THE OPTIMAL SELECTION OF VEHICLE SIZES OF MASS RAPID TRANSIT SYSTEMS FOR DEVELOPING COUNTRIES

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INTRODUCTION

To select the basic units for Mass Rapid Transit (MRT) systems is an important process in planning rolling-stock, since it deeply affects system performance and operational efficiency. The basic units of rapid transit are called "operating units" and generally classified into Single-Unit(SU), Married-Pair(MP), Three-Car Unit(3C), Articulated Unit(AU), etc. For efficient operation, choosing the optimal vehicle-type can reduce the operating cost, provide for sufficient capacity, increase the demand and comfort of passengers, and it can even derive the whole optimal operation plans in rapid transit lines. Therefore, how to accurately evaluate and choose the optimal vehicle-type is an urgent study when building a MRT system.

There are few reports focusing on choosing the optimal vehicle-type for MRT systems. In 1988, Klein, J. employed a discounted cash flow on analyzing a model to compare the costs between single and married-pair transit cars. He used a simple arithmetic equation to generate the net present value (NPV) between single and married-pair vehicals. As a result, for small scheduled configuration of a transit line, the costs of a single car are less than those of married-pair. Whereas, costs are obviously lower for a married-pair in a large scheduled configuration of transit line.

Secondly, Bugarcic,H. and Chin, Ch'êng-K'ai(1989) used an absolute value in comparison with vehicle depreciation cost, maintenance and repair cost of operating vehicles, and energy cost. They found that the three-car unit had the lowest average expenditure. This paper tries to develop a mathematical linear programming model to select the optimal MRT vehicle-type. And it practically studies the applied possibility, the economic analysis of the model.

1.MODEL FORMULATION

1.1.The objective function of this model

1. Total ownership costs of units per day (TC_o) (\$/day)

In accordance with the difference of vehicle-type, unit size, number of driver control sets per units. The total ownership costs of units are:

$$TC_o = C_{ni} \times n_i \qquad \forall\, i \text{ ---------------------------- } (1)$$

C_{ni} : purchasing cost per units of vehicle-type i per day (\$/units-day)

n_i : total number of units of vehicle-type i (units)

2. Total costs of storage facility per day (TC_s) (\$/day)

For efficient management and operation, when the system closes every day, all ownership units have to be located in the storage spaces. Thus, the formula is:

$$TC_s = \sum_{i=1}^{I} E_{si} \times e_{si} \qquad \forall\, s \text{ -------------------- } (2)$$

E_{si} :per storage spaces cost of vehicle-type i at terminal s per day (\$/spaces-day)

e_{si} :number of storage spaces per vehicle-type i at terminal s (spaces)

3. Total operating costs of units per day (TC_P) (\$/day)

Operating cost(not including labor costs) is mainly related to the energy consumption cost, units joined or disjoined cost, the variable cost of material and accessory (such as electric component), etc.

$$TC_P = \sum_{t=1}^{T} \sum_{r=1}^{R} P_t \times L_r \times \left(\sum_{i=1}^{I} C_{vi} \times c_{tri}\right) \text{ --------- } (3)$$

P_t :duration of time period t (hour)

L_r :length of route r (kilometer)

C_{vi} :operating cost per units-kilometer of vehicle-type i per day (\$/units-km/day)

c_{tri} :flow rate of units on route r during time period t for vehicle-type i (units/hour)

4. Total operating costs of a train per day (TC_t) (\$/day)

The difference between the operating cost of a train and the units relies on that the former is increased along with the increased number of units per train, whereas the latter is related to the frequency. Different frequency or units type on different route during different time period affects the difficulty and easiness of train management and operation.

$$TC_t = \sum_{t=1}^{T} \sum_{r=1}^{R} T_r \times \left(\sum_{i=1}^{I} C_{hi} \times o_{tri}\right) \text{ ------- } (4)$$

T_r :one-way running time on route r (hour)
C_{hi} :operating cost per units-hour of vehicle-type i ($/units-hr/day)
o_{tri} :number of units per train of vehicle-type i on route r during time period t (units/train)
T :number of time period in operating day
R :number of route

5.Total administration costs of units per day (TC_m)

Without relation to running kilometers of fleets, the administration costs are only concerned with the quantity and size of units.

$$TC_m = \sum_{s=1}^{S} C_{ai} \times v_{tsi} \quad \forall\ i,\ t\text{:last time period} \quad \text{--- (5)}$$

