# THE DESIGN OF HIGH CAPACITY BUS SYSTEMS: THE USE OF FUZZY SUPPORTS TO REPRESENT EXPERT OPINION

**Nicholas TYLER**
**Lecturer in Transport Studies**
**Transport Studies Group**
**University College London**
**London - United Kingdom**

## 1. INTRODUCTION

The research from which this paper is derived considers the use of artificial intelligence (AI) techniques to assist the design of high capacity bus systems (Tyler 1992). During the course of the research it was established that the engineers responsible for the design of existing systems relied to a considerable extent on their judgement and opinion at various stages in the design process. In order to represent the decisions in which such decision-making methods were used, it was necessary to incorporate them within the computer model being developed to assist the design process. The approach used in the research, and described briefly in this paper, was to derive a model to represent opinion within the decision process.

## 2. THE USE OF OPINION IN THE DECISION PROCESS

The design engineers consulted during this research used their opinion and judgement skills within the design process in situations where there was some doubt about the data being used for a particular decision. Such data may be known imprecisely, or with some degree of uncertainty, or may be the result of the application of a model to data which fall into these categories. Indications that such interpretative approaches include the description of a decision rule which includes a qualitative description such as "*if the level of parking is low, the bus stop may be positioned at the kerbside*". The use of the word "*low*" in this context implies that some evaluation has been made of the level of parking in order to determine whether this could be reasonably described as "*low*".

Another point to draw from this simple example is that there exists a significant difference between the type of input to the decision rule and the type of output. The input is derived from some quantitative estimation, such as the number of parked cars per kilometre which is likely to be a continuous function. The output, however, is a set of discrete decisions: the bus stop could be at the kerbside, or it could be positioned in the median. It is not possible to locate the stop at some compromise position between these two alternatives. The decision rule must choose a result from a set of discrete options. The engineer, in using descriptors such as "*low*", is converting the input data from its continuous state to a discrete state in order to facilitate the decision.

The concept of linguistic variables introduced by Zadeh (1965) seemed to be appropriate for the representation of these descriptors and so fuzzy logic was examined during the research in order to establish whether this could be used to represent the descriptors within the decision process.

## 3. THE USE OF FUZZY LOGIC

### 3.1 Fuzzy support values

The use of fuzzy logic within a decision process was considered by Bellman and Zadeh (1970). While the propagation of support values throughout a decision process is well documented, the derivation of the initial values of the fuzzy supports is less well described. The values of the fuzzy supports are constrained by three factors:

1 a fuzzy set can contain element values that appear in other sets and set membership is not exclusive;

2 full membership (a support value of 1) of a set is transitory for any given element value; and

3 the support values are assigned arbitrarily.

While the first two conditions are concerned with the principal tenets of fuzzy set theory (that set membership may be characterized by support values between and including 0 and 1), the third condition is important where the theory is intended to be used in practice.

Since it was necessary to obtain from the engineers their estimates of support values for various linguistic descriptors, it was important to establish a consistent method for the derivation of the initial support values for the fuzzy sets used to describe these descriptors. It was established very early in the process that the calculations of qualifiers as proposed by Zadeh tended to provide problems, and therefore different descriptors were described by opinion distributions which were of the same mathematical form, but functionally independent.

Two characteristics were identified which were concerned with the interpretation of approximate data. First, for some large ranges of data values there is no significant alteration in the resulting decision and second, for some small ranges a marginal change in the data value results in a major change in the ensuing decision.

It is therefore necessary to encapsulate these two concepts of the interpretation process within any interpretative model. These characteristics are related to the degree of precision with which the data are known. If the data imply that it is possible that a decision threshold might be breached, then greater precision could be helpful. In other cases this is less important. It is therefore necessary to know the approximate thresholds: a model which is intended to represent this process should be able to identify the thresholds and to relate these to the use of the data.

