# A D.S.S. FOR THE EVALUATION OF TACTICAL AND STRATEGIC PLANNING OF TRANSIT LINES

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## 1. INTRODUCTION.

For a transit system working on a corridor, several management operations can be performed or a new system can be used in order to obtain an improvement in the level of service. Each strategy leads to variations in terms of system performances that may affect both investment and maintenance costs and demand level.

This paper describes a D.S.S. for the evaluation and the comparison of effects in terms of cost, revenue and service quality, effectiveness and efficiency among several alternative system configurations. The model, called T-COST and created by IVECO-FIAT in collaboration with C.S.S.T. (Centro Studi sui Sistemi di Trasporto), is able to evaluate and compare, for several transportation systems, cost, revenues, quality of service, and demand fluctuation due to the new system configuration. It consists of a data base of system characteristics, a simulative model and a man-machine interface.

The core of the simulative module is an analytical-probabilistic model for the simulation of operations performed by the convoys of a line. By means of this procedure it is possible to compute averages and variances of some significant variables such as travel time, passenger waiting time at stops, number of passenger on board along each link. Line performance is evaluated through indicators such as waiting time, running and travel time, over-crowding rate and service failure probability.

Below is a brief description of the D.S.S. structure and the simulation module with the analytical-probabilistic flow model (Section 2) and the general structure of the simulation procedure (Section 3). Lastly, a case-study applying T-COST to a corridor of an Italian urban area is reported (Section 4).

## 2. THE D.S.S STRUCTURE AND THE SIMULATION MODULE.

The D.S.S. essentially consists of a data base, a simulation module and a man-machine interface.

The data base contains data related to demand and system characteristics, model parameters and outputs. The man-machine interface makes the I/O process possible and enables results to be represented. The simulation module will be described in greater detail in Section 3.

The D.S.S. structure is shown in fig. 1. Input data consist of demand (for the first attempt system) and of system characteristics. The main variables determined in the simulation module are Level of Service indicators and commercial speed. Thus it is possible to evaluate demand, the number of convoys and staff needed, as well as efficiency and productivity indicators.

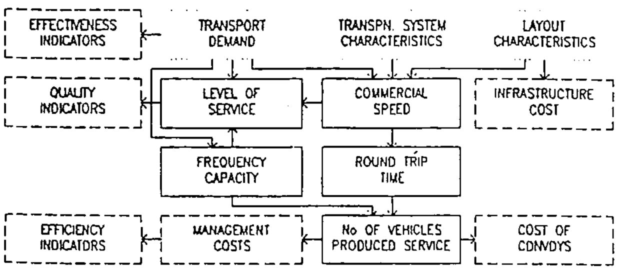


FIG. 1. General structure of the D.S.S.

## 2.1. The flow model.

The core of the simulation module consists of a flow model able to describe the whole set of operations made by a generic convoy.

An overview of the literature shows the existence of three methodological categories for simulating the operations of a transit line: micro-simulative models, analytical models and probabilistic models.

Micro-simulative models, as exemplified by Anderson et al. (1979) and Turnquist and Bowman (1979), whilst having the advantage of a limited number of simplificatory assumptions, entail a certain complexity of use and pose various statistical problems, besides requiring high calculation times. They are therefore mainly suitable for in-depth analyses of specific situations.

Analytical models, on the other hand, utilize mathematical relations obtained by resorting to extremely simplifying assumptions. This is illustrated by Newell (1971), who advances the hypothesis that the boarding and alighting of all passengers occurs at a single stop and that the arrival process is deterministic. This procedure was later generalized to the case of several stops by Sheffi and Sugiyama (1981). Single convoy dynamics were also studied by Newell (1977) without being tied in any way to capacity , and not envisaging either the boarding of passengers or interaction between convoys.

