# EMPIRICAL ANALYSIS OF TRAFFIC VOLUME FOR THE APPLICATION OF THE OPTIONS THEORY TO HIGHWAY CONCESSIONS

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#### **ABSTRACT**

Investment projects in the field of transportation infrastructures have a high degree of uncertainty and require an important amount of resources. In highway concessions in particular, the calculation of the Net Present Value (NPV) of the project by means of the discount of cash flows, may lead to erroneous results when the project incorporates certain flexibility. In these cases, the theory of real options is an alternative tool for the valuation of concessions. When the variable that generates uncertainty (in our case, the traffic) follows a random walk (or Geometric Brownian Motion), we can calculate the value of the options embedded in the contract starting directly from the process followed by that variable. This procedure notably simplifies the calculation method.

In order to test the hypothesis of the evolution of traffic as a Geometric Brownian Motion, we have used the available series of traffic in Spanish highways, and we have applied the Augmented Dickey-Fuller approach, which is the most widely used test for this kind of study. The main result of the analysis is that we cannot reject the hypothesis that traffic follows a Geometric Brownian Motion in the majority of both toll highways and free highways in Spain.

Key words: Unit root test; Traffic; Real Options; Highway.

#### INTRODUCTION: REAL OPTIONS IN HIGHWAY CONCESSIONS

In a strict sense, the real options method is the extension of the theory of financial options to the valuation of options on real assets or projects. This approach is particularly appropriate in

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the context of strategic decision-making and in the evaluation of investment opportunities under conditions of uncertainty.

An investment project has many similarities with an option to buy stock on the financial markets. Both imply a right, but not an obligation, to acquire an asset by paying a specific amount of money at a certain moment in the future, or before a certain moment in the future. The right to buy a stock is known as a call option; when the stock is actually acquired, it is said that the option is exercised. Similarly, the decision to invest (to buy an existing asset or to build a new one) can be treated as the exercise of a call option.

Real options can be used to design and manage strategic investments in an active way. An option provides the opportunity of taking a decision after watching how events develop. On the exercise date, if everything has happened as expected, one kind of decision will be taken, but if an unforeseen, or unlikely event has taken place, another decision can be taken. This means that the return obtained is not strictly lineal, as it depends on the decision that is adopted. Non-lineal returns can be a tool to reduce the exposure to uncertainty, and this is extremely useful for managers.

From a traditional point of view, the higher the level of uncertainty, the lower the value of a project. Under the real options approach, a higher level of uncertainty can mean a higher value of the project if managers identify and use their options to meet the future and sometimes unexpected evolution of events with flexibility. Options that are inherent to investments allow managers to reduce the exposure to bad results and increase the ability to exploit good results, as and when they occur, and, thereby increase the value of the investment project.

In the field of transportation infrastructure, investment projects have some specific features, different to other types of investment. Generally speaking, infrastructure transportation projects have a high degree of uncertainty, they require an important amount of resources and are, to a large extent, irreversible. These features are clear in highway concessions, where there is a wide potential for the application of the real options approach.

Real options arise in a natural way from the interpretation of the clauses established in the actual contracts of highway concessions. After all, the real options approach contributes a valuation tool, but the terms of the options are embedded in the contracts that regulate rights and obligations for both contracting parties.

In the case of highways, some of the existing or possible real options in the projects are as follows:

- 1. Minimum traffic guarantees (traffic floors) or maximum traffic limitations (traffic caps)
- 2. Public subsidies when traffic is lower than expected
- 3. Investment in new highway stretches

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- 4. Early abandonment of the concession
- 5. Anticipated reversion of the concession
- 6. Public participation loans
- 7. Extension of the concession period

These mechanisms reduce cash-flow volatility, add flexibility to the project and allow a better management of the concession based on the occurrence of future events. Some of these mechanisms have been used one way or another in toll highway programs in different countries. In Spain, for example, public participation loans have been granted in numerous highway projects since 1996 (Vassallo and Sánchez Soliño, 2007).

The possible exercise of this series of rights represents an added value for the project which is not captured by the traditional procedures of valuation. So, the lack of an appropriate quantitative tool has prevented the effectiveness of these mechanisms from reaching their maximum potential. The habitual practice of calculating the net present value (NPV) of the project by means of the discount of cash flows, leads to erroneous results when the project incorporates a certain flexibility.

