MICRO-SIMULATION OF HOUSEHOLD LOCATION CHOICE WITH MATCHING BASED HOUSING MARKET MODEL

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ABSTRACT

The objective of this study is to develop housing market model for household location choice incorporating transaction process of housing market into spatial micro-simulation by focusing on the matching between households and houses to which they need to move. Boston mechanism was applied to the matching algorithm for our housing market model. Preferences for houses by households were estimated in the way of choice-based conjoint analysis with the data of over 5,000 households collected by our survey in Toyama city. After the trial simulation, our model was confirmed to produce useful and credible output for the better planning and policy assessment.

Keywords: Matching, Micro-Simulation, Housing Market, Choice-Based Conjoint Analysis

1. INTRODUCTION

Several types of land-use micro-simulation models such as UrbanSim (Waddell(2002), Waddell et.al.(2003)), ILUTE (Miller and Salvini(2001),Salvini and Miller(2005)) , PECAS (Hunt and Abraham(2003),(2005)), ILUMASS(Wagner and Wegener (2007)) have been developed. In most of the micro-simulation models, household location choice or housing choice has been modelled. For example UrbanSim, one of the most popular models ever built for practical use models household location choices to zones by the discrete choice model and allocation of each household to a parcel in the zone by Monte-Carlo simulation. As the consequence, the correlation between households and specific locations or parcels where they choose to live cannot be expressed for its randomness. For the problem of uncertainty generated in the micro-simulation, Ševčíková et al.(2007) assessed uncertainty with Bayesian modelling. In the most existing land-use micro-simulation, determination of land price has not been modelled in directly connection with location or housing choice of household in general. In the modelling of micro-simulation such as UrbanSim, land prices are estimated by hedonic regression with explanatory valuables such as land and neighbour environment. This method is easy to estimate land price but the price determination in the land or housing market and competition between demanders (households) in the market cannot be explicitly expressed and incompatible with location or housing choice of demanders (households).

For addressing these problems, Wang and Waddell (2009) proposed extended random bidding model (Ellickson (1981) and Lerman and Kern (1983)) to represent the location choice of individual households by Bayesian approach. For the problems, we have developed the housing market model which focused on the matching between households and houses through the price competition process in housing market on the micro-simulation model (Suzuki et.al.(2010),(2011)) . Housing market can be considered as matching process where various types of households propose their favourite houses while housing suppliers select preferable household such as those who has highest bid price among them. We have developed theoretical frame of matching model applying matching algorithm by Gale and Shapley (1962) to housing market for forecasting future household location and housing market price in the existing work (Suzuki et.al.(2010),(2011)). However, our existing model has been rather theoretical and goodness-of-fit with the real data was not so high.

Therefore, two main improvements are conducted in our housing matching model. The first is that Boston Mechanism (Abdulkadiroğlu and Sönmez(2003), Ergin and Sönmez (2006)) which has been used in a school choice problem is applied to the housing market model to improve our matching algorithm. The second is to adopt the choice-based conjoint analysis to estimate utility function of household for houses which they need. After theoretical development, large scale questionnaire survey was also conducted in the Toyama city in Japan from December 2011 to February 2012 to get data for calibrating our model as well as other information about households. The objective of this study is to apply this brand-new matching model to the housing market in the actual urban area to make sure that this model is useful to simulating future household location and housing market price for planning and assessing policy measures for urban sustainability.

The structure of this paper is as follows. In Chapter 2 after Introduction, theoretical framework of the model is explained. In Chapter 3, data and methodology of estimation is shown. In Chapter 4, estimation results for housing choice by households are shown. In Chapter 5, results of the matching simulation are shown. Chapter 6 is conclusion.

2. THEORETICAL FRAMEWORK OF THE MODEL

2.1 Preference of Household for Housing

The matching model we are developing is composed of a set of households *I* and a set of houses *H*. A set of *I* and *H* are formulated as follows,

$I = \{1_1, 2, \cdots, i, \cdots\}$ is a set of households
$H = \{1_1, 2, \cdots, h, \cdots\}$ is a set of houses

where number of households and houses are not necessarily same. Sets of households and houses are divided into some groups as $\{I_1, I_2, \cdots, I_k, \cdots\}$ and $\{H_1, H_2, \cdots, H_k, \cdots\}$. A set of households is a group of those who could have intension to change their houses and move to new ones. A set of houses include vacant and newly constructed ones at that time period.

