INVESTIGATING LAND-USE AND TRANSPORT INTERACTION WITH AN AGENT-BASED MODEL

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ABSTRACT

We present an agent-based model grounded in the standard urban economics framework, using microeconomic interactions between heterogeneous agents who trade-off between housing costs and transport costs to access to job centers. This model is shown to reproduce the standard urban equilibrium with two income groups. Two job centers at various distances from each other are introduced, and the economic, social and environmental outcomes of these various polycentric spatial structures are presented. Then the model is calibrated on Lyon urban area. The results of the simulation show a good fit to observed density and a reasonably good but perfectible fit of simulated housing rent to observed rent.

Keywords: agent-based model, urban economics, location choice, polycentric city, Lyon

INTRODUCTION

The earth population is now predominantly urban, and urban areas are expanding fast (Seto et al, 2011). Beside social and economic issues, this process raises environmental concerns regarding biodiversity conservation, loss of carbon sinks and energy use. Empirical evidence shows that various urban development patterns significantly influence carbon dioxide emissions (Glaeser and Kahn, 2010). Low density brings about increasing vehicle usage while both low density and increased vehicle usage bring about increasing fuel consumption (Brownstone and Golob, 2009). Compact urban development would be the natural answer to these issues but the debate regarding welfare, distributive and environmental aspects is fierce between opponents and promoters of compact cities (see e.g. Gordon and Richardson, 1997; Ewing, 1997). The issue of spatial and social structure and operation of cities has never been so acute, and there is an obvious need to better understand city spatial development (Anas et al, 1998).

A good starting point is the well-known urban economics framework: the Alonso-Muth-Mills (AMM) model is derived from Alonso (1964) monocentric model of land market, Muth's introduction of housing industry (1969) and Mills (1967, 1972) model (see Fujita, 1989). This analytical framework has proved its robustness in describing the higher densities, land and housing rents in city centers (Spivey, 2008; Mills, 2000), despite its limitations – among others, the monocentric assumption.

Polycentrism (or multiple centers) is indeed a reality, as shown by empirical evidence (Giuliano and Small, 1991). However, introducing polycentrism in the AMM model proves difficult from the point of view of analytical tractability. Wheaton (2004) also challenges monocentrism, based on empirical evidence on US cities, which shows that employment is almost as dispersed as residences. However, in this work, simplifying hypotheses are needed for analytical tractability, such as an exogenous density (consumption of land per worker is fixed and independent of location). In other approaches, centers (and sometimes subcenters) are given exogenously (Hartwick and Hartwick, 1974; White, 1976, 1988; Sulliva, 1986; Wieand, 1987; Yinger, 1992). In Fujita and Ogawa (1982) no centers are specified a priori and multiple equilibria are shown (monocentric, multicentric or dispersed patterns). Here again, since the model is not analytically tractable, simplifying hypotheses are required (e.g. lot size is fixed). Lucas and Rossi-Hansberg (2002) go further into endogenous polycentrism. A well shared conclusion of these papers is that numerical simulations are needed.

Regarding income heterogeneity of residents, Straszheim (1987) points out that with multiple classes of bidders it is difficult to find realistic specifications of distribution functions (of income) which yield tractable results and, again, this requires numerical solutions. Fujita (1989) describes a principle of numerical resolution when the population is divided into several income groups.

Such difficulties in extending the AMM model while retaining analytical solutions, have brought us around another modeling tool, i.e. agent-based models. Agent-based models in urban economics are still in the infancy (putting aside the extensive literature on the Schelling model). Caruso et al (2007) integrating urban economics with cellular automata in order to simulate peri-urbanization, may be considered as a first example.

This tool has an interesting flexibility and seems adequate to model the behavior of interacting economic agents, especially in a spatial context such as the one provided by the urban space of the AMM model. However, before exploiting the full range of possibilities offered by agent-based modeling, and in particular, before the study of dynamic phenomena, we want to show its relevance to model urban equilibria, within the AMM framework. This is the first aim of this paper. The agent-based simulation framework is used to define microeconomic interactions between agents (households) and shown to reproduce the results of the AMM model. The simulated model is dynamic: starting from a random initial state, interactions between agents on an urban housing market lead progressively the whole system to an equilibrium state. Illustrations of the time evolution of the urban system are presented. However, the present work focuses rather on the equilibria of the models.