The main limitation of the NPV is that it assumes a static view of the investment project, without considering the value of possible decisions which may be made in the future and, subsequently, does not fully take into account changes in the variables that affect the project. On the other hand, the use of a decision tree technique, in combination with the NPV method, has an important drawback: the results depend on the discount rate used in the analysis, but, at the same time, the relevant discount rate depends on the options that are present in the project.

With respect to these limitations, some authors, such as Myers (1984) and Kester (1984), have suggested that investment analysis should benefit from the use of options valuation techniques, to take into account the true investment opportunities and eliminate the distance between financial theory and strategic planning. In other words, the NPV of projects, obtained through the traditional method, should be supplemented with the value of options that stem from strategic flexibility. In order to value these options in investment projects, the financial options theory can be used, when taking into consideration the following premises.

Samuelson's theorem (Samuelson, 1965) proved that the rate of return on any security follows a random walk, as long as investors have complete information about those cash flows. This means that all the information about the expected future cash flows is already backed into the current stock price. The extension of these concepts for real assets markets would make it possible to consider an investment project as an asset with a value that follows a stochastic process similar to that of a financial asset.

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Copeland and Antikarov (2001) have applied Samuelson's theorem to investment projects, and have concluded that, regardless of the pattern of cash flows that the project is expected to generate in the future, the variations of its present value will also follow a random walk. As a result, all the uncertainty sources of the project can be combined into one (the present value of the project), whose parameters can be estimated using a Monte Carlo simulation.

However, when the variable that generates uncertainty (in our case, the traffic on a highway) itself follows a random walk, we can calculate the value of the derivative assets (the options) starting directly from the process followed by that variable, which is considered the underlying asset determining the value of the options. This procedure notably simplifies the calculation method.

Therefore, an essential assumption in the theoretical model adopted by analogy with the financial options theory is that traffic growth follows a random walk. This is equivalent to assuming that variations of traffic on a highway can be modeled as a stochastic process known as Geometric Brownian Motion (GBM), which can be described in the following way:

$$\theta_{t} - \theta_{t-1} = \mu \cdot \theta_{t-1} \cdot \Delta t + \sigma \cdot \theta_{t-1} \cdot \Delta z \tag{1}$$

Or:

$$\frac{\theta_{t} - \theta_{t-1}}{\theta_{t-1}} = \mu \cdot \Delta t + \sigma \cdot \Delta z \tag{2}$$

Where, in our case:

 $\theta_t$ : traffic volume

μ: traffic growth rate

σ: traffic volatility

 $\Delta z = \xi_t \cdot \sqrt{\Delta t}$ : a Wiener process, where  $\xi_t$  is a normally distributed variable with zero mean and unit standard deviation (Dixit & Pindyck, 1994).

The equation (1) could be expressed as follows:

$$d\theta = \mu \cdot \theta \, dt + \sigma \cdot \theta \, dz \tag{3}$$

Where:

 $\theta$ : traffic volume

μ: traffic growth rate

dt: differential period of time

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σ: traffic volatility

dz: increment of a Wiener process

Starting from equation (3), and applying Itô's lemma, we can find the process followed by the natural logarithm of  $\theta$  (Itô, 1951):

$$d(\ln \theta) = \mu' \cdot dt + \sigma \cdot dz \tag{4}$$

Where  $\ln \theta$  is the natural logarithm of traffic and  $\mu' = \mu - \frac{\sigma^2}{2}$ .

On the right hand of equation (4), the parameter  $\mu'$  is a constant drift term or growth parameter. It means that the logarithm of traffic has a growth of  $\mu'$  per unit of time.

The assumption of a GBM (non-stationarity hypothesis) is frequently made for economic and financial variables. For stock prices, for example, this hypothesis is generally accepted, and it has been used for the development of the theory of option valuation, since the initial works carried out by Black and Scholes (1973) and Merton (1973). A GBM has also been applied for certain real variables. In the field of road traffic, this assumption has been made by Zhao et al. (2004) to analyze the decision-making process in highway development, and by other authors (Rose, 1998; Irwin, 2003, 2007; Huang and Chou, 2006; Wei-hua and Da-Shuang, 2006; Lara Galera, 2006 and Lara Galera and Sánchez Soliño, 2010).