Preferences of households for houses are derived from the utility function. Given utility optimization of a household under budget constraint, indirect utility function of household *i* can be expressed as a function of attributes of houses and location and price of house as follows,

$$V_i^t = u_i(\mathbf{X}_h^t, \mathbf{X}_l^t, p_h^t) \qquad (1)$$

where $\mathbf{X}_h^t = \{X_{h1}^t, X_{h2}^t, \cdots\}$ is a set of attributes of houses *h*
$\mathbf{X}_i^t = \{X_{il}^t, X_{l2}^t, \cdots\}$ is a set of attributes of location *i*
p_h^t : is a price of house *h*

Variables in a set of attributes of houses could include floor space, number of rooms, age of the house and so on. Variables in a set of attributes of location could include accessibility or proximity to transport facilities, environment of neighbourhood and so on. Households rank houses not only they want to move but also those where they live at present according to their own preferences because they could have an option not to move in this term if utility to move to a new house would be lower than that to continue to live in present house.

2.2 Preference of Housing Supplier for Household

Bid price of household *i* is defined as a maximum value of price for the house *h* under the level of indirect utility $V_i^t = u_i(\mathbf{X}_h^t, \mathbf{X}_l^t, r_h)$ is V_i^*. Bid price of household i for house *h* is expressed as follows,

$$p_{ih}^* = p(\mathbf{X}_h^t, \mathbf{X}_l^t, V_i^*) \qquad (2)$$

Preferences of housing suppliers for households who propose to their housing are assumed to depend on the bid price that households present. Therefore, ranks of preferences of housing suppliers for households are not decided until they receive proposal of households who want their houses. The market prices of houses are decided through the negotiated transaction between households and housing suppliers. Rubinstain (1982),(1985) has theoretically clarified price determination in bargaining process between demander and supplier. In our model for housing market, housing suppliers gives right to contract to the households who have applied for the house following their priority order (bid price of households) until there are no rooms left. If housing suppliers have just one house, they choose just one household who has higher bid price among households who applied their houses. This framework is same as auction.

2.3 Matching Algorithm for Housing Market

In the existing study, we (Suzuki, Kitazume and Miyamoto (2010), Suzuki, Kitazume and Miyamoto (2011)) have proposed matching algorithm which two-sided matching market (Gale and Shapley (1962)) is applied to the housing market. Two-sided matching by Gale and Shapley has been known as a stable matching model for marriage problem in which there are two-sided players: n men and n women. In this model, each person ranks those of the opposite sex in accordance with his or her preferences for a marriage partner. They described a simple algorithm that always finds a stable matching. Here, definition of "stable" is that a set of matching is called unstable if under it there are a man and a woman who are not married to each other but prefer each other to their actual mates. Gale and Shapley's two-sided matching is useful for finding stable matching. But it is a little unrealistic when it is applied to the housing market because the procedure that woman (housing supplier) does not accept the men (households) yet, but keeps them on a string to allow for the possibility that someone better may come along later is not necessarily the case in housing market. Instead of that, Boston mechanism is applied to our matching model in the housing market. Boston Mechanism is one of the matching procedure that is most widely used as a student assignment mechanisms in real-life applications of school choice problems (Ergin and Sönmez (2004)) . The procedure applied to the housing market model is as follows,

Step1. Each household proposes to their favourite house (first choices).

Step2. Each housing supplier gives right to contract to the households who have listed it as their first choice one at a time following their priority order until either there are no rooms left or there is no household left who has listed it as his first choice.

Step3. Those households who were rejected by first choices propose to their second choices.

Step4: For each house with still available seats, consider the households who have listed it as their 2nd choice and assign the remaining seats to these households one at a time following their priority order until either there are no seats left or there is no household left who has listed it as his 2nd choice.

Step3 and Step4 are repeated in the same manner until every household would get new houses or give moving up as their proposal to all houses which they can accept would have been rejected by housing suppliers.

In our housing market model, housing supplier's priority order for households is assumed to be determined by their bid prices which are willingness to pay for the houses.

3. DATA AND METHODOLOGY OF ESTIMATION

For practical use of our housing market model, a large numbers of individual data of households was collected by conducting large scale survey in a Japanese local city to estimate utility functions for several types of households and to do trial simulation.

3.1 Survey in Toyama city

Survey by questionnaire was conducted in Toyama city and a part of neighbouring area from December 2011 to January 2012 to collect information about the history of location and housing choices by the households who live in the survey area. The total amount of questionnaires distributed to them by mail is 14,073 which is 10% for all households in this area and the amount of returned to answer are 5,089 which is equivalent to 36.2% for the total distribution. Parameters of utility functions of households which express the preferences of households toward houses can be calibrated with these data.

3.2 Contents of Questionnaire

Questionnaire to send households in Toyama city contains questions such as 1) attributes of households, 2) attributes of houses where they live now and before, 3) intension of moving and 4) preference of housing to move. Questionnaire 4) preference of housing is designed as some housing profiles to use conjoint analysis mention below.

3.3 Conjoint Analysis

Choice-based conjoint analysis is used to estimate the parameters of utility function of households and willingness to pay for houses which they need. Conjoint analysis is a generic term used to describe several ways to elicit preferences. It has been developed originally in psychology and applied to various disciplines such as marketing analysis, environmental evaluation. Choice-based conjoint analysis is one of the approaches of conjoint analysis (Louviere (1994)). In this approach, respondents chose one alternative from each of several sets of full profiles and aggregate multinomial logit model is used to estimate their preferences. This approach is appropriate for calibrating choice model which has multiple choice sets with different profiles such as housing choice as well as its easiness for respondents to answer.

