A Needs-based Approach to Activity Generation for Travel Demand Analysis

February 27, 2013

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Number of Words: 6067

Abstract

The specification of activity generation models in operational activity-based travel demand models is based on empirical considerations and is weakly founded in a behavioral theory. Based on the theory of needs, this paper presents an analytical random utility model to describe the choice of activity location, duration, and frequency as one that aims to maximize human need satisfaction. Every need is associated with a psychological inventory that reflects the level of satisfaction of the need. When an activity that satisfies a need is conducted, the need is satisfied and the corresponding psychological inventory is replenished by a quantity called the activity production. Over time, this inventory gets consumed and the need builds up. Using a saw-toothed inventory model, an optimization problem that seeks to maximize the average level of need satisfaction or psychological inventory subject to time and budget constraints is formulated under steady-state conditions. The problem is solved in two stages, for discrete (location) and continuous (duration and frequency) decision variables. The properties of the general solution are studied, and then explored for a translog form of the activity production function. Finally, an empirical estimation method that can be applied to single day travel diary data is proposed and validated using Monte-Carlo experiments.

Keywords: Needs, activity-based model, activity generation, psychological inventory, time use, travel demand

Introduction

The theory and practice of the activity-based approach to travel demand modeling are well developed. This approach, particularly in a form known as the day activity schedule approach (Ben-Akiva et al., 1996), is increasingly being adopted by transportation planning organizations especially in the United States. The activity schedule approach first generates a set of activities an individual performs on a day (number of activities performed on tours and stops by purpose), and then models the travel dimensions including destination, mode, and time-of-travel given an activity pattern.

Several modeling developments have been incorporated into these models over the last decade or so, including better representations of time-of-travel choices and household joint trip-making. Yet, in Abou-Zeid and Ben-Akiva (2012), we argue that the specification of the activity pattern model in operational activity-based model systems is weakly founded in a behavioral theory, and combines a number of socio-economic, demographic, lifestyle, and accessibility variables based on empirical considerations. We have proposed two extensions to enhance the specification of the activity pattern model. The first extension maintains the standard activity pattern utility specification but adds information about the utility using well-being measures. By using individuals' self-reported satisfaction levels with their chosen activity patterns as indicators of the utility of these patterns (through measurement equations), it is anticipated that the resulting model will be more efficient than one without well-being measures (see Abou-Zeid (2009), for an example in a mode choice context). The second extension aims at enhancing the behavioral richness of the activity pattern model by explicitly modeling the drivers of activity participation. This extension is based on the premise that individuals pursue different activities to satisfy their needs. The objective of this paper is to develop a conceptual and analytical framework of this second extension and specify an empirical estimation procedure.

Various studies in the literature have discussed the relationship between needs, activities, time use and travel. According to the theory of needs (Maslow, 1943), human activities are motivated by a set of needs. Most studies that explore this relationship between needs and activities are conceptual, rule-based, or generally do not develop the needs-activity relationships into an analytical model (Adler and Ben-Akiva, 1979; Marki et al., 2011; Nijland et al., 2010; Westelius, 1972). Another approach models the relationship between time use and activities using a utility maximizing framework that allows for substitution between activities with a binding time constraint, but without modeling the motivation for activities (Eluru et al., 2010). Arentze et al. (2009) developed an analytical model of needs and activity generation where the utility of an activity is affected by the satisfaction of the need and an activity is performed if its utility exceeds a certain threshold (representing time pressure). The model predicts which activities are performed on a given day, but not their sequence, location, duration, start times, and travel modes. A method to estimate the model using one-day household travel survey data is proposed, which, however, requires knowledge of the last time an activity was conducted before the survey day, either based on a random draw (Arentze et al., 2011) or based on an extended travel survey (Nijland et al., 2012).

In this paper, a general framework of a needs-based approach to activity generation is presented. The framework draws on ideas from inventory theory (as in some other studies on needs) to conceptualize the evolution of the need. This framework allows for the formulation of needs-based utility maximizing models of activity choices, including frequency, sequence, location, duration, expenditure, etc., for the general case of multiple needs and activities. In this paper, an analytical formulation of a needs-based utility maximizing model of activity location, duration, and frequency is developed for a single need, and the activity that satisfies the need. It is formulated as a steady-state optimization model that determines the location, duration, and frequency of activity participation that maximize the level of need satisfaction subject to time and budget constraints. A solution of the model is presented and its properties are studied. An example using a specific functional form of need satisfaction (or "activity production") is discussed. Finally, an estimation procedure that can be applied to single day travel diary data with no knowledge about the last time an activity was conducted is presented and tested on synthetic data.

The remainder of the paper is organized as follows. Section 1 presents the model formulation. Section 2 develops a solution procedure, analyzes its properties, and illustrates a specific functional form of activity production. Section 3 presents an estimator and results of a Monte-Carlo experiment to validate the estimator. Section 4 concludes the paper.

1 Theoretical Model

1.1 Definitions

1. **Needs:** According to the theory of needs (Maslow, 1943), human activities are motivated by a set of different and distinct needs. There is a finite set of needs that motivate all human activities, and these needs coexist. A need may be satisfied by several activities, and conversely, an activity may satisfy several needs. Needs are unobserved or latent; only the activities that satisfy the needs are observed.

2. **Psychological inventory:** We associate a need with a "psychological inventory", denoted as I, which can be interpreted as the level of need satisfaction at a certain point in time (see Figure 1). When the need is low, the psychological inventory is high and vice versa. Over time, the need builds up and so the inventory gets depleted. The inventory is replenished once the individual performs an activity that satisfies the need. In other words, the level of psychological inventory corresponds to the level of satisfaction of the needs. A gain in the psychological inventory of a need may be viewed as being similar to the utility gained by performing activities that satisfy this need.