## 3.2 Decision thresholds - data values between thresholds

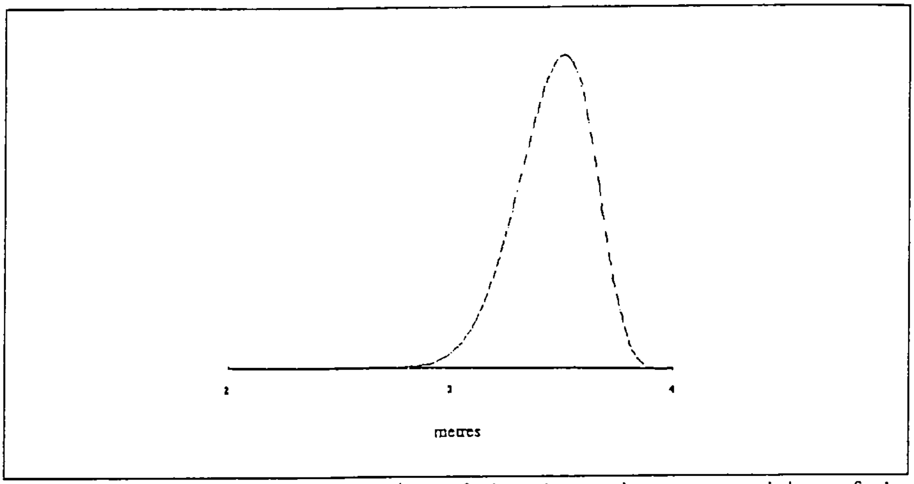


**Figure 1** A graphical representation of the change in expert opinion of the satisfactory width of a bus lane.

A decision based on imprecise knowledge of the data should be treated with more caution than one in which the data are more precisely known. The degree of caution necessary is not constant: if the data concerned are far from any threshold (for example near the centre of a range between two threshold values), then the degree of caution could be less than would be the case if knowledge of the data suggested that these could encompass values near or across a threshold.

The width of a bus lane serves as an example. During this research, design experts were asked about the ideal width for a bus lane. A common response to this question was that the bus lane should be 3.5 metres wide. When the engineers were asked whether a bus lane which was 3.4 metres wide would be acceptable. Again a common response was that this would be possible, but the engineer had a lower opinion of this width than with 3.5 metres.

This change in the opinion level could be represented in a graphical form as a uni-modal distribution which is bounded at the lower and upper limits. Such a distribution is shown in Figure 1. Between the bounds of the data values, which are determined by the feasibility of such values (for example, a lane width of less than 0 is not feasible), the distribution reaches a maximum at a point between these limits.

### 3.3 Decision thresholds - data values near to thresholds

The question about the width of a bus lane could be repeated with different widths, until the opinion changed sufficiently to alter the decision from:-

*"this width of the bus lane is satisfactory"*

to:-

*"if this is the only way to have the bus lane then this width is possible, but it is not desirable"*

and from this to:-

*"this width is too narrow to be used as a bus lane".*

Thus the experts find it easier to establish thresholds at which their chosen decision changes for a marginal change in the given data values than to quantify their opinion values. Near to these thresholds, the opinion level of each of the two competing decisions tends to equality: it is very difficult to choose between the two options, and knowledge of a slight change in the data could alter the decision. The opinion of the new decision is, however, only marginally different from the opinion of the previous option.

It therefore appears to be possible to use data which are only known imprecisely if it is also possible to know the relevant thresholds at which the decision options change, and around which the data carry more information for the decision process. The next section discusses how these thresholds may be identified.

### 3.4 Expert evaluation of decision thresholds

As has been discussed in Section 2, the design engineers involved in the design of high capacity bus systems appear to be able to use qualitative interpretations of data rather than actual values. The reasons for this have been discussed in Section 3.2 along with some of the issues surrounding the use of imprecise data. This section discusses how these qualitative interpretations might be modelled in order to represent this aspect of the design decision process.

Consider the question of the width of a bus lane, raised in Section 3.2. Figure 1 illustrates a distribution of expert opinion of the concept of '*satisfactory*' when related to the width of a bus lane. Obviously, there are widths for which the opinion of the descriptor '*satisfactory*' is very low, and thus where a different description might be better. The concepts identified in Section 3.2 above could each be evaluated by a distribution similar to that for '*satisfactory*', but with a different descriptor. The constraints on such distributions would be the same as those for '*satisfactory*', although they would have a different shape.