3.4 Estimation of households' preferences

Households' preferences for houses where they want to move next can be estimated in the way of choice-based conjoint analysis as mentioned above. Multinomial logit model is used to estimate parameters in choice-based conjoint. Random utility U_{ij} which household i get when they choose profile of house j is expressed as equation (3). V_{ij} and ε_j are respectively called the systematic component and the random component of the utility.

$$U_{ij} = V_{ij} + \varepsilon_j = \boldsymbol{\beta}\mathbf{x}_{ij} + \varepsilon_j \tag{3}$$

where x_{ij} : is the vector of the profile of each house option
β : is the vector of parameter for the profile of house options

If we assume that random component in equation (3) ε_j is Gumbel distributed with a location parameter η = 0, and a positive scale parameter μ =1, the probability any housing profile *j* is chosen is given by

$$P_j = \frac{\exp(V_j)}{\sum_k \exp(V_k)} \tag{4}$$

In the equation of utility of households (3), there is a variable about housing price p. Marginal willingness to pay (*MWTP*) related to a particular attribute in the profile x_1 that is the additional amount respondents are willing to pay for one more unit of a particular attribute x_1 is expressed as equation (5). This is marginal utility in monetary terms.

$$MWTP_{x_1} = \frac{dp}{dx_1} = -\frac{\partial V}{\partial x_1} \Big/ \frac{\partial V}{\partial p} = -\frac{\beta_1}{\beta_p} \tag{5}$$

Bid price of household i for house h p_{ih} can be calculated by sum of values for attributes of each house multiplying the MWTP for each attribute.

3.5 Profile of Housing Options

Households were to answer the question through the questionnaire as “Which type of houses would you choose in the following lists of housing candidates? “ 9 options of houses (A to I) have profiles such as floor space, age of house, price, time to nearest station by walk and short comment as a supplementary explanation. Those profiles of houses were shown to respondents separately three times by 3 profiles in order to choose easily. Here, profile of each option was designed as orthogonal main effects properly considered. The prices of houses were set by taking market price in Toyama city into consideration. Table 1 represents profiles of housing options for conjoint analysis.

Table 1 – Housing profile for choice-based conjoint analysis

housing options	A	B	C	D	E	F	G	H	I
Floor space(m^2)	160	80	80	120	120	120	160	160	80
Age of house	30	10	30	0	10	30	0	10	0
Price (million yen)	30	30	20	30	20	10	20	10	10
Time to nearest station by walk(min)	5	15	30	30	5	15	15	30	5

4. ESTIMATION RESULTS FOR HOUSING CHOICE

In this chapter, coefficient values in the utility function of households defined above and the results of test are explained.

4.1 Classification of Households

Households are segmented into twelve groups by a number of members in the household, whether they have children or not and age of the head of households. Table 2 shows the definition of the household segmentation. Single households are divided into three types 1,2 and 3 by age of household head. Households who have two or three members are divided by having children or not as well as age of the head of household. Households with four and over members are divided into three groups by only age of the head of household.

Table 2 – Household segmentation

A number of household	Children	Age of head of household		
		20s and 30s	40s and 50s	Over 60s
1	Without	Type 1	Type 2	Type 3
2 and 3	Without	Type 4	Type 5	Type 6
	With	Type 7	Type 8	Type 9
≧ 4	With or Without	Type 10	Type 11	Type 12

4.2 Results of Parameter Estimation and MWTP

Parameters β in the utility function shown in the equation (3) were estimated by maximum likelihood estimation. The results of estimation for the housing choice are shown in the Table 3 and 4. The results of the parameters for the explanation variables for each type are appropriate. The sign of coefficients for all explanation variables but floor space are negative which is consistent with our priori expectation because increase of the all valuables but floor space would decrease utility of households. Most of the results in the t test and the likelihood ratio index (rho-squared) are also considered to be good in general.

From the equation (5) and the value of parameters estimated in the Table 3 and 4, marginal willingness to pay (MWTP) can be estimated. The results of estimation for MWTP are shown in the following Table 5 and 6. The values of MWTP for the floor space are positive while the values for the other two valuables are negative that is straightforward from the equation (5) and the sign of parameters estimated in the Table 3 and 4.