3. **Activity production:** The quantity of psychological inventory generated by performing an activity is referred to as the activity production and denoted as Q (see Figure 1). It is a non-negative function of the various inputs that are expended to perform the activity, namely, activity duration T_a, activity expenditure C_a, and activity location attractiveness A, where a denotes an activity. The following properties are desired for the activity production function:

   (a) Monotonicity: With a monotonic production function, the extent to which an individual's psychological inventory is replenished by performing an activity is greater when more time or money is spent performing the activity, or when it is performed at a more attractive location (e.g. shopping at a location with larger retail space).

   (b) Concavity: A concave activity production function has the property of decreasing marginal returns with respect to inputs. Consequently, the additional benefit (inventory) gained from utilizing extra resources (time, money, attractiveness) to perform the activity is decreasing. This property captures satiation in activity production.

1.2 Problem Formulation

Given an individual with known socio-economic characteristics and fixed mobility status (e.g. residential location, vehicle ownership), the problem to be addressed is how the individual chooses the location, duration, expenditure, and frequency of activities to be performed such that his/her need satisfaction is maximized over time. While some activities (e.g. work, school, picking up a child from daycare) are rigid and need to be performed at fixed locations with fixed durations, expenditures, and frequencies, other activities are flexible (e.g. shopping, recreation). However, the choice of location, duration, expenditure, and frequency available for performing these flexible activities is constrained by the amount of time and money available after allocating these resources to the rigid activities.

1.3 Assumptions

The following simplifying assumptions are employed in formulating the model.

1. Single need and single activity: The model considers one need and the activity that satisfies this need. The need is satisfied only by this activity, and conversely the activity satisfies only this need.

2. Constant rate of depletion: The level of psychological inventory depletes at a constant rate λ, which may vary across individuals.

3. Steady-state conditions: The model is formulated for steady-state conditions wherein an individual performs the activity at a fixed location i for a fixed duration T_a and spends a fixed amount of money C_a at constant intervals of time. In Figure 1, which illustrates the evolution of the psychological inventory of a need over time, the individual conducts the activity at regular intervals at times T_1, T_2, T_3, etc. with the same level of activity production Q each time the activity is conducted. Clearly, in reality individuals do not conduct activities at regular intervals, at the same location, for the same duration and spend fixed amounts of money. However, travel surveys usually collect data on a random weekday. Therefore, this steady-state model does not capture the short term dynamic activity choices, but instead describes long term stationary patterns to predict the probability of an individual conducting an activity on a random weekday.

4. Minimum cycle time: The activity is performed at most once in a day. Consequently, the cycle time for the activity, defined as the time between successive performances of the activity, is at least one day. It can be seen from Figure 1 that the cycle time is given by $\frac{Q}{\lambda}$. The average frequency is the inverse of the cycle time, i.e. $\frac{\lambda}{Q}$.

5. Minimum and maximum levels of psychological inventory: The individual performs the activity when the level of the psychological inventory drops to a minimum threshold value denoted as I_{min}, which can be interpreted as a safety stock for the need. The maximum level of inventory that the individual can attain by performing an activity is limited to I_{sat}, the satiation limit, beyond which it is not possible for the individual to increase his/her level of inventory. It is assumed that the maximum level of inventory is a characteristic of the individual since it reflects satiation, while the minimum level of inventory is a decision that the individual makes (i.e. when to "restock").

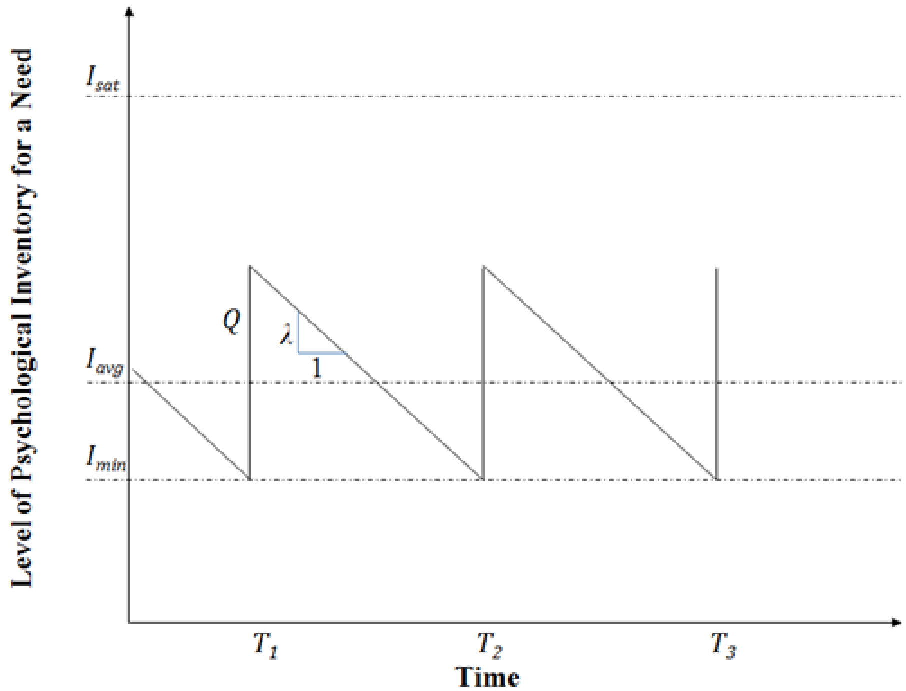

Figure 1: Illustration of the psychological inventory for a need

1.4 Mathematical Formulation

The individual chooses a location i, activity duration T_a, activity expenditure C_a, activity frequency, and a minimum level of inventory to be maintained I_{min} for performing the activity such that the individual's need satisfaction, measured by the average level of psychological inventory over time I_{avg} (see Figure 1), is maximized over time. The maximum level of inventory attained by an individual by conducting an activity is limited to I_{sat}. It is clear that the variation in the inventory over time, and not just the average level over time, also affects the individual's need satisfaction and activity choices (see, for example, evidence from the psychology literature reported in Ariely and Loewenstein, 2000; Kahneman et al., 1993; Redelmeier et al., 2003). However, under the steady state assumption, the proposed formulation is equivalent to a model that maximizes the minimium level of inventory, since the maximum level (I_{sat}) of inventory is fixed (and is reached under the steady state formulation, see Section 2.1.1) and the minimum level (I_{min}) is a decision variable. Therefore, this model accounts for the variation in the inventory over time by fixing the maximum level as a characteristic, selecting the minimum level as a decision variable, and maximizing the average level of inventory over time under steady state conditions. In addition, the individual's choices are also subject to time and monetary budget constraints. Let TT_i and TC_i denote the travel time and travel cost, respectively, associated with performing the activity at location i. The optimization model is formulated as follows for a given individual:

$$\underset{i, T_a, C_a, I_{min}}{Maximize} \quad I_{avg} = \quad I_{min} + \frac{1}{2}Q_i \tag{1}$$

