OPTIMAL LINK TOLLS FOR MULTI-NODE AND MULTI-LINK TRANSPORTATION NETWORKS TAKING INTO ACCOUNT THE WELFARE COST OF FUND PROCUREMENT

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ABSTRACT

This study discusses the optimal link toll, which maximizes social surplus under a user equilibrium condition, with imperfect substitution assumption for route choice in a transportation network with many nodes and links, as well as taking into account the welfare cost of funds procurement. In contrast to previous studies, this study formulates optimal link tolls, taking into account the marginal cost of public funds (MCF), which is the marginal welfare loss of taxpayers due to a marginal tax raise. The formula for optimal tolls on links is derived from the following conditions. One is MCF classified into two, not taking into account funding (MCF equal to -1) and pricing for funding (MCF does not equal -1), respectively. Another is tolls classified into two cases, pricing on all links (full link pricing), and pricing a specific link (partial link pricing). Following the above conditions, this study succeeds in deriving the formula for optimal tolls on a full network with many links and nodes. Furthermore this study indicates two calculation methods: one is to solve analytically or numerically for when the functional form of link flow demand is known. When the functional form is unknown, such as a perfect substitution case, it is necessary to carry out iteration until convergence: with the traffic assignment given the price level and with a change in price level based on the traffic assignment.

Keywords: Optimal tolls, Congestion, MCF, Procurement of funds

1. INTRODUCTION

This study tries to formulate the optimal toll level on links for multi-node and multi-link transportation networks taking into account the welfare cost of funds procurement by maximizing a social welfare defined by the road users' utility level minus the welfare cost of taxpayers for public funds procurement. This study addresses the pricing and the financing of transportation infrastructure; in other words, road pricing and fund procurement. Many studies provide the theory and practice of road pricing for congestion control. Furthermore, fund procurement for roads is discussed in relation with taxation issues. However, little is available on the study of road pricing which takes into account the welfare cost of fund procurement.

The welfare cost of funds procurement is measured as the marginal cost of public funds (MCF) multiplied by the subsidy necessary to cover the shortage of toll revenues for the link construction cost. The marginal cost of tax or toll is defined and measured by the marginal loss of consumers' surplus divided by marginal net tax or toll revenue increase. According to Dahlby (2008), "The MCF is a summary measure of the additional distortion in the allocation of resources that occurs when a government raises additional revenues" (p. 1). The value of MCF for income tax, consumption tax, and fuel tax in Japan is -1.1 to -1.5. The MCF of lump sum tax is -1.0, which means the marginal revenue is equal to consumers' surplus. Conventional marginal cost pricing theory supposes explicitly and implicitly that MCF is -1 because it assumes the toll revenues are distributed as a lump sum rather than decreasing the tax level. On the other hand, optimal tax theory or pricing for funding the construction explicitly take into account the MCF.

This study formulates optimal link toll level to maximize social welfare for multi-node and multi-link transportation networks on the following steps and conditions:

(1) User equilibrium is formulated as user's utility maximization under budget and time constraints. Users' welfare is measured by the indirect utility function.
(2) The social welfare function is the sum of the indirect utility function minus MCF multiplied by the subsidy necessary to cover the shortage of funds for the construction cost.
(3) The optimal link pricing level is obtained by maximizing the social welfare function with respect to prices rather than traffic volume.
(4) MCF classified into two cases; first, not taking into account funding (MCF equal to -1), and second, pricing for funding (MCF does not equal -1).
(5) Tolling classified into two cases; first, pricing all the links (full link pricing), and second, pricing a specific link (partial link pricing).

This study will show, first, when MCF equals -1, a full link optimal toll level implies that the optimal toll level on each link can be levied by observing traffic volume and taking into account how durations change depending on traffic volume of that link only. This coincides with the simplest optimal toll solution of a simple link. However, this fact is already well known in the previous studies even for multi-node and multi-link transportation networks. But previous studies derived it by what they call system optimization for the equivalent optimization problem with respect to traffic volume rather than price in the context of the conventional traffic assignment.

Second, when MCF equals -1, it is shown that the optimal toll level for partial links needs to depart from the full link pricing by the distortions on other links. In this case, information on all links is needed. Many previous studies also showed similar formulas, but all are for two simple parallel links. For multi-node and multi-link transportation networks, previous studies adopting system optimization for the equivalent optimization problem did not succeed in the derivation.

Third, when MCF does not equal -1, a full link optimal toll level implies that the pricing is the sum of the marginal congestion externality modified by MCF plus distortion modification on all the links due to saving the public funds of construction costs taken from tax revenue. The latter term says that the optimal pricing, even for no congestion, is not zero, unlike the marginal cost pricing theory, due to saving the public funds of construction costs taken from ordinary tax revenues. Some previous studies also showed similar formulas, but all are for two simple parallel links. For multi-node and multi-link transportation networks, it seems that previous studies did not succeed in its derivation.

Fourth, when MCF does not equal -1 and while the other link toll remains at the present price level, partial link pricing for a given single link shows that optimal single link toll is not zero, even when there is no congestion. This is the same as the full link pricing for when MCF does not equal -1. Full link pricing has the entire link distortion due to the tax burden effect on its link flow. Partial link pricing modifies the price level on links by a non-optimizing price for when congestion exists on other links. Therefore it may be said that the optimal toll on a link is the marginal congestion externality deviated by the distortion in all other links due to the price level departing from the marginal congestion externality. We believe this is the first success in deriving a toll level formula for a full network with many links and nodes.

The remainder of the paper is as follows. Section 2 reviews previous studies. Section 3 describes the road users' behavior and Section 4 describes the social welfare function used. Following these sections, the formulations and the solution for optimal tolls on links are highlighted in Section 5 and 6, respectively. Finally, Section 7 concludes the paper and discusses future research issues.

2. PREVIOUS STUDIES

It seems that previous studies on developing a formula for the optimal link toll have five aspects of networks, route choice substitution, user equilibrium, tolled links, and MCF, as shown in Table I. The network conditions considered are simple or full networks. The route choices have three types of substitution, perfect substitution, logit type substitution, and imperfect substitution. The formulation of user equilibrium is benefit function or utility function approach. The toll included full link or partial link pricing. MCF equals -1 or does not equal -1.