However, the GBM hypothesis is not always evident. Pindyck and Rubinfeld (1998), for example, have analyzed whether commodity prices follow this process. They found that, for very long time series (more than 100 years), detrended prices of crude oil and copper do not follow a random walk, but a mean-reverting process. However, and to the contrary, the hypothesis of a random walk cannot be rejected for the detrended prices of lumber.

Given the relevance of this question for the application of the real options approach, in this paper we perform a test for the hypothesis of a GBM for the evolution of traffic volume on highways. We have used the series available for Spanish highways, which, in most cases, cover a thirty-year period. In the following section a description is given of the methodology used for the analysis of the existence of unit roots in time series in general. We have used the Dickey-Fuller approach, which is the most widely used test for this kind of analysis. We have then applied this methodology for traffic series to Spanish highways and examined the results obtained. Finally, we discuss the possible application of the results.

#### DICKEY-FULLER AND AUGMENTED DICKEY-FULLER TEST

In order to perform the unit root (or non-stationarity) test of time series, we start from an autoregressive model like the following:

$$y_t = \alpha + \rho y_{t-1} + \varepsilon_t \tag{5}$$

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Where:

 $y_t$ : random variable

α : intercept (constant)

 $\rho$ : constant parameter

 $\varepsilon_t$ : white noise

Subtracting the term  $y_{t-1}$  from both sides of equation (5), we obtain:

$$y_t - y_{t-1} = \alpha + (\rho - 1)y_{t-1} + \varepsilon_t$$
 (6)

In order to make the calculation easier, we can call  $\beta = \rho - 1$ , so equation (6) can be written as follows:

$$y_t - y_{t-1} = \alpha + \beta \cdot y_{t-1} + \varepsilon_t \tag{7}$$

Then, we could try to estimate the parameter  $\beta$  by using Ordinary Least Squares (OLS), and calculating the t-statistic to test whether  $\beta$  is significantly different from 0. If we cannot reject the hypothesis that  $\beta=0$ , then we say that the process has a unit root, and cannot reject that the  $y_t$  variable is non-stationary. However, if the true value of  $\rho$  is 1 ( $\beta=0$ ), then the OLS estimator of  $\rho$  is biased towards zero (Pindyck & Rubinfield, 1998). Then the use of OLS could lead us to incorrectly reject the non-stationarity hypothesis.

To solve this problem, Dickey-Fuller (1979, 1981) used a Monte Carlo simulation to calculate the correct critical values for the distribution of the t-statistic when  $\rho = 1$ .

Additionally, other authors have obtained these critical values, such as McKinnon (McKinnon, 1990, 2010).

In Table 1, there are shown the McKinnon critical values for the 1%, 5% and 10% significance level depending of the sample size. Shown values have been used as a reference in the unit root test.

Table 1: Mckinnon's critical values

Mckinnon's critical values								
N (simple size) 1% 5% 10								
5	-5,6046	-3,6949	-2,9828					
10	-4,2971	-3,2127	-2,7477					
15	-3,9591	-3,0810	-2,6810					
20	-3,8085	-3,0207	-2,6504					
25	-3,7241	-2,9862	-2,6326					
30	-3,6701	-2,9640	-2,6210					
35	-3,6329	-2,9484	-2,6129					

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N (simple size)	1%	5%	10%
40	-3,6056	-2,9369	-2,6069
45	-3,5847	-2,9281	-2,6022
50	-3,7896	-2,9212	-2,5986

If the t-statistic obtained in our estimation is greater than the critical value, we cannot reject the null hypothesis,  $\beta = 0$ , and we cannot reject that the traffic time series is non-stationary.