Table 3 – Estimation results for housing choice (1/2) (upper: coefficient estimate, lower: t statistic)

	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6
Price	-0.0025 (-3.571)**	-0.0033 (-4.467)**	-0.0023 (-3.532)**	-0.0024 (-3.826)**	-0.0022 (-4.924)**	-0.0011 (-2.598)**
Floor space	0.0251 (2.873)**	0.0244 (2.889)**	0.0207 (2.482)*	0.0217 (2.860)**	0.0147 (2.672)**	0.0054 (1.012)
Age of houses	-0.2055 (-4.582)**	-0.2338 (-5.073)**	-0.1838 (-4.716)**	-0.2126 (-5.252)**	-0.1786 (-6.629)**	-0.0947 (-3.915)**
Time to nearest station by walk	-0.0266 (-2.243)*	-0.0232 (-1.803)*	-0.0639 (-6.085)**	-0.0352 (-3.375)**	-0.0464 (-6.537)**	-0.0537 (-8.725)**
Number of observations	37	39	58	61	124	157
ρ^2	0.22	0.29	0.33	0.30	0.33	0.22

**1% significance level, *5% significance level

Table 4 – Estimation results for housing choice (2/2) (upper: coefficient estimate, lower: t statistic)

	Type 7	Type 8	Type 9	Type 10	Type 11	Type 12
Price	-0.0018 (-4.107)**	-0.0019 (-4.511)**	-0.0022 (-5.049)**	-0.0017 (-4.325)**	-0.0019 (-6.377)**	-0.0018 (-5.416)**
Floor space	0.0172 (3.032)**	0.0146 (2.792)**	0.0241 (4.302)**	0.0218 (4.185)**	0.0202 (5.285)**	0.0248 (5.469)**
Age of houses	-0.1695 (-5.917)**	-0.1632 (-6.361)**	-0.1989 (-7.521)**	-0.1757 (-6.804)**	-0.1667 (-9.283)**	-0.1635 (-7.736)**
Time to nearest station by walk	-0.0268 (-3.609)**	-0.0403 (-5.993)**	-0.0561 (-7.763)**	-0.0299 (-4.644)**	-0.0520 (-11.093)**	-0.0510 (-8.965)**
Number of observations	92	134	136	110	250	160
ρ^2	0.21	0.26	0.31	0.21	0.25	0.20

**1% significance level, *5% significance level

Table 5 – Estimation results for MWTP (1/2)

	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6
Floor space	102.01	73.14	90.82	91.31	67.80	50.46
Age of houses	-833.68	-700.55	-805.02	-894.93	-826.39	-877.07
Time to nearest station by walk	-107.74	-69.50	-279.73	-148.39	-214.60	-497.48

Table 6 – Estimation results for MWTP (2/2)

	Type 7	Type 8	Type 9	Type 10	Type 11	Type 12
Floor space	94.64	77.91	111.09	132.07	108.76	138.30
Age of houses	-933.77	-869.65	-915.71	-1064.06	-899.13	-910.54
Time to nearest station by walk	-147.78	-214.64	-258.21	-180.99	-280.38	-283.89

5. RESULTS OF THE MATCHING SIMULATION

5.1 Sample of Households and Houses for Simulation

200 households and 250 houses are chosen as samples for matching simulation. Samples of 200 households are randomly chosen from 5,089 households who returned to answer the questionnaire survey in Toyama city. A number of the households for each type and average attributes of their houses where they live at present are shown in Table 7.

Table 7 – Samples of households and attributes of their houses

Type of household	1	2	3	4	5	6	7	8	9	10	11	12	Total
Number of samples	5	6	8	9	18	27	13	19	20	16	36	23	200
Floor space(m^2)	81.0	114.3	113.3	102.0	109.8	126.7	99.9	123.6	96.6	114.6	114.9	115.1	112.5
Age of House	8.6	10.8	13.2	8.1	16.1	22.5	11.1	21.4	16.2	22.6	24.1	40.5	21.4
Time to nearest station by walk(min)	13.8	15.7	6.5	9.9	10.9	8.7	6.9	9.2	7.6	7.6	13.5	6.6	9.6

Figure 1 shows the relation between a number of members in the households and attributes of their houses where they live at present, floor space, age of houses and the time to nearest station by walk. The group of single person household contains type1, type2 and type3 of household, 2 and 3 persons household contains from type4 through type9 of household and 4 persons household contains type10, type11 and type12 of household. Floor space and age of houses are in direct proportion to a number of members in households. While time to nearest station by walk for 2 and 3 member households is shortest in all groups.
In Figure 2 shows the relation between the age of head of households and attributes of their houses. The group of “young” households contains type1, type4, type7 and type10 of household, “middle” households contains type2, type5, tyype8 and type11 of household and “old” household contains type3, type6, type9 and type12 of household. Floor space and age of houses are in direct proportion to age of head of households. On the other hand, time to nearest station by walk for middle age is longest in all groups.

