. The driver, who enter the expressway, should pay the same amount of toll fare in all the Osaka area (i.e. five hundred yen in current). For determination of membership functions, the survey with Fuzzy Delphi Method (F.D.M.) was done[3], and the linguistic labels are derived as shown in Figure 3.

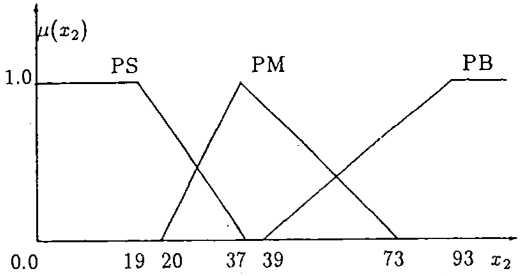


Figure 3 The Membership Functions Derived from F.D.M.

c) Distance between origin and destination

The value of the variable is measured by spatial distance. The drivers make decision of route choice mainly through the consideration of the previous factors. However, when their trip length are not so

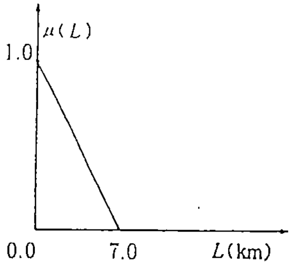


Figure 4 M.F. of OD distance

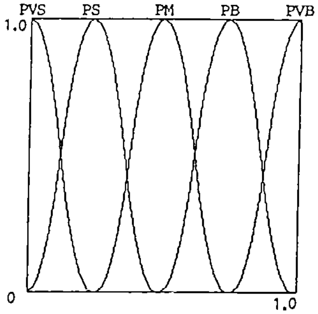


Figure 5 M.F. of Diversion rate

large, drivers feel no need to use the expressway any more. Under the circumstances, linguistic variables "short" is only defined on the OD distance. It is shown in Figure 4.

d) Diversion rate

According to the concluding part of the IF/THEN rule, linguistic variables are prepared to give meanings of the diversion rate between zero percent and hundred percent. The diversion rate, the conclusion of fuzzy inference, is only defined as a nonlinear curve based on Pi function (quadratic function) as shown in Figure 5.

1.2.3. Inference rules

Inference of route choice consists of ten rules in Fig.5. Distance of OD related with diversion appears only in Rule-1. This rule fires only when OD distance is very short (i.e.'PVS'). It means many drivers never use the expressway when the trip distance is very short. Main concept of the reasoning is denoted on Rule-2 to Rule-10. That is, the diversion rate becomes large as time difference (x_1) become large or unit cost for travel (x_2) becomes small.

Rule 1 : IF 1 is PS THEN y is PVS
Rule 2 : IF x_1 is PVS THEN y is PVS
Rule 3 : IF x_1 is PS and x_2 is PS THEN y is PM
Rule 4 : IF x_1 is PS and x_2 is PM THEN y is PS
Rule 5 : IF x_1 is PS and x_2 is PB THEN y is PS
Rule 6 : IF x_1 is PM THEN y is PM
Rule 7 : IF x_1 is PB and x_2 is PS THEN y is PB
Rule 8 : IF x_1 is PB and x_2 is PM THEN y is PB
Rule 9 : IF x_1 is PB and x_2 is PB THEN y is PM
Rule10 : IF x_1 is PVB THEN y is PVB

Figure 6 Fuzzy reasoning rules

Five labels to the time difference and three to the cost are prepared. If we employed non-fuzzy rules, fifteen rules (i.e. =3x5) at least should be prepared to exhaust collectively all conditions.

On the other hand, each fuzzy rules has the interval of condition, therefore, smaller number of rules can work under the same conditions. The fact implies a advantage of using fuzzy rules. In actual inference, the ten rules mentioned above simultaneously work to estimate the diversion rate on every OD pairs.