Table I – Pricing models

		MCF equal to −1						MCF not equal to −1					
		Full link pricing		Partial link pricing		Investment		Full link pricing		Partial link pricing		Investment	
		1. Benefit Function Approach	2. Utility Function Approach	3. Benefit Function Approach	4. Utility Function Approach	5. Benefit Function Approach	6. Utility Function Approach	7. Benefit Function Approach	8. Utility Function Approach	9. Benefit Function Approach	10. Utility Function Approach	11. Benefit Function Approach	12. Utility Function Approach
Simple network	Route choice of perfect substitution	Sheffi(1985) Yang and Huang (1998) Dial (1999a) Dial (1999b)		Yang and Zhang (2003) Yang and Huang (2005)	McDonald (1995) Verhoef, Nijkamp and Rietveld (1996) Verhoef (2002a) Verhoef (2002b) Mun (2005) Takeuchi (2006)		Kidokoro (2006)		Verhoef and Rouwendal (2004)		Verhoef and Rouwendal (2004)		Verhoef and Rouwendal (2004)
	Route choice of Logit type substitution	Sheffi (1985) Akamatsu and Kuwahara (1989) Dial (1999a) Dial (1999b) Ying and Yang (2000) Ying and Miyagi (2000) Yang, Meng and Hau (2004)		Yang and Huang (2005)			Kidokoro (2006)						
	Route choice of imperfect substitution				Levy-Lambert (1968) Marchand (1968) Arnott and Yan (2000) Verhoef (2000) Rouwendal and Verhoef (2004)		Parry and Bento (1999) Kidokoro (2005) Kidokoro (2006) Kidokoro (2010)		De Borger, Mayeres, Proost and Wouters (1996) Mayeres and Proost (1997)		Mayeres and Proost (1997) Morisugi and Kono (2012)		Mayeres and Proost (1997) Kidokoro (2005) Kidokoro (2010) Calthrop, Borger and Proost (2010)
Full network	Route choice of perfect substitution	Oppenheim (1995) Yang and Huang (2005)	Verhoef, Koh and Shepherd (2010) This study		Verhoef, Koh and Shepherd (2010) This study		Verhoef, Koh and Shepherd (2010)		This study		This study		
	Logit type stochastic equation	Oppenheim (1995) Maruyama, Harata and Ohta (2003) Yang and Huang (2005)	This study		This study				This study		Palma and Lindsey (2006) Palma, Lindsey, Proost and Loo (2007) This study		
	Imperfect substitute equation	This study	This study		This study				This study		This study		

First, we pay attention to the first left column 1 of Table I, which shows studies using MCF equals -1, for full link pricing with benefit function approach (Yang and Huang, 2005). The benefit function is defined as the consumers' surplus. The network equilibrium is obtained by maximizing benefit function with respect to route flow with a given price (congestion) level due to small contribution of each user. So far this is the same formulation even for the utility function approach. The difference rises in obtaining an optimal price level. In case of benefit function approach, the full link optimal pricing is obtained by maximizing with respect to route flow with endogenous price instead of by price itself, which is called system optimization. On the contrary, the utility function approach maximizes the indirect utility function (net of the welfare cost of fund procurement) with respect to the price.

The formulation for optimal pricing on entire links for multi-node and multi-link transportation networks by benefit function approach has already been accomplished. Sheffi (1985), Akamatsu and Kuwahara (1989), Oppenheim (1995), Yang and Huang (1998), Dial (1999a, 1999b), Yang, Meng and Hau (2004), Yang and Huang (2005), Ying and Yang (2005), Maruyama, Harata and Ohta (2003) succeeded in a benefit function approach on perfect substitution (Wardrop equilibrium) and logit type substitution (Stochastic equilibrium). This approach is well known as having been originally invented by Beckmann, McGuire and Winston (1956). Yang and Huang (1998) make a theoretical investigation into how the classical principle of marginal-cost pricing would work in a general congested network. They derived the optimal link pricing for a system optimization, which equals the marginal congestion externality. According to Yang and Huang (2005), "a toll that is equal to the difference between the marginal social cost and the marginal private cost is charged on each link, so as to internalize the user externalities and thus achieve a system optimum flow pattern in the network" (p. 47). This is established as congestion pricing theory in general transport networks.

The formulation for optimal pricing on a specific link for multi-node and multi-link transportation networks (see column 3) has not yet been clearly derived, except for very simple networks such as those with single OD parallel link(s). The benefit function approaches are Yang and Zhang (2003) and Yang and Huang (2005). On the contrary, the utility function approach has been conducted by Lévy-Lambert (1968), Marchand (1968), McDonald (1995), Verhoef, Nijkamp and Rietveld (1996), Liu and McDonald (1999), Arnott and Yan (2000), Verhoef (2002a, 2002b), and Rouwendal and Verhoef (2004). After these studies, Ubbels and Verhoef (2008) developed a simple two-link serial roads network model. They reviewed the economic literature on road pricing and network interactions. According to their review, most studies targeted parallel or serial networks, except Verhoef (2002a), who studied generalized networks of under-determined size and shape. In fact, the study derived a general analytical solution of second best optimal toll with elastic origin-destination (OD) demand. Based on the Verhoef (2002a) proposed solution, Verhoef (2002b) focused on practical aspects when applying this general solution in the larger transport network model and his proposed solution was validated. Verhoef, Koh and Shepherd (2010) succeeded in deriving partial link pricing with only perfect substitution on a full network.

The even-numbered columns of Table I show studies with a utility function approach instead of a benefit function approach. Kidokoro (2005, 2006, 2010) and Parry and Bento (1999) succeeded in a utility function approach on a simple network, noting that perfect substitution and logit type substitution are a special form of a utility function (see column 6 of Table I). In particular, Kidokoro (2006) deals with a homogeneous consumer model, which is a quasi-linear utility function. However, Kidokoro (2005, 2006, 2010) does not deal with full networks. Verhoef, Koh, and Shepherd (2010) extended Kidokoro's contribution, as will the present study.

