Risk Analysis Approach to System Optimum Flow Considering Losses Caused by Travel Time Variation

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INTRODUCTION

Traffic congestion frequently occurs on city-area road networks, disturbing citizens' lives, their social and economic activities. Relieving congestion is an emerging issue in road transport service. Although road network capacities should be expanded as a fundamental measure with the construction of new roads, construction requires an excessive amount of time. Thus, demand controlling measures become necessary in allowing for efficient use of existing road networks. For this reason, some software-like countermeasures such as route guidance through providing information [Iida, 1989] or staggered working hours [Iida et. al., 1991] have drawn attention. These countermeasures disperse traffic demand spatially and temporally reducing the demand peak. They aim to prevent or reduce traffic congestion without cutting down the total traffic demand. Assuming these measures are actually adopted, to what degree should traffic be dispersed; also, what is the ideal attainable goal?

This study proposes an ideal traffic state which is regarded as the goal of demand controlling measure, with a restriction of spatial dispersion. It is a state of traffic flow which prevents the occurrence of traffic congestion and reduces losses resulting from any congestion. This study considers travel time variation, which depends on the frequency of congestion, and proposes a system-optimized traffic assignment, here designated **Risk Assignment**, which minimizes losses caused by travel time variation.

If traffic congestion is liable to occur and one cannot predict travel time exactly, the driver must depart early allowing additional time in order to arrive at a destination by a scheduled time. This margin time and effective travel time (the time between the departure time and scheduled arrival time) increase as variations in travel time increase. Focusing on this behavior and presuming travel time variation to be a risk, this study evaluates drivers' travel costs. Risk Assignment is a prior or pre-posterior measure to minimize expected total travel cost.

We can determine or evaluate measures that cope with uncertain events, such as traffic congestion and travel time variation, using a risk analysis framework. This paper, in section 1, first discusses concepts of Risk Assignment based on risk analysis. In section 2, Risk Assignment is mathematically formulated. The ensuing section 3 illustrates the performance and efficiency of Risk Assignment with a numerical example. Lastly, in section 4, concluding remarks are made.

1. RISK ASSIGNMENT

1.1. Traffic Congestion and Risk Assignment

Rapidity and travel time stability are of great concern to drivers as socioeconomic activities increase dramatically. Improved rapidity and travel time stability can be achieved by 1) reducing the mean travel time by lightening traffic volume, and 2) lessening the frequency of traffic congestion.

Traffic congestion is never an unusual phenomenon in urban areas, and persistently causes a great loss of time. For example, on the Hanshin Expressway Network in Japan, congestion occurred 44.2 times per day on the average with a mean congestion interval of 2.08 hours and queue lengths of 4.5 kilometers (in 1987). Preventing congestion and improving rapidity/travel time stability are important issues.

Traffic volume on intra-urban expressways can be controlled at on-ramps to prevent congestion. On-ramp control is usually applied in order to terminate a queue quickly after congestion has occurred since the relationship between traffic volume and the occurrence of congestion is stochastic. Considering the losses that may be caused by probable congestion, it is more profitable to control traffic volume in advance than to take posterior countermeasures. Risk Assignment provides an ideal traffic flow goal for traffic control.

1.2. Risk Analysis Approach

Risk analysis is the process of scientifically determining measures for coping with potentially serious circumstances such as traffic congestion which occurs un-deterministically causing great losses. Risk analysis denotes possible risk agents. An agent who endures losses is called a "risk-suffering agent." An "action-taking agent" determines an action that will minimize losses. These two agents are not necessarily identical.

"Risk" consists essentially of uncertainty and losses caused by random events. Therefore, a problem of determining action to cope with risk is generally formulated as follows:

$$\min_{A} R = R(C(H,A|\theta),\ P(H|A,\theta)) \tag{1}$$

where, R: risk,

H: peril,

A: action to cope with risk circumstance,

θ: circumstance condition,

$C(H,A|\theta)$: loss caused by peril conditioned with action A,

$P(H|A,\theta)$: occurrence probability of peril H.