Thus another distribution could be derived to describe another concept with respect to lane widths, for example where the lane width is not ideal, but if the choice were between having no bus lane and having a bus lane which is a little narrow it would be possible. Figure 2 shows such a distribution plotted with the previous distribution. As expected there is a point at which the two distributions intersect. At this point a

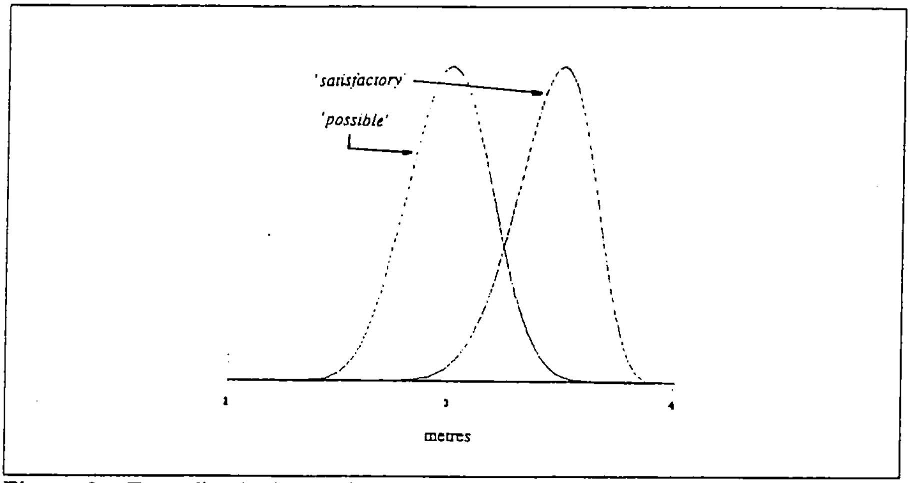


**Figure 2** Two distributions of expert opinion about the width of a bus lane: '*satisfactory*' and '*possible but really too narrow*', showing the intersection which identifies the decision threshold.

threshold is identified at which the chosen description changes as the width increases, from '*possible but really too narrow*' to '*satisfactory*'.

The relationship between these two distributions is a matter of some importance. If they are mathematically related, this could save some computational effort. On the other hand, even if they are mathematically independent, they are related semantically in the sense that they are merely describing graphically opinions of two different descriptions of the same object. There are two aspects to be considered concerned with any mathematical relationship between two distributions. The first aspect to consider is whether there is any commonality between the form of the distribution in each case. The second consideration is whether there is any mathematical dependence between the two distributions (whether of the same form or not).

In this case, however, there is more than a mathematical dependence to be considered. The distributions are not intended to describe mathematically some functional relationship between two parameters. Each distribution is a representation of the opinion held about a description of some values of a particular variable. Opinion is not measurable in the normal way, and therefore any mathematical function can only approximate the changes in opinion as the variable changes in value. Thus the mathematics is used to describe - rather than to define - opinion.

## 4. THE OPINION MODEL

The derivation of the opinion model is presented in Tyler (1992). The model has two parts: the function which describes the opinion and the process with which this function is used in order to represent the decision process. The mathematical function will be derived first.

### 4.1 The mathematical representation of opinion

The mathematical function used to represent opinion is continuous, unimodal, and bounded at both the upper and lower limits. Equivalence between the opinion distributions of different variables can be achieved by scaling the distribution so that the mode is equal to one. The β distribution is a function that is continuous, unimodal and bounded at both the lower and upper limits. This distribution can also be scaled so that the mode is equal to 1. The β distribution also has three major advantages within this context:-

1 **it is flexible**: as the distribution is used to describe the 'shape' of the opinion distribution (rather than just its numerical values), a single function which can provide many such shapes is computationally convenient;

2 **it is easy to compute**: the form of the function requires only one dependent variable;

3 **it has a small number of parameters to evaluate**: only three parameters are required, and it can be shown that these are mathematically related (Tyler 1992).

The β distribution is now described. It is given by:-

$$\beta(x) = K\, x^{\alpha}\,(1 - x)^{\gamma} \tag{1}$$

where

| | | |
|---|---|---|
| $\beta(x)$ | = | the value of β given the value of $x$ |
| $x$ | = | independent variable |
| K | = | constant scalar |
| $\alpha,\gamma$ | = | parameters to be calibrated |

The β distribution is defined only for values of $x$ between 0 and 1, so values of a variable must be scaled so that they fall in the range [0,1]. Decisions are represented using an adaptation of the process of composition as defined by Zadeh (1965). This is only used to process two fuzzy relations at a time. The adaptation of Jowitt and Lumbers (1983) allows composition between one fuzzy set and a relation. In the decision model presented here, there is one fuzzy set for each sub-range of the data, each consisting of the supports evaluated for each descriptor. Composition must therefore be performed on the relation between each sub-range of the data and each descriptor. The adaptation of the composition process in order to accommodate this is now described.