Probabilistic models come half-way between micro-simulative and analytical models. For example, in Powell and Sheffi (1983) the procedure calculates the arrival and departure distributions of each bus at each stop, and, according to some theories of independence, proposes relations to obtain the spatio-temporal trajectory of each convoy. It also provides us with distributions of passenger waiting times and link loads in addition to parameters for the evaluation of service reliability. The main limitation of this type of model is that results relating to each convoy and each stop are expressed in terms of the status both of the previous convoy and the previous stop.

The model used in the D.S.S. (Nuzzolo and Di Gangi, 1990 and 1991) is a cross between analytical and probabilistic models. The objective of this model is to determine some laws of probability, or at least the main moments, of those base variables (travel time from the terminus to the generic stop, number of passengers on board the convoy, passenger waiting time at each stop) which are essential in order to evaluate line performance. With this aim the above base variables have been considered as linear combinations of variables whose distribution laws (main moments) may be determined quite straightforwardly in relation to the system's characteristics (fig. 2).

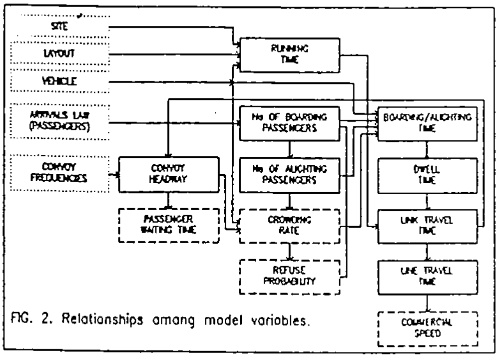


FIG. 2. Relationships among model variables.

So as to allow for the specific characteristics of each link of the line and of each stop or station, use is made of a meso-simulation of the operations performed by the generic convoy along the line itself.

Below are listed the criteria adopted for flow models and the techniques utilized. For further details, especially with regard to the derivation of the distribution laws of headways and number of passengers waiting, see the article by Nuzzolo and Di Gangi (1990) cited above.

The transit line is divided into a sequence of links, with the generic link x included between stop x and stop x+1. If a junction exists within a link, links are divided into sub-links bounded by two junctions or by a junction and a stop. The generic junction on link x is here indicated as $y^{x}_{1}$.

2.1.1. Travel time.

The generic convoy leaves stop x after having stopped for an interval $t_{sx}$ referred to here as dwell time, due to the boarding and alighting of passengers. The time needed to reach the next stop is called running time related to the link and expressed as $t_{cx}$. A link travel time is also defined, $t_{px}$, as the sum of dwell time and running time:

$$t_{px} = t_{sx} + t_{cx} \qquad (1)$$

The running time of the link $t_{cx}$ is calculated by hypothesizing a trapezium vehicle motion law and that all along the link the vehicle can make $n_y$ stops, each of length $t_y{}^1$, due to the presence of junctions, besides those corresponding to the stops. Hence:

$$t_{cx} = \Sigma_{j=1,ny+1} \; [l_j/v_j + \tfrac{1}{2} v_j(1/a_m + 1/d_m)] + \Sigma_{k=1,ny} \; t_y{}^k$$

where am and dm are, respectively, the mean acceleration and deceleration of the vehicle , $v_j$ its operating speed in the link fraction between two successive stops, $l_j$ the length of the fraction and $t_y{}^k$ the duration of the dwell-time corresponding to the k th stop.

The presence of a junction implies the existence of a delay, which depends on the type of site, link priority level and traffic-induced interference. For an evaluation of such a delay, see Nuzzolo and Di Gangi, 1991.

According to the type of site and interference of the system with the external environment, the model allows for the following four cases: i) Reserved site with no interference; ii) Reserved site with junctions; iii) Mixed site without junctions; iv) Mixed site with junctions. They differ in the approach adopted in evaluating the mean value and variance of travel time.

Where the means under consideration has two separate doors for boarding and alighting, the dwell time is supposed equivalent to the maximum between boarding time and alighting time. The relations used are as follows:

$$t_{sx} = a + b \cdot [NP^{s(d)}{}_x/np]$$

where a = dead time; b = mean marginal boarding (alighting) time; $NP^{s(d)}{}_x$ = number of passengers boarding (alighting) at stop x; np = number of doors used for boarding (alighting).