The loss C is a function not only of peril H, which is a direct cause of loss, but also of risk-coping action A. Risk uncertainty is expressed separately with $P(H|A,\theta)$ and $C(H,A|\theta)$. $P(H|A,\theta)$ denotes the uncertainty of the cause of the event and $C(H,A|\theta)$ denotes the uncertainty in a performance of risk-coping action. "Aversion" and "elimination" which intend to control P and C, respectively, and "reduction" which controls both P and C can be regarded as risk coping actions.

1.3. Interpretation based on Game Theory

The agents in Risk Assignment are not identical. The risk-suffering agent is the driver and the risk-taking agent is the traffic manager. So, equation (1) should be reformulated as the following recursive equations:

$$\begin{aligned} &\min_{A_1} R_1 = R_1(C_1(H,A_1|\tilde{A}_2,\theta'), P(H|A_1,\tilde{A}_2,\theta')) \\ &\min_{A_2} R_2 = R_2(C_2(H,A_2|\tilde{A}_1,\theta'), P(H|A_2,\tilde{A}_1,\theta')) \end{aligned} \tag{2}$$

where, A_i: action of agent i,

$\tilde{A}_i$: agent i's action surmised by agent j ($\neq$i).

The action-taking agent (agent 1) decides on an action A_1 upon surmising the action of agent 2 as $\tilde{A}_2$. On the other hand, the loss-suffering agent (agent 2) also takes risk-coping action A_2 upon considering the action of agent 1 $\tilde{A}_1$.

Under these circumstances, it is natural that decision problem (2) is interpreted based on Game Theory. This study considers the traffic manager as agent 1 and drivers as agent 2. No binding agreement for action can be made between these agents, and information about travel time variations and the action of the opposing agent is not equal for each agent. It is adequate, therefore, that the problem is interpreted as an un-cooperating two-players game of asymmetrical information, i.e. a Stackelberg problem, in which the traffic manager is the leader and the driver is the follower.

The Stackelberg problem is mathematically formulated as a bilevel optimization problem. It consists of an upper level problem which determines the action of agent 1, and a lower level problem which describes behavior of agent 2.

2. FORMULATION OF RISK ASSIGNMENT

This study considers the time that driver consumes for travel as the substance of risk. It is assumed that the traffic manager intends to minimize the temporal risk of total drivers in reducing the frequency of congestion and travel time variation by controlling route traffic volume. Drivers are assumed to change their departure time in order to cope with risk circumstances by themselves. Risk Assignment is defined as that determining path flow by this manner or obtained path flow.

2.1. Framework

The framework of Risk Assignment is summarized in Fig. 1 and follows:

[contents of risk circumstance]

* agent 1 (action–taking agent): traffic manager
* agent 2 (risk–suffering agent): drivers
* performance of facility: travel time of link

[substance of risk]

* loss: travel time delay, effective travel time
* uncertain event: 1. congestion (peril)
  2. travel time variation

[**risk–coping action**: reduction strategy]

* agent 1: controlling link volume (share of travel demand)
* agent 2: changing departure time

[objective for deciding action]

* agent 1: minimize expected value of drivers' total travel cost
* agent 2: minimize expected sum of travel time and penalty for late arrival.

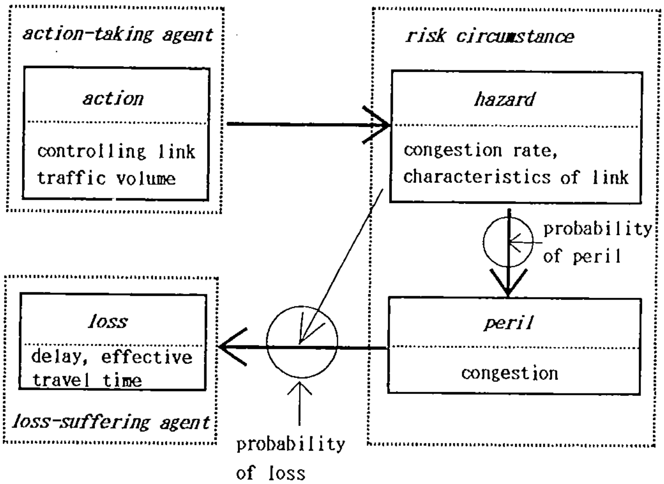

Fig. 1 Risk Assignment Framework.