As the results of the agent-based model are validated by comparison with the analytical model, the model can be made more complex by adding various ingredients. This is done here in keeping with a parsimony principle. The first ingredient is heterogeneity through income groups. The second ingredient is multiple centers, shown here with two job centers at various distances from each other. Then we introduce two-worker households whose partners may work in different job centers. Economic, social and environmental outcomes of these various polycentric spatial structures are studied, and this is the second aim of this work. The economic outcome of the introduction of two centers is shown to be positive, as agents' utility increases when the distance between centers increases. However, pollution linked to commuting distances decreases first when centers are taken away from each other but then increases again. At the same time, the decreasing competition for land results in increasing housing surfaces and thus city size. Moreover, the existence and uniqueness of equilibria in these polycentric models are discussed and various arguments are elaborated upon to support these features.

The remainder of the paper is structured as follows. Section 1 describes the agent-based implementation with the microeconomic behavior of agents. Section 2 compares the simulations results with the analytical ones of the AMM model and illustrates the dynamic feature of the model. Section 3 presents the polycentric urban forms and their economic, social and environmental outcomes. Section 4 presents the calibration of the model on Lyon urban area. Research perspectives of this work are discussed in section 5.

1 DESCRIPTION OF THE FRAMEWORK

1.1 Urban economics model

The AMM model was developed to study the location choices of economic agents in an urban space, with agents competing for housing (identified with land in the simplest version of the model). Agents have a transport cost to commute for work. Their workplace is located in a central business district (CBD), which is represented by a point in the urban space. Agents usually represent single workers, but they can also be used to describe households, which can be made more complex in further versions of the model. Housing is rented by absentee landowners who rent to the highest bidder, which introduces a competition for housing between agents. They also compete with an agricultural use of land, which is represented by an agricultural rent R_a.

Agents have a utility function describing their welfare, which is here a Cobb-Douglas function $U=z^{\alpha}s^{\beta}$, where z is a composite good representing all consumer goods except housing and transport (whose price is the same everywhere in the city), s is the surface of housing, α and β are parameters describing agents' preferences for composite good and housing surface, with $\alpha+\beta=1$. Agents also have a budget constraint $Y=z+tx+ps$, where Y is their income, t the transport cost per unit distance, x the distance of their housing location to the CBD and p the price of a unit surface of land at location x. See Fujita (1989) for a detailed presentation of a more general model which includes this one.

The analytical model reproduced in this work with agent-based simulations is a closed city model, where the population size N is chosen exogenously and remains constant during a simulation.

1.2 Agent-based implementation

Let us describe in this section the agent-based implementation of the standard monocentric AMM model. In the agent-based system, the simulation space is a two-dimensional grid where each cell can be inhabited by one or several agents, or used for agriculture. These cells have a fixed land surface s_{tot}. The unit of distance is taken as the side length of a cell. The CBD is a point at the center of the space.

At the initialization, a population of N agents is created. These agents are placed at random locations. The initial land price in each cell p_0 is equal to the agricultural rent R_a. At a given location x, agents occupy a quantity of land which is the optimal consumption of land conditional on price p: $s=\beta\frac{Y-tx}{p}$. This determines the quantity of composite good they consume, and their utility.

1.2.1 Dynamics of moves

The main feature of the model consists in agent-based dynamics of moves and bids in the urban space. The rules defining agents' moves are suggested by the competition for land in the analytical model. They can be related to general ideas of the literature on price adjustment processes, for instance Smale (1976). However, the framework here is more complex, which leads us to use numerical simulations.

Agents move with no cost, as in the AMM model. Let us describe an iteration n of the simulation, changing the variables from their value at step n to their value at step $n+1$. An agent which will be candidate to a move and a cell are chosen randomly. The price of this cell, located at a distance x of the center, is p_n at step n. The optimal housing surface that the agent can choose in the candidate cell is $s_n=\beta\frac{Y-tx}{p_n}$, which allows us to compute her composite good consumption and the utility that she would get thanks to the move.

If the agent candidate can have a utility gain $\Delta U>0$ by moving into the candidate cell, then she bids for renting at the price $p_{n+1}=p_n(1+\varepsilon\frac{s_{occ}}{s_{tot}}\frac{\Delta U}{U})$, where ε is a parameter that we introduce to control the magnitude of this bid. Since landowners rent to the highest bidder the price of the candidate cell is raised. The higher parameter ε is, the faster cell prices evolve. s_{occ} is the surface of land occupied by other agents in the cell and s_{tot} the total land surface of the cell. The factor $\frac{s_{occ}}{s_{tot}}$ makes the bid higher if the cell is more occupied, that is to say, more attractive. Because of this factor, the first agent to move in an empty cell does not raise the price.