2. COMPARISON OF FUZZY REASONING

2.1. Triangular Norms

Description of human perception in fuzzy reasoning can be altered by the defined form of implication. In general, fuzzy implication is defined by T-norm (Triangular Norm). T-norm is a function from [0,1] x [0,1] to [0,1] under following conditions[4]:

(1) $T(x,1)=x,\ T(0,x)=0$; boundary condition
(2) $x_1<x_2,\ y_1<y_2 \ \to\ T(x_1,y_1) < T(x_2,y_2)$; monotonousness
(3) $T(x,y) = T(y,x)$; convertibility
(4) $T(x,T(y,z)) = T(T(x,y),z)$; associative law

There are many kinds of T-norms to satisfy with these four conditions. Typical t-norms usually applied to practical fuzzy reasoning are as follows[5]:

(a) Logical Product (LP) $x \wedge y = \min\,[x,y]$
(b) Algebraic Product (AP) $x \cdot y = xy$
(c) Bounded Product (BP) $x \odot y = \max\,[0,\ x+y-1]$
(d) Drastic Product (DP)

$$x \dot{\wedge} y = \begin{cases} x, & \text{if } y=1 \\ y, & \text{if } x=1 \\ 0, & \text{if } x,y < 1 \end{cases}$$

As mentioned above, the basic diversion model is constructed with LP implication. This type of reasoning is so called Mamdani's method. These four implications are to be arranged in order of the magnitude of inference result as DP < BP < AP < LP. This observation is very important because every other T-norms are in the interval between DP and LP. Therefore, the proposed T-norms are served to be typical examples.

In fuzzy reasoning, the estimated values come from the result of inference related with the truth value of antecedent and latter conclusion. The inference result is displayed as a distribution of membership function.

For example, if we employ the algebraic product in place of logical product, the distribution of the estimated diversion rate changes. As the results of Logical Product and Algebraic Product are shown in Figure 7, the distribution has upper bound in case of LP. It was reasonable result to consider minimized composition. In this constant interval, the degree of truth on each fired rule becomes to equal.

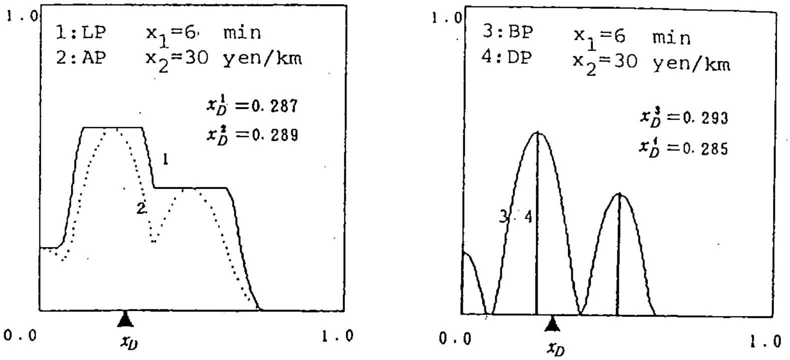


Figure 7 The Inference Results to the Example Case

On the other hands, Algebraic Product is defined as xy. Information involved in antecedent and conclusion are used uniformly, and the reasoning result depends on both. In this case, the truth value of the condition is preserved to produce the result proportionally.

By using boundary product and dynamic product are both degree of fuzziness on the result becomes small. This situation is observed in Figure 7 (right diagram). Because of the magnitude of each method, the result becomes close to crisp information. BP(Bounded Product) and DP(Drastic Product) are understood to special case of human reasoning even their definition are little complicate.

2.2. Properties of each implication

To perform the estimation of diversion rate, three explanatory variables were defined. Major parts of reasoning consist of the rules related with x_1 (time difference) and x_2 (cost for unit length travel). Three dimension diagram is utilized to illustrate non-linear relationship. The relationship is shown in Figure 8 under the condition that OD distance is fixed at seven kilometer.