In this kind of test, we assume that there is no serial correlation in the error term  $\epsilon_t$ . However, the process described before could be non-stationary, even when serial correlation exists. So, the same authors (Dickey-Fuller, 1979, 1981), proposed an extended method which is known as Augmented Dickey-Fuller test (ADF). In this test, the model is expanded by adding the lagged dependent variable to the right side of the equation, as follows:

$$\Delta y_t = \alpha + \beta \cdot y_{t-1} + \sum_{j=1}^m \lambda_j \cdot \Delta y_{t-j} + \varepsilon_t$$
(8)

where  $\lambda_j$  represents the m parameters obtained in the regression analysis between the dependent variable  $\Delta y_t$  and the same dependent variable with a lag of j periods. The accurate number of lags is calculated in this paper using both the Akaike Info Criterion (AIC) and the Schwarz Info Criterion (SIC), taking the highest number of lags between them as the optimum.

#### RESULTS

We have analysed the main highways in Spain, both toll highways and free ones. Data have been taken from the concessionaries (in the case of toll highways) and from the traffic maps published by the Spanish Ministry of Public Works (in case of free highways).

We have used for the research the Annual Average Daily Traffic (AADT) in each highway in order to avoid the seasonality problem in traffic volumes.

In order to perform both the DF test and ADF test, we have called:  $y_t = \ln(\theta_t)$  and  $\Delta y_t = \ln(\theta_t/\theta_{t-1})$ , where  $\theta_t$  is the traffic volume in the year t in terms of AADT. With these considerations, equations (7) and (8) take the form:

$$\ln\left(\frac{\theta_{t}}{\theta_{t-1}}\right) = \alpha + \beta \cdot \ln(\theta_{t-1}) + \varepsilon_{t} \tag{9}$$

$$\ln\left(\frac{\theta_{t}}{\theta_{t-1}}\right) = \alpha + \beta \cdot \ln(\theta_{t-1}) + \sum_{j=1}^{m} \lambda_{j} \Delta \ln(\theta_{t-j}) + \varepsilon_{t}$$
 (10)

We have applied a regression analysis, using OLS to obtain the estimation of the parameter  $\beta$  and the t-statistic for that estimation. We have used the program "Eviews 7" in order to perform the analysis.

Out of the twenty seven toll highways existing in Spain, we have analyzed only those ones where traffic time series are long enough (around thirty years).

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The results of the unit root test for toll highways are shown in Table 2:

Table 2: Results of the Unit Root test for toll highways' traffic time series

Highway	Period	Dickey- Fuller test	AIC	SIC	Number of lags according to AIC/SIC criterion	ADF test t- statistic
Barcelona - Tarragona	1974 - 2010	-2,5294	9	1	6 (*)	-1,9198
Bilbao – Zaragoza	1978 – 2010	-0,2129	1	0	1 (AIC)	-0,7713
Burgos – Armiñón	1978 – 2010	-2,8290	1	1	1	-1,4551
León – Campomanes	1978 – 2010	-0,6453	0	0	0	-0,6453
Montmeló – La Junquera	1983 – 2010	-0,4721	1	1	1	-1,5181
Montmeló - Papiol	1978 – 2010	-0,5123	0	0	0	-0.5123
Sevilla – Cádiz	1974 - 2010	0,1321	1	1	1	-0,4094
Tarragona – Valencia	1974 - 2010	-1,4902	2	2	2	-1,1973
Villalba –Adanero	1974 - 2010	-0,4646	1	1	1	-0,3903
Zaragoza - Mediterráneo	1976 - 2010	-1,9539	1	0	1 (AIC)	-1,6393
Valencia - Alicante	1976 - 2010	-2,5949	1	1	1	-1,5220

(\*) We have used the Durbin-Watson (D-W) test in order to check if the number of lags adopted by AIC and SIC are correct (testing the non-autocorrelation of errors hypothesis). In this case, the D-W statistic shows that with nine lags, the non-autocorrelation of errors hypothesis is not conclusive, so a reduction in the number of lags should be done. With six lags the non-autocorrelation of errors hypothesis cannot be rejected, and the t-statistic value is -1,9198, which is higher than the McKinnon critical values.

For toll-free Spanish highways, data have been taken from traffic maps published by the Spanish Ministry of Public Works between 2000 and 2010, starting from the vehicle counting devices (stations) installed along the corridors.

Due to the fact that a lot of stations have changed their category (secondary to primary or primary to permanent), and their location as well, it has been necessary a research work for all the stations along the years.

Once all data were collected, a lot of stations did not have complete traffic time series. Because of that, only those stations with complete traffic time series were considered. The following toll-free highways were analysed:

A-1 Highway.