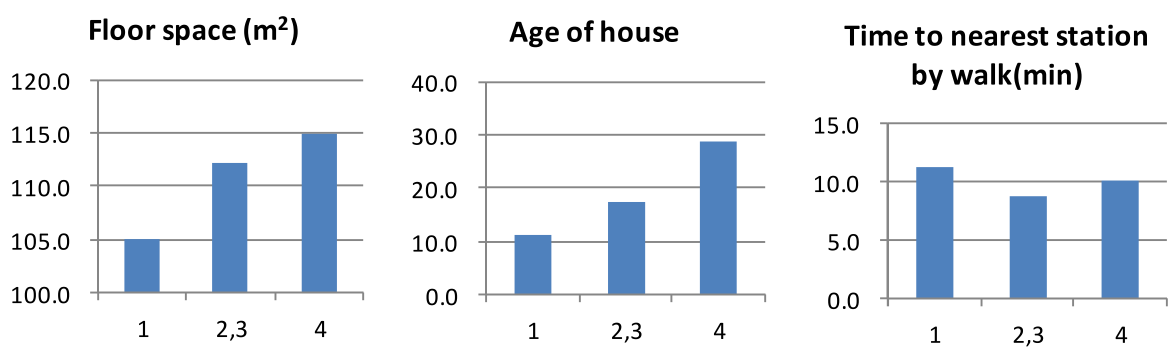

Figure 1 – Relation between a number of household members and the attributes of their present house

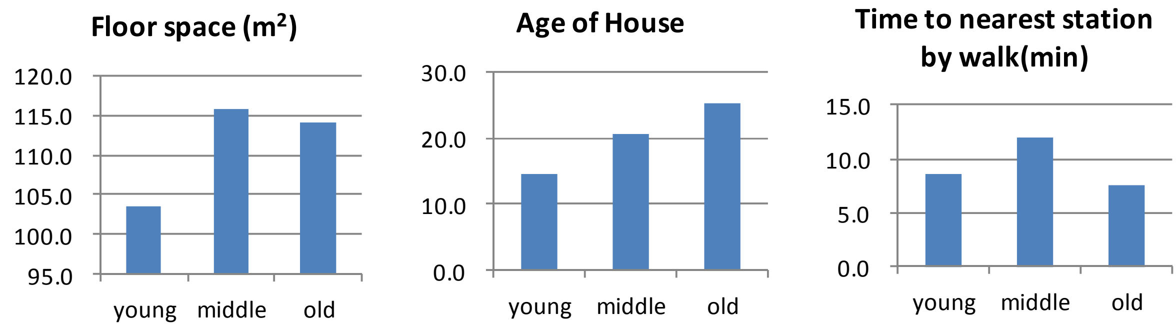

Figure 2 – Relation between age of households head and the attributes of their present house

Samples of 250 houses were chosen randomly from the website of the real estate ad in Toyama city which were for sale in August 2011. Average and distribution for the attributes of the houses, floor space, age of house and time to nearest station by walk are shown in Figure 3. The types of houses in all Samples are detached houses which contains both new-built and secondhand houses.

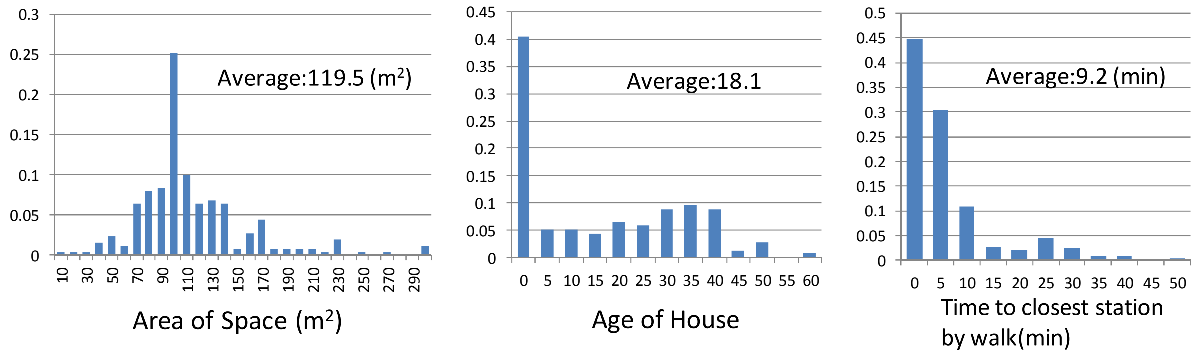

Figure 3 – Figure legend (Use Caption Figure style to insert the figure legend)

5.2 results of matching

Matching simulation based on the algorithm explained in the Chapter 2 was conducted with the sample data of 200 households and 250 houses mentioned above. Preference orderings for 250 houses by 200 households can be determined with the utility function estimated in the Chapter 4. Three sets of Gumbel distributed random numbers with location parameter $\eta = 0$ and positive scale parameter $\mu = 1$ named “random1”,”random2” and “random3” were given as sets of random components in the random utility function of 200 households for housing choice shown in the equation (3) . On the other hand, preference orderings for 200 households by 250 housing suppliers were determined by Willingness to pay (WTP) by households for houses expressed in the equation (5). For the suppliers’ preferences as well as households’, random components are considered because households who have same ordering or same WTP come out unless some random components considered. A set of random number between 0 and 1 is considered as a random component of suppliers’ preferences which can be interpreted as a lot or first come, first served rule for the case that there are some candidates for the house.