C_{ai} :administration cost per units of vehicle-type i per day ($/units-day)
v_{tsi} :number of operating units stopped in terminal(or station) s at the ending of last time period t (units)

6.Total maintenance and repair costs of units per day (TC_r) ($/day)

Different vehicle-type needs different maintenance and repair equipment, and it also affects the maintenance and repair expenditure of units.

$$TC_r = C_{mi} \times r_{si} \quad \forall\ i \quad \text{---------------------- (6)}$$

C_{mi} :maintenance and repair cost per units of vehicle-type i per day($/units/day)
r_{si} :number of units of vehicle-type i to be remained in repair shop(or storage) s for repairing or clearing

1.2.The constraints of this model

1.Vehicle conservation of flow constraint

The conservation of flow means that the number of operating units into terminal(or station) during a time period plus the number of units staying in terminal at the beginning of that time period should be equal to the number of units staying in terminal at the ending of that time period plus the number of units out of the terminal. Thus, the conservation of flow equation would be :

$$\sum_{r \in R_{tz}} X_i^{us} \times P_{us} \times c_{uri} + v'_{usi} - v_{usi} = 0 \quad \forall\ u,s,i \text{ -- (7)}$$

X_i^{us} { +1:vehicle-type i during time period u into terminal s; -1:vehicle-type i during time period u out of terminal (or station) s }
P_{us} :duration of time period u at terminal(or station) s (hours)
c_{uri} :number of units of vehicle-type i at time period u

on route r into(or out of) terminal(or station) s per hour (units/hr)

v'_{usi} :number of units of vehicle-type i staying in terminal (or station)s at the beginning of time period u(units)

v_{usi} :number of units of vehicle-type i staying in terminal (or station) s at the ending of time period u (units)

2.Number of units for preparation,maintenance constraint

In order to maintain normal operation and emergent aid for systems, we must appropriate few units of each vehicle-type in storage for preparation and maintenance. Assuming that there is α to α' percent number of units stayed in the storage. For the rapid transit systems, α and α' are general about 5～10%.

$$\alpha \times n_i <= \sum_{s=1}^{S} r_{si} <= \alpha' \times n_i \qquad \forall\ i \quad \text{---------} \quad (8)$$

3.Storage spaces constraint

During operating period every day , we must provide sufficient storage spaces (e_{si}) for units stopping at each terminal.

$$v_{usi} <= e_{si} \qquad \forall\ u,\ s,\ i \quad \text{--------------------} \quad (9)$$

On the other hand, cars have to be in storage(n_i) at the end of the operating time every day. So, we must build adequate storage spaces.Therefore,

$$n_i <= \sum_{s=1}^{S} e_{si} \qquad \forall\ i \quad \text{---------------------} \quad (10)$$

4.Demand of passenger volume constraint

Providing sufficient capacity of units to satisfy the passenger's demand is an important premise, especially in Maximum Load Section(MLS). Assume that D_{tz} is the passenger volume through MLS between zone pair z during time period, then the transit capacity [$(1+k)s_i \times c_{tri}$] should be greater than or equal to D_{tz}.

$$\sum_{r \in R_{tz}} \sum_{i=1}^{I} (1+k)S_i \times c_{tri} >= D_{tz} \qquad \forall\ t,\ z \quad \text{--} \quad (11)$$

where,

k :proportion of standing to seating (0.5～1.0 times in off-peak and 1.0～2.0 times in peak)

S_i :number of seat per units of vehicle-type i(seat/units)

C_{tri} :flow rate of operating car units(units/hr)

D_{tz} :passenger volume passing through zone pair z during time period t(person/hr)

R_{tz}:set of routes through zone pair z during time period t

5.Scheduled line capacity constraint

For transfering the maximum number of passengers under the lowest threshold of safety, we must design the maximum offered line capacity (C_{max},person/hr).

$$\sum_{r \in R_{tz}} \sum_{i=1}^{I} (1+k)S_i \times c_{tri} <= C_{max} \quad \forall\, t,\, z \quad ---- \quad (12)$$

6.Train length constraint

The train length is limited the platform length.Theorically, train length is shorter than the minimum platform length ($\overline{L_r}$), and the platform length should fit the maximum length of units($\underline{L_{ri}}$).

