AN EFFICIENT RELIABILITY ANALYSIS METHOD FOR LARGE SCALE ROAD NETWORKS

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INTRODUCTION

A high reliability road network provides sure and unfluctuating traffic service by offering drivers alternative routes even when traffic accidents occur, road maintenance is taking place or disaster such as an earthquake occurs. Thus, reliability is an important indicator reflecting present road network quality for strategies of traffic management and future network construction.

This paper presents an efficient reliability analysis method combined with network aggregation technique for large scale road networks. Conventional reliability analyses which require all the minimal path sets and/or cut sets are impractical for a large system since the system involves a great number of path sets and cut sets. Although many approximation methods have been proposed so far, they also require all the minimal path sets and/or cut sets. The methods proposed by the authors are quite useful since they require only partial minimal path sets and cut sets[1,2]. When the network expands, however, even the numbers of partial minimal path sets and cut sets become large. Therefore, it is important to develop an efficient method for large scale road network reliability analysis. Reliability analysis is costly for a large system, but is more easily carried out for a small system. This paper proposes new reliability analysis methods combined with network aggregation techniques. These methods involve dividing the original network into small subnetworks based on decomposition; the path sets and cut sets in every subnetwork are aggregated into surrogate path sets and cut sets through intermediate reliability analysis, and a simplified master network is created for final reliability analysis.

1. DEFINITION AND SIGNIFICANCE OF RELIABILITY

According to the mathematical definition, reliability is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered[3]. In a more general sense, reliability is the probability that a system or a unit performs a given function for a certain time period. When a system composed of many units is described by means of graph theory with a set of links and nodes, the system reliability is defined as the probability that the input node and the output node are connected. This is called terminal reliability.

Similarly, the road network reliability can be treated as the probability of connectivity between two given nodes. In this paper, the terminal reliability of road networks is defined as the probability that two given nodes are connected for certain traffic service levels for given time periods. This reliability is an indicator of alternativity and redundancy.

For example, when the service level is a simple physical connection between two nodes under unusual conditions, a road network at this reliability level provides traffic service by offering vehicles alternative routes even when some part of the network is damaged. Thus, this indicator can be used for network invulnerability in cases of disaster such as earthquakes or unusual weather. However, since this indicator illustrates only simple connectivity, traffic conditions and travel distance are not taken into consideration. Thus, if the traffic conditions do not matter, at least minimum transport service is available for relief goods transportation.

When the service level is smooth traffic without a heavy delay, a road network at this reliability level provides sure and unfluctuating traffic service by offering drivers alternative routes even when a certain route is down. Thus, this reliability indicator illustrates the stability of the road network performance in case of not only disaster but also traffic accidents or maintenance construction. At the same time, this indicator can express the network conditions where detour traffic due to damage of some part of the network affects the congestion of another part of the network. In other words, it represents the network service level which follows the failure of a network by disaster. This indicator, therefore, can be used for daily traffic management to enable smooth transportation. In addition, a high reliability network can provide the traffic service that ambulances and fire engines need in order to arrive at their destinations smoothly for administering quick medical care and preventing the expansion of disasters.

Therefore, especially in today's cities of heavy traffic congestion, it is important to increase the reliability of road networks. In order to construct a high reliability road network, road planning requires a road network reliability analysis. Reliability analysis, however, requires in general a great amount of computation work and the computational procedure is very complicated. Therefore, the next section will identify the outline of the existing methods for obtaining a reliability value and present a new efficient and practical method for estimating an approximate value of terminal reliability.

2. INTERSECTION METHOD: AN EFFICIENT RELIABILITY ANALYSIS METHOD

2.1. The Intersection Method

As mentioned above, the terminal reliability of a road network is defined as the probability that two given nodes are connected for certain traffic service levels for given time periods. Reliability analysis for a network system, such as a telecommunication network system, should involve in general all the minimal path sets and cut sets regardless of their length or cost, since the connectivity between two given nodes is more important than the time or distance between the nodes. In the reliability analysis of a road network system, however, routes expected to be used by drivers should be taken into major consideration.