Composition consists of two operations: the minimum of the supports for the input and those in the relation (decision matrix), and the maximum of these minima to determine the fuzzy supports for the outputs. Since a descriptor encompasses a range of data values, it is necessary to determine which of the set of fuzzy supports defined within the data range should be compared with the relation. The first operation of the composition process is therefore to take the minimum of the value of the opinion distribution ($B$) for each data value $x_i$ within the descriptor range and the value of the relation at this data point:-

$$\mu_{(A', B)}(x_i) \mid (d, x_i) = \min \{ \mu_A (d, v), \mu_B (v, x_i) \} \quad (2)$$

where

$\mu_A(d,v)$ = the supports for the decision output $d$ given the variable sub-range descriptor $v_{1 \dots 4}$ and decision descriptors (expertise) $d_{1 \dots 4}$

$\mu_B(v,x)$ = the supports for the variable sub-range descriptor $v$ given the data range $x_{1 \dots n}$ and sub-range descriptors $v_{1 \dots 4}$

$\mu_{A,B}(d,x)$= the fuzzy set of supports for the decision output $d$ (given the data range $x_{1 \dots n}$ and decision descriptors (expertise) $d_{1 \dots 4}$)

and the relations $A$ and $B$ are given by:-

$A(d_j v_i)$ = expertise relation between the fuzzy supports for the variable sub-range descriptor $v_i$ and decision output $d_j$

$B(v_j x_i)$ = data relation between the fuzzy supports for the variable value $x_i$ and the sub-range descriptor $v_j$

Having obtained the minima for all data ranges and all descriptors, the maximum support is obtained for each descriptor $j$:-

$$\mu_j = \max_{i,j} \{ \mu_{i,j} \mid d_j, x_i \} \quad (3)$$

This results in a decision set, which is a fuzzy set of four ordered pairs, where each pair is the support for a particular descriptor:-

$$\Omega_{d,x} = [ \mu_1 \mid d_1, x_1, \mu_2 \mid d_2, x_2, \mu_3 \mid d_3, x_3, \mu_4 \mid d_4, x_4 ] \quad (4)$$

The subsequent decision is then defined by the descriptor with the maximum support within this fuzzy set:-

$$\mu \mid \Omega = \max_{1 \le i \le 4} [ \mu_i \mid ( d_i, x_i ) ] \quad (5)$$

### 4.2. Using the opinion model

Three possible conditions exist with respect to the final decision set. First, where the values in the set are obtained from different descriptor ranges. In this case, the chosen result depends on the 'true' values of the variable. If the data value were in fact below the threshold, the decision would be different than if the data value were above the threshold. The degree by which this might be a problem is indicated by the decision factor, which is the ratio of the maximum value in the set to the next highest value. The second condition is where the decision set emanates from the same descriptor range. In this case, descriptor with the maximum opinion value is the chosen decision, although the degree of ambivalence associated with this decision can also be estimated by the decision factor. The last condition is where the data are known precisely, and the decision matrix consists of only one row. Once again ambivalence about the decision can be estimated with the decision factor.

In this case the final decision is bifurcated because the final fuzzy set of opinions of the decision outputs contains elements from different sub-ranges of the data. This occurs when the distribution of opinion of the data intersects with the distributions of different descriptors in different sub-ranges of data values. This is most likely to occur if the data are known only very approximately (thus the variance is very large), but it can happen if the data range crosses a decision threshold even where the variance is small.

There are two possibilities arising from this result. The first is that the data could be re-assessed in order to reduce the variance (and thus to reduce the possibility of this bifurcation). The second is to decide whether the appropriate sub-range of the data is above or below the threshold, and to act accordingly. Where data quality is poor it is extremely useful to have this check on the reliability of a decision based on poor quality data. In some cases, even very poor quality data may result in a unique decision set, while in others data which are known with a reasonable degree of precision may produce a bifurcation.

## 5. AN EXAMPLE

The expertise obtained in the form of opinion distributions was calibrated with the experts to ensure that the distributions represented correctly the changes in opinion, particularly near to the decision thresholds. This section describes part of an experiment to validate the decision model.