Values of parameters a and b have been deduced in experimental studies (Doras, 1975; Lin and Wilson, 1991).

The variance of the dwell time is a function of the variance of the number of passengers boarding (alighting) according to the relation $var[t_{si}] = (b/n)^2 \cdot Var[NP^{s(d)}_i]$.

Where there is interference between the two flows, mean values of boarding and alighting times are evaluated while also allowing for the degree of vehicle crowding and utilizing non-linear models in the parameters (Andersson and al., 1979; Lin and Wilson, 1991).

All things considered, means and variance of link travel time can be defined as:

$$E[t_{px}] = E[t_{cx}] + E[t_{sx}]$$

$$Var[t_{pi}] = Var[t_{cx}] + Var[t_{sx}] + 2 \cdot Cov[t_{cx}, t_{sx}]$$

In the model described it is supposed that the two components of (1) are independent. Thus the variance is obtained simply as the sum of the variances of dwell time and travel time.

The mean value and the variance of travel time required to reach the generic bus-stop z from the initial terminus are given by:

$$E[t_p^{1z}] = \Sigma_{i=1,z-1} E[t_{pi}]$$

$$Var[t_p^{1z}] = \Sigma_i Var[t_{pi}] + \Sigma_i \Sigma_j Cov[t_{pi}, t_{pj}] \quad i=1,z-1, \; j=1,i-1$$

2.1.2. Headway distribution.

The time interval between the arrival of two successive convoys at a generic stop is defined as headway $I_x$. The headway value at stop x is thus given by the difference of the times of two successive transits. Thus, by defining $O_p(h)$ and $t_p^{1x}(h)$ respectively as the departure time from the terminus and the time taken to reach stop x, relative to the h-th convoy, it may be supposed that:

$$I_k = O_p(h) + t_p^{1x}(h) - \{O_p(h-1) + t_p^{1x}(h-1)\} \qquad (2)$$

Hence, supposing $I_o = O_p(h) - O_p(h-1)$ is the time interval between two successive convoys at the terminus, equation (2) becomes:

$$I_k = I_o + t_p^{1x}(h) - t_p^{1x}(h-1)$$

The mean value of the headway $E[I_x]$ is considered constant for each stop and equal to the reciprocal of the service frequency $f_e$ of the system:

$$E[I_x] = I^* = 1/f_e \quad \forall x$$

The variance of the time interval at stop x, assuming that terminus departures and travel times are independent, can be expressed as:

$$Var[I_x] = Var[I_o] + Var[t_p^{1x}(h)] + Var[t_p^{1x}(h-1)] + 2 \cdot Cov[t_p^{1x}(h), t_p^{1x}(h-1)] \qquad (3)$$

and by denoting as $\Theta_t$ the autocorrelation between travel times , equation (3.12) may be expressed as:

$$Var[I_x] = Var[I_o] + 2 \cdot Var[t_p^{1x}] \cdot (1 - \Theta_t)$$

On the basis of experimental distributions reported in Doras (1979), the distribution of the headways is likened to Erlang's law of probability, namely:

$$f(I;\ \mu,k) = [\mu^K \cdot I^{K-1}]/(K-1)! \cdot \exp\{-\mu I\}$$

with parameters $\mu>0$ and K integer determined according to the means and variance of the headways.

2.1.3. <u>Number of passengers waiting at a stop</u>.

By equating the law of passenger arrivals at a stop to a Poisson process at rate $\sigma$, the law of probability of arrival numbers at stop x during the headway $I_x$ is expressed by $p(n,\ \sigma I_x) = [(\sigma I_x)^n]/n! \cdot \exp\{-\sigma I_x\}$.