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- A-2 Highway.
- A-3 Highway.
- A-4 Highway.
- A-5 Highway.
- A-6 Highway.

The results of the unit root test for the A-1 Highway are shown in Table 3. Each station analysed is named after the distance (in kilometres) to the origin of the road (column P.K.):

Table 3: Results of the Unit Root test for A-1 Highway traffic time series

P.K.	Period	Dickey- Fuller test	AIC	SIC	Number of lags according to AIC/SIC criterion	ADF test t- statistic
P.K.32,00	1975 - 2010	-0,8458	2	2	2	-0,4650
P.K.98,40	1960 – 2010	-2,8342	0	0	0	-2,8342
P.K.135,90	1975 – 2010	-1,3413	6	0	6 (AIC)	-1,2230
P.K.199,00	1989 – 2010	-2,1743	0	0	0	-2,1743
P.K.230,86	1988 – 2010	-1,1623	0	0	0	-1,1623

In this highway only in the station located in the P.K.98,40, the result of the analysis is that we can reject the null hypothesis of the existence of a unit root.

In the Figure 1, the A-1 Highway layout is shown:



Figure 1 – A-1 Highway

In order to know why in the station located at the P.K.98,80, we can reject the null hypothesis of the existence of a unit root, the evolution of the AADT is shown in Figure 2:

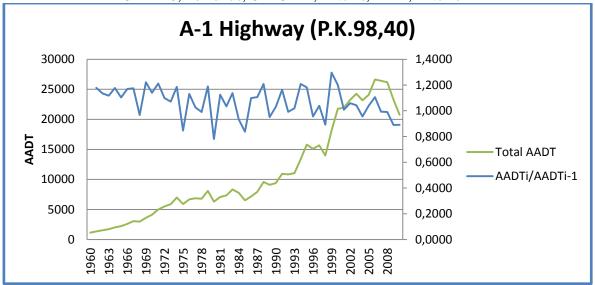


Figure 2 - Traffic evolution in the station located in the P.K.98,40 in the A-1 Highway

As we can see in Figure 2 there is an important break in the trend between the years 1999 and 2002 (see green line). This break could affect the analysis of the existence of a unit root.

The results of the unit root test for the A-2 Highway are shown in Table 4:

Table 4: Results of the Unit Root test for A-2 Highway traffic time series

P.K.	Period	Dickey- Fuller test	AIC	SIC	Number of lags according to AIC/SIC criterion	ADF test t- statistic
P.K.35,90	1975 - 2010	-0,4718	0	0	0	-0,4718
P.K.70,40	1988 – 2010	-2,1929	0	0	0	-2,1929
P.K.116,20	1975 - 2010	-1,3339	2	0	2 (AIC)	-1,0331
P.K.148,10	1988 – 2010	-2,3833	4	4	4	-0,3743
P.K.190,60	1988 – 2010	-2,0808	1	1	1	-2,0812
P.K.229,00	1960 - 2010	-3,3858	1	1	1	-3,8269
P.K.258,15	1988 – 2010	-2,9424	0	0	0	-2,9424
P.K.277,23	1988 – 2010	-2,7362	1	1	1	-3,4300
P.K.317,15	1988 – 2010	-1,6004	4	4	1 (*)	-1,7428
P.K.334,70	1988 – 2010	-2,1275	1	1	1	-1,9092
P.K.373,85	1960 - 2010	-3,5014	3	0	3 (AIC)	-2,9021
P.K.402,00	1988 – 2010	-2,3933	2	0	2 (AIC)	-1,3641
P.K.510,70	1988 – 2010	-0,6744	0	0	0	-0,6744
P.K.563,70	1990 – 2010	-1,7810	0	0	0	-1,7810
P.K.608,30	1988 - 2010	-2,0935	0	0	0	-2,0935

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(\*) In this case, the D-W statistic shows that with four lags, the non-autocorrelation of errors hypothesis is not conclusive, so a reduction in the number of lags should be done. With one lag the non-autocorrelation of errors hypothesis cannot be rejected, and the t-statistic value is -1,7428, which is higher than the McKinnon critical values.

In this highway, in the stations located in the P.K.229,00 and P.K.277,23, the result of the analysis is that we can reject the null hypothesis of unit root existence.