The result of Algebraic Product is so closed to the result of Logical Product that only LP case appears in this figure. Some properties of estimation are observed. First, as x_1 becomes large, the diversion rate have no relation with the variable x_2. In this situation, all the driver move to the expressway (i.e. diversion rate is almost equal to be 1.)

Second turning point is observed in the area that x_1 is approximately fifteen minutes and x_2 stays around forty yen/km. As x_1 becomes large, the influence of value of x_2 tend to be small. In addition, every part of the diagram, non-linear and different slope is observed. This property is a most different point from formulation with function.

The results on implications, Bounded Product and Drastic Product, are classified to almost the same cluster. Figure 8 shows the result in case of DP(Drastic Products). The tendency of estimation is not very different from Logical Product (or Algebraic Product). However,

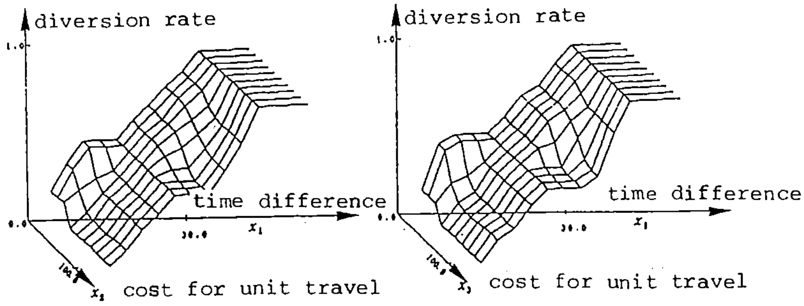


Figure 8 Three Estimation Space of x_1 and x_2

little difference is found out only when x_1 becomes smaller than thirty minutes. In this case, vibration of estimated value is larger.

2.3. Comparison with the estimation

For further discussion of the techniques, the case study with realistic data has been carried out. The shape of membership function seems to give influence to the estimated value. In this section, membership functions of time difference and diversion rate are put into consideration. The width of membership function (e.g. base of Triangular Fuzzy Number) will be changed in order to define various kinds of functions. In the case study, the width of membership function on time difference (W_1) is changed from 7.5 to 22.5 minutes at 1.5 minutes interval. The width of membership function on diversion rate (W_2) is also change from 12.5 to 37.5 percent at 2.5 percent interval. In this manner, one hundred and twenty one cases are prepared as numerical example.

Table 1 Estimation results with each implication

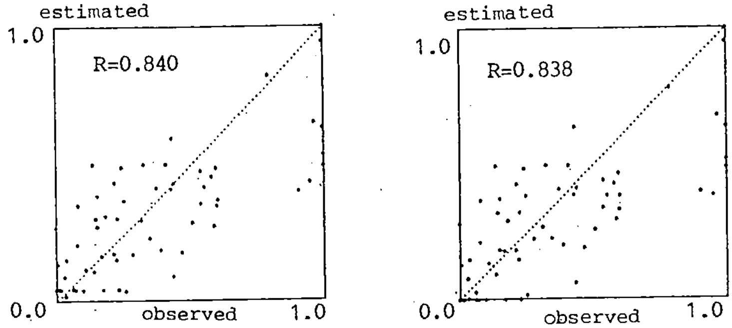
implication	W_1	W_2	RMSE	r
Logical Product	15.0	32.5	0.1636	0.840
Algebraic Product	18.0	37.5	0.1636	0.838
Bounded Product	15.0	12.5	0.1627	0.839
Drastic Product	15.0	25.0	0.1656	0.833

Table 1 displays the statistics for estimation with each implication. The combination of widths shown in the table are selected according to minimum value of RMSE (root mean square error). Correlation coefficient, r between actual value and estimated value is additionally displayed.

This observation concludes the following consideration. First, the optimal values of the widths of membership functions depends on which form of implication is in use for formulation. In particular, the membership function of diversion rate can easily be altered by change of alternative implication products.