Therefore it is necessary to choose the significant path sets and cut sets which contribute greatly to the determination of the reliability value in the road network reliability analysis. To attain this, a partial network actually used by vehicles should be excerpted from the road network. Through this excerpt technique, the number of path sets and cut sets can be reduced.

In this paper, we present a new approximation method using partial minimal path sets and cut sets. We introduce the two reliability functions as

$$R_p = 1 - \prod_{s=1}^{p'} \left(1 - \prod_{a \in P_s} r_a \right), \tag{2.1}$$

and

$$R_k = \prod_{s=1}^{k'} \{ 1 - \prod_{a \in K_s} (1 - r_a)\} , \tag{2.2}$$

where r_a is link reliability, and

$$r_a = \mathrm{E}[X_a]. \tag{2.3}$$

X_a is a binary indicator variable for link a, as follows:

$$X_a = \begin{cases} 1, & \text{if link } a \text{ provides given traffic service level,} \\ 0, & \text{otherwise.} \end{cases} \tag{2.4}$$

"Functioning" or "failure" of links is assumed to be independent of each other, where "failure" is caused by natural disaster, traffic accidents, maintenance of link, and traffic congestion; and where every link is sufficiently long so as not to affect another link. P_s represents s-th minimal path set and K_s denotes s-th minimal cut set, and the subscripts p of R_p and k of R_k stand for "path" and "cut". And, p' and k' are the number of partial minimal path sets and cut sets respectively and,

$$p' \leq p \ , \ k' \leq k , \tag{2.5}$$

where p and k are the total number of minimal path sets and minimal cut sets respectively.

The value of R_p is characterized as follows: R_p is the increasing function of the number of path sets. When the number of the minimal path sets p' is small, Eq.(2.1) provides the lower bound value of reliability. When all the minimal path sets are employed in Eq.(2.1), the value R_p yields the upper bound value of reliability. This upper bound is known as Esary and Proschan's upper bound[3], and is

$$U = 1 - \prod_{s=1}^{p} (1 - \prod_{a \in P_s} r_a) . \tag{2.6}$$

Hence, the value R_p increases monotonically with an increase in the number of path sets from the lower bound of reliability to Esary and Proschan's upper bound of reliability.

Similarly, the value of R_k decreases monotonically with the increase in the number of cut sets from the upper bound of reliability to the lower bound of reliability. This lower bound of reliability is known as Esary and Proschan's lower bound and is

$$L = \prod_{s=1}^{k} \{ 1 - \prod_{a \in K_s} (1 - r_a)\} , \tag{2.7}$$

and

$$L \leq R \leq \mathrm{U} . \tag{2.8}$$

Therefore, the two functions will cross at a certain point between Esary and Proschan's upper and lower bounds. We propose the value at the intersection as an approximation for the reliability value.

The intersection method provides sound approximations and the necessity for computational work is extremely limited due to the incorporation of non-Boolean absorption[1] and the use of partial minimal path sets and cut sets. The CPU-TIME is 1/200 to 1/20,000 times more efficient than the exact method and Monte Carlo method[2]. Since it is simplistic and easily calculable, the intersection method can be applied to a large scale road network.

In addition, the partial path sets and cut sets employed in the reliability analysis procedure correspond to the existing traffic routes and screen lines. Thus, we can use the reliability analysis results in connection with actual road planning and traffic management.

2.2. Selection of the Partial Minimal Path Sets and Cut Sets: The N-th Shortest Route Search Problem

An approximate value of reliability depends greatly on how to select the partial minimal path sets and cut sets. This section describes an efficient way for selecting the partial minimal path sets and cut sets for a good approximation.