Having assumed for $I_x$ Erlang's distribution with parameters K e $\mu$, the probability that the number of passengers arriving at stop x during the interval $I_x$, corresponding to the number of passengers waiting NPA, is N is given by (Nuzzolo and Di Gangi, 1990)

$$\mathrm{Prob}[NP^A=N] = (N+K-1)!/[N!\cdot(K-1)!]\cdot[\mu/(\sigma+\mu)]^K\cdot[\sigma/(\sigma+\mu)]^N$$

and the moments are (Kendall and Stuart, 1963):

$$E[NP^A{}_x] = \sigma\cdot E[I_x]$$

$$\mathrm{Var}[NP^A{}_x] = \sigma^2\cdot\mathrm{Var}[I_x] + \sigma\cdot E[I_x]$$

If all the passengers waiting were to board the convoy,the number of passengers alighting at stop x $NP^D{}_x$ would be given by:

$$NP^D{}_x = \Sigma_{j=1,x-1}\Omega_{jx}\cdot NP^A{}_j$$

where $NP^A{}_j \equiv NP^S{}_j$ = Number of passengers boarding at stop j; $\Omega_{jx}$ = Percentage of those boarding at j who alight at x. The moments of the v.a. $NP^D{}_x$ are the following:

$$E[NP^D{}_x] = \Sigma_{j=1,x-1}\Omega_{jx}\cdot E[NP^A{}_j] = \Sigma_{j=1,x-1}\Omega_{jx}\cdot\sigma E[I_x]$$

$$\mathrm{Var}[NP^D{}_x] = \Sigma_{j=1,x-1}\Omega^2{}_{jx}\cdot\mathrm{Var}[NP^A{}_j] + \mathrm{Cov} =$$

$$= \Sigma_{j=1,x-1}\Omega^2{}_{jx}\cdot\{\sigma^2\cdot\mathrm{Var}[I_x]+\sigma\cdot E[I_x]\} + \mathrm{Cov}$$

where $\mathrm{Cov} = 2\cdot\Sigma_{i=1,x-2}\Sigma_{j=i+1,x-1}\mathrm{Cov}[NP^A{}_i, NP^A{}_j]$

2.1.4. <u>Number of passengers on board</u>.

The passengers-on-board random variable along the x-th link, between stop x and stop x+1 inclusive, is given by:

$$NP^B{}_x = \Sigma_{i=1,x}\ \Sigma_{j=x+1,nf}\ \Omega_{ij}\cdot NP^S{}_i$$

or

$$NP^B{}_x = NP^B{}_{x-1} + NP^S{}_x - \Sigma_{j=1,x}\Omega_{jx}\cdot NP^S{}_j \qquad (4)$$

with:

$$E[NP^B{}_i] = E[NP^B{}_{i-1}] + E[NP^S{}_i] - \Sigma_{j=1,i}\Omega_{ji}\cdot E[NP^S{}_j] \qquad (5)$$

$$\mathrm{Var}[NP^B{}_i] = \mathrm{Var}[NP^B{}_{i-1}] + \mathrm{Var}[NP^S{}_i] + \Sigma_{j=1,i}\Omega^2{}_{ji}\cdot\mathrm{Var}[NP^S{}_j]$$

where $NP^B{}_i$ = Number of passengers on board along the i-th link; $NP^S{}_i$ = Number of passengers boarding at the i-th stop; $\Omega_{ji}$ = Percentage of those boarding at j who alight at i; having supposed the components of (4) to be independent.

The random variable passenger on board, being a linear combination of variables distributed according to an Erlang distribution, may be compared, in an initial approximation to a normal random variable with moments expressed by (5).

2.1.5. <u>Passengers waiting times at stops</u>.