$$\underline{L_{ri}} <= \sum_{i=1}^{I} L_i \times o_{tri} <= \overline{L_r} \quad \forall\, t,\, r \quad ----------- \quad (13)$$

$\underline{L_{ri}}$:maximum length of units in all vehicle-type i (meter)
L_i :length of units per vehicle-type i (meter/units)
$\overline{L_r}$:minimum platform length in all platforms on route r(m)

7.Level of service constraint

Level of service constraint means that we must offer minimum policy frequency ($\underline{F_{tz}}$).

$$\frac{\sum_{r \in R_{tz}} \sum_{i=1}^{I} c_{tri}}{\sum_{r \in R_{tz}} \sum_{i=1}^{I} o_{tri}} >= \underline{F_{tz}} \quad \forall\, t,z \quad ------ \quad (14)$$

8.Station capacity constraint

Due to the differente driving control ways (such as ATO,ATP, ATS,etc.), the station capacity is different, and it even affects the headway. So, we must limit the frequency ($\overline{F_{tz}}$) to avoid accidents and ensure service reliability.

$$\frac{\sum_{r \in R_{tz}} \sum_{i=1}^{I} c_{tri}}{\sum_{r \in R_{tz}} \sum_{i=1}^{I} o_{tri}} <= \overline{F_{tz}} \quad \forall\, t,\, z \quad ----- \quad (15)$$

9.The number of units of each vehicle-type constraint

The required number of cars of vehicle-type i stored in storage at the ending of the operating time is equal to the amount of cars that the system owns.

$$\sum_{s=1}^{S} v_{tsi} = n_i \quad \forall i, \quad -------- \quad (16)$$

t:the ending of last time period

10.Equivalency of passenger volume constraint

The passenger volume carried by the system must be equivalently same for each different vehicle-type. So, we can compare with the number of supplied seats among different vehicle-type.

$$(1+k)S_i x c_{tri} = (1+k)S_j x c_{trj} \quad \forall\ t,r,i,j,i\neq j \text{ -- } (17)$$

11.Equivalency of train length constraint

The constraint is based on providing the same capacity for various vehicle-types. It means the train length must be equivalent among each vehicle-type.

$$L_i \times o_{tri} = L_j \times o_{trj} \quad \forall\ t,r,i,j,i\neq j \text{ -------- } (18)$$

where,

L_i : Length of units per vehicle-type i

O_{tri}: number of car units pen train

2.MODEL APPLICATION

To interpret the applications of this model, we collect some data from Taipei Department of Rapid Transit Systems (DORTS), Taiwan, R.O.C..Including passenger volume and four vehicle-types. Then we establish the model step by step and use the linear programming package (LINDO package) to solve the optimal solution.

2.1 Data for operating network

According to Taipei DORTS' report, the total line length of Taipei-Tamshui rapid transit line is 22.8km. And there are 21 stations (including 2 terminals). It takes 15 seconds for a train to stop at every station. One-way travel time between Taipei and Tamshui is approximately take 0.58hr. In accordance with an operating forecast in 2001, the operating time will be about 19 hr per day. It is separated by 5 time periods as shown in Tab. 1.

Table 1. Operating time period.

period	beginning～ending	peak/off-peak	time period (Pt)
1	5:30 ～ 7:00	off-peak	1.5 hr
2	7:00 ～ 9:00	peak	2.0 hr
3	9:00 ～ 16:00	off-peak	7.0 hr
4	16:00 ～ 18:30	peak	2.5 hr
5	18:30 ～ 0:30	off-peak	6.0 hr

The maximum passenger volume (D_{tz}) through Maximum Load Section (MLS) during this time period is shown in Table 2.

Table 2 Maximum passenger volume through MLS

time period number	route A Taipei ―> Tamshui	route B Tamshui ―> Taipei
1	1341	1697
2	8503	18412
3	3419	4269
4	14733	6802
5	2729	3421

2.2.Data of costs for each vehicle unit-type

The capital cost and operating cost for each type of vehicle are shown Table 3.