Subject to:

$$Q_i = q(T_a - T_0, C_a, A_i) \tag{2}$$

$$T_a + TT_i \leq t(\frac{Q_i}{\lambda}) \tag{3}$$

$$C_a + TC_i \leq c(\frac{Q_i}{\lambda}) \tag{4}$$

$$I_{min} + Q_i \leq I_{sat} \tag{5}$$

Constraint (2) expresses the activity production Q_i at a location i as a function of the inputs that are invested in conducting the activity. These include the effective activity duration $(T_a - T_0)$, expenditure (C_a) and location attractiveness (A_i). The effective duration that produces psychological inventory is less than the actual amount of time spent conducting the activity by a quantity T_0, referred to as the set-up time. T_0, a psychological characteristic of an individual, accounts for the inefficiency involved with starting up the activity each time it is conducted, and may be viewed as the minimum time an individual must invest in conducting the activity each time before any inventory is generated. Constraints (3) and (4) ensure that the total amount of time and money that the individual spends on performing the activity per cycle are at most equal to the amount of time and money available for this activity, given that the individual has made decisions about all other activities. Note that the amount of time and money available depends on the cycle time $(\frac{Q_i}{\lambda})$, given the fraction of time (t) and income (c) available (per unit time) for this activity. Therefore, if the activity is performed less frequently, the amount of time and money available per cycle is higher (since the cycle time is higher). Conversely, if the activity is performed more frequently (with a lower cycle time), lesser time and money are available to perform the activity per cycle. Constraint (5) ensures that the replenished level of inventory after the activity is performed does not exceed the satiation limit for the individual.

2 Solution Procedure and Properties

2.1 Solution Procedure

For mathematical simplicity, we assume that the budget constraint (4) is not binding. In reality, time is more often a binding constraint that affects the choice of activities and therefore the simplification has little effect on the behavioral realism of the model. Therefore, the decision on activity expenditure and the corresponding budget constraint are ignored hereafter. Since the optimization problem has discrete (location) and continuous (duration, frequency, and minimum level of inventory) decision variables, a solution may be obtained in two stages. First, conditional upon a location i (and thus given TT_i and A_i), the optimal value $\widetilde{T}_{ai}$ of activity duration T_a for each location i that maximizes the objective function ($I_{avg,i} = I_{min} + \frac{1}{2}Q_i$) is computed. Thus, the optimal values of the activity production, $\tilde{Q}_i$, and the average level of inventory at each location, $\widetilde{I}_{avg,i}(= I_{min} + \frac{1}{2}\tilde{Q}_i)$, can be computed. The optimal frequency of performing the activity at this location is given by $\frac{\lambda}{\tilde{Q}_i}$. In the second stage, the location i that has the highest value of $\widetilde{I}_{avg,i}$ is found to be the optimal location.

2.1.1 First Stage Optimization Model

The first stage optimization model at a given location i can be formulated as follows:

$$\max_{T_a, I_{min}} \quad I_{avg,i} = I_{min} + \frac{1}{2}Q_i \tag{6}$$

Subject to:

$$Q_i = q(T_a - T_0, A_i) \tag{7}$$

$$T_a + TT_i \leq t(\frac{Q_i}{\lambda}) \tag{8}$$

$$I_{min} + Q_i \leq I_{sat} \tag{9}$$

The Lagrangian function can be written as follows, with Q_i defined by Equation (7):

$$L_i = I_{min} + \frac{1}{2}Q_i + \mu_1(T_a + TT_i - t(\frac{Q_i}{\lambda})) + \mu_2(I_{min} + Q_i - I_{sat}) \tag{10}$$

This is a continuous optimization problem that can be solved by writing the first order conditions of the Lagrangian, along with the Kuhn-Tucker conditions for the constraints. The inventory constraint (9) becomes an equality on applying the first order condition to the decision variable I_{min}. The first order condition, along with the corresponding Kuhn-Tucker condition, is written as follows:

$$\frac{dL_i}{dI_{min}} = 1 + \mu_2 = 0 \Rightarrow \mu_2 = -1 \tag{11}$$

$$\mu_2(I_{min} + Q_i - I_{sat}) = 0 \ ; \ \mu_2 \leq 0 \Rightarrow I_{min} = I_{sat} - Q_i \tag{12}$$

We may resubstitute the value of I_{min} obtained in Equation (12) to formulate the optimization problem with the objective as shown below with constraints (7) and (8):

$$\begin{matrix} Max \\ T_a \end{matrix} \quad I_{avg,i} = \quad I_{sat} - \frac{1}{2}Q_i \tag{13}$$

This new formulation requires Q_i to be minimized, in order to maximize $I_{avg,i}$. Intuitively, this new model may be interpreted as trying to minimize the depletion from the maximum level of inventory (I_{sat}), thereby maximizing the average level of satisfaction, subject to a time constraint (8). The Lagrangian can now be expressed as:

$$L_i = I_{sat} - \frac{1}{2}Q_i + \mu_1(T_a + TT_i - t(\frac{Q_i}{\lambda})) \tag{14}$$

As noted earlier, the activity production at any location is a function of the duration and location attractiveness ($Q_i = q(T_a - T_0, A_i)$). To find the optimal value of T_a at location i, the first order condition in T_a is written as:

$$\frac{dL_i}{dT_a} = -\frac{dq(T_a - T_0, A_i)}{dT_a}(\frac{1}{2} + \mu_1\frac{t}{\lambda}) + \mu_1 = 0 \tag{15}$$

The Kuhn-Tucker condition for the time constraint is written as:

$$\mu_1(T_a + TT_i - t(\frac{q(T_a - T_0, A_i)}{\lambda})) = 0 \ ; \ \mu_1 \leq 0 \tag{16}$$

Equation (16) may be satisfied when either $\mu_1 = 0$ or the time constraint is an equality. Each of these cases is considered separately, and the optimal solution to the first stage optimization problem is obtained.