The evaluation of the waiting time $t_{wx}$ incurred by users at the generic stop x is effected while bearing in mind the law of distribution of the arrival of users and of the irregularity of the service with the well-known relations (Doras, 1979; Welding, 1963):

$$E[t_{wx}] = E[I_x^2]/2E[I_x]$$

A particular value of waiting time taken into consideration is that tolerated by users arriving at the stop straight after the departure of a convoy, which is indicated as maximum waiting time $t_{wx}^M$. This is also a v.a. whose distribution is equivalent to that of the headways Ix. Therefore, the probability that a user waits a maximum time greater than a pre-fixed value $\varepsilon$ is given by:

$$\text{Prob}\ [t_{wx}^M > \varepsilon] = 1 - F(\varepsilon)$$

where $F(\varepsilon)$ is the distribution function of $t_{wx}^M$.

2.1.6. Probability of not boarding the next vehicle.

The probability $P^{ns}_x$ that the generic user at stop x is unable to board the convoy, it having reached saturation point , if C is the capacity of the convoy, is given by:

$$P^{ns}_x = \text{Prob}[NP^B_x \geq C] \qquad (6)$$

where $NP^B_x$ is the number of passengers on board along the link having x as initial stop. The probability expressed by (6) may be calculated once the distribution of the v.a. $NP^B_x$ is known.

## 3. GENERAL ARTICULATION OF SIMULATION PROCEDURE.

The procedure proposed for the simulation of the line may be subdivided into three distinct inter-connected blocks. As illustrated in fig. 3, the three sections are as follows: input and data control, system operations analysis and performance evaluation, and management of the alternative system.

The first block consists in the acquisition and control of data. This is effected interactively so as to identify and correct in real time any possible incongruencies (insufficient frequency) without having to interrupt the work session in order to correct the input file.

Following the input and the various tests on the congruence of data, the procedure sets about determining the levels of service of the various alternatives considered. This essentially amounts to an evaluation of the components of travel times and indicators of regularity, reliability and comfort of the service offered by the system under examination.

Lastly we have the block related to the evaluation of alternative systems, which consists of a procedure of input and data control analogous to the one described above. It is a routine matching of demand to the new configuration of the system, able to determine the evolution of demand by connecting it to the variation of the performance offered by the alternative considered.

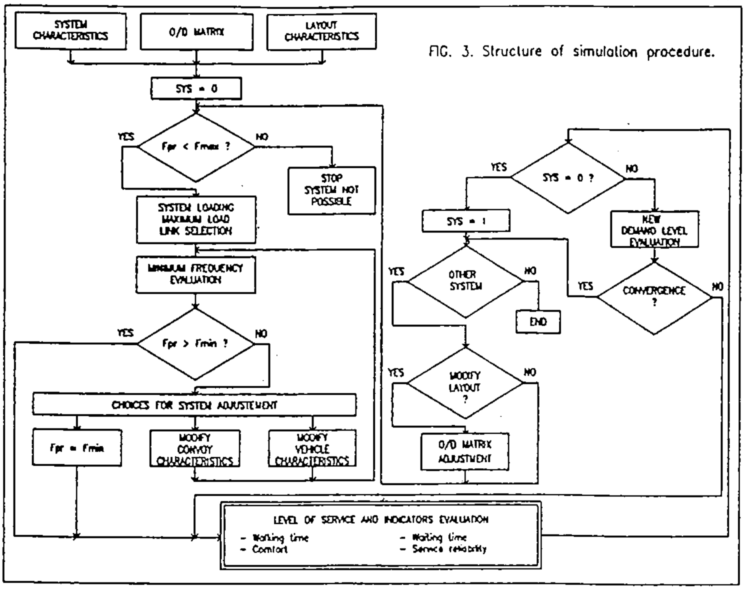


FIG. 3. Structure of simulation procedure.

### 3.1. Input data and their treatment.

Data input section may be subdivided into two parts: the first concerns the acquisition of data while in the second the data acquired undergo tests designed to guarantee their congruence and, in the case of data related to demand , procedures are applied so as to improve their precision.

The data that characterise the transport systems under consideration are as follows: vehicle and/or convoy capacity, operative starting and braking acceleration, cruising speed, number of doors for boarding and alighting, maximum service frequency, project frequency, irregularity of terminus departures.