The price is a price per unit surface, linked to a cell. When an agent bids higher, the price is changed for all agents in the target cell. Their consumption of land is also changed according to $s=\beta\frac{Y-tx}{p_{n+1}}$ and their utility is computed again. This feature of the model defines a competition for land between agents and a market price, as in the standard analytical model[1].

1.2.2 Surface constraint, time decrease of price

We described how prices increase in the model. Due to the stochastic choice of agents and cells, prices can rise above their equilibrium level at some locations, making some cells unattractive. Indeed, the price of a cell where several agents move in successively can increase so much that it reaches a value which makes the cell unattractive. In this case, agents living there will progressively leave the cell for more attractive locations.

Therefore we choose to decrease exponentially the price of cells where there is free place to accommodate one or more agents. This is done according to $p_{n+1}=p_n-(p_n-R_a\times 0.9)/T_p.s_{av}/s_{tot}$, where T_p is a parameter determining the speed of decrease of prices, $s_{av}=s_{tot}-s_{occ}$ and s_{tot} are the available (non occupied) surface and the total

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[1] Specific situations arise which do not appear in an equilibrium (static) model. For instance, an agent may want to move into a candidate cell that is already full, proposing a higher bid on the price of housing there. Then we make the following choice: the price of housing is raised for all agents living in the cell to the level of this new bid, but the agent candidate does not move in. Then agents' surfaces of housing and utilities are computed again. As the price is raised, housing surfaces are decreased and there is a chance that enough space is freed for the candidate agent to move in, in which case she does. Else, she has to wait until she is proposed another move.

surface of the cell respectively[2]. If no agent moves in, the price decreases according to this formula until it reaches the agricultural rent, which occurs after a finite time because of the form used. Then the decrease stops, and the cell is used for agriculture. The factor $\frac{s_{av}}{s_{tot}}$ makes this time decrease quicker as the cell is emptier and thus less attractive.

1.2.3 Parameters

The different parameters of the model are listed in Table 1. Most parameters belong to the AMM model itself: α, β, Y, t, N, R_a, s_{tot}. Their values are chosen arbitrarily, as the model is not calibrated on real data for the moment. However, it can be noted that a higher population N could have improved the agreement between the analytical and the agent-based model, but it would have slowed down the simulations. Parameters ε and T_p are specific to the agent-based model. Their values have been chosen such that the competition between agents on the housing market is efficient and the system reaches the equilibrium in the whole city.

The present study focuses on the equilibrium of the agent-based model, which is shown to correspond to the equilibrium of the analytical AMM model, so that the agent-based dynamics is only presented in this work as a resolution method. Comparison with the dynamics of real urban housing markets is beyond the scope of this paper. At first sight, these parameters ε and T_p do not seem to have an immediate correspondence with relevant or measurable variables explaining the dynamics of urban housing markets.

Parameters	Description	Default value
α, β	Preferences for composite good and housing	0.75; 0.25
Y_p,Y_r	Incomes of poor and rich agents	300, 450
t	Transport cost (unit distance)	10
N	Total population size	10000
R_a	Agricultural rent	10
s_{tot}	Surface of a cell	30
□	Bidding parameter	0.5
T_p	Time decrease of the price of non-full cells	30

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[2] With two income groups, it is difficult to determine whether a cell is full or not: we choose to let the price decrease if the smallest optimal surface of housing s_{min} of agents in this cell is smaller than the available surface of the cell s_{av}.

Table 1: Parameters of the simulations

1.3 Socioeconomic outcomes

To study the urban social structure and the socioeconomic outcomes of the various models developed here, we focus on some variables of the model, which characterize these outcomes. Our benchmark is a reference simulation of a monocentric city.

Three kinds of outcomes are studied. The utility of individuals is associated to their welfare and gives an economic outcome of the models. The cumulated distances of agents' commuting to work, associated with housing surfaces, give their environmental outcome, which could be conveyed for instance in terms of greenhouse gases emissions associated to transport, land use and housing (heating and cooling). The evolution of social inequalities can be given by the difference in the utility of individuals belonging to different income groups. The agent-based framework gives an easy access to any other individual or global variable of the model, such as land rents for instance.