The present study will show the formula of optimal pricing for multi-node and multi-link networks. It successfully derived an imperfect substitution of partial link pricing for a full network. Notice that the full link pricing formula can be derived from both approaches of network equilibrium. However, for expressing partial link pricing the formulation can be derived only by a utility function approach. When MCF does not equal -1, there are no studies on a benefit function approach. But all focus on the utility function approach, as few studies investigate simple parallel link network with both perfect and imperfect substitution. Mayeres and Proost (1997) and Morisugi and Kono (2012) assumed imperfect substitution and Verhoef and Rouwendal (2004) assumed perfect substitution. For a full network, Palma and Lindsey (2006) made a simulation model for partial link pricing with logit type substitution (stochastic equilibrium). Their study takes into account the welfare loss of public funds, calculating efficient road pricing in the Paris region, although an efficient road pricing formula was not indicated. Morisugi and Kono (2012) derived the optimal highway toll level on parallel links, taking into account welfare loss associated with fund procurement and estimated efficient toll levels. But their study could not derive the efficient toll levels on transportation networks with many nodes and links.

For investment issues, Calthrop, De Borger and Proost (2010) developed a general equilibrium model to explore the impact of transport infrastructure investment in distorted economies and in endogenously determined MCF. The present study assumes exogenously determined MCF.

Looking at those previous studies, not all of which derived a formula for optimal pricing on links in a full network, this study tries to formulate the optimal road pricing of a single link and entire network link for multi-node and multi-link transportation networks, respectively. We believe that the present study succeeded in deriving an optimal full and partial link pricing formula for a general entire network with imperfect substitution.

3. ROAD USERS' BEHAVIOR

This section formulates the user equilibrium for three types of utility functions, general imperfect substitution, perfect substitution (Wardrop equilibrium), and logit type substitution (Stochastic equilibrium) by assuming a socio-economic environment, as shown below.

1. The planner may impose the toll fee of each link to road users.
2. Road users implement traffic volume assignment of path flow to maximize their utility under budget and time constraints.
3. Road users recognize the impact of their behavior on traffic congestion as negligible.
4. The link duration is described as a monotonically increasing convex function of link traffic volume.
5. The planner may take into account the MCF for covering the shortage to construction cost.

3.1 Imperfect Substitution

Under the above assumption, to derive optimal pricing for general multi-node and multi-link networks, we assume that the number of homogeneous road users behave according to the utility maximization principle *U* of (1) under the budget and time constraint of (2) and (3), and link route flow relationship of (4).

$$\max_{l,f_k^{rs},x_a} U = z + u\left(.....f_k^{rs}....,l\right) \tag{1}$$

subject to

$$z + \sum_a P_a x_a = wL + y, \quad a \in A, \tag{2}$$

$$l + \sum_a t_a\left(\overline{x_a}\right)x_a + L = T, \quad a \in A, \tag{3}$$

$$x_a = \sum_{rs}\sum_k \delta_{a,k}^{rs} f_k^{rs} \quad a \in A, k \in K, \tag{4}$$

$$f_k^{rs} \geq 0 \quad k \in K, rs \in R. \tag{5}$$

where on budget constraints, z is composite goods with unitary price, P_a is the price of the link *a*, w is the wage rate, L is the labour hours, y is asset income. For time constraints, l is leisure time, t_a is the duration of link *a*, $\overline{x}_a$ is the total traffic volume of link *a*, given equilibrium of traffic flow in the full network while x_a is the users' traffic volume of link *a*, T is the total available time. For link flow relationship, $\delta_{a,k}^{rs}$ is equal to *1* if link *a* is on path *k* and *0* otherwise, and f_k^{rs} is path flow traffic volume of the path *k* between the OD pair *rs*. $\overline{x}_a$ of $t_a\left(\overline{x}_a\right)$ is the total traffic volume, which is given from the viewpoint of individuals, that is, it assumes that they disregard the impact their traffic has on other's traffic. This treatment is described as the externality of road congestion.
From equation (2), with substitute L and x_a, we obtain

$$\begin{aligned} z &= wL + y - \sum_a P_a x_a \\ &= w\left(T - l - \sum_a t_a\left(\overline{x_a}\right)x_a\right) + y - \sum_a P_a x_a \\ &= wT + y - wl - \sum_a \left(P_a + wt_a\left(\overline{x_a}\right)\right)x_a \end{aligned} \tag{6}$$

The Lagrangian for (1), (4) and (5) is

$$\begin{aligned} L &= wT + y - wl - \sum_a \left(P_a + wt_a\left(\overline{x}_a\right)\right)x_a + u\left(.......f_k^{rs}......,l\right) \\ &+ \sum_a \lambda_a (x_a - \sum_{rs}\sum_k \delta_{a,k}^{rs} f_k^{rs}) + \sum_{rs,k} \mu_k^{rs} f_k^{rs} \end{aligned} \tag{7}$$

The first order condition is

$$\frac{\partial L}{\partial l} = -w + \frac{\partial u}{\partial l} = 0 \tag{8}$$

$$\frac{\partial L}{\partial x_a} = -P_a - wt_a\left(\overline{x}_a\right) + \lambda_a = 0 \tag{9}$$

$$\frac{\partial L}{\partial f_k^{rs}} = \frac{\partial u}{\partial f_k^{rs}} + \lambda_a \delta_{a,k}^{rs} + \mu_k^{rs} = 0 \tag{10}$$

$$If\ \ f_k^{rs} > 0,\ \mu_k^{rs} = 0$$
$$If\ \ f_k^{rs} = 0,\ \mu_k^{rs} > 0$$

We assume the imperfect substitution between any route traffic, therefore, (7) has a positive inner solution. Therefore the demand functions on leisure and route traffic are

$$\therefore\quad l = l\left(1, w, \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k=1}^{rs=1}, \cdots, \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k}^{rs}, \cdots, \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k=n}^{rs=m}\right) \quad (11)$$

$$f_k^{rs} = f_k^{rs}\left(1, w, \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k=1}^{rs=1}, \cdots, \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k}^{rs}, \cdots, \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k=n}^{rs=m}\right) \quad (12)$$

Substituting (11) into (4), the traffic link demand function can be derived as

$$X_a = X_a\left(1, w, \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k=1}^{rs=1}, \cdots, \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k}^{rs}, \cdots, \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k=n}^{rs=m}\right) \quad (13)$$

And the indirect utility function is

$$V = wT + y + v\left(1, w, \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k=1}^{rs=1}, \cdots, \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k}^{rs}, \cdots, \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k=n}^{rs=m}\right) \quad (14)$$

Note that $\sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k}^{rs}$ is the cost of path *k* and is the function of only link flow x_a.