For the same corridor it is possible to define various transit lines which have the two outermost termini in common; the two lines may be differentiated by number of stops, length of links, degree of interference present on the links, location of the stops.

To determine the various indicators, the model uses an Origin-Destination matrix between the stops, referring to a pre-established time band. Should the matrix not be available, it is generated by using estimate procedures which efficiently exploit the available information on the demand structure (Nuzzolo and Di Gangi, 1990). The estimate procedure requires at least one set of data consisting of number of passengers boarding and alighting at each stop or load on each link.

### 3.2. Evaluation of Level of Service.

The Level of Service (L.o.S.) offered to users is evaluated on the basis of values assumed by indicators described below which take into consideration the performance of the system in terms of walking times, waiting times at stops, travel times, travel comfort, service reliability.

The evaluation of comfort takes into account the capacity of the system and the degree of saturation it has reached. Thus, along the generic link i, we have an evaluation of the probability that the number of passengers on board exceeds a certain degree of filling $G_r$ of the convoy, having hypothesized for the number of passengers on board the law of distribution described in Section 2.

Therefore, the probability that the number of passengers on board along the i-th link exceeds the fixed degree of filling is given by:

$$\mathrm{Prob}\ [NP^B{}_i > G_r \cdot C] = 1 - F(G_r \cdot C)$$

where $F(G_r \cdot C)$ is the value of the distribution function of $NP^B{}_i$ calculated in correspondence with $G_r \cdot C$.

Likewise, by making reference to the number of seats, the probability of standing while travelling along the i-th link may be evaluated.

Service reliability has been related to the probability that the user waiting at the stop does not succeed in boarding the first available convoy. This happens when the number of users waiting at stop x is greater than the number of places available on the convoy once passengers have alighted at the same stop.

Defining the "residual capacity" of the convoy (the number of places available at stop x) as CRx, we obtain:

$$CR_x = NP^B{}_{x-1} - NP^D{}_x$$

where the probability of not boarding is evaluated by calculating $\mathrm{Prob}[NP^A{}_x > CR_x]$.

### 3.3. Evaluation of alternative systems.

The procedure enables us to evaluate, besides the so-called first attempt system, some alternative systems intended as any type of configuration, both in terms of means of transportation and of layout topology, provided it is inserted into the same corridor and bounded in both directions by the same termini. The only possible variation is thus given by the number of stops and/or their positioning, with a consequent variation of the characteristics of the links.

If the transit line is modified either with regard to the number of stops or to their positioning, the Origin-Destination matrix of departure must be transformed so as to adapt the original relations to the new characteristics of the transit line. To do this, it is supposed that the demand in every o,d relation is distributed according to a generic function f(o,d) in the domain defined by the Cartesian product of the two regions of influence of stops o and d.

The function $z = f(x,y)$ may therefore be defined as:

$$z = f(x,y) \; : \; \forall \; x \in [A_x, B_x], \quad \forall \; y \in [A_y, B_y]$$

$$\longrightarrow \quad z = \tau_{xy} = \frac{d_{xy}}{(B_x - A_x)\cdot(B_y - A_y)}$$

The redefinition of the stops leads to a redefinition of the regions of influence, and thus the new generic element of the O/D matrix is obtained as:

$$d_{od} = \int_{A_o}^{B_o}\int_{A_d}^{B_d} \tau_{uv} \; du \; dv$$

The operating conditions of the alternative system, in terms of performance variations, lead to fluctuations in demand which are evaluated with a Multinomial Logit model.

$V_j$ being the average value of systematic disutility, the probability of choosing the alternative j is given by:

$$\text{prob}(j) = \exp\{V_j\} \; / \; \Sigma_{i=1,na} \exp\{V_i\} \qquad (7)$$

where $n_a$ represents the number of alternatives available.