2.2. Formulation

The following is presumed for formulating the problem:

[assumption]

1. An interval of time (e.g. one hour) is considered. Macroscopic and static evaluations are made.
2. OD traffic demand and traffic flow patterns are given as constant values.
3. Drivers take an optimum reacting action in responding to the traffic manager's action.
4. The traffic manager knows that assumption 3 is true.

Risk Assignment is formulated as a bilevel optimization problem as follows:

[**Upper Level Problem**: determining the action of the traffic manager]

$$\min_{Q} EC(Q) = \sum_i \sum_x \sum_t q_i t_i P(t_i|X,Q) P(X|Q) \quad (3)$$

$$\text{s.t. } \sum_i q_i = \text{const.}$$

where, EC: expected cost,

t_i: travel cost of route i (given from lower problem),

q_i: traffic volume of route i,

Q: vector of q_i,

x_i: traffic state of route i,

X: vector of x_i,

$P(t_i|X,Q)$: probability of t_i conditioned with Q, X,

$P(X|Q)$: probability of X conditioned with Q.

[**Lower Level Problem**: describing driver behavior]

$$\min_{t_0} L = \beta(t_d - t_0) + \gamma(1 - F(t_d|t_0)) \quad (4)$$

where, β: value of time (yen/minute),

γ: penalty for late arrival (yen),

t_d: scheduled arrival time,

t_0: departure time,

$F(t_d|t_0)$: cumulative probability of arriving at destination by t_d, conditioned with departure time t_0.

The lower level problem depicts driver behavior as a departure time decision problem that copes with the risk of late arrival, which is a result of travel time variation or congestion. Under such a behavioral assumption, the time interval between the scheduled arrival time t_d and the departure time chosen t_0^* is called the Effective Travel Time t_e (= $t_d - t_0^*$) [Hall, 1983]. The effective travel time is a virtual travel time that the driver estimates beforehand. The lower level problem is also interpreted as a model of a driver's risk evaluation since the effective travel time can be considered an index of travel time reliability [Kato et al., 1986].

There is a mutual determining relationship between the upper level problem and the lower level problem. In the upper level problem, the traffic manager chooses an action to minimize expected total travel cost which is defined through the lower level problem. In the lower level problem, the probability of arriving $F(t_d|t_0)$ depends on the traffic manager's action controlled in the upper level problem.

2.3. Calculation Method

Under assumptions #3 and #4, the bilevel optimization problem stated above can be regarded as a Stackelberg problem. In the case of Risk Assignment, the bilevel optimization problem is reduced to an ordinary single-level optimization problem, since the lower level problem can be solved analytically, considering that network flow is an exogenously given value for the lower level problem.

If a perceived distribution of travel time conforms to a normal distribution $N[\mu_T^c, \sigma_T^{c2}]$, travel cost (effective travel time) t_e is derived as follows:

From eq. (4),

$$\frac{dL}{dt_0} = -\beta + \gamma\frac{1}{\sigma_T^c}\phi\left(\frac{t_d-t_0-\mu_T^c}{\sigma_T^c}\right) = 0 \qquad (5)$$

where, μ_T^c: mean of perceived travel time distribution,

σ_T^c: standard deviation of perceived travel time distribution,

$\phi(\cdot)$: standard normal probability density function.

then,

$$t_e = \mu_T^c + \sigma_T^c\phi^{-1}(\sigma_T^c / \gamma) \qquad (6)$$

$$\sigma_T^c / \gamma < \phi(0),$$

where, $\phi^{-1}(\cdot)$: inverse of $\phi(\cdot)$.

This effective travel time t_e is utilized in the upper level problem.

If the functions and parameters shown in Table 1 are determined for any feasible traffic flows, Risk Assignment can be derived from eqns.(3) and (6). Figure 2 depicts the procedure applied, for simplicity, to a single OD pair / two link network. Firstly, from (A) probability of congestion and (B) travel cost distribution conditioned with the traffic state, (C) travel cost distribution is attained and effective travel time is calculated. Then, (D) expected values of travel cost which are independent from the traffic state are determined for the traffic volume. The solution reached is the set of traffic volumes that minimize the sum of expected travel costs.