Case 1: Constraint is not binding and $\mu_1 = 0$

Substituting $\mu_1 = 0$ in the first order condition, Equation (15), the value of optimal activity duration at location i, denoted by $\widetilde{T}_{ai}$, is computed by solving the following equation:

$$\frac{dq(T_a - T_0, A_i)}{dT_a} \mid_{\widetilde{T}_{ai}} = 0 \tag{17}$$

The value of duration obtained by solving Equation (17) is optimal if the time constraint for this value of $\widetilde{T}_{ai}$ is satisfied, and the second order condition of L_i is satisfied as:

$$\frac{d^2 L_i}{dT_a^2} = -[\frac{d^2 q(T_a - T_0, A_i)}{dT_a^2}(\frac{1}{2})] \mid_{\widetilde{T}_{ai}} < 0 \tag{18}$$

Equation (18) can only be satisfied if $\frac{d^2 q(T_a - T_0, A_i)}{dT_a^2}$ is positive. However, the assumption of concavity of the activity production function with respect to inputs requires the second derivative (total, not partial) to be negative. Therefore, a solution to this case would maximize Q_i, and consequently minimize L_i. However, a maximum of the objective function is obtained when L_i is maximized, and hence, since the solution to this case minimizes the objective function, it is rejected.

Case 2: Constraint is binding and $\mu_1 < 0$

In this case, the time constraint is an equality and is an equation with a single unknown variable. In other wordes, the value of duration ($\widetilde{T}_{ai}$) that maximizes L_i and the objective function $I_{avg,i}$ at location i is found by solving the following equation, where the slack in the time constraint for a duration T_a is referred to as $s(T_a)$:

$$s(\widetilde{T}_{ai}) = \widetilde{T}_{ai} + TT_i - t(\frac{q(\widetilde{T}_{ai} - T_0, A_i)}{\lambda}) = 0 \tag{19}$$

For a general function $q(T_a - T_0, A_i)$, this equation is transcendental and does not have a closed form solution for the duration. However, knowing that the function is non-negative, monotonic, and concave in T_a, the generic shape of $s(T_a)$, given the travel time, attractiveness, time availability and the function q, is as shown in Figure 2, which also illustrates the variation of activity production as a function of activity duration. Note that depending on the values of the parameters in the constraint equation, the $s(T_a)$ curve may always be increasing (i.e. if $\frac{ds(T_a)}{dT_a} > 0 \; \forall \; T_a \geq 0$). However, this case is not illustrated since it always corresponds to infeasibility of the constraint equation (i.e. $s(T_a = 0) > 0, \frac{ds(T_a)}{dT_a} > 0 \; \Rightarrow s(T_a) > 0 \; \forall \; T_a \geq 0$).

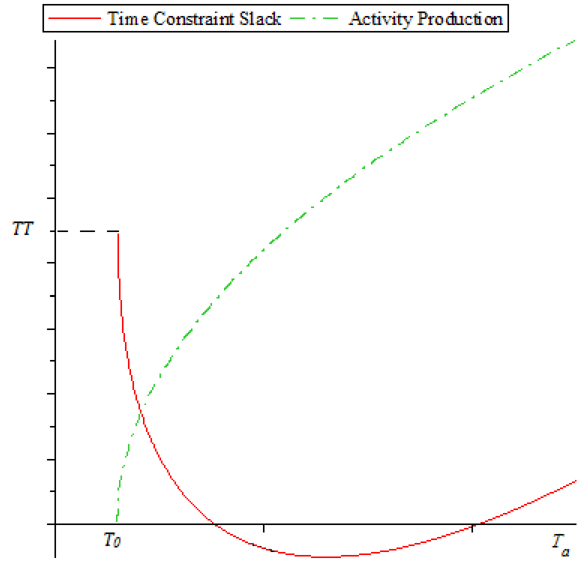

Figure 2: Variation of Activity Production and the Constraint Slack with respect to Activity Duration

Depending on the values of the various parameters in the constraint equation, the actual slack curve may be either shifted upward or downward from the one shown in Figure 2. Consequently, the constraint equation may have two solutions (as shown in the figure, or when the slack curve shifts down), one solution (when the slack curve shifts slightly upward), or no solution (when the slack curve shifts further upward). In each of these cases, the following procedure is used to select the optimal solution:

1. Two Solutions: In this case, the value of the objective function $I_{avg,i}$ is computed at both solutions and the solution that maximizes $I_{avg,i}$ is accepted as the optimal solution. Since maximizing $I_{avg,i}$ corresponds to minimizing Q_i at a location, and since q is a monotonically increasing function of T_a, the solution that is selected is one that has a smaller value. Behaviorally, this indicates that by performing an activity for a shorter duration of time more frequently, an individual maintains a higher average level of need satisfaction since the depletion from the satiation limit is minimized.

2. One Solution: In case the constraint equation is satisfied as an equality at exactly one value of duration, then this value is accepted as the optimal duration.