2 COMPARISON WITH THE ANALYTICAL MODEL AND TEMPORAL EVOLUTION

2.1 Results with two income groups: model 1

The simulations allow us to reach the equilibrium of the AMM model (or more precisely the Muth-Mills equilibrium, see Brueckner, 1987), as can be seen on Figure 1. This equilibrium corresponds to a configuration where no agent can raise her utility by moving, and therefore no agent has an incentive to do so. In each income group, individuals have an identical utility across the city. With two income groups, the utility of "rich" agents is still higher than that of "poor" agents, because they do not have the same exogenous parameters (they have different incomes, see Table 1). A gap can be observed on the density and housing surface curves, because of this discrete difference in income. As in the analytical equilibrium, rich agents are located at the periphery of the city, where they pay lower land prices and have higher housing surfaces, but also with higher transport costs. This is the standard result of the AMM model and it reproduces the pattern observed in most North-American cities (Fujita, 1989).

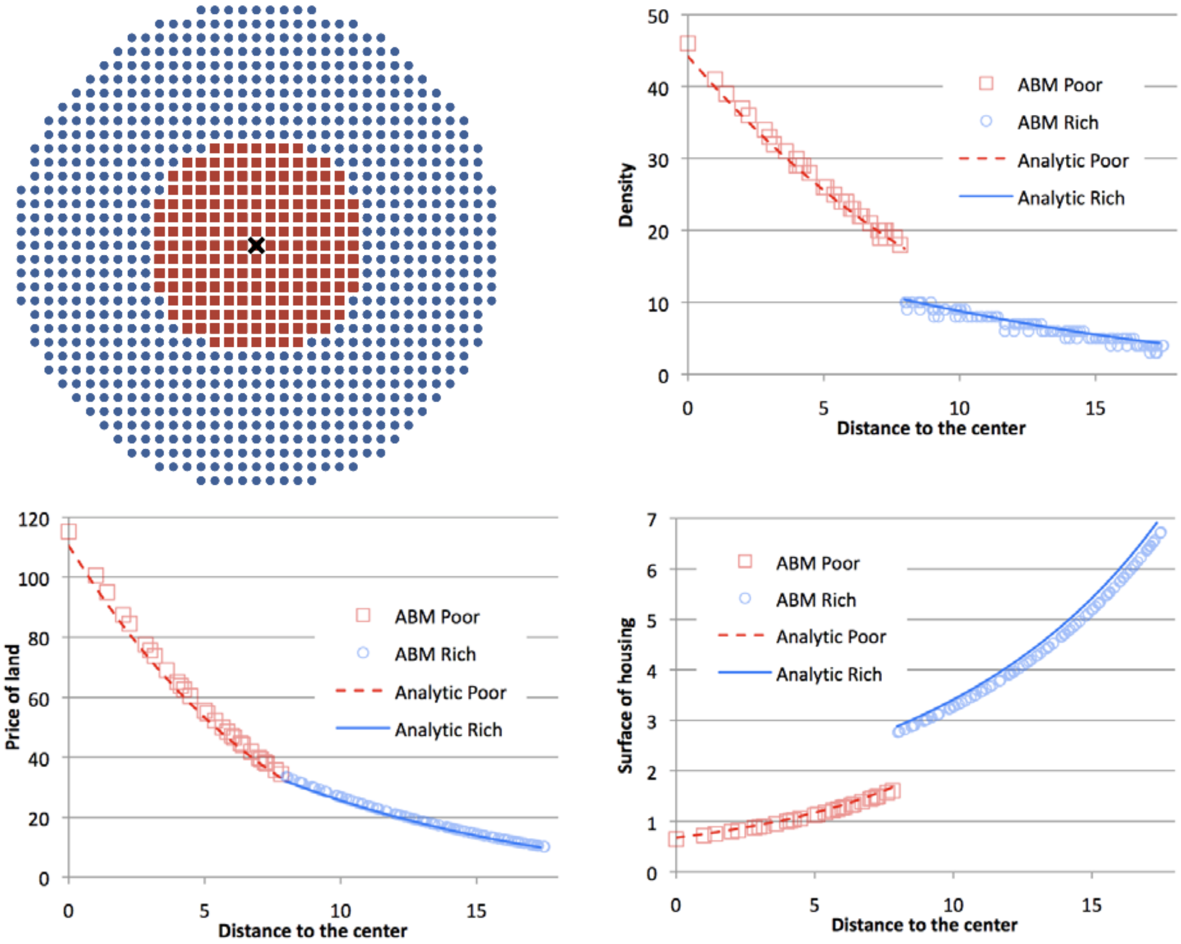

Figure 1: Top left panel, shape of the city. Poor agents are represented by squares, rich agents by circles. Other panels: comparison of the results of the agent-based model and the analytical results. Density, land rents and housing surfaces as functions of the distance to the center. The lines represent the analytical results, whereas the symbols indicate the results of the agent-based model.