In the Figure 3, the A-2 Highway layout is shown:



Figure 3 – A-2 Highway

The results of the unit root test for the A-3 Highway are shown in Table 5:

Table 5: Results of the Unit Root test for A-3 Highway traffic time series

P.K.	Period	Dickey- Fuller test	AIC	SIC	Number of lags according to AIC/SIC criterion	ADF test t- statistic
P.K.30,10	1988 – 2010	-0,5067	4	4	4	-1,0740
P.K.60,20	1988 – 2010	-1,8222	2	0	2 (AIC)	-2,3823
P.K.86,90	1988 – 2010	-1,4884	0	0	0	-1,4884
P.K.130,00	1988 – 2010	-2,2033	0	0	0	-2,2033
P.K.150,80	1988 – 2010	-2,6525	0	0	0	-2,6525
P.K.261,30	1988 - 2010	-1,4314	0	0	0	-1,4314
P.K.306,30	1988 – 2010	-1,9869	0	0	0	-1,9869
P.K.334,70	1988 – 2010	-0,9614	4	4	2 (*)	-0,7283

<sup>(\*)</sup> In this case, the D-W statistic shows that with four lags, the non-autocorrelation of errors hypothesis is not conclusive, so a reduction in the number of lags should be done. With two lags the non-autocorrelation of errors

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hypothesis cannot be rejected, and the t-statistic value is -0,7283, which is higher than the McKinnon critical values.

In the Figure 4, the A-3 Highway layout is shown:



Figure 4 - A-3 Highway

The results of the unit root test for the A-4 Highway are shown in Table 6:

Table 6: Results of the Unit Root test for A-4 Highway traffic time series

P.K.	Period	Dickey- Fuller test	AIC	SIC	Number of lags according to AIC/SIC criterion	ADF test t-statistic
P.K.19,00	1975 - 2010	-1,0240	1	0	1 (AIC)	-0,5641
P.K.60,00	1960 – 2010	-3,7896	1	0	1 (AIC)	-4,2855
P.K. 95,80	1960 – 2010	-3,2552	0	0	0	-3,2552
P.K. 122,80	1960 – 2010	-2,8010	1	0	1 (AIC)	-3,3635
P.K. 196,90	1988 – 2010	-2,2423	0	0	0	-2,2423
P.K. 266,00	1974 – 2010	-1,3287	3	0	3 (AIC)	-0,7280
P.K. 325,20	1988 – 2010	-2,9034	0	0	0	-2,9034
P.K. 369,80	1988 – 2010	-2,3123	0	0	0	-2,3123
P.K. 435,10	1988 – 2010	-1,7338	0	0	0	-1,7338
P.K. 505,00	1988 – 2010	-1,8382	0	0	0	-1,8352
P.K. 549,80	1960 - 2010	-4,0372	0	0	0	-4,0372
P.K. 588,00	1975 – 2010	-2,4782	0	0	0	-2,4782
P.K. 628,30	1988 – 2010	-4,1026	0	0	0	-4,1026

In this highway, in the stations located in the P.K.60,00; P.K.95,80; P.K.122,80; P.K.549,80 and P.K.628,30 the result of the analysis is that we can reject the null hypothesis of the existence of a unit root.

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In three out of five stations where the result of the analysis is that we can reject the null hypothesis of the existence of a unit root, the traffic evolution shows singular elements that can influence the result, as we can see in Figures 6, 7 and 8.

In the Figure 5, the A-4 Highway layout is shown:



Figure 5 – A-4 Highway

In order to know why in the station located at the P.K.60,00, we can reject the null hypothesis of the existence of a unit root, the evolution of the AADT is shown in Figure 6:

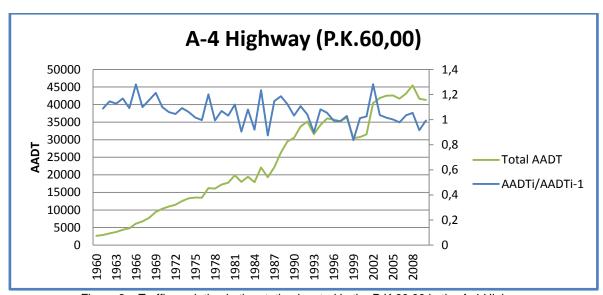


Figure 6 – Traffic evolution in the station located in the P.K.60,00 in the A-4 Highway

As we can see in Figure 6 there is an important break in the trend between the years 2001 and 2002. There is an evolution from 31.544 vehicles in 2001 to 40.428 in 2002. This break could affect the analysis of the existence of a unit root.