Applying the envelope theorem to (14),

$$\begin{aligned}
\frac{\partial V}{\partial P_a} &= \sum_{rs}\sum_k \frac{\partial V}{\partial\left(\sum_{a'}\left(P_{a'} + wt_{a'}\left(\bar{x}_{a'}\right)\right)\delta_{a',k}^{rs}\right)} \frac{\partial\left(\sum_{a'}\left(P_{a'} + wt_{a'}\left(\bar{x}_{a'}\right)\right)\delta_{a',k}^{rs}\right)}{\partial P_a} \\
&= \sum_{rs}\sum_k -f_k^{rs}\bullet\left(\delta_{a',k}^{rs} + w\sum_{a'}\delta_{a',k}^{rs}\frac{\partial t_{a'}\left(\bar{x}_{a'}\right)}{\partial \bar{x}_{a'}}\frac{\partial \bar{x}_{a'}}{\partial P_a}\right) \\
&= -\sum_{rs}\sum_k f_k^{rs}\delta_{a',k}^{rs} - w\sum_{a'}\sum_{rs}\sum_k f_k^{rs}\delta_{a',k}^{rs}\frac{\partial t_{a'}\left(\bar{x}_{a'}\right)}{\partial \bar{x}_{a'}}\frac{\partial \bar{x}_{a'}}{\partial P_a} \\
&= -x_a - w\sum_{a'} x_{a'}\frac{\partial t_{a'}\left(\bar{x}_{a'}\right)}{\partial \bar{x}_{a'}}\frac{\partial \bar{x}_{a'}}{\partial P_a}
\end{aligned} \quad (15)$$

In equilibrium, $\bar{x}_a = x_a$, therefore $\frac{\partial V}{\partial P_a}$ is

$$\frac{\partial V}{\partial P_a} = -x_a - w\sum_{a'} x_{a'}\frac{\partial t_{a'}\left(x_{a'}\right)}{\partial x_{a'}}\frac{\partial x_{a'}}{\partial P_a} \quad (16)$$

Note that the change in users' welfare (= consumers' surplus) of price change is expressed by only traffic links, therefore it does not need the route traffic for calculating the welfare change.

3.2 Perfect Substitution (Wardrop Equilibrium)

For perfect substitution of the route choice, we assume that the number of homogeneous road users behave according to the utility maximization principle *U* of (17) under budget and time constraint, and link route relationship of flow.

$$\max_{l, f_k^{rs}, x_a, d^{rs}} U = wT + y - wl - \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)x_a + u\left(........d^{rs}.......,l\right) \tag{17}$$

subject to

$$x_a = \sum_{rs}\sum_k \delta_{a,k}^{rs} f_k^{rs}, \quad a \in A, k \in K, \tag{18}$$

$$d^{rs} = \sum_k f_k^{rs}, \quad k \in K, rs \in R, \tag{19}$$

$$f_k^{rs} \geq 0, \quad k \in K, rs \in R. \tag{20}$$

where on budget constraints and time constraints are treated in the same manner as the above imperfect substitution. d^{rs} is the total path flow traffic volume between the OD pair *rs* which is the sum of route flow. A variable of sub-utility function is not route flow but distribution trips. This is distinguished from imperfect substitution; this assumption assures that there is perfect substitution.

The Lagrangian is:

$$\begin{aligned} L = {} & wT + y - wl - \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)x_a + u\left(.......d^{rs}......,l\right) \\ & + \sum_a \lambda_a \left(x_a - \sum_{rs}\sum_k \delta_{a,k}^{rs} f_k^{rs} \right) \\ & + \sum_{rs} \eta^{rs} \left(d^{rs} - \sum_k f_k^{rs} \right) \\ & + \sum_{rs}\sum_k \mu_k^{rs} f_k^{rs}. \end{aligned} \tag{21}$$

The first order condition is:

$$\frac{\partial L}{\partial l} = -w + \frac{\partial u}{\partial l} = 0 \tag{22}$$

$$\frac{\partial L}{\partial x_a} = -\left(P_a + wt_a\left(\bar{x}_a\right)\right) + \lambda_a = 0 \tag{23}$$

$$\frac{\partial L}{\partial d^{rs}} = \frac{\partial u}{\partial d^{rs}} + \eta^{rs} = 0 \tag{24}$$

$$\frac{\partial L}{\partial f_k^{rs}} = -\sum_a \lambda_a \delta_{a,k}^{rs} - \eta^{rs} + \mu_k^{rs} = 0 \tag{25}$$

Substituting η^{rs} and λ_a in (25) into (23) and (24), it can be shown as below.

$$\frac{\partial L}{\partial l} = -w + \frac{\partial u}{\partial l} = 0 \tag{26}$$

$$\frac{\partial L}{\partial f_k^{rs}} = -\sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k}^{rs} - \frac{\partial u}{\partial d^{rs}} + \mu_k^{rs} = 0 \tag{27}$$

$$\mu_k^{rs} f_k^{rs} = 0, \ \mu_k^{rs} \geq 0 \tag{28}$$

From the relationship of equation (27) and (28), it can be expressed as equation (29).

$$-\min_k \left(\sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right) \right) \delta_{a,k}^{rs} + \frac{\partial u}{\partial d^{rs}} = 0 \tag{29}$$

Therefore the demand functions on leisure and distribution trip demand functions are

$$\therefore \quad l = l\left(1, w, \cdots, \min_k \left(\sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\right)\delta_{a,k}^{rs}, \cdots\right) \tag{30}$$

$$d^{rs} = d^{rs}\left(1, w, \cdots, \min_k \left(\sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\right)\delta_{a,k}^{rs}, \cdots\right) \tag{31}$$

f_k^{rs} is not solved uniquely, and therefore, nor for x_a. But the latter is determined by the equilibrium condition $\bar{x}_a = x_a$. Therefore the indirect utility function can be expressed as

$$V = wT + y + v\left(1, w, \cdots, \min_k \left(\sum_a \left(P_a + wt_a\left(x_a\right)\right)\right)\delta_{a,k}^{rs}, \cdots\right) \tag{32}$$

which is a special form of the general imperfect substitution type of (14).