The equilibrium of the agent-based model is described in more detail in the following sections. Let us first depict rapidly the temporal evolution of the simulation.

2.2 Dynamics

The evolution of the agent-based model shows how a “city” emerges from the interactions between individuals during a simulation. Initially agents are located at a random and all prices are equal to the agricultural rent. Density is quite low as agents are dispersed over the simulation space. Then agents move mainly towards the CBD as shown on Figure 2 and bid higher, so that the rent curve evolves from a flat rent to the equilibrium rent.

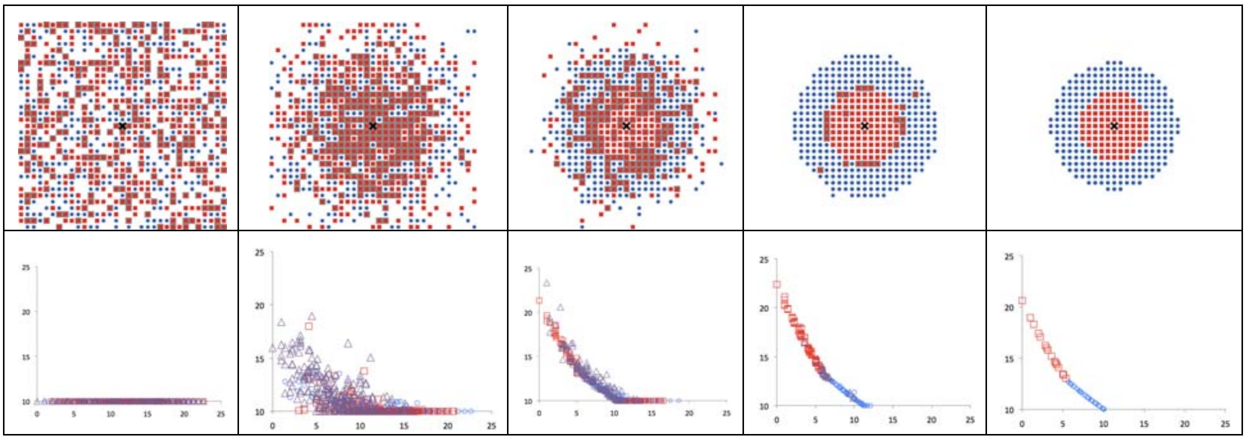

Figure 2: Evolution of the shape of the city (first line) and of the price of land as a function of the distance to the center (second line) during a simulation. On the first line, cells whose background is grey indicate that poor and rich agents live there; these cells are displayed as triangular symbols on the second line. At the equilibrium, the city is completely segregated and there are no more such cells. *n* indicates the mean number of moves per agent since the beginning of the simulation.

At the beginning of the simulation (Figure 2a and Figure 2b), agents gather at the city center without competing much for land, because many cells close to the center are still not full. But when all agents are concentrated around the center (from Figure 2d on), most bids do not result in an agent moving, for few cells have a sufficient available surface to allow an agent to move in with an interesting utility.

The main variable which indicates the proximity to the equilibrium is the homogeneity of the utility of agents. To describe this homogeneity, we use the relative inhomogeneity of the utility defined as $\Delta U_{max}=(U_{max}-U_{min})/U_{max}$. With two income groups, we compute this variable within each income group and keep the highest of both values. During a simulation, ΔU_{max} has a decreasing value. We choose to stop the simulations when the relative variations of utility within income groups are smaller than 10^{-6}, which means that ΔU_{max} has decreased by approximately five orders of magnitude.

The equilibrium of the agent-based model is described in more detail in the following section.

3 POLYCENTRIC CITY

The agent-based mechanism introduced in this work is robust enough for us to study phenomena which are difficult to treat analytically. For instance, several employment centers can be introduced to deal with polycentric cities.

The simplest way to study a polycentric city consists in defining two employment centers, separated by a distance *d*. Agents work at the center which is the closest to their housing, and as a consequence, can change jobs as they move. This last feature seems unrealistic but prevents market frictions, in order to reach the equilibrium more rapidly. The results of such a model are presented in Figure 3, keeping only one income group for simplicity.