If T is the global demand, the demand using mode j may therefore be obtained as follows:

$$d_j = T\cdot\text{prob}(j) \qquad (8)$$

Now suppose that the performance offered by mode j varies; the system disutility relative to the mode will assume the value $V_{j*}$ and thus the probability of choice associated with the above alternative and the relevant demand will be expressed as:

$$\text{prob}^*(j) = \exp\{V^*_j\} \; / (\Sigma_{i=1,na \; i\neq j} \exp\{V_i\} + \exp\{V^*_j\}) \qquad (9)$$

$$d^*_j = T\cdot\text{prob}^*(j) \qquad (10)$$

From a comparison of (8) and (10) we obtain:

$$d^*_j/\text{prob}^*(j) = d_j/\text{prob}(j) \qquad (11)$$

and therefore the demand expressed by (10) may also be obtained by (11) as follows:

$$d^*_j = d_j\cdot\{\text{prob}^*(j)/\text{prob}(j)\} \qquad (12)$$

In a first approximation, the difference between the denominators of (7) and (9) may be considered negligible, and consequently (12) may be expressed as:

$$d^*_j = d_j\cdot\exp\{V^*_j\}/\exp\{V_j\}$$

The disutility perceived by users depending on the system can be formalized as a linear combination of the various components of travel time and monetary cost. Assuming that the monetary cost does not vary with alternative systems of public transport and therefore, remaining constant, it does not affect comparative evaluations, the disutility can be expressed as:

$$V^{od} = w_f \cdot (t_f^{o} + t_f^{d}) + w_w \cdot t_{wx} + w_p \cdot t_p^{od} + w_c \cdot C_{od}$$

where $t_f^{o}$ = walking time from origin to first stop; $t_f^{d}$ = walking time from last stop to destination; $t_{wx}$ = passenger waiting time; $t_p^{od}$ = total travel time for considered o/d pair; $C_{od}$ = monetary cost; $w_k$ = coefficients of reciprocal substitution.

## 4. A CASE STUDY

The procedure illustrated above was applied to a corridor of approximately 20 km in the urban area of Padua, in the North-East of Italy. Three possible transport solutions were examined on this transit line:

- a bus line with a capacity of 100 places and with the most reserved route possible (system only considered as a term of reference);
- a tram line with a capacity of 120, with a protected site but also with level junctions;
- an automatic light underground line with a convoy capacity of 210 and with routes entirely in tunnels.

By means of a system of demand models and an assignment model, the potential number of users was estimated for the rush-hour (8.00-9.00), as set out in tab.1. The determination of the characteristics of the service and the comparison between the alternatives, in terms of investment costs, running costs and level of service offered, were carried out by means of the procedure described above. The results relating to performance are reported in tab. 2, those relating to costs in tab. 3.

TAB. 1. Demand composition and load condition.

| | BUS | TRAM | MLA |
|---|---|---|---|
| Urban lines passengers | 8091 | 8653 | 9345 |
| Extra-urban lines passengers | - | 2035 | 2035 |
| Car users (Urban) | - | 1034 | 2068 |
| Car users (Extra-urban) | - | 460 | 850 |
| Total no of users | 8091 | 12182 | 14298 |
| Maximun load | 3167 | 4504 | 5293 |

TAB. 2. Systems performances

| | BUS | TRAM | MLA |
|---|---|---|---|
| Capacity of the system (pass/h) | 3000.0 | 5000.0 | 6000.0 |
| Commercial speed (km/h) | 14.0 | 20.0 | 30.0 |
| Round trip time (min) | 89.0 | 72.0 | 42.0 |
| Headway (min) | 2.0 | 2.6 | 2.1 |
| No of vehicles needed | 52.0 | 33.0 | 23.0 |
| Max prob. of not boarding | 0.97 | 0.36 | 0.31 |

TAB. 3. Transportation systems costs (Lit.).

| | BUS | TRAM | MLA |
|---|---|---|---|
| Total cost of investment (Billions) | 22.5 | 198.0 | 720.0 |
| Unit operating cost (Lit/p•km) | 44.3 | 33.3 | 15.0 |
| Total annual operating cost (billions) | 8.5 | 13.6 | 6.0 |

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