The reliability of s-th minimal path set P_s in Eq.(2.1) is

$$\Pr\{P_s\} = \prod_{a \in P_s} r_a. \tag{2.9}$$

Similarly, the unreliability of s-th minimal cut set K_s in Eq.(2.2) is

$$\Pr\{K_s\} = \prod_{a \in K_s} (1 - r_a). \tag{2.10}$$

R_p in Eq.(2.1) and R_k in Eq.(2.2) are increasing and decreasing functions respectively with increase in the number of path sets and cut sets as stated above. Thus it is expected that we can quickly get a good approximation of reliability for a small number of the choice of path sets and cut sets, if we successively select the minimal path sets and cut sets which contribute greatly to the reliability value.

A logarithm of Eq.(2.9) leads to

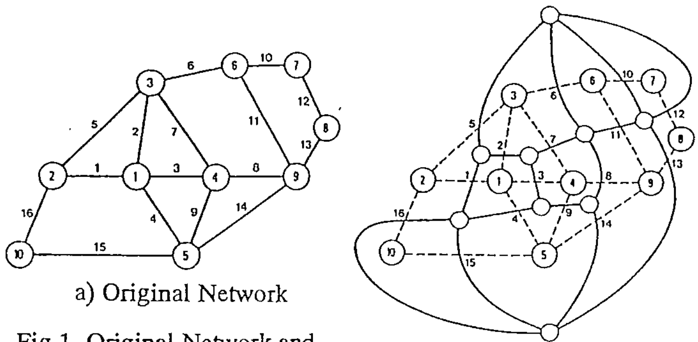
$$\log(\prod r_a) = \log r_{a_1} + \log r_{a_2} + \ldots + \log r_{a_m} , \tag{2.11}$$

where m is the number of links included in this minimal path set. From the inequality,

$$0 \leq r_a \leq 1 , \tag{2.12}$$

$-\log r_a$ can be regarded as the virtual link length. Thus the problem of finding the minimal path set with the highest reliability is treated equivalently as the problem of finding the shortest route over the network. Once the shortest route is chosen, next, the second-shortest route should be chosen. Therefore the problem of selecting the partial minimal path sets is reduced to the problem of finding the N-th shortest route over the network successively.

In the case of minimal cut sets, a dual network representation is used since the minimal cut sets in the original network are exactly equivalent to corresponding minimal path sets in the dual network (See Fig.1). Here, link reliability is replaced by $(1-r_a)$ as the virtual link length for convenient computation work. For the selection of minimal cut sets, the same procedure is followed for minimal path sets over a dual network.



a) Original Network

Fig.1 Original Network and Dual Network which is used for finding minimal cut sets

b) Dual Network

3. RELIABILITY ANALYSES COMBINED WITH NETWORK AGGREGATION

Although the intersection method is an efficient reliability analysis method, it becomes time-consuming when the network expands. This is caused by the increase in the number of path sets and cut sets, in the computational work done in searching path sets and cut sets and in CPU-time for reliability analysis itself. For example, because there are many inter-nodal path sets between origin and destination nodes, the actual number of path sets becomes huge, since the path sets between origin-destination nodes are given as a combination of inter-nodal path sets. Reliability analysis for small size networks can be, however, carried out with less difficulty. The efficiency of the reliability analysis can be improved through decomposing networks into small subnetworks.

Many methods for network aggregation have been developed for simplifying computational work in the field of traffic assignment[4-11]. However, no network aggregation method for road network reliability exists since road network reliability analysis is a newly developing field. The differences between traffic assignment and reliability analysis are summarized as below.

(1) For traffic assignment, the matching of traffic volume and chosen routes between original networks and aggregated networks or subnetworks should be considered. Here, the aggregation of link flow and the composition of travel time functions which are updated step-by-step in the course of assignment procedure are important. In the reliability analysis, however, the matching of link reliability between original network and aggregated network after traffic assignment is important. In other words, the conservation of minimal path sets and cut sets in the course of aggregation should be considered.

(2) In the process of traffic assignment, all OD pairs should be strictly focused on at the same time. In reliability analysis, however, specific OD pairs are considered since the reliability measure is used for road network evaluation. In addition, the reliability analysis of specific OD pairs contributes to the efficiency of the calculation. Thus, road network reliability analysis yields ununiform network aggregation while traffic assignment requires uniform network aggregation at the focus area of the network.