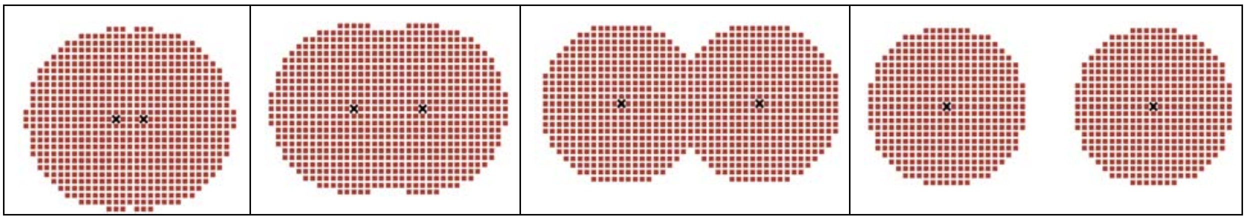

Figure 3: Shape of the city with households (model 2), with *m*=0 (first line), *m*=0.2 (second line) and *m*=1 (third line). The different columns correspond to different values of the distance *d* between centers: *d*=4,10,20,30 from left to right. Agents of the "common" group have a darker color than agents of the "split" group.

Figure 4 shows the evolution of different variables characterizing the outcomes of this polycentric model, such as agents' utility U_{mean}, the total commuting distance of agents D_{tot}, the total rent R_{tot}, the mean price P_{mean} and the total surface of the city S_{tot}, compared with the reference monocentric configuration given by the case *d*=0. The mean density is given by the inverse of the total surface, as the population is fixed.

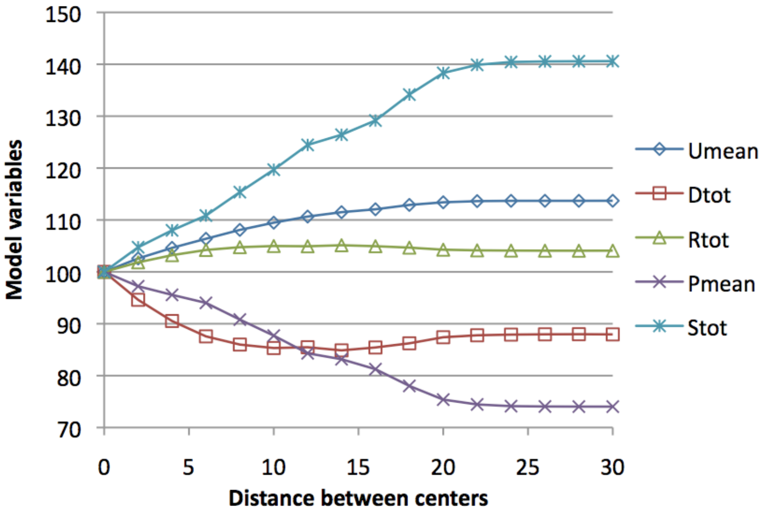

Figure 4: Evolution of the variables characterizing the simplest polycentric model as a function of the distance between centers

Raising the number of centers and the distance between them amounts to raising the surface available at a given commuting distance in the city, thus simultaneously reducing competition for land and transport costs. Agents have greater housing surfaces (S_{tot}), which spreads the city, smaller commuting distances (D_{tot}) and a higher utility (U_{mean}). The total rent R_{tot} increases, which seems surprising but is explained by the fact that housing surfaces are greater, which offsets the decrease in prices (P_{mean}).

The economic outcome of the introduction of two centers in this model is positive, as agents' utility increases when the distance between centers increases. However, the environmental outcome is more difficult to assess. Indeed, as Figure 4 shows, commuting distances D_{tot} decrease first when centers are taken away from each other, which means a reduction of pollution linked to commuting, but then they increase again. At the same time, the decreasing competition for land results in increasing housing surfaces, and thus city size. Bigger housing surfaces result in greater heating (and cooling) needs, which are a major source of energy needs and greenhouse gases emissions.

In this simple model, both halves of the city stop interacting if the centers are far away from each other. This can be seen on Figure 4, where all curves are flat for distances greater than approximately 25 cells.

4. CALIBRATION

The model calibrated here implements the Muth model of housing. The budget constraint is

$$Y = z + tx + p_H q$$

where p_H is the floor price of housing and q the quantity (area) of housing floor.

The housing production function writes $q = As^a k^b$

where k is the input of capital and s the input of land surface to produce the quantity q of housing surface.