3. No Solution: When the constraint slack is always positive, the constraint equation does not have a solution. Given limited availability of time (t) and the inventory consumption rate (λ), there are two situations that lead to infeasibility of the time constraint. First, if a location is far off (very high TT_i), the total time spent on conducting the activity (i.e., the sum of activity duration and travel time) is high, and is likely to exceed the time available per cycle. Second, when a location has very

low attractiveness (A_i), the activity production at this location (Q_i) is low, and the cycle time for conducting an activity at this location ($\frac{Q_i}{\lambda}$) is also low. Consequently, the time available to conduct the activity at this location during one cycle ($t(\frac{Q_i}{\lambda})$) is low, and so time available to conduct the activity at this location is likely to be lower than the time required to conduct the activity at this location. Therefore, locations which do not have a real solution to the constraint equation are considered infeasible, and are eliminated from the choice set for the second stage location choice optimization.

Given the nature of the equation, Brouwer's fixed point theorem may be used to obtain a sufficient condition for the existence of a solution over a range of values of T_a, say $T_a \in (x, y)$. If the constraint slack function $s(T_a)$ has different signs at values x and y, then there is at least one solution to this equation over this range. Mathematically, this may be stated as:

$$s(x)s(y) \leq 0 \Rightarrow \exists\ T_a \in (x, y) \text{ such that } (s(T_a) = 0) \tag{20}$$

It must be noted, however, that this is not a necessary condition and its ability to discover a solution is sensitive to the length of the search interval.

At the end of the first stage optimization, the feasibility of every location is determined. Further, for all feasible locations, the optimal solution may be computed as:

1. Duration ($\widetilde{T}_{ai}$) that satisfies the constraint: $\widetilde{T}_{ai} + TT_i - t(\frac{q(\widetilde{T}_{ai} - T_0, A_i)}{\lambda}) = 0$
2. Activity production ($\widetilde{Q}_i$) , knowing the activity duration: $\widetilde{Q}_i = q(\widetilde{T}_{ai} - T_0, A_i)$
3. Frequency ($\widetilde{f}_i$) defined as the inverse of the cycle time: $\widetilde{f}_i = \frac{\lambda}{\widetilde{Q}_i}$
4. Average level of inventory ($\widetilde{I}_{avg,i}$), knowing the activity production: $\widetilde{I}_{avg,i} = I_{sat} - \frac{1}{2}\widetilde{Q}_i$

2.1.2 Second Stage Optimization Model

The second stage optimization model is a discrete optimization problem that finds the optimal location. Given a set of feasible locations, and the optimal duration, frequency, and average level of inventory to perform the activity at each location, the second stage optimization problem selects the solution that maximizes the level of need satisfaction across all these locations. Mathematically, this problem may be formulated as:

$$\underset{i}{Maximize} \quad \widetilde{I}_{avg,i} = I_{sat} - \frac{1}{2}\widetilde{Q}_i \tag{21}$$

At the end of the second stage optimization, the set of activity dimensions that maximize an individual's level of need satisfaction or average level of psychological inventory is given by the optimal location (i), duration ($\widetilde{T}_{ai}$), and frequency ($\widetilde{f}_i$).

2.2 Solution Properties and the Activity Production Function

In this section, the behavioral properties that are supported by, and desired of this model are presented. A translog form for the activity production function is verified to support the desired properties.

2.2.1 Solution Properties

Three behavioral properties of the optimal solution are discussed in this section. While the first property follows from the mathematical derivations presented in Section 2.1, the second and third properties are desirable. The mathematical conditions desired of the optimal solution are presented here.

Property 1: Resource constraints dictate activity choices

The optimal solution is one where the peak of the psychological inventory saw-tooth reaches I_{sat}. This follows from Equation (12) and may be visualized as shown in Figure 3. At optimality, an individual chooses to maintain a high level of need satisfaction by minimizing the depletion from I_{sat} before performing the activity each time. While theoretically this could be achieved by performing the activity continuously in very small quantities, this is not possible due to limited availability of time. Thus, the solution is in line with behavioral expectation that resource (time, money) constraints limit the level of need satisfaction that can be achieved and necessitate an individual to perform activities at discrete intervals of time.

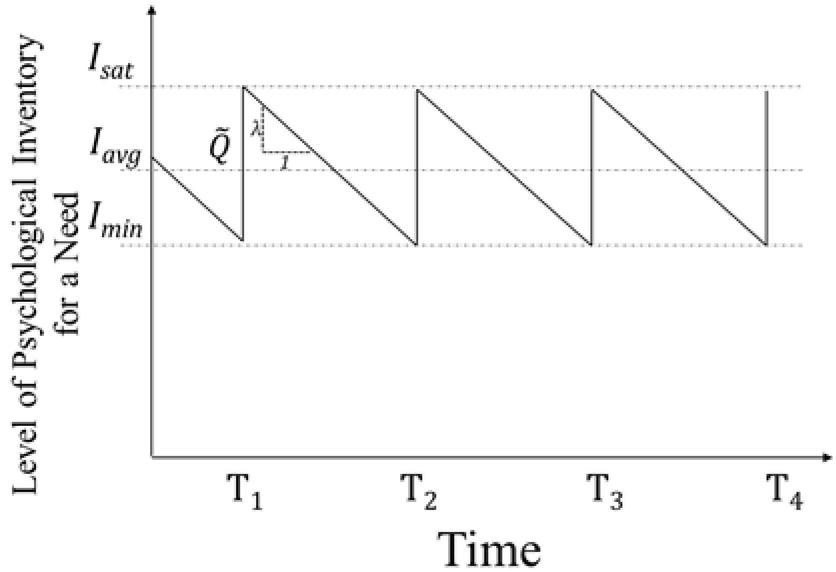

Figure 3: Psychological inventory in the optimal solution

Property 2: Given equal travel times, a more attractive location is preferred

Given two locations with the same travel time (TT), we expect that an individual will choose a location with higher attractiveness. Mathematically, this requires the optimal average level of inventory at the more attractive location to be higher. Based on Equation (13), this requires the more attractive location to have a lower value of optimal activity production. Further, since the activity production function is monotonically increasing in the activity duration, this property is satisfied when the more attractive location has a lower value of optimal activity duration ($\widetilde{T}_a$). Mathematically, this property may be stated as satisfy the property $\frac{d\widetilde{T}_a}{dA} < 0$. Differentiating Equation (19), we obtain the following simplified condition:

$$\frac{d}{dA}(\widetilde{T}_a + TT - t(\frac{\widetilde{Q}}{\lambda})) = 0 \Rightarrow \frac{d\widetilde{T}_a}{dA} = \frac{\frac{t}{\lambda}\frac{d\widetilde{Q}}{dA}}{1 - \frac{t}{\lambda}\frac{d\widetilde{Q}}{d\widetilde{T}_a}} < 0 \tag{22}$$