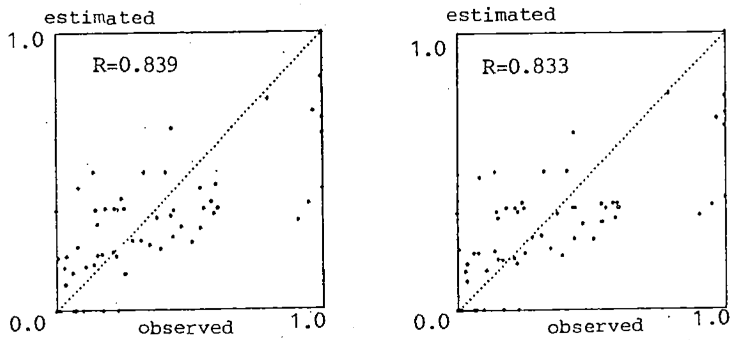
Second, considering the goodness of fit, the value of RMSE is smallest in case of bounded product. The difference among the products, however, is very small and the fitness is best in case of logical product from the point of correlation. Though implication of drastic product gives inferior results than other ones, the difference in deviation of estimation is not so large.

The comparison will be carried out from the correlation diagram on Figure 9. In case of algebraic product and boundary product, many points lie in the area uniformly around the diagonal line . On the contrary, many points tend lie in the lower area in case of Logical Product. It may be suggested that slight underestimation may be carried out by using logical product. Nevertheless, both logical product (Mamdani's method) and algebraic product are still recommended from the simplicity of modeling.



(logical product)

(algebraic product)



(bounded product)

(drastic product)

Figure 9 The Estimation Result of Each Implication

3. ADVANCED TECHNIQUES FOR FUZZY REASONING

Influence of the change of implication products was discussed in the previous section. To construct advanced technique, however, inference rule combining technique as well as implication should be investigated. All methods mentioned previously involves maximum combination for union of the rules. The focus of the section was on the advanced formulation and the performance of fuzzy reasoning model.

3.1. Modified technique for inference

In this section, two types of reasoning are introduced. They are min-sum-gravity and product-sum-gravity methods. Essential difference of these techniques to previous is to use the summation in place if supreme operation at second stage of fuzzy reasoning.

The advantage of this change is to make a reasonable combination of the fuzzy conclusion with the results of every rule. In general, the last conclusion of fuzzy reasoning is determined as a combination of results on plural rules which fires at once. As mentioned above, the (*)-max-gravity formed method neglects the doubly defined part of concluding membership function. However, the magnitude of these section should be doubly counted as much as the inference results are accumulated.

This definition of inference is applicable to cased which the T-norms are used such as logical product (minimum) and normal product (algebraic product). The inference result of the modified algorithm are depicted in Figure 10.

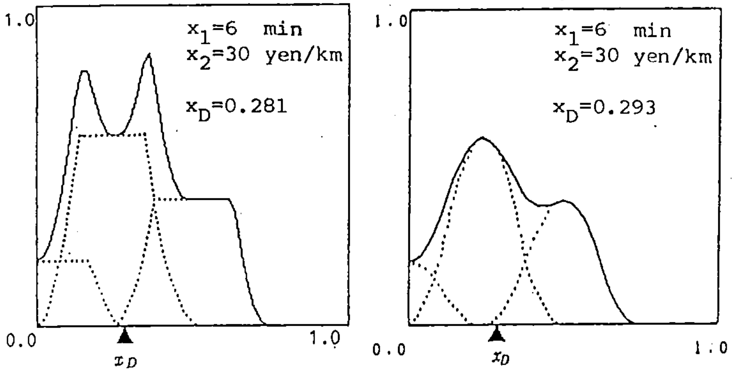


Figure 10 The Inference Results with MSG and PSG Methods

The concluding distributions in solid line are obtained from the sum of all result in dotted line. The difficulty in interruption is the fact that final fuzzy number is not sometimes normal (i.e. height of membership distribution sometimes exceed to one.) The estimated value, however, is scarcely affected by using gravity of distribution at defuzzification stage[4],[5].