3.3 Logit Type Substitution (Stochastic Equilibrium)

For logit type substitution of the route choice, it is well known that the number of homogeneous road users behave according to the utility maximization principle *U* of (33) under the budget and time constraint and link route relationship of flow.

$$\max_{l, f_k^{rs}, x_a} U = wT + y - wl - \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)x_a + u_1(l) - \frac{1}{\theta}\sum_{rs} \bar{d}^{rs} \sum_k \frac{f_k^{rs}}{\bar{d}^{rs}} \ln \frac{f_k^{rs}}{\bar{d}^{rs}} \tag{33}$$

subject to

$$x_a = \sum_{rs}\sum_k \delta_{a,k}^{rs} f_k^{rs}, \quad a \in A, k \in K_0 = (0, K), \tag{34}$$

$$\bar{d}^{rs} = \sum_k f_k^{rs}, \quad k \in K_0 = (0, K), rs \in R, \tag{35}$$

$$f_k^{rs} \geq 0, \quad a \in A, k \in K_0 = (0, K). \tag{36}$$

Where we introduce an artificial route *k = 0* with zero price and zero time cost for each OD pair *rs* and given total OD traffic $\bar{d}^{rs}$ including artificial route traffic f_0 in order to make real OD traffic $\sum_{k \neq 0} f_k^{rs}$ endogenous (Kidokoro, 2006). On budget constraints and time constraints are treated in the same manner as the above imperfect substitution.

It is also well known that solution of logit model as

$$l = l(w) \tag{37}$$

$$f_0^{rs} = \frac{\bar{d}^{rs}}{1 + \sum_{k' \neq 0} \exp[-\theta \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k'}^{rs}]} \tag{38}$$

$$f_k^{rs} = \frac{\bar{d}^{rs} \exp[-\theta \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k}^{rs}]}{1 + \sum_{k'} \exp[-\theta \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k'}^{rs}]} \tag{39}$$

Indirect sub-utility function for each OD is log sum as

$$V^{rs} = -\frac{\bar{d}^{rs}}{\theta} \ln\left[1 + \sum_{k'} \exp[-\theta \sum_a \left(P_a + wt_a\left(\bar{x}_a\right)\right)\delta_{a,k'}^{rs}]\right] \tag{40}$$

The indirect utility function is

$$V = Tw + y + v(w) + \sum_{rs} V^{rs}, \tag{41}$$

which is a special form of the general imperfect substitution type of (14).

4. SOCIAL WELFARE FUNCTION

The social welfare function is defined by equation (42). This is the sum of consumers' surplus, which is the quasi-linear indirect utility function of road users and welfare loss of taxpayers for the subsidy of a construction cost minus toll charge revenue.

$$W = V + MCF\left[\sum_{a'}\left(I_{a'} - P_{a'}x_{a'}\right)\right] \tag{42}$$

where *V* is the indirect utility function of users of (14), and *MCF* is the marginal cost of fund procurement. The pricing issues two aims, funding the construction cost of links and regulatiing by toll charge. The construction cost of the link *a* is I_a, and its fund comes from toll charge revenue of link *a* and from taxes such as fuel tax, income tax, consumption tax, etc. of which MCF is assumed constant because fund procurement is a very small portion of total public expenditure.

The optimal road toll level of link *a*, which maximizes social welfare function *W*, satisfies the equation (43).

$$\begin{aligned}\frac{\partial W}{\partial P_a} &= \frac{\partial V}{\partial P_a} - MCF\left(x_a + \sum_{a'} P_{a'}\frac{\partial x_{a'}}{\partial P_a}\right)\\ &= \left(\frac{\dfrac{\partial V}{\partial P_a}}{x_a + \sum_{a'} P_{a'}\dfrac{\partial x_{a'}}{\partial P_a}} - MCF\right)\left(x_a + \sum_{a'} P_{a'}\frac{\partial x_{a'}}{\partial P_a}\right) = 0\end{aligned} \tag{43}$$

Where the first term of the right hand side of the first equation of (43), which shows the marginal change of consumers' surplus due to the price change. And $\left(x_a + \sum_{a'} P_{a'}\frac{\partial x_{a'}}{\partial P_a}\right)$ is the marginal revenue derived from toll on entire links. Therefore, their ratio of the first term of the second equation is the marginal cost of pricing by definition. Accordingly, (43) says that the marginal cost price is equal to marginal cost of public fund procured at the optimal pricing.

This study considers the case of imperfect substitution case without generality. Substituting Roy's identity (16) into (43),

$$\begin{aligned}\frac{\partial W}{\partial P_a} &= \frac{\partial V}{\partial P_a} - MCF\left(x_a + \sum_{a'} P_{a'}\frac{\partial x_{a'}}{\partial P_a}\right)\\ &= -x_a - w\sum_{a'} x_{a'}\frac{\partial t_{a'}\left(x_{a'}\right)}{\partial x_{a'}}\frac{\partial x_{a'}}{\partial P_a} - MCF\left(x_a + \sum_{a'} P_{a'}\frac{\partial x_{a'}}{\partial P_a}\right)\\ &= -(1+MCF)x_a - MCF\sum_{a'}\left(P_{a'} + \frac{w}{MCF}\frac{\partial t_{a'}\left(x_{a'}\right)}{\partial x_{a'}}x_{a'}\right)\frac{\partial x_{a'}}{\partial P_a} = 0\end{aligned} \tag{44}$$

Where

$$\frac{\partial V}{\partial P_a} = -x_a - w\sum_{a'} x_{a'} \frac{\partial t_{a'}\left(x_{a'}\right)}{\partial x_{a'}} \frac{\partial x_{a'}}{\partial P_a} \tag{16}$$

5. OPTIMAL ROAD PRICING

We shall derive the optimal link pricing level that maximizes the previously mentioned social welfare function *W*. We start with MCF equal to -1. In both cases we derive full link pricing on entire links and partial link pricing on a single link *a* for the general imperfect substitution.