Therefore, network aggregation in reliability analysis for road networks involves the problem of combining characteristics of the existing path sets and cut sets in the original network into surrogate path sets and cut sets. In this paper, we use the technical terms "original network", "subnetwork" in reference to the divided original network, and "master network" in reference to the reconstructed aggregation network.

4. NETWORK RELIABILITY ANALYSIS WITH NODES BUNDLING METHOD

4.1. Network Decomposition and the Combining of Nodes into Surrogate Nodes: The Concept

When the network is divided into subnetworks on the boundary links, many boundary nodes are generated there. To reduce the increase in the number of boundary nodes, bundling method for combining the boundary nodes into surrogate nodes has been proposed[12]. For example, consider the network shown in Fig. 2a which has been divided into four subnetworks in Fig.2b. Here, every three existing links on the boundary are combined into one surrogate node. This surrogate node is, at the same time, the combined node of the boundary nodes and its role is to be the connector between two adjacent subnetworks. We call these combined nodes "bundling nodes", and every link between a bundling node and a real node is a dummy link.

Bundling the nodes reduces the number of starting points of the same directional path sets in a subnetwork, and also allows the path sets in the subnetwork into a smaller number of path sets. As a result, the master network reconstructed by the aggregated path sets has a quite simple structure. In Fig.2b, the path sets between origin node and bundling nodes, between themselves, and between the bundling nodes and destination node are aggregated into one path set. Then the master network can be reconstructed quite simply as shown in Fig.2c. Link reliability transformation from real links in the original network into dummy links in the subnetworks(i.e. from Fig.2a to Fig2b) requires further consideration.

This method is insufficient in aggregating cut sets from the original network into subnetworks, especially around bundling nodes and OD nodes, since this method focuses mainly on path sets aggregation.

The key in bundling manipulation is to determine the link reliability for dummy links that connect to bundling nodes. For example, consider the transformation from Fig.3a to Fig.3b using the bundling method. It is difficult to determine the link reliability of dummy link l, m, n and o. A sample case assumes that link l and link n in a series are substituted for link 6, link m and link o in a series for link 7; and that link m and n, and link l and o are disconnected. Then the following simultaneous equations hold:

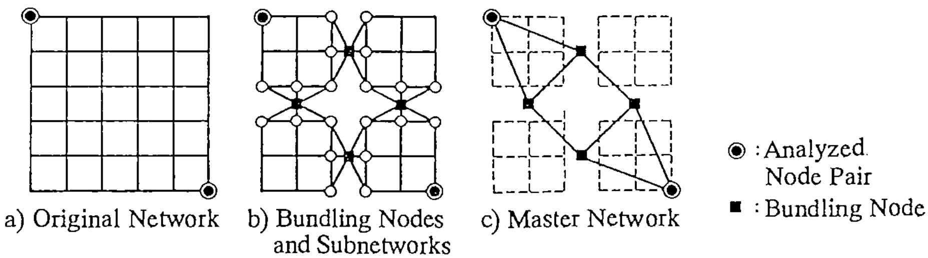
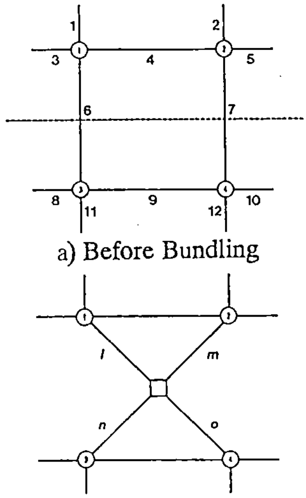