In order to know why in the station located at the P.K.122,80, we can reject the null hypothesis of the existence of a unit root, the evolution of the AADT is shown in Figure 7:

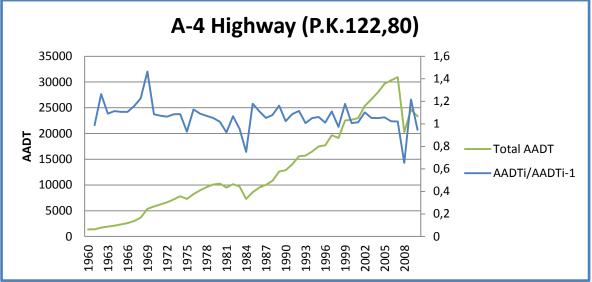


Figure 7 - Traffic evolution in the station located in the P.K.122,80 in the A-4 Highway

As we can see in Figure 7 there is an important traffic loss between the years 2007 and 2008. The traffic evolution goes from 30.960 vehicles in 2007 to 20.273 in 2008 (see green line). This break could affect the analysis of the existence of a unit root.

In order to know why in the station located at the P.K.549,80, we can reject the null hypothesis of the existence of a unit root, the evolution of the AADT is shown in Figure 8:

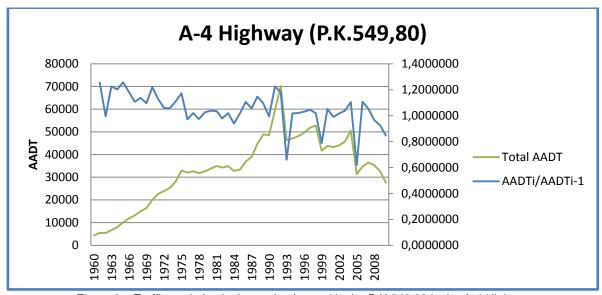


Figure 8 – Traffic evolution in the station located in the P.K.549,80 in the A-4 Highway

As we can see in Figure 8 there are two important traffic losses. The first one, between the years 1992 and 1993 (from 70.190 vehicles to 46.344 vehicles). The second one, between the years 2004 and 2005 (from 50.562 vehicles to 31.347 vehicles). The traffic evolution evolves from 30.960 vehicles in 2007 to 20.273 in 2008 (see green line). This break could affect the analysis of the existence of a unit root.

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The results of the unit root test for the A-5 Highway are shown in Table 7:

Table 7: Results of the Unit Root test for A-5 Highway traffic time series

P.K.	Period	Dickey- Fuller test	AIC	SIC	Number of lags according to AIC/SIC criterion	ADF test t-statistic
P.K.37,00	1988 – 2010	-0,5011	0	0	0	-0,5011
P.K.62,70	1975 – 2010	-0,9683	1	0	1 (AIC)	-0,1879
P.K. 109,60	1960 – 2010	-2,7664	0	0	1 (*)	-2,1814
P.K. 153,00	1988 – 2010	-1,0553	0	0	0	-1,0553
P.K. 230,10	1967 – 2010	-2,4284	0	0	0	-2,4284
P.K. 265,90	1988 – 2010	-0,5559	4	0	4 (AIC)	-0,7380
P.K. 295,00	1988 – 2010	-0,9863	3	0	3 (AIC)	-0,1199
P.K. 329,20	1988 – 2010	-3,7349	2	2	3 (**)	-2,2439
P.K. 377,00	1975 - 2010	-0,7524	0	0	0	-0,7524

- (\*) In this case, the D-W statistic shows that with zero lags, the non-autocorrelation of errors hypothesis is not conclusive, so an increase in the number of lags should be done. With one lag the non-autocorrelation of errors hypothesis cannot be rejected, and the t-statistic value is -2,1814, which is higher than the McKinnon critical values.
- (\*\*) In this case, the D-W statistic shows that with two lags, the non-autocorrelation of errors hypothesis is not conclusive, so an increase in the number of lags should be done. With three lags the non-autocorrelation of errors hypothesis cannot be rejected, and the t-statistic value is -2,2439, which is higher than the McKinnon critical values.