5.1 Optimal Link Tolls with MCF equal to -1

5.1.1 Full link pricing with MCF equal to -1

Full link pricing formulation for all links, which maximize the social welfare function *W*, can be obtained by solving the following formula:

$$dW = \sum_a \frac{\partial W}{\partial P_a} dP_a = 0 \tag{45}$$

Applying the equation (44), (16) and MCF equal to -1, we find

$$\begin{aligned} dW &= \sum_a \left(\frac{\partial V}{\partial P_a} + \left(x_a + \sum_{a'} \left(P_{a'} \frac{\partial x_{a'}}{\partial P_a} \right) \right) \right) dP_a \\ &= \sum_a \sum_{a'} \left(P_{a'} - w \frac{\partial t_{a'}(x_{a'})}{\partial x_{a'}} x_{a'} \right) \frac{\partial x_{a'}}{\partial P_a} dP_a \\ &= \sum_{a'} \left[\left(P_{a'} - w \frac{\partial t_{a'}(x_{a'})}{\partial x_{a'}} x_{a'} \right) \left(\sum_a \frac{\partial x_{a'}}{\partial P_a} dP_a \right) \right] \end{aligned} \tag{46}$$

Using $\sum_a \frac{\partial x_{a'}}{\partial P_a} dP_a = dx_{a'}$, we obtain

$$dW = \sum_{a'} \left(P_{a'} - w \frac{\partial t_{a'}(x_{a'})}{\partial x_{a'}} x_{a'} \right) dx_{a'} = 0\,, \tag{47}$$

Thus it can be shown that toll $P_{a'}$ on any link *a*’ equals

$$P_{a'} = w \frac{\partial t_{a'}(x_{a'})}{\partial x_{a'}} x_{a'} \tag{48}$$

This study assumes that the network consists of many links and many nodes. However, equation (48) implies that optimal road pricing on each link can be levied by observed traffic volume, and shows how the durations change depending on traffic volume on that link only. It coincides with the simplest optimal pricing solution of a simple link. Formula (48) and those above facts are already well known in the previous studies for full networks with many links and nodes, but in the previous studies using MCF equals -1, full link pricing adopted a benefit function approach (e.g., Yang and Huang, 2005). The benefit function is defined as the consumers’ surplus. The network equilibrium is obtained by maximizing benefit function with respect to route flow with a given price (congestion) function due to small contribution of each user. So far this is the same formulation even for the utility function approach. The difference rises in obtaining an optimal price level. In the case of benefit function approach the full link optimal pricing is obtained by maximizing with respect to *link flow* with endogenous price in spite of externality instead of by price itself, which is called system optimization. On the contrary, the utility function approach maximizes the indirect utility function (net of the welfare cost of fund procurement) with respect to the *price*. Benefit function approach can hardly be applied to the partial link pricing due to the core technical point of endogenous price level of the link to be optimized for pricing. But it has a merit in that it directly calculates optimal link flow.

5.1.2 Partial link pricing with MCF equal to -1

In this study, partial link pricing means to optimize pricing on a specific single link and other links that are set with the given price level. Partial link pricing formulation of link *a*, which maximizes the social welfare function *W*, can be obtained by solving (44) with respect to pricing *P* with *MCF* equal to -1:

$$P_a = w\frac{\partial t_a(x_a)}{\partial x_a}x_a - \sum_{a'\neq a} x_{a'}\left(P_{a'} - w\frac{\partial t_{a'}(x_{a'})}{\partial x_{a'}}x_{a'}\right)\frac{\dfrac{\partial x_{a'}}{\partial P_a}}{\dfrac{\partial x_a}{\partial P_a}} \tag{49}$$

The first term of the right hand side of (49) is a marginal congestion externality, which is equal to the full link pricing case. The second term is the distortion of all other link pricing when $P_{a'}$ does not equal the best level. The link *a* pricing needs to depart from the full link pricing by the distortions on other links. In this case, it needs information on all links. Many previous studies also showed formulas similar to the formula (49) but all are for two simple parallel links. So we believe this is the first successful derivation of a formula (49) for a full network with many links and nodes. Finally, note that partial link optimal pricing is expressed by only link traffic, therefore it does not need the route traffic for calculation. Also note that (49) is not perfect closed form because link traffic is a function of P_a. This matter will be discussed later in section 6.

5.2 Optimal Link Tolls with MCF not equal to - 1

5.2.1 Full link pricing with MCF not equal to -1

Full link pricing formulation of entire links for any network which maximizes the social welfare function *W* can be obtained by solving (44).
It is expressed as the following system of equations by supposing $a = 1,\cdots,A$,

$$\begin{aligned}
&\left(\frac{\partial x_1}{\partial P_1}\right)\left(P_1+\frac{w}{MCF}\frac{\partial t_1}{\partial x_1}x_1\right)+\cdots+\left(\frac{\partial x_A}{\partial P_1}\right)\left(P_A+\frac{w}{MCF}\frac{\partial t_A}{\partial x_A}x_A\right)=-\left(1+\frac{1}{MCF}\right)x_1\\
&\left(\frac{\partial x_1}{\partial P_2}\right)\left(P_1+\frac{w}{MCF}\frac{\partial t_1}{\partial x_1}x_1\right)+\cdots+\left(\frac{\partial x_A}{\partial P_2}\right)\left(P_A+\frac{w}{MCF}\frac{\partial t_A}{\partial x_A}x_A\right)=-\left(1+\frac{1}{MCF}\right)x_2\\
&\vdots\\
&\left(\frac{\partial x_1}{\partial P_A}\right)\left(P_1+\frac{w}{MCF}\frac{\partial t_1}{\partial x_1}x_1\right)+\cdots+\left(\frac{\partial x_A}{\partial P_A}\right)\left(P_A+\frac{w}{MCF}\frac{\partial t_A}{\partial x_A}x_A\right)=-\left(1+\frac{1}{MCF}\right)x_A
\end{aligned} \tag{50}$$