Figure 4 represents ranks for the houses which households finally got in the process of matching for each random set (in the left figure) and ranks for households to whom housing supplier finally sold their houses in the process of matching for each random set (in the right figure). In the left figure, almost half of households got their new houses which they rank as the first and 3/4 of households could get new houses which are ranked in their top 10 for any random set. In the other right figure, ranks are almost on the straight line for any random set.

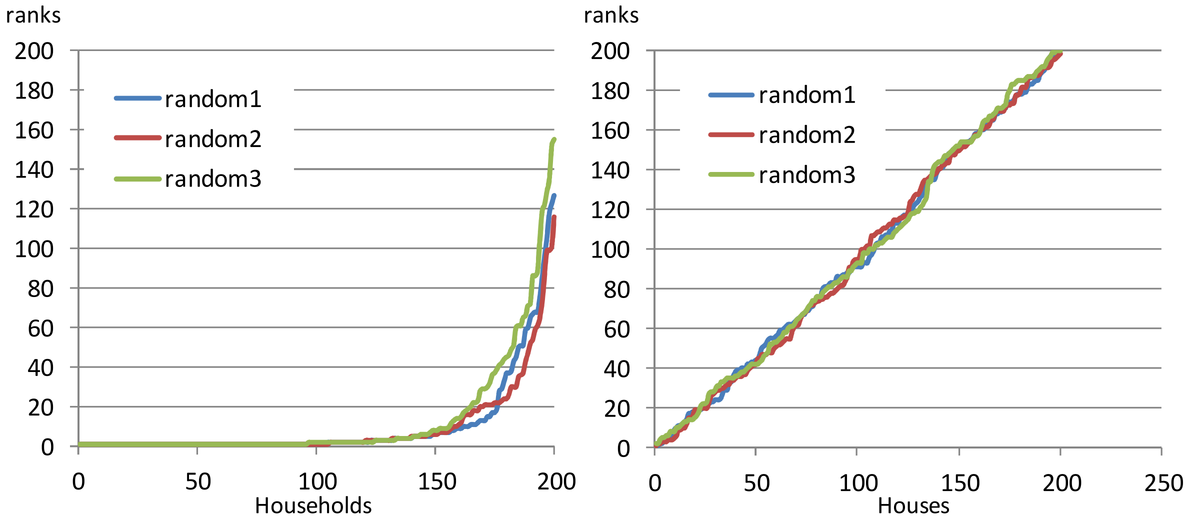

Figure 4– Ranks for houses and households at the matching results

In Figure 5, relation between a number of members in the households and attributes of matched their new houses such as floor space, age of houses, time to nearest station by walk and estimated price of houses are shown as one of the results for matching simulation. Horizontal axis represents categories of a number of household members for three sets of random component, random1, random2 and random3. Floor space and housing price estimated in the matching simulation are in proportion to a number of household members.

These results are appropriate for the reality and robust for the randomness. On the contrary, age of houses and time to nearest station by walk are decreasing as a number of household members increase that is not necessarily accordance with the present situation of their houses. The reason of the results can be considered that sensitivity of parameters of utility for age of houses and time to nearest station fewer member households have are likely to be smaller than those of households which have more members.