The data used for the calibration relates to the agglomeration of Lyon containing 303 communes. The study area contains 1,770,000 habitants with an average density of 798 habitants/km^2 in 2008. The housing price for each commune is collected from a private company[3]. The agricultural land price in Rhône county surrounding Lyon urban area is about 500 euro/hectare[4], i.e. 0.05 euro/m^2.

The agent-based model is calibrated to meet the density and housing price data at its equilibrium urban structure. The size of Lyon agglomeration is about 43 km from the CBD[5]. As shown in Figure 5 the communes with higher density are located within 5 km from the CBD. Most communes have relative low density (less than 1000 habitant/km2) in the suburb or rural area of the agglomeration. For the housing price, it ranges from 1200 to 4300 euro/m2 with an average of 2293 euro/m2. The equivalent housing rent per month is calculated by estimating an annual mortgage payment with an annual rate of 3%, assuming a loan duration of 25 years.

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[3] Agence immobilier EffiCity. http://www.efficity.com, 2010

[4] DDAF Rhone 2009

[5] The CBD is assumed as the geometrical center of the commune located Lyon City Hall

The key determinants of the model are the transport cost, agricultural land rent and household income. For the household income, we assume that each household has one worker commuting between his/her home and the CBD. The household income is assumed fixed as 30000 euro/year (2500 euro/month). For the transport cost, the estimated transport cost is around 220 euro/km/year. The parameters used in the model are shown in Table 2.

Table 2: Parameters of the calibrated model

Parameters	Description	Value
α, β	Preference for composite good and housing	0.81, 0.19 [1]
a, b	Land and capital exponents of the production function	0.20, 0.80 [2]
A	Multiplicative of the production function	0.0115
Y	Income	30000 euro/year/household[3]
T	Transport cost	220 euro/km/year[4]
N	Population and household size	$1.771*10^6$ inhabitants, 2.3 person/household
R_a	Agricultural land rent	0.05 euro/m^2/year
S_{tot}	Surface of a cell	900m*900m
ε	Bidding parameter	10
T_p	Time decrease of the price of non-full cells	20

Remark : 1. Preference for composite good and housing is based on the ratio of the expenditure of “housing, water and energy” (277151 million euros) on actual expenditure of households by current prices (1437165 million euros) in 2010 (INSEE, 2013).

2. There is a great dispersion in the share of land cost in total housing cost. According to CERTU (2011) and CGDD (2011) it varies between 14% and 30%. We take an average of 20%.

3. The average household income of two workers

4. According to Armoogum et al (2010), the average travel distance is 25 km/day. The annual transport cost is estimated by 25km/day*365day*0.3euro/km=2737.5 euros. The annual return trip cost per km to city center is 2737.5/12.5= 219 euros.

Figure 5 shows observed population density (measured on a 1km² grid), the mean of observed density at each kilometer from the city centre and the density simulated with the agent-based model. There is of course a great dispersion in densities at every distance from the city centre, which may be attributed to other factors (discussed below). However, when compared to the mean observed density at each kilometer from the city centre, the simulation results show a good fit.

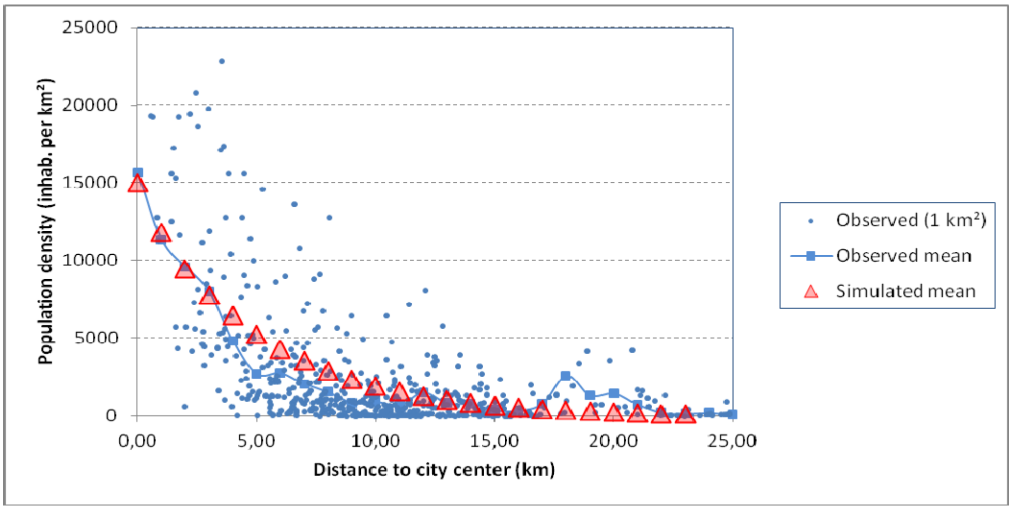

Figure 5: Observed population density and density simulated by the agent-based model