Fig.2 Network Aggregation with Nodes Bundling Method



a) Before Bundling

b) After Bundling

Fig.3 Reliability of Dummy Link

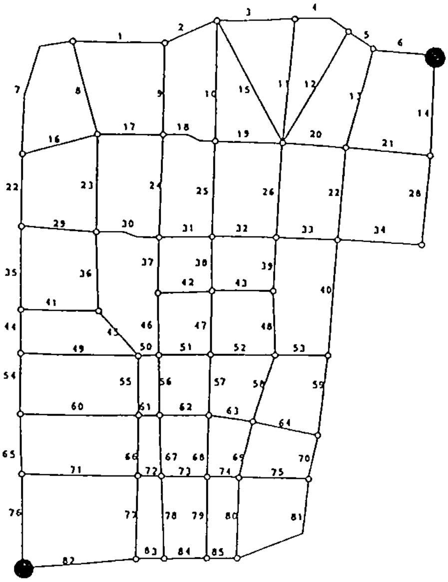


Fig.4 Analyzed Network and Node Pair

$$r_l * r_n = r_6, \quad r_m * r_o = r_7, \tag{4.1a}$$

$$r_m * r_n = 0, \quad r_l * r_o = 0. \tag{4.1b}$$

Since these equations cannot be solved, we consider (4.1a) only, and the solution becomes

$$r_l = r_n = r_6^{1/2} . \tag{4.2}$$

However, in Fig.3, link 6 is replaced by two parallel link sets, that is link l and, link 4 and m with the bundling method. Consequently, terminal reliability is overestimated. In addition, since $r_l > r_6$ from (2.12) and (4.2), terminal reliability will again be slightly overestimated. Here we find the problem with the bundling method. In this paper, for avoiding overestimation of terminal reliability while retaining simple calculation, we set the reliability of dummy links at

$$r_l = r_n = r_6, \quad r_m = r_o = r_7 . \tag{4.3}$$

4.2. Application of Bundling Method to An Actual Network

We will test the bundling method for the diagonal OD pair shown in Fig.4 for three bundling cases. The approximation of terminal reliability calculated using the original intersection method is 0.87083. The exact value of terminal reliability used as the reference standard is incalculable since the CPU–time exceeds the achievable CPU–time even with a super computer. Thus the reference standard is calculated using the Monte Carlo method in conjunction with a restricted sampling technique[13].

The actual network is divided into four subnetworks depicted by dotted lines shown in Fig.5. A reconstructed master network is plainly represented by solid lines. Link reliability for the master network is given by path aggregation using the intersection method in each subnetwork. The result is shown in the second row of Table 1. The approximated value of reliability in this case is 0.92463 and has a relatively larger error than both the reference standard and the original intersection method's values. This error is caused by the lack of cut sets conservation. For example, one of the cut sets in the original network around the north-eastern node in Fig.4 is {6,14}. However, a cut set around the same node in the master network of Fig.5 is only {1,2} and the probability of the connection around this node is larger than for the original network. The master network, therefore, has a tendency to connect rather than to disconnect.

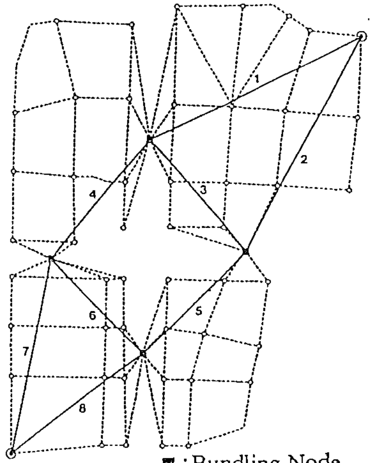


Fig.5 Master Network with Bundling Method

It is found that the terminal reliability depends a great deal upon the network expression around the targeted OD nodes. Thus the improved bundling method is achieved with conservation of links around the targeted OD nodes. In the case 2 of the bundling method, no aggregation of path sets is carried out in either of the subnetworks including the OD nodes. In case 3, only links connecting with the OD nodes are conserved and the rest are aggregated. The results are improved and shown in the third and fourth rows of Table 1.