Table 3. capital cost and operating cost for each vehicle unit-type

relative cost (NT$)	item	type 1 (SU)	type 2 (MP)	type 3 (3C)	type 4 (AU)	unit
vehicle cost	C_{mi}	19037.2	29155.1	49530.8	35226.3	$/units
storage facility cost	E_{si}	414.9	851.3	2009.1	1240.1	$/space
vehicle operating cost	C_{vi}	15.04	24.83	43.23	30.96	$/units-km
train operating cost	C_{hi}	17.34	16.11	18.11	16.81	$/units-hr
vehicle administration cost	C_{ai}	99.3	194.5	449.3	277.6	$/units
vehicle maintenance and repair cost	C_{ri}	2165.4	3851.4	8439.7	5347.9	$/units

source:[1]

There are 15 combinations or modes among the four vehicle-types of this paper, as follows:

Example I

model no.	sole type
1	SU
2	MP
3	3C
4	AU

Example II

model no.	twin type
5	SU.MP
6	SU.3C
7	SU.AU
8	MP.3C
9	MP.AU
10	3C.AU

Example III

model no.	triad type
11	SU.MP.3C
12	SU.MP.AU
13	SU.3C.AU
14	MP.3C.AU

Example IV

model no.	four type
15	SU.MP.3C.AU

3.RESULTS AND ANALISIS

From Table 4, model 3 has the minimum total costs. It's lower than any other models. In Example I, model 3 saves NT$ 956,358 more in one day than model 1, and saves NT$ 301,509 more in one day than model 2, and saves NT$ 89,302 more in one day than model 4. Totally, NT$ 32~349 million is saved in one year. (Note that 1 US$ ≒ 25 NT$) From Table 4, we can also obtain the number of cars needed either in operation or in storage or repair and the optimal vehicle types.

In Table 5, we know the ratio of maintenace and repair units to operating units. The ratio of model 1 is the lowest. It means that model 1 has the highest vehicle usage ratio of then models 2, 3, and 4.

The value of δ refers to the proportion of factual loading passenger-km to supplied seat-km, as Table 6. Model 1 has the highest ratio no matter if the route is A or B. But model 3 has the lowest. It means 3-car unit has more capacity to satisfy the demand under the volume just now.

Table 4 Results of fifteen models

exam. no.	model no.	veh. type	total costs	number of cars	number of storage spaces term. 1	number of storage spaces term. 2	number of maint. and repair cars	number of cars in operation
I	1	SU	3,239,364	137	26	112	7	130
	2	WP	2,584,515	70	13	57	4	66
	3	3C	2,283,006	36	3	34	2	34
	4	AU	2,372,308	53	10	43	3	50
II	5	SU		69	13	56	4	65
		WP	2,911,960	35	7	29	2	33
	6	SU		69	13	56	4	65
		3C	2,762,230	18	4	15	1	17
	7	SU		69	13	56	4	65
		AU	2,805,880	27	5	22	2	25
	8	WP		35	7	29	2	33
		3C	2,435,300	18	4	15	1	17
	9	WP		35	7	29	2	33
		AU	2,478,420	27	5	22	2	25
	10	3C		18	4	15	1	17
		AU	2,329,150	27	5	22	2	25
III	11	SU		64	17	61	4	60
		WP	3,653,188	33	9	31	2	31
		3C		17	5	16	1	16
	12	SU		64	26	62	4	60
		WP	3,762,385	33	14	32	2	31
		AU		25	10	24	2	23
	13	SU		65	30	63	4	61
		3C	3,947,039	17	8	17	1	16
		AU		25	12	24	2	23
	14	WP		33	9	31	2	31
		3C	3,263,929	17	5	16	1	16
		AU		25	7	24	2	23
IV	15	SU		48	14	46	3	45
		WP		25	7	24	2	23
		3C	3,542,095	13	4	12	1	12
		AU		19	6	18	1	18

Example I: sole type; Example II,III and IV: mixed type

Table 7 lists the number of operating units for model 1~4 at each time period. In model 3, it only needs two basic operating units during the period to satisfy the passenger volume. As to the other peak period, just need one units. So, 3-car unit has the highest units shifting movability.

In Table 8, the higher frequency of these four models is approximate in peak. Whereas, model 3 needs only 4 trains/hr to satisfy the passenger volume in off-peak. Hence, model 3 has fewer empty car than any other model. It will decrease energy consumption and operating cost. For results of above, model 3 (3C type) is the most economic vehicle-type.