By matrix form,

$$\begin{pmatrix}\frac{\partial x_1}{\partial P_1} & \cdots & \frac{\partial x_A}{\partial P_1}\\ \vdots & \ddots & \vdots\\ \frac{\partial x_1}{\partial P_A} & \cdots & \frac{\partial x_A}{\partial P_A}\end{pmatrix}\begin{pmatrix}P_1+\frac{w}{MCF}\frac{\partial t_1}{\partial x_1}x_1\\ \vdots\\ P_A+\frac{w}{MCF}\frac{\partial t_A}{\partial x_A}x_A\end{pmatrix}=-\left(1+\frac{1}{MCF}\right)\begin{pmatrix}x_1\\ \vdots\\ x_A\end{pmatrix} \tag{51}$$

The matrix on the left hand side of (51) is a substitution effect matrix because we assume the quasi-linear utility function shown as (1), therefore there is no income effect; we assume its inverse matrix exists. Then, by using Cramer's formula, we obtain

$$P_{a'}=-\frac{1}{MCF}w\frac{\partial t_{a'}(x_{a'})}{\partial x_{a'}}x_{a'}-\frac{\left(1+\frac{1}{MCF}\right)\begin{vmatrix}\frac{\partial x_1}{\partial P_1}, & \cdots & \frac{\partial x_{a-1}}{\partial P_1}, & x_1, & \frac{\partial x_{a+1}}{\partial P_1}, & \cdots & ,\frac{\partial x_A}{\partial P_1}\\ \vdots & & & \vdots & & & \vdots\\ \frac{\partial x_1}{\partial P_A}, & \cdots & \frac{\partial x_{a-1}}{\partial P_A}, & x_A, & \frac{\partial x_{a+1}}{\partial P_A}, & \cdots & ,\frac{\partial x_A}{\partial P_A}\end{vmatrix}}{\begin{vmatrix}\frac{\partial x_1}{\partial P_1} & \cdots & \frac{\partial x_A}{\partial P_1}\\ \vdots & \ddots & \vdots\\ \frac{\partial x_1}{\partial P_A} & \cdots & \frac{\partial x_A}{\partial P_A}\end{vmatrix}} \tag{52}$$

Note that if MCF equals -1, then the second term of the right hand side vanishes, and the pricing is exactly the marginal congestion externality of the first term of the right hand side, which is identical to (48). When MCF does not equal -1, the first term is the marginal congestion externality modified by MCF. This modification is necessary because the distortion on a congestion derived from a market tax should reflect in the optimal pricing level. The second term of the right hand side of (52) says that the optimal pricing, even for no congestion, is not zero, unlike the marginal cost pricing theory, due to saving the public funds of construction costs taken from tax revenue.

For the sake of completeness, the cases of simple link and two links are shown below, respectively.

When a = 1, (52) becomes

$$P=-\frac{1}{MCF}w\frac{\partial t(x)}{\partial x}x-\left(1+\frac{1}{MCF}\right)\frac{x}{\frac{\partial x}{\partial P}} \tag{53}$$

This is how Morisugi and Kono (2012) succeeded in the formulation and calculation.

When *a* = 1, 2, the denominator is,

$$\begin{vmatrix} \frac{\partial x_1}{\partial P_1} & \frac{\partial x_2}{\partial P_1} \\ \frac{\partial x_1}{\partial P_2} & \frac{\partial x_2}{\partial P_2} \end{vmatrix} = \frac{\partial x_1}{\partial P_1}\frac{\partial x_2}{\partial P_2} - \frac{\partial x_2}{\partial P_1}\frac{\partial x_1}{\partial P_2}$$

The numerator of P_1 is,

$$\begin{vmatrix} x_1 & \frac{\partial x_2}{\partial P_1} \\ x_2 & \frac{\partial x_2}{\partial P_2} \end{vmatrix} = \frac{\partial x_2}{\partial P_2} x_1 - \frac{\partial x_2}{\partial P_1} x_2$$

And the numerator of P_2 is,

$$\begin{vmatrix} \frac{\partial x_1}{\partial P_1} & x_1 \\ \frac{\partial x_1}{\partial P_2} & x_2 \end{vmatrix} = \frac{\partial x_1}{\partial P_1} x_2 - \frac{\partial x_1}{\partial P_2} x_1$$

Therefore, we obtain

$$P_1 = -\frac{1}{MCF} w \frac{\partial t_1(x_1)}{\partial x_1} x_1 - \frac{\left(1+\frac{1}{MCF}\right)\left(\frac{\partial x_2}{\partial P_2} x_1 - \frac{\partial x_2}{\partial P_1} x_2\right)}{\frac{\partial x_1}{\partial P_1}\frac{\partial x_2}{\partial P_2} - \frac{\partial x_2}{\partial P_1}\frac{\partial x_1}{\partial P_2}},$$

$$P_2 = -\frac{1}{MCF} w \frac{\partial t_2(x_2)}{\partial x_2} x_2 - \frac{\left(1+\frac{1}{MCF}\right)\left(\frac{\partial x_1}{\partial P_1} x_2 - \frac{\partial x_1}{\partial P_2} x_1\right)_1}{\frac{\partial x_1}{\partial P_1}\frac{\partial x_2}{\partial P_2} - \frac{\partial x_2}{\partial P_1}\frac{\partial x_1}{\partial P_2}}$$

We believe there are no previous studies that derived the above full link pricing for when MCF does not equal -1, except for Morisugi and Kono (2012) for a simple link.