3.2. The results of estimation

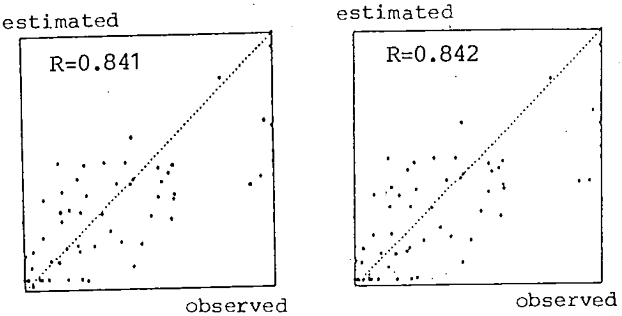
In order to consider the advantage of the each method, RMSE and correlation coefficients are obtained to the same one hundred and twenty one (=11x11) example as the previous section. The results are shown in table 2.

The reliable estimation of diversion rate was done using these methods. It is concluded that PSG (Product-sum-gravity) give the best estimation compared with the other methods. It has been also reported that PSG method shows good performance when it is used in the field of control.

Table 2 Estimation results with advanced methods

method	W_1	W_2	RMSE	r
min-max-gravity	18.0	32.5	0.1624	0.841
product-sum-gravity	16.5	12.5-38.5	0.1616	0.842

In the same manner, the scatter diagrams for estimation are shown in Figure 11. Estimation errors on each case are observed to be little smaller than the previous case.



(min-sum-gravity) (product-sum-gravity)

Figure 11 The Estimation Results with MSG and PSG Methods

The optimal width of spread of membership function on diversion rate was determined with 26% (12.5% to 38.55) interval in case of using product-sum-gravity method. This kind of robustness in parameters is required to model improvement. Therefore, product-sum-gravity approach will be recommended to give better performance to the model.

4. CONCLUDING REMARKS

The diversion modeling with fuzzy reasoning is proposed in the research. In particular, it focused how the decision of driver is simulated and the the model could be improved to have high accuracy of estimation by change of techniques of reasoning. In addition, advanced techniques of fuzzy reasoning were introduced. The concluding results are summarized as follows:

First, the concept of general fuzzy reasoning model was mentioned. The basic model is easily developed by the procedure, (*)-max-gravity method. In addition, the graphic results of reasoning, which represents perceptions of drivers, can be observed to consider the travel behavior.

Second, triangular norms, which works as a the implication, were compared each other from the point of logical meanings and accuracy of estimation. No one have the special advantage to the estimation of the diversion rate. Logical product and Algebraic product, however, were recommended to use in the model because of their simplicity.

Third, consideration of the way to define the union of inference results make improvement of fuzzy reasoning. The methods based on sum union, min-sum-gravity and product-sum-gravity, were proposed. In particular, the latter method was recognized to suite human perception and it gave better performance of the model.

In order to consider practical traffic assignment on urban networks including expressway, the better performed diversion model will be required. The further studies are recommended: The first problem occurs when the incremental assignment is utilized for forecasting the realistic large scale transport networks. The problem is how to estimate the dynamic change of diversion rate at the time iteration is proceeded. It means the combined traffic assignment with estimation of diversion should be carried out[6].

Second, the advantage of the fuzzy reasoning approach is summarized as its nonlinear estimation. This feather, however, sometimes decline to worse estimation. The difficulty is exists in calibration of membership functions. The combined model between fuzzy reasoning and neural network to have learning process is now in consideration.

ACKNOWLEDGMENTS

The authors wishes to express their appreciation to Dr. Shao Chunfu the advice and encouragement received in the execution of this work. The regional and traffic planning laboratory co. is also gratefully acknowledged.

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