Table 1. Exogenous Variables and Parameters of Risk Assignment.

performance of facility	probability of congestion	P(X|Q)
	travel time distribution (un-congested)	$N[\mu_T, \sigma_T^2]$
	(congested)	$N[\mu_T', \sigma_T'^2]$
driver behavior	perceived travel time distribution	$N[\mu_T^c, \sigma_T^{c2}]$
	penalty for late arrival	γ

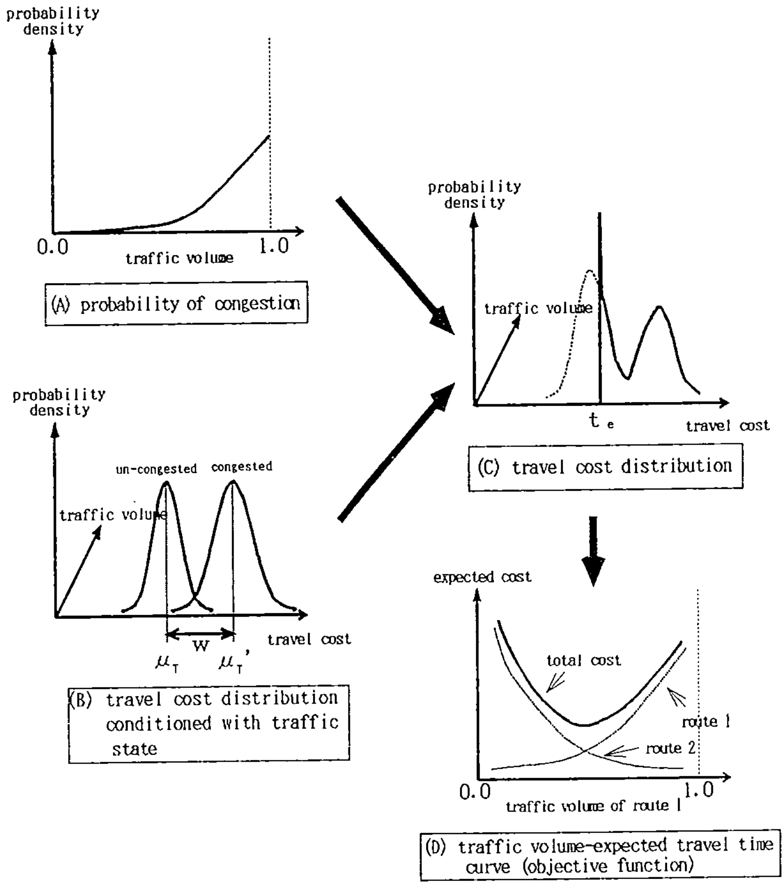

Fig. 2 Risk Assignment Calculation Procedure
(for a 1-OD 2-link network).

3. NUMERICAL EXAMPLE

3.1. Model Data

A numerical example is performed in order to illustrate the performance of Risk Assignment. For simplicity, the network considered has only one OD pair, origin and destination is connected by two parallel links. One link is a toll and access-restricted intra-urban expressway, the other is a free-access arterial road.

The functions and parameters shown in Table 1 should be determined statistically throughout the survey on an actual road network. In this study, they are given arbitrarily as in a previous study [Iida et al., 1990].

3.1.1. Link travel time performance

Actual travel time of link i is assumed to conform to normal distribution $N[\mu_T, \sigma_T^2]$ and $N[\mu_T', \sigma_T'^2]$ depending on the traffic state, which is classified either as un-congested or congested. For an un-congested state, the relationship between the traffic volume and mean travel time μ_T is determined by a modified BPR function. Travel time variance σ_T^2 is given as a function of mean travel time μ_T as follows:

It is known that following relationship exists between mean traffic volume μ_Q and variance σ_Q^2:

$$\sigma_Q^2 = r\mu_Q^s \quad (7)$$

where, r,s: parameters.