5.2.2 Partial link pricing with MCF not equal to -1

Partial link pricing for a given single link, while the other link toll remains at the given price level, is obtained from the first order condition of (44) as

$$P_a = -\left(1+\frac{1}{MCF}\right)\frac{x_a}{\frac{\partial x_a}{\partial P_a}} - \frac{1}{MCF}\left(w\frac{\partial t_a(x_a)}{\partial x_a}\right)x_a - \sum_{a'\neq a}\left(P_{a'} + \frac{w}{MCF}\frac{\partial t_{a'}(x_{a'})}{\partial x_{a'}}x_{a'}\right)\frac{\frac{\partial x_{a'}}{\partial P_a}}{\frac{\partial x_a}{\partial P_a}} \quad (54)$$

If MCF equals -1 for (54), we obtain (49). The equation (54) shows that the optimal single link toll is not zero, even when there is no congestion, which is the same as the full link pricing for when MCF does not equal -1. Full link pricing has the entire link distortion due to the tax burden effect on its link flow. Partial link pricing modifies the price level on links by non-optimizing price for when congestion exists on other links. Therefore equation (54) indicates that the optimal toll on a link is the marginal congestion externality deviated by the distortion in all other links due to the price level departing from the marginal congestion externality.
We believe that no previous studies derived the above partial link pricing for when MCF does not equal -1, except for Morisugi and Kono (2012) for parallel links.

6. METHOD FOR SOLUTION

This section will briefly discuss the method for a solution to calculate the optimal link pricing level focusing a partial link pricing optimization of (54) with MCF not equal -1. Note that (54) includes the unknown variable P_a and equilibrium link traffic flows $X_1, \ldots, X_a, \ldots, X_A$ which satisfy (13) given the functional form derived by specification of the utility function. Here we write those simultaneous equations as follows.

$$P_a = -\left(1 + \frac{1}{MCF}\right)\frac{x_a}{\frac{\partial x_a}{\partial P_a}} - \frac{1}{MCF}\left(w\frac{\partial t_a(x_a)}{\partial x_a}\right)x_a - \sum_{a' \neq a}\left(P_{a'} + \frac{w}{MCF}\frac{\partial t_{a'}(x_{a'})}{\partial x_{a'}}x_{a'}\right)\frac{\frac{\partial x_{a'}}{\partial P_a}}{\frac{\partial x_a}{\partial P_a}} \quad (55)$$

for specific link *a*.

$$x_{a'} = x_{a'}\left(1, w, \sum_{a'}\left(P_{a'} + wt_{a'}\left(x_{a'}\right)\right)\delta_{a',k=1}^{rs=1}, \cdots, \sum_{a'}\left(P_{a'} + wt_{a'}\left(x_{a'}\right)\right)\delta_{a',k}^{rs}, \cdots, \sum_{a'}\left(P_{a'} + wt_{a'}\left(x_{a'}\right)\right)\delta_{a',k=n}^{rs=m}\right) \quad (13)$$

for entire link $a' \in A$.

Therefore one of the methods is to analytically solve the above nonlinear simultaneous equations: The other way is to solve it by a numerical calculation, such as the Newton method or GAMS program for when the functional form of link flow demand function is known. If the functional form is unknown, such as a perfect substitution case, it is necessary to carry out the following iteration until convergence:

Step 1: carry out the traffic assignment by using user equilibrium model;

Step 2: substitute the results into the right hand of pricing formula (54) and check whether or not the newly obtained value of price increases the social welfare function;

Step 3: modify the price level based on step 2;

Step 4: insert the new price level to the traffic assignment problem;

Step 5: check for convergence and go back to step 1 if the system has not yet converged.

7. CONCLUSION

This study formulates the optimal link toll level to maximize social welfare for multi-node and multi-link transportation networks on the following conditions:

(1) MCF classified into two cases; first, not taking into account funding (MCF equal to -1), and second, pricing for funding (MCF does not equal -1),

(2) Toll classified into two cases; first, pricing all the links (full link pricing), and second, pricing a specific link (partial link pricing).

When MCF equals -1, full link optimal toll level implies that the optimal road toll level on each link can be levied by observing traffic volume and taking into account how durations change depending on traffic volume of that link only. This coincides with the simplest optimal toll solution of a simple link. But this fact is already well known in the previous studies.

In contrast, the optimal toll level for partial link pricing needs to depart from the full link pricing by the distortions on other links. In this case, information on all links is needed. Many previous studies also showed similar formulas, but all are for two simple parallel links. So we believe this is the first success in deriving a toll level formula for a full network with many links and nodes.

When MCF does not equal -1, which means to take into account funding for construction of links, full link pricing is characterized as follows: The first term is the marginal congestion externality modified by MCF. This modification is necessary because the distortion on a congestion derived from a market tax should be reflected on the optimal pricing. The second term on the right hand side states that the optimal pricing, even when there is no congestion, is not zero, unlike the marginal cost pricing theory, due to saving the public funds of construction costs coming from the general tax revenue.

When MCF does not equal -1, which means to take into account funding for construction of links, partial link pricing on a specific link is characterized as follows: First, optimal single link toll level is not zero, even with no congestion, which is the same as full link pricing. Full link pricing reflects give the entire link distortion due to the tax burden effect on the link flow itself. On the contrary, partial link pricing is a modification of price on that link *a* because the price is not at the optimal level for other links when congestion exists. Therefore partial link pricing indicates that optimal single link toll is the marginal congestion externality deviated by the distortion in all other links due to the price level departing from the marginal congestion externality. We believe that no previous studies derived the above partial link pricing for when MCF does not equal -1, except for Morisugi and Kono (2012) for two parallel links.

Finally, we proposed two analytical and one iterative methods to calculate the optimal pricing. One way is to solve analytically the nonlinear simultaneous equations of price formula and equilibrium link flow with respect to price and link flow: The other way is to solve it by a numerical calculation, such as the Newton method or GAMS program for when the functional form of link flow demand function is known.

If the functional form is unknown, such as a perfect substitution case, it is necessary to carry out the following iteration until convergence:

Step 1: carry out the traffic assignment by using user equilibrium model;

Step 2: substitute the results into the right hand of pricing formula (54) and check whether or not the newly obtained value of price increases the social welfare function;

Step 3: modify the price level based on step 2;

Step 4: insert the new price level to the traffic assignment problem;

Step 5: check for convergence and go back to step 1 if the system has not yet converged.

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