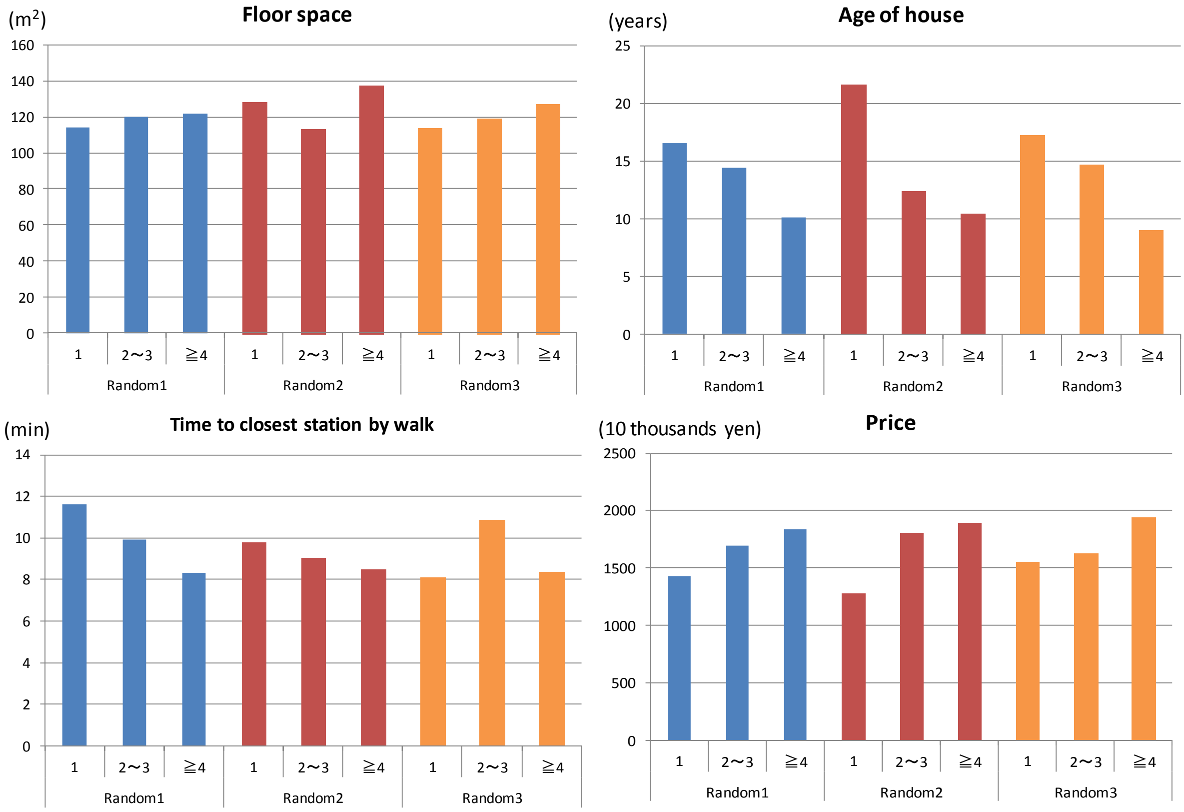

Figure 5– Relation between a number of household members and the attributes of matched their houses

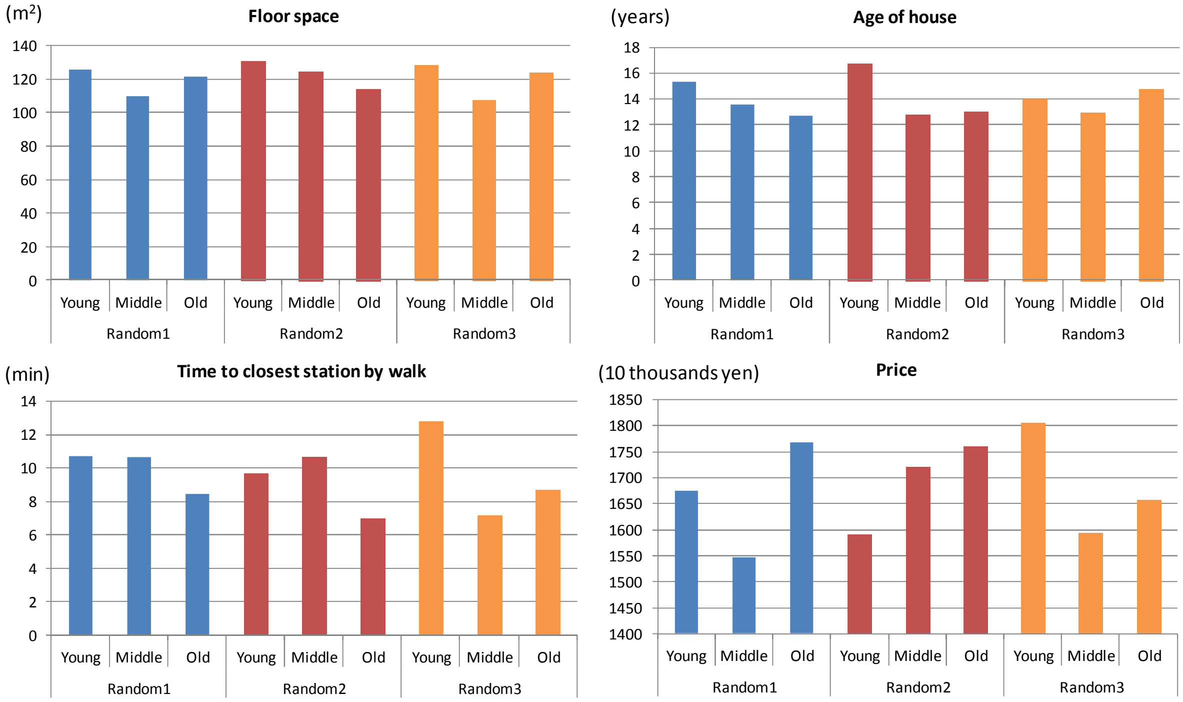

Figure 6– Relation between age of households head and the attributes of matched their new houses

In Figure 6, the relation between the age of head of households and attributes of matched their new houses such as floor space, age of houses, time to nearest station by walk and estimated price of houses are shown. These results were simulated for three sets of random component, random1, random2 and random3.
For results of Figure 6, attributes but housing price have tendency to be in reverse proportion to age of the head of household. These results suggests that younger households hope to have larger houses for next their houses because they might be going to have more family such as their children while they have to compromise newness and accessibility for their budget constraint. On the contrary, the results also suggest that older households tend to emphasize accessibility to station than younger households because they are going to be difficult to drive by themselves.