Table 5 Ratio of maintenance and repair units to operating units

model no.	percentage (%)
1	5.38
2	6.06
3	5.88
4	6.00

Table 6 Maximum supplied usage ratio (δ)

(1) route A (Taipei → Tamshui)

model no.	supplied seat-km(1) (space-km)	factual loading passenger-km(2) (passenger-km)	δ=(2)/(1)
1	2,800,980	1,385,772.6	0.4947
2	2,895,372	1,385,772.6	0.4786
3	2,948,040	1,385,772.6	0.4701
4	2,897,697	1,385,772.6	0.4782

(2) route B (Tamshui → Taipei)

model no.	supplied seat-km(1) (space-km)	factual loading passenger-km(2) (passenger-km)	δ=(2)/(1)
1	2,820,542	1,577,805.6	0.5594
2	2,850,022	1,577,805.6	0.5536
3	3,009,600	1,577,805.6	0.5243
4	2,911,651	1,577,805.6	0.5419

Table 7 Number of operating units

model no.		number of units 1		2		3		4	
route		A	B	A	B	A	B	A	B
time period	1	1	1	1	1	1	1	1	1
	2	4*	4	3**	3	2	2	2	2
	3	1	1	1	1	1	1	1	1
	4	4	3	2	2	1	1	2	2
	5	1	1	1	1	1	1	1	1

* :SU+SU+SU+SU (multi-type)
**:MP+MP+MP (multi-type)

Table 8 Frequency

model no.		frequency 1		2		3		4	
route		A	B	A	B	A	B	A	B
time period	1	24	11	13	6	4	4	10	5
	2	30	33	21	21	16	16	23	23
	3	24	28	13	14	7	8	10	11
	4	24	29	25	22	25	24	19	17
	5	18	22	10	12	6	6	7	9

4.SENSITIVITY ANALYSIS OF THE MODEL

4.1.Effects of units purchasing cost

In Graph 1, the effects of total costs in model 1 is the highest, then model 2, 4, and 3. It means that the sensitivity is lowest for large units when the units ownership cost was varied.

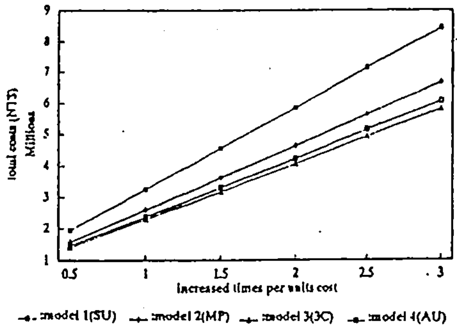
4.2 Effects of the number of vehicle units

In Graph 2, model 3 it has the highest sensitivity against any other models. Because it contains three cars per unit and has a higher unit operating cost, train operating cost, and units storage spaces cost, etc. So, we must consider the effects of the number of vehicle units are varied when the system was extended among the four types.

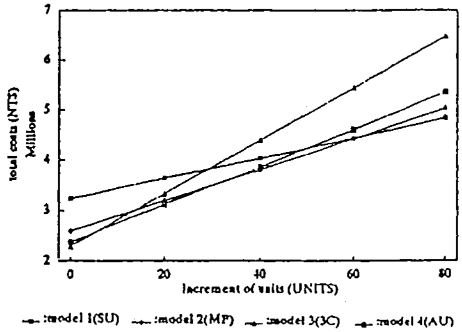
4.3.The influence of the number of storage spaces.

In Graph 3, model 3 has the highest slope to the right when the number of storage spaces were increased, but has the lowest

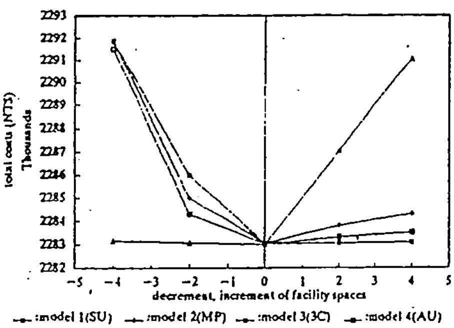
Graph 1 Effects of total costs by varing vehicle purchasing cost



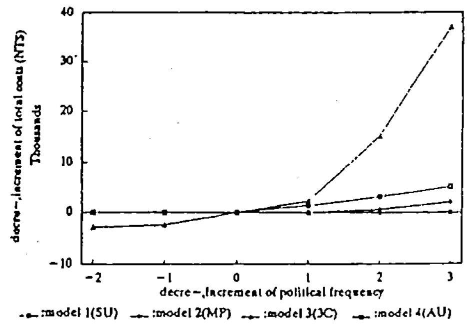
Graph 2 Effects of total costs by increasing the number of units