5. NETWORK RELIABILITY ANALYSIS WITH PATH SETS AND CUT SETS AGGREGATION NETWORK METHOD

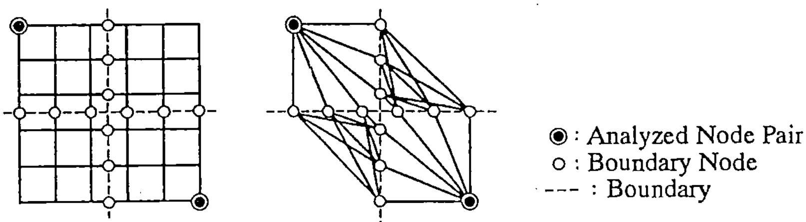
5.1. Network Decomposition and Independent Network Aggregation of Path Sets and Cut Sets: The Concept

The bundling method focuses mainly on path sets aggregation and is insufficient in the aggregation of cut sets. In this section, for the rigorous conservation of network structure, path sets and cut sets are independently aggregated into two master networks. In this method, plural path sets between boundary nodes of every subnetwork are aggregated into one path set. With cut sets aggregation, we use a dual network since path sets in the dual network are strictly equivalent to the cut sets of the original network. The dual network is divided into subnetworks and the same procedure is followed.

The procedure is explained in Fig.6. There are 252 minimal path sets from the north-western origin node to the south-eastern destination node in Fig.6a. Every path set includes 10 links, which is the smallest number for this node pair. We divide this network into four subnetworks and aggregate path sets between the boundary node and the origin/destination node or between the boundary nodes. With this procedure, we obtain the path sets-aggregated master network shown in Fig.6b. The number of path sets

Table.1 Reliability Values of Network Aggregation Methods and Original Intersection Method

Approximation Methods of Reliability Analysis	Reliability Value	No. of Path/Cut Sets
Original Intersection Method	0.87038	22
Bundling Method : Case 1	0.92463	3
Bundling Method : Case 2	0.81999	14
Bundling Method : Case 3	0.85524	6
Path Sets/Cut Sets Aggregation Network Method	0.87933	3
Monte Carlo method with Restricted Sampling (used as Reference Standard)	0.82696	-



a) Original Network   b) Aggregated Network

Fig.6 Path Sets Aggregation Network

between OD nodes can be reduced to only 18. Since aggregation is given for each path set only once, rigorous aggregation is achieved between the original network and master network for the path sets with the smallest amount of links. There are path sets, however, which exist in the original network but not in the master network. These path sets are zigzag or detour path sets including more than 10 links, and the information gets lost. However, since the intersection method characteristically uses partial path sets and cut sets whose contribution is large, there is no problem when the approximated value of reliability is obtained with the use of these partial path sets and cut sets which include smaller amount of links.

With this method, two different master networks for path sets aggregation and cut sets aggregation are constructed. During the course of reliability analysis, we can regard the reciprocal characteristics of path sets and cut sets independently. We call these master networks "path sets aggregation network" and "cut sets aggregation network".

5.2. Application of Path Sets/Cut Sets Aggregation Network Method to An Actual Network

We calculate terminal reliability for the same network and the same node pair shown in Fig.4. The original network is divided into two subnetworks for path sets aggregation (Fig.7). The dual network for the original network is also divided into two subnetworks for cut sets aggregation(Fig.8). Here, nodes②~⑨ in Fig.7 and nodes [2]~[8] in Fig.8 denote the boundary nodes. A path sets aggregation network and cut sets aggregation