Property 3: Given equal attractiveness, a closer location is preferred

Given two locations with the same attractiveness (A), we expect that an individual will choose a location with lower travel time. Mathematically, this requires the optimal average level of inventory at the closer location to be higher. Based on Equation (13), this requires the closer location to have a lower value of optimal activity production. Further, since the activity production function is monotonically increasing in the activity duration, this property is satisfied when the closer location has a lower value of optimal activity duration, or conversely when a location that is farther away has a higher value of optimal activity duration ($\widetilde{T}_a$). Mathematically, this property may be stated as satisfy the property $\frac{d\widetilde{T}_a}{dTT} > 0$. Differentiating Equation

(19), we obtain:

$$\frac{d}{dTT}(\widetilde{T}_a + TT - t(\frac{\widetilde{Q}}{\lambda})) = 0 \Rightarrow \frac{d\widetilde{T}_a}{dTT} = \frac{1}{\frac{t}{\lambda}\frac{d\widetilde{Q}}{d\widetilde{T}_a} - 1} > 0 \tag{23}$$

Properties 2 and 3 described above are desired and satisfied by the solution when Equations (22) and (23) are satisfied. However, verifying these constraints requires knowledge of the functional form of the activity production to describe the optimal solutions $\widetilde{T}_a$ and $\widetilde{Q}$. Given the transcendental nature of this solution, a specific functional form is chosen here to empirically verify these properties.

2.2.2 Translog form of the activity production function

To verify that the optimal solution satisfies the properties described in the preceding section, a translog functional form as shown in Equation (24) is chosen for the activity production function.

$$\begin{aligned} Q &= q(T_a - T_0, A) \\ &= exp(q_o + q_1 ln(T_a - T_0) + q_2 ln(A) + q_3 ln(T_a - T_0) ln(A) + q_4(ln(T_a - T_0))^2 + q_5(ln(A))^2) \end{aligned} \tag{24}$$

This functional form allows for flexibility in the relationship between $Q, T_a,$ and A and ensures that the activity production function is non-negative. It may be noted that while it is possible to impose monotonicity and concavity globally to the translog function, this greatly reduces the flexibility of the function (Terrel, 1996). Imposing monotonicity and concavity over the realistic range of values of T_a and A provides a good trade-off between flexibility of the function and the desired properties. The realistic range of these variables (e.g. 1 hour to 14 hours for out of home activity durations and 1 to 100 persons per square mile for retail employment density in the locations for shopping activity) can be scaled without loss of generality.

For this function q to be monotonically increasing, the first derivative of the production function with respect to T_a and A should be positive as shown below:

$$\frac{dQ}{dT_a} = \frac{dq(T_a - T_0, A)}{dT_a} = \frac{1}{T_a - T_0}(q_1 + q_3 ln(A) + 2q_4 ln(T_a - T_0))Q > 0 \tag{25}$$

$$\frac{dQ}{dA} = \frac{dq(T_a - T_0, A)}{dA} = \frac{1}{A}(q_2 + q_3 ln(T_a - T_0) + 2q_5 ln(A))Q > 0 \tag{26}$$

Similarly, for the concavity condition to hold, the second derivative of the production function with respect to T_a and A should be negative as shown below:

$$\frac{d^2Q}{dT_a^2} = \frac{d^2q(T_a - T_0, A)}{dT_a^2} \tag{27}$$
$$= \frac{1}{(T_a - T_0)^2}(-q_1 - q_3ln(A) - 2q_4ln(T_a - T_0) + 2q_4 + (q_1 + q_3ln(A) + 2q_4ln(T_a - T_0))^2)Q < 0$$
$$\frac{d^2Q}{dA^2} = \frac{d^2q(T_a - T_0, A)}{dA^2} \tag{28}$$
$$= \frac{1}{A^2}(-q_2 - q_3ln(T_a - T_0) - 2q_5ln(A) + 2q_5 + (q_2 + q_3ln(T_a - T_0) + 2q_5ln(A))^2)Q < 0$$

While it is possible to impose restrictions to attain global monotonicity and concavity of these functions, doing so restricts the flexibility of the translog function. Instead, a simple procedure to impose these monotonicity and concavity restrictions over a range of values of T_a and A as described by Terrel (1996) may be applied.

The optimal duration at a location obtained by solving the time constraint (19) using a translog production function does not have a closed functional form and is analytically intractable. Empirical analysis of the optimal solution over a range of values of the parameters of the model verified that Properties 2 and 3 described in Section 2.2.1 are satisfied by the optimal solution.

3 Estimation Method

In this section, a method to estimate the model from standard travel diary data is developed. The empirical model additionally contains stochasticity to account for the various sources of error and heterogeneity, and accounts for the effect of aggregate representation of location alternatives (e.g. use of Traffic Analysis Zones instead of shopping malls). Finally, a maximum likelihood estimator is developed and tested on synthetic data to verify that the estimator can recover the true model parameters from observable data.

3.1 Travel Diary Data

In a typical travel survey, respondents record details about the various trips and activities they conducted on a given day. For the activity of interest (e.g. shopping), the following data are available for an individual n. First, an indicator δ_n is available, which is defined as:

$$\delta_n = \begin{cases} 1 & \text{if activity was performed on observed day} \\ 0 & \text{otherwise} \end{cases} \tag{29}$$

For an individual who performed the activity on the observed day, his/her chosen location i_n and chosen duration $T_{obs,n}$ are also available.