On the other hand, the following relationships are derived from Taylor expansion of the modified BPR function;

$$\mu_T = a + b\mu_Q$$
$$\sigma_T^2 = b^2\sigma_Q^2 \quad (8)$$

where, a,b: parameters.

Substituting eqn. (8) for eqn. (7), we get

$$\sigma_T^2 = rb^{2-s}(\mu_T - a)^s \quad (9)$$

For a congested state, delay time w is introduced and mean travel time μ_T' is given as $\mu_T' = \mu_T + w$. Variance $\sigma_T'^2$ is derived by substituting μ_T' for μ_T in eqn. (9). Delay time w is assumed to be an increasing function of congestion rate (traffic volume / capacity) for the link.

3.1.2. Link characteristics

In order to clarify the performance of Risk Assignment, the links are characterized as follows:

1. Travel time of free flow is set at 1.0 hour for the expressway and 1.33 hours for the arterial road.
2. Delay time and probability of congestion increase as the link traffic volume increases. The degree of increase is smaller for expressway than for arterial road.
3. In order to consider the effects of a toll charge, the charged amount is given in terms of time. The sum of travel time and toll charge is used for assignment. The charge for the expressway is set at 0.5 hours.

Expressway is advantageous, in short, in terms of travel time and risk of congestion. Arterial road, on the other hand, is preferable because of the lack of a toll charge. Each link has the same capacity.

3.1.3. Characteristics of driver behavior

The parameter γ, i.e. penalty for late arrival, which determines driver behavior, is given as ten times the value of the toll charge (5 hours). Perceived distribution of travel time is assumed to coincide with the distribution of an un-congested state. In other words, the situation considered is one in which drivers have little information about traffic congestion while the traffic manager has a perfect information.

3.1.4. Normal assignment for comparison

In order to examine the efficiency of Risk Assignment, a comparison is made with the uncontrolled flow. The uncontrolled flow is given with user equilibrium (UE) assignment. In UE assignment, travel time distribution of an un-congested state is used. Some actual drivers may have some information about traffic congestion, the UE assignment in this example gives an extreme flow. The real flow may be located between the UE flow and Risk Assignment flow.

3.2. Example Results

The results of the numerical example are illustrated in Figures 3 to 6. As mentioned above, Risk Assignment assumes that traffic demand stays constant. In order to see the effective influence exerted on the change in demand, however, traffic demand is treated as a parameter. In these figures, traffic demand is standardized, divided by the sum of the capacity of both links.

3.2.1. Link flow

Figure 3 plots link traffic volume, which is standardized according to link capacity, and Figure 4 plots the expressway share of traffic demand, which is in proportion to traffic demand. They show that, while traffic demand is small, no flow is assigned to the expressway in either assignment because of opposition to the toll charge.

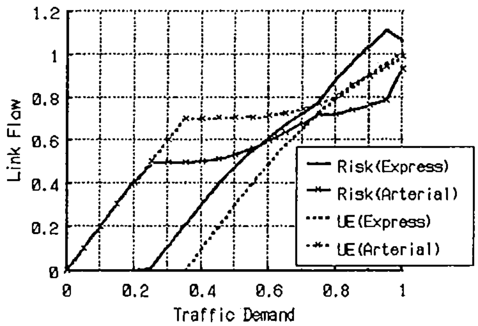

Figure 3. Link Flow of Respective Route.

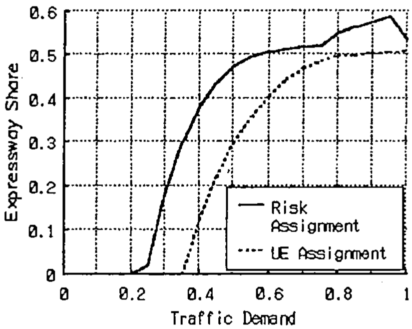

Figure 4. Expressway Share of Traffic Demand.