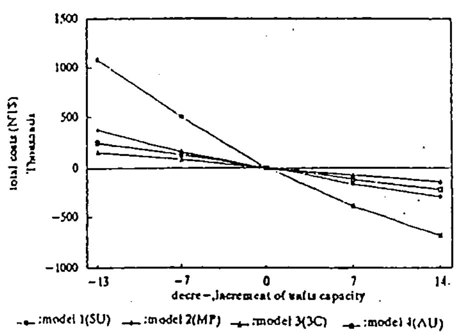
Graph 3 Effects of total costs by varing the number of the storage spaces



Graph 4 Effects of total costs by varing policy frequency



Graph 5 Effects of required number of units by varing units capacity



slope to the left, So, we can decrease the number of storage spaces in model 3, so as to decrease the total costs.

4.4. The influence of the level of service.

As the policy frequency was varied, the total costs and total desired number of units will simultaneously be varied. In Table 9 and Graph 4, the frequency increases from 2 to 7 trains/hr, The total costs were varied among the four models (although the desired number of units were not varied). So, we can adjust the policy frequency to satisfy the passenger demand but we can't increase the desired number of units. In Graph 4 (see model 3), when the frequency increases to 6 trains/hr (4 is initial), the total costs speedly increases. The reason is that model 3 has a larger capacity, and higher vehicle. cost. So, we know that large operating units are the most suitable vehicle-type for a heavy passenger transit line where the policy frequency is not so important.

Table 9 Effects of policy frequency

model no.	policy frequency	number of units	total cost (NT$)
1 (SU)	2	137	3.239.364
	3	137	3.239.364
	4	137	3.239.360
	5	137	3.239.364
	6	137	3.239.364
	7	137	3.239.364
2 (MP)	2	70	2.584.515
	3	70	2.584.515
	4	70	2.584.520
	5	70	2.584.515
	6	70	2.585.137
	7	70	2.586.507
3 (3C)	2	36	2.280.185
	3	36	2.280.598
	4	36	2.283.010
	5	36	2.285.413
	6	36	2.298.209
	7	36	2.320.004
4 (AU)	2	53	2.372.308
	3	53	2.372.309
	4	53	2.372.310
	5	53	2.373.745
	6	53	2.375.453
	7	53	2.377.405

4.5. The influence of vehicle capacity

Graph 5 shows the economic effectiveness between capacity of units and desired number of units. Model 1 has the highest sensitivity, whereas model 3 has the lowest sensitivity. However, we know that to increase the capacity of small vehicle-types is more difficult than larger ones. So, we only need to increase the number of small vehicle units to satisfy the increased passenger volume. But in larger ones this may not be needed.

5.CONCLUSIONS AND SUGGESTIONS

1.The traditional selection methods of the best vehicle-type use the heuristic method. People cannot obtain a truely optimal vehicle-type using this way. This paper develops a mathematical linear programming model to choose the best vehicle-type. Moreover, this model is based on a linear function and it processes a series of analysis about economic effectiveness, sensitivity, etc. Also, this model can simultaneously change frequency and train length for satisfying the different passenger volume. The tranditional model cannot do this.

2. There are fifteen combinations in our examples. Among them, model 3 is the most economic ones. In comparison with the model 13, it can save NT$ 1,664,029 in one day. Totally, it can save about NT$ 600,000,000 in one year.

3. If we adopt two or more vehicle types in these systems, the larger vehicles will have lesser operating and maintenance costs in the off-peak hours than the small vehicles do in the peak hours. The reason is that if we adopt small vehicle during the peak hours, they will be used more often and that will cause the operating and maintenance costs to rise. Also , assembling a train with one vehicle-type is more economical than mixing vehicle types together.The main reason is that mixed vehicle-type will raise the administration cost and increase the difficulty of management, maintenance and repairing.

4. The data of labor costs is difficult to collect. This factor would deeply affect the calculation of total costs. Thus, we suggest that labor costs should be put into this model, so as to obtain a more complete and accurate solution. On the other hand, only the items of quantifiable costs were considered in this model,it suggests to make a whole consideration including the unquantifiable factors to match the real world.

5. This model is a linear function, and the solving process is a LP problem. It has some differences with a practical situation. Therefore, we suggest to modify this model into a nonlinear form.

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