3.2 Introducing Stochasticity and Size Variables

1. Location Choice: The choice of location is subject to optimization errors on the part of the decision maker, and measurement errors in recording the chosen location. For an individual n with an optimal average level of inventory $\widetilde{I}_{avg,in}$ at location i, an error term, ϵ_{in}, with an Extreme Value Type I distribution (i.i.d., with scale parameter μ) is added to the location choice optimization model. The location choice model transforms into a logit model under this assumption. Additionally, since the model aggregates elemental alternatives (e.g. shopping malls) into aggregate alternatives (Traffic Analysis Zones), a size measure (M_i), that reflects the size of location i, is included as a sum of non-negative measures of size (e.g. retail employment, area of TAZ, see Ben-Akiva and Lerman (1985)). The second stage optimization problem for location choice, described in Section 2.1.2, along with stochasticity and size variables is written as:

$$\underset{i}{Max} \quad \widetilde{I}_{avg,in} + ln(M_i) + \epsilon_{in}, \epsilon_{in} \quad \sim \quad \text{Extreme Value Type I } (0, \mu) \tag{30}$$

$$M_i = \sum_{k'} \beta_{k'} x_{ik'n};\ \beta_{k'} \geq 0,\ x_{ik'n} \geq 0,\ \forall i, k', n;\ \beta_{K'} = 1 \tag{31}$$

In Equation (31), k' indexes the set of size variables, $x_{ik'n}$ denotes the value of the k'^{th} size variable of alternative i for individual n, and $\beta_{k'}$ denotes the parameter of the k'^{th} size variable. If K' size variables are included in the specification, only $K' - 1$ parameters are identified (i.e. normalize one parameter, say, $\beta_{K'} = 1$).

2. Duration Choice: While the optimal duration at a location i is given by one that satisfies the time constraint, Equation (19), as an equality, the observed duration may contain measurement errors. A lognormally distributed multiplicative error term whose underlying normal distribution has a mean 0, and variance σ_ν^2 is introduced into the model. Since the error term is always positive, the observed duration is positive. The duration choice model is written as:

$$T_{obs,n} = \widetilde{T}_{a,n} exp(\nu_n)\ ,\ \nu_n \sim N(0, \sigma_\nu^2) \tag{32}$$

3. Heterogeneous Consumption Rate: The rate of consumption of psychological inventory λ is heterogeneous in the population, and assumed to be distributed with a lognormal distribution, whose underlying normal distribution has a mean μ_λ and variance σ_λ^2.

$$\lambda \sim LN(\mu_\lambda, \sigma_\lambda^2) \tag{33}$$

4. Heterogeneous Set-up Time: The set-up time to conduct an activity, which is the minimum activity duration required to generate psychological inventory, is heterogeneous in the population, and assumed to be distributed with a lognormal distribution, whose underlying normal distribution has a mean μ_{T_0} and variance $\sigma_{T_0}^2$.

$$T_0 \sim LN(\mu_{T_0}, \sigma_{T_0}^2) \tag{34}$$

The model can be further enhanced by allowing for heterogeneous availability of time.

3.3 Maximum Likelihood Estimator

The sample of respondents is divided into two groups of people, based on whether or not they performed the activity on the observed day. The likelihood functions for these two groups are developed in this section.

3.3.1 Likelihood Function for Individuals Who Performed the Activity on the Observed Day

For the group of individuals who performed the activity on the observed day, their activity location and duration are known. The joint likelihood for the activity location, duration and frequency for an individual belonging to this group is written as:

$$l(\delta_n = 1, i_n, T_{obs,n}) = \int_{T_0} \int_\lambda R(\delta_n = 1 | i_n, \lambda, T_0, \theta) f(T_{obs,n} | i_n, \lambda, T_0, \theta) P(i_n | \mu, \lambda, T_0, \theta) h(\lambda) g(T_0) d\lambda dT_0 \tag{35}$$

$$R(\delta_n = 1 | i_n, \lambda, T_0, \theta) = \frac{\lambda}{\tilde{Q_{i_n}}} \tag{36}$$

$$f(T_{obs,n} | i_n, \lambda, T_0, \theta) = \frac{1}{T_{obs,n}\sigma_\nu} \phi(\frac{ln(T_{obs,n}) - ln(\widetilde{T}_{a,n})}{\sigma_\nu}) \tag{37}$$

$$P(i_n | \mu, \lambda, T_0, \theta) = \frac{exp(\mu \widetilde{I}_{avg,in} + ln(M_i))}{\sum_j exp(\mu \widetilde{I}_{avg,jn} + ln(M_j))} \tag{38}$$

The conditional probability (R) of observing the activity on a random day is given by the frequency at which the individual performs the activity as shown in Equation (36). Note that this likelihood relies on the steady-state formulation of the model and does not make any assumption on the last time the activity was performed. Based on the error term distribution assumed in Equation (32), the conditional probability (f) of observing a duration ($T_{obs,n}$) is given by Equation (37). Similarly, based on the error term distribution assumed in Equation (30), the conditional probability (P) of observing the activity at a location (i_n) is given by Equation (38). The density function of the consumption rate λ is given by h, and the density of T_0 is given by g. In Equations (35) to (38), θ denotes a vector of unknown parameters including those in the activity production function, and the fraction of time available t.