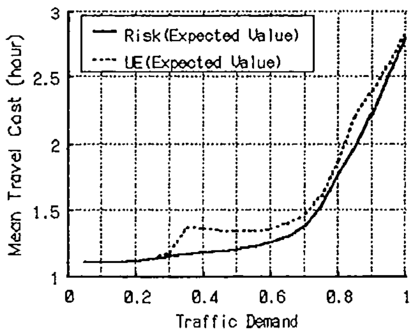

Figure 5. Expected Value of Mean Travel Time.

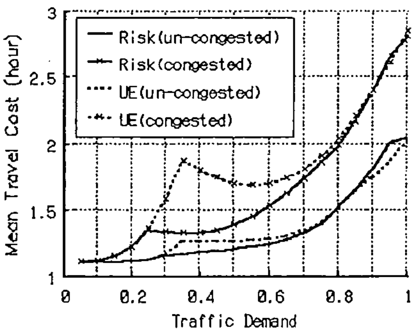

Figure 6. Mean Travel Time Conditioned by Traffic State.

If traffic demand grows, however, Risk Assignment assigns flow to the expressway under the conditions that there is a smaller demand than for the UE Assignment (some flow is assigned to expressway by Risk Assignment or UE Assignment when the demand exceeds 0.25 or 0.4, respectively). The expressway share is always higher for Risk Assignment than for UE Assignment.

It is interesting to note that the expressway flow value assigned by Risk Assignment protrudes when traffic demand exceeds 0.85. This is because the probability of congestion for the arterial road reaches 1.0. Traffic volume is fairly large and difference in travel cost between the congested and un-congested state. Thus, Risk

Assignment assigns flow to the expressway for which congestion probability is smaller.

For UE Assignment, these curves change smoothly because the risk of congestion is not considered.

3.2.2. Expected travel cost

In order to assess the efficiency of Risk Assignment, driver travel costs for Risk Assignment and UE Assignment are compared in Figures 5 and 6. These figures show the mean travel cost per vehicle. Because travel cost varies depending on the traffic state, Figure 5 depicts the expected value, and Figure 6 illustrates the values when traffic states of both links are congested and un–congested, providing maximum and minimum values of mean travel cost, respectively. These curves consist of two parts; one corresponds to the phase in which no flow is assigned to the expressway, and the other represents the phase in which each link is assigned with flow. The inflection points (the points at which traffic demands are 0.25 for Risk Assignment and 0.35 for UE Assignment) reflect the phase shifts.

Comparing the expected values in Figure 5, although Risk Assignment always yields a lower cost (because it gives solutions for minimizing expected costs), the difference between both assignments varies greatly depending on traffic demand. When traffic demand is small (0.25–0.35), ignoring the risk of congestion, UE Assignment assigns no flow to the expressway because of opposition to the toll charge. The risk of congestion is higher when traffic converges to one link, consequently, Risk Assignment which distributes demand to both links produces a lower cost. As traffic demand increases, UE assignment also assigns flow to the expressway and the difference in **mean** travel cost decreases. The difference in **total** cost remains large, however, because traffic demand is great.

Comparing the cost under the un–congested state, Figure 6 shows that Risk Assignment generates a higher cost when traffic demand exceeds 0.9. In other words, the drivers who drove under an un–congested state, as a result, endure unnecessary cost under Risk Assignment. But the probability of un–congested state is very low.

As for congested state, while Risk Assignment is highly efficient when roads have sufficient margins (e.g. the traffic demand equals 0.3), when traffic demand is large, the efficiency of Risk Assignment declines.

4. CONCLUDING REMARKS

This paper proposes a new system optimum traffic assignment concept based on risk analysis, i.e., Risk Assignment. A numerical example was also conducted to assess its performance and efficiency. In assessing efficiency, the Risk Assignment traffic flow was compared with the flow given under the assumption that drivers have no information about congestion. Drivers in the real world might have some information about traffic congestion, but it is not very feasible that they would know the objective distribution of travel time with precise accuracy. Consequently, the results of the example may not be unrealistic and provide proof that less information may cause greater loss.

This study is still at its preliminary stages. Further refinement and modifying of the model's concepts and formulas are necessary. We plan to:

1. describe driver behavior more precisely in the lower level problem,
2. construct the methodology for applying Risk Assignment to actual scaled road networks.

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