Figure 6 illustrates the observed housing rent per commune, the mean rent according to distance and the housing rent simulated by the agent-based model. The simulation result of the housing price is reasonably well fitted with the observed data, although there is an underestimation of housing rent farther than 15km from the city centre.

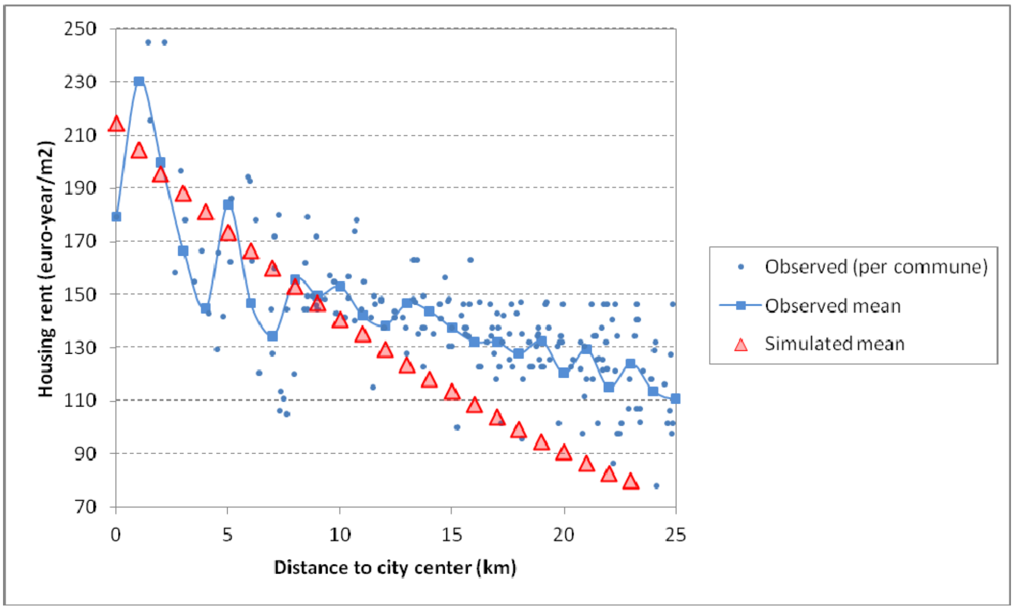

Figure 6: Observed housing rent and rent simulated by the agent-based model

5. DISCUSSION AND PERSPECTIVES

Building on the standard urban economics theory, we run simulations of a microeconomic model with agents interacting in an urban area. The dynamics of the model consist mainly in agents moving and bidding on the urban housing market. This pushes our system into the direction of the equilibrium. This equilibrium corresponds to a discrete version of the analytical equilibrium of the AMM model. A comparison shows the very good agreement of the analytical and the agent-based monocentric models with two income groups.

Then we study the evolution of this equilibrium when the monocentric hypothesis is abandoned to explore polycentric cities. Our results present economic and environmental outcomes of simple polycentric forms within the agent-based model.

The introduction of several centers, when compared to the monocentric city model, has a positive impact on agents' welfare, as transport expenses and competition on the housing market decrease. Commuting distances are reduced, which gives a positive environmental outcome of the polycentric city in this model. However, the increase of housing surfaces may counterbalance this decrease of greenhouse gases emissions. Although the global effect of a reduction of competition for land between agents is clear, its impact on the different variables of this simple urban model and on different income groups is not obvious, as the results show.

Then the model is calibrated on Lyon urban area. The results of the simulation show a good fit to observed density and a reasonably good but perfectible fit of simulated housing rent to observed rent.

This calibration is expected to be improved, using the power of agent-based microsimulation by reintroducing heterogeneity of income in households as in the theoretical model, and also heterogeneity in transport costs by the means of various transport modes and infrastructures.

Overall we believe that this microsimulation platform is a robust basis upon which simulation models can be designed which could be used by city planners to study economic, environmental and social consequences of different urban planning policies and thus help decision-making.

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