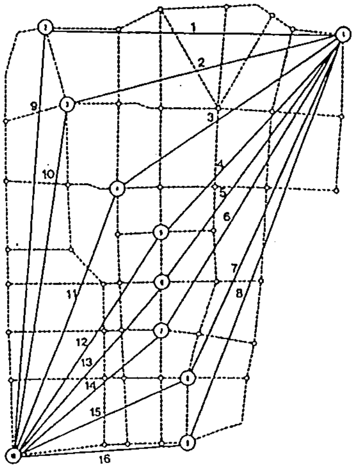


Fig.7 Path Sets Aggregation Network

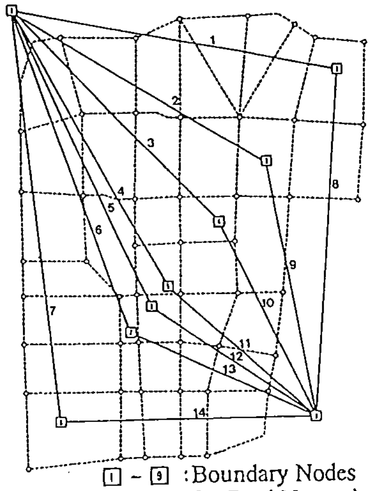


Fig.8 Cut Sets Aggregation Network

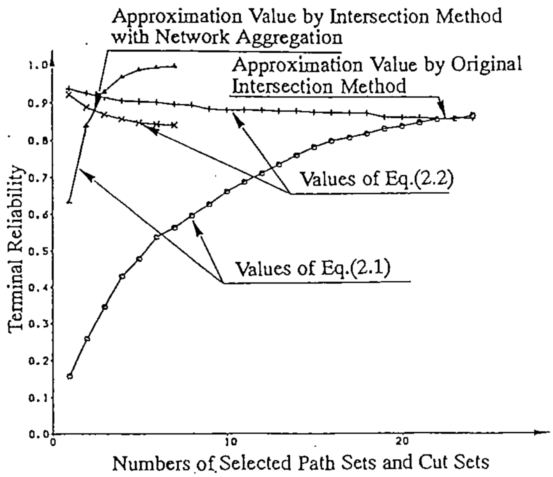


Fig.9 Facilitation of Generation of Intersection in
the Intersection Method with Network Aggregation

network have been constructed, and they are parallel structure networks of eight and seven exclusively aggregated path sets respectively. The result of the intersection method using path sets/cut sets aggregation network is shown in fifth row of Table 1, and in Fig.9

illustrating the result of the original intersection method without aggregation technique. The number of path sets and cut sets required for generating the intersection is 22 for the original intersection method while only three path sets and cut sets are needed for the intersection method with aggregation technique. The network aggregation facilitates generation of the intersection. The error from the reference standard is as small as that of the original intersection method, which is sufficient for practical use of actual network evaluation.

This method's shortcoming is that it requires much reliability analyses for subnetworks in the course of constructing two master networks. Despite the reduction of computational work required for each reliability analysis, the number of reliability analyses increases. In addition, the preparation for network decomposition, construction of a dual network, and preparation for reliability analysis of a subnetwork, are given much weight in the whole reliability analysis procedure as the results of dramatic reduction of reliability analysis. The development of a computer aided reliability analysis for network expression that includes this work as well is anticipated.

6. CONCLUSION

This paper proposes applying reliability analysis methods combined with network aggregation techniques to the intersection method proposed by the authors, in analyzing larger scale road networks. Two aggregation methods have been presented. One is the network decomposition and node bundling method. This method is quite effective in transforming the original network into a simply aggregated network. Although the computational work is simplified, the approximate value of reliability obtained is rough. However, the accuracy of the approximate value can be improved by conserving the links around the terminal nodes. Another method is the network decomposition and the constructing of path sets and cut sets aggregation network method. In this method, since path sets aggregation and cut sets aggregation are carried out independently, the path sets and cut sets which are important for traffic analysis in the original network can rigorously be conserved. The accuracy of the approximate value is sound.

Both of the methods prove to accelerate the generation of intersections with this network aggregation technique. Therefore, these methods are effective in reliability analysis for large scale road networks. With the dramatic reduction of reliability computational work combined with the development of the intersection method, preparation work for network decomposition, construction of dual networks and preparation for reliability analysis in subnetworks will have a large proportion in reliability analysis work. When a computer aided reliability analysis system including these anticipatory investigations for reliability analysis is developed, the system will be quite a strong weapon in large scale reliability analysis.

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