#### In the Figure 9, the A-5 Highway layout is shown:



Figure 9 – A-5 highway

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The results of the unit root test for the A-6 Highway are shown in Table 8:

Table 8: Results of the Unit Root test for A-6 Highway traffic time series

P.K.	Period	Dickey- Fuller test	AIC	SIC	Number of lags according to AIC/SIC criterion	ADF test t- statistic
P.K.21,00	1966 – 2010	-2,6693	0	0	0	-2,6693
P.K.112,20	1988 – 2010	-2,1552	0	0	0	-2,1552
P.K.131,0	1975 – 2010	-1,2491	0	0	0	-1,2491
P.K.173,10	1988 – 2010	-2,3691	1	0	1 (AIC)	-3,1233
P.K.255,30	1970 – 2010	-1,0569	0	0	0	-1,0569

In this highway, in the stations located in the P.K.21,00 and P.K.173,10, the result of the analysis is that we can reject the null hypothesis of unit root existence.

In the Figure 10, the A-6 Highway layout is shown:



Figure 10 – A-6 Highway

In order to know why in the station located at the P.K.21,00, we can reject the null hypothesis of the existence of a unit root, the evolution of the AADT is shown in Figure 11:

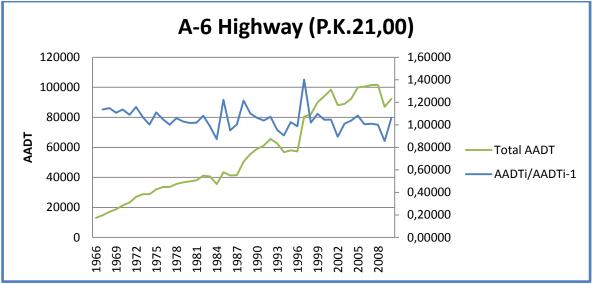


Figure 11 – Traffic evolution in the station located in the P.K.21,00 in the A-6 Highway

There is an important break which can affect to the result of the analysis. Between the years 1996 and 1997 the traffic evolves from 57.312 vehicles to 80.366.

#### CONCLUSION

The main result of our analysis is that we cannot reject the hypothesis that traffic follows a Geometric Brownian Motion (GBM), in all of analysed toll highways and in the majority of free highways (we have analysed 55 stations and only in the 14,55% we can reject the hypothesis that traffic follows a GBM) in Spain. In other words, we cannot reject the non-stationarity hypothesis for the evolution of detrended series of traffic volume in periods of around thirty years. Therefore, the GBM hypothesis can be applied to the valuation of highway concessions, where both the forecast of future traffic and the measure of the risk involved are essential for the appraisal of the business.

On the other hand, we consider that those stations where the result of the analysis is that we can reject the null hypothesis of the existence of a unit root are influenced by anomalous breaks in the evolution of traffic. In most cases, this may be due to problems in the collection of data.

The results obtained in this paper are useful for the valuation of highway concessions. The terms of the contracts in highway concessions often contain certain clauses that allow for a degree of operational flexibility in the management of the business. The valuation of these kinds of clauses in contracts can be carried out using a real options approach, a methodology based on the development of the theory of financial options. The full description of this methodology is beyond the scope of this paper (see Lara Galera and Sánchez Soliño, 2010), but some of the options that usually appear in concession contracts have been quoted: minimum traffic guarantees (traffic floors), maximum traffic limitations (traffic caps),

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extension of the concession, anticipated reversion, granting of public subsidies, public participation loans, etc.

Under this approach, and taking into account the results of this paper, traffic volume on the highway is used as the underlying asset in an option contract. Options that are embedded in the concession agreement are then calculated as a derivative of the traffic volume. This means that traffic is treated as the source of uncertainty that determines the value of the options, and the results of our research allow for the application of this methodology under the assumption that the evolution of traffic volume follows a Geometric Brownian Motion.

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