Figure 7 represents the comparison between estimated floor space of the house which households got in the matching simulation and that of their present houses both of which are standardized to 1. Horizontal axis shows category of households which is divided by age of the household head and with or without child. Furthermore the comparison has been done for three cases for different random components of utility. This result represents that estimated floor space by matching simulation of housing market can get the feature of the reality well. However, estimated values of floor space for younger households are likely to larger than those of present houses and vice versa for old and middle without child households. This result suggests that our model can express well tendency that younger households want to live in more spacious houses and old and middle without child households don't want so.

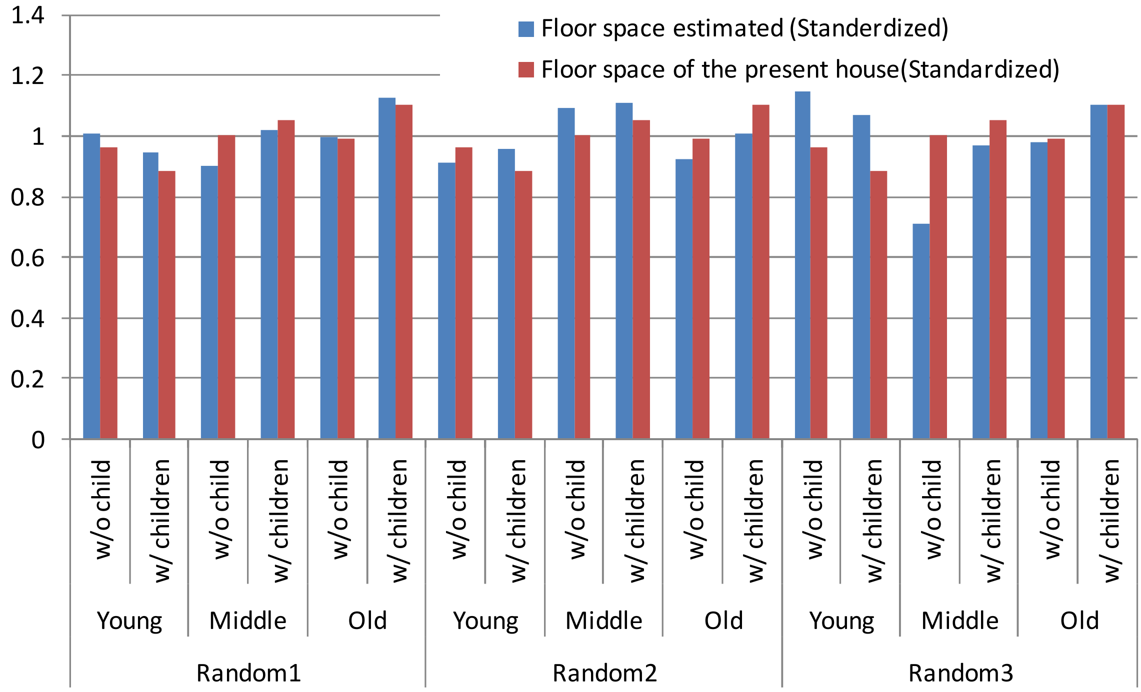

Figure 7– Comparison of floor space for between simulated and present houses

6. CONCLUTION

The housing market model we have developed is explained in this paper. The main concept of our model is incorporating transaction and competition process of housing market into spatial micro-simulation by focusing on the matching between households and houses to which they need to move. Three main contributions to theory and practice are shown in this paper. The first thing is that Boston Mechanism which has been used in a school choice problem is applied to the housing market model to improve our matching algorithm. This mechanism is consistent with housing market process as well as making sure a stable matching. The second thing is improving the way of estimation for household preferences by applying choice based conjoint analysis that also makes consistency between matching algorithm and household behaviour better. The third contribution is that huge numbers of micro-data of households were collected by conducting questionnaire survey in Toyama city by ourselves that made possible effective parameter estimation and practical simulation. These results from the housing market model based on matching are able to offer useful implications for the planning of the cities such as most Japanese local cities where structure of households and their locations are getting to change as a result of declining birth rate and aging population even if population wouldn't be changed as well as other growing cities. The reason is that our model can effectively deal with complicated relation between various types of households and houses or locations. The implication from our model could also offer useful information about policies for the problem of mismatch between households and houses or locations where they live. That is just a point for better planning and assessing policies for the cities or regions where spatial change could be caused by not only population change but also the change of household structure.

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