3.3.2 Likelihood Function for Individuals Who Did Not Perform the Activity on the Observed Day

For the group of individuals who did not perform the activity on the observed day, no information is available on their chosen location or duration. Therefore, the likelihood of not observing the activity is written for an individual belonging to this group as follows, where all terms are as defined earlier in Section (3.3.1):

$$l(\delta_n = 0) = 1 - \int_{T_0} \int_{\lambda} \sum_{i_n} R(\delta_n = 1 | i_n, \lambda, T_0, \theta) P(i_n | \mu, \lambda, T_0, \theta) h(\lambda) g(T_0) d\lambda dT_0 \tag{39}$$

Note that this likelihood is unconditioned on location, since this information is unknown. Moreover, since the probability of the individual conducting the activity on the observed day depends only on the chosen location and the optimal duration, this equation does not uncondition over the unobserved duration. In other words, while the chosen location and the optimal duration at that location affect the probability of the individual conducting the activity on the observed day, the unobserved (chosen) duration is considered to differ from the optimal value only due to measurement errors. Consequently, we do not uncondition over the unobserved (chosen) duration.

3.3.3 Likelihood Function for the Entire Sample

The likelihood function over the full sample of respondents can be expressed as:

$$\mathcal{L}^* = \prod_n (l(\delta_n = 1, i_n, T_{obs,n}))^{\delta_n} (l(\delta_n = 0))^{1-\delta_n} \tag{40}$$

This likelihood function may now be maximized to estimate the set of unknown parameters based on the observed data. It may be noted that in developing this empirical model, no assumption is made about the last time an activity was conducted before the observed day. The model relies on the steady-state assumption to develop the relationship between activity location, duration, and frequency choices and need satisfaction.

3.4 Monte-Carlo Experiment

The estimator developed in Section 3.3 was tested on a synthetic sample generated using Monte-Carlo simulation. A sample of 2,000 individuals was created with a choice set consisting of 20 location alternatives (TAZs) for shopping activity. The retail employment in these zones was randomly generated between 1 to 100 employees per zone, and the area of the TAZs randomly from 0.1 to 2 mile^2. While the retail employment and area comprise the size variables that affect location choice, retail employment density, defined as the number of employees per unit area, was used as a measure of attractiveness in the activity production function. The travel times between these different zones were chosen to be uniformly distributed between 15 mins and 2 hours. Individuals in the sample were randomly assigned a home location from one of the 20 alternatives. Additionally, given the distribution of the rate of consumption of psychological inventory (λ) and a set-up time (T_0) in the population, a value of λ and T_0 was assigned to every individual by simulating from their distribution. Their choice of shopping activity location, duration, and frequency was generated using the model. The resulting activity durations were in the range of 10 mins to 2 hours, with cycle times in the range of 3 to 7 days. Second order terms in the translog function were set to true values of zero for the synthetic data generation process. For each individual, the data contains an indicator of whether or not the activity was conducted on the observed day, and the location and duration if the activity was conducted.

A maximum likelihood estimation was performed to estimate 10 parameters with a log-likelihood of -2677.77 at convergence. Two parameters, namely q_o and $\beta_{RetailEmp}$ had to be fixed (arbitrary normalization, to their true values in this case) to make the model identifiable. The estimation results shown in Table 1 indicate that the estimates are significantly different from 0 and are not significantly different from their true values. This shows that the model can estimate true parameters from observable data.

Table 1: Estimation Results Using Synthetic Data

Coefficient	True Value	Estimate	Standard Deviation	t-stat (against 0)	t-stat (against true value)
q_0	0	0	Fixed	-	-
q_1	5.0000E-01	4.9149E-01	1.3368E-01	3.68	-0.06
q_2	5.0000E-01	4.9973E-01	8.0961E-02	6.17	0.01
t	1.0000E-01	1.0164E-02	3.0170E-04	33.69	0.54
μ_λ	-5.0000E-00	-5.0013E+00	4.7247E-01	-10.59	0.03
σ_λ	1.0000E-01	8.8216E-02	1.4576E-02	6.05	-0.81
μ_{T_0}	1.3863E+00	1.3534E+00	5.0066E-01	2.70	-0.06
σ_{T_0}	1.0000E-01	1.0591E-01	2.1684E-01	0.49	0.03
$\beta_{RetailEmp}$	1.0000E+00	1	Fixed	-	-
β_{Area}	7.0000E-01	6.3804E-01	1.5092E-01	4.23	-0.41
μ	1.0000E+00	1.0043E+00	4.7215E-01	2.13	0.01
σ_ν	2.0000E-01	2.1539E-01	1.7039E-02	12.64	0.90

4 Conclusion

This paper presented a framework for needs based models of activity generation and developed an analytical model of activity location, duration, and frequency choice based on a needs-based utility maximization approach. Activity participation is driven by the desire to satisfy various needs like shopping, recreation, etc. Every need is associated with a level of psychological inventory, which reflects the level of need satisfaction at any point in time. As the need builds up, the inventory gets depleted. Each time an individual conducts an activity that satisfies the need, the inventory is replenished by a quantity called the activity production, that is a function of the activity duration, expenditure, and location attractiveness. Individuals choose locations, durations, and frequencies of activities so as to maximize their psychological inventory of needs subject to time and budget constraints. A solution procedure was proposed and its properties were studied and then tested for a translog functional form of activity production which exhibited desirable properties governing the relationships between activity participation and satisfaction of needs. An empirical model was developed to illustrate how the theoretical model can be estimated using standard one-day travel diary data with no knowledge of the last time the activity was performed. A Monte-carlo experiment was conducted which demonstrated that the model can recover true parameters from observable data.

The initial development in this paper was done for the case of a single need and a single activity that satisfies this need under steady-state conditions. The next steps of this work include estimating the proposed model using standard travel survey data, modeling heterogeneity in the activity production function, and modeling the effect of socio-economic characteristics. Future extensions include modeling multiple needs and multiple activities including trip chaining behavior and developing a dynamic activity participation model. The model can also be improved by incorporating measures of needs and satisfaction, obtained through surveys, and using them as indicators of utility/psychological inventory in the model.

Acknolwledgements

This research was funded by the University Transportation Center of New England and the Singapore MIT Alliance for Research and Technology. An earlier version of this paper was presented at the 91^{st} Annual Meeting of the Transportation Research Board held in Washington, D.C. in January 2012.

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