The objective of the experiment was to see whether the decision model, with the use of the opinion model, could produce a design for a high capacity bus system which was similar to the system produced by the experts from whom the expertise was obtained. Accordingly the data used by the experts in the design of a high capacity bus system in Sao Paulo (the Santo Amaro/9 de julho corridor) were used. For the purposes of this paper, the design of a particular bus stop is considered.

This bus stop is one of a particular type of design used for this corridor. This involves the use of platforms to accommodate waiting passengers, and an overtaking lane for buses to pass stopped buses. Critical aspects of the design are that the stop should be located in the median and that the stop platforms should accommodate two groups of double-berth stops for each direction. An obvious constraint is also that the

Table 1 Comparison of bus stop elements

| | MODEL DESIGN | IMPLEMENTED DESIGN |
|---|---|---|
| Number of overtaking lanes (buses) | 1 | 1 |
| Number of groups | 2 | 2 |
| Platform length | 74 | 75 |
| Number of berths | 2 | 2 |
| Number of traffic lanes | 2 | 2 |
| Width of traffic lanes | 3.11 | 2.8 |
| Width of bus lanes | 3.5 | 3.5 |
| Width of footways | 1.055 | 1.675 |

Table 2 Decision sets for the element widths obtained from the computer model

| ELEMENT WIDTHS | | | | | |
|---|---|---|---|---|---|
| parameters | *too narrow* | *narrow but possible* | *satisfact-ory* | *too wide* | decision factor (DF) |
| traffic lanes | 0.55 | 0.99 | 0.61 | 0.42 | 1.62 |
| footways | 0.20 | 0.56 | 0.79 | 0.19 | 1.41 |
| bus lanes | 0.25 | 0.02 | 1.00 | 0.48 | 2.08 |

lane and footway widths should be satisfactory. Table 1 shows the relevant elements of the bus stop as designed by the model (column 1) and as implemented in the corridor (column 2). The sets of opinion values for these elements are shown in Table 2, together with the decision factors which indicate the ambivalence held about each description.

In this case, it can be seen that the decisions taken by the model are fairly close to those implemented, although the traffic lanes are considered by the model to be a little narrow, those provided in the implementation are even narrower. On the other hand, the decisions with respect to the footways are the reverse: the design by the model gives narrower footways than the implementation. Nevertheless, the designs were sufficiently close to bear a reasonable comparison.

Of course, in the consideration of the widths of these design elements the opinion sets are derived from precise data values. This is because the measurements are obtained by an iteration process where the width of each element is calculated and the set of widths for all the elements is then tested against the opinion distributions obtained from the experts as described in Section 4.2.

The decision to locate the bus stops in the median is the result of the analysis of the level of parking, and the effectiveness of the enforcement of parking measures. These can both be described qualitatively using the opinion model. The resulting decision matrices are shown in Table 3. It is clear that the imprecision with which the parking levels are known has produced three possible decision outputs from the composition procedure, with the interpretative possibilities described in Section 4.2. The descriptor which is chosen in this case is "*fairly high*", a description which with a decision factor of 1.56 is considered to be reasonably clear.

**Table 3** Decision sets describing opinions of the level of parking and degree of enforcement

| PARKING | | | | | |
|---|---|---|---|---|---|
| data range | *very low* | *fairly low* | *fairly high* | *very high* | decision factor (DF) |
| 1 | 0.12 | 0.57 | | | 1.70 |
| 2 | | | 0.97 | | 1 |
| 3 | | | | 0.62 | 1.56 |
| overall | 0.12 | 0.57 | 0.97 | 0.62 | 1.56 |
| ENFORCE MENT | | | | | |
| 1 | 0.97 | | | | 1 |
| 2 | | 0.59 | 0.14 | 0.08 | 1.64 |
| overall | 0.97 | 0.59 | 0.14 | 0.08 | 1.64 |

## 6. CONCLUSIONS

This research has shown that it is possible to represent the use of opinion and judgement within the decision process concerned with the design of high capacity bus systems. The use of opinion can be modelled using some aspects of fuzzy set theory, although mathematical dependence between fuzzy supports for descriptors has not been established. This can be alleviated by using a set of distributions where each distribution provides the opinion values for a particular descriptor.

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