

# ECONOMIC CHARACTERISTICS AND THE MINIMUM EFFICIENT SCALE OF TAIPEI MASS RAPID TRANSIT

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## **ABSTRACT**

It has been well recognized that Mass Rapid Transit (MRT) has better performance on the environmental sustainability and social sustainability than private motorized vehicles. However, the capital investment of MRT is usually considerably high which may result in financial burdens on operators and government. Therefore, in order to improve the economic and financial performance of MRT systems, the operating performance and the optimal network scale of MRT should be evaluated cautiously while taking account of capital costs. This paper analyzes the cost characteristics and the long-run minimum efficient scale of Taipei Mass Rapid Transit (TMRT) by using a Translog cost function methodology. The research result indicates that TMRT has strong characteristics of economies of density, and the elasticity of substitution is not significant. It has also shown that the minimum efficient scale of TMRT occurs when the network length is 140 km with an average ridership of 2.36 million trips per day. The long-run average cost will be increased by 7% after the future network is completed as compared to the average cost at the minimum efficient scale. This implies that the government and the operator need to carefully evaluate the financial plan of the future network of 280 km rail-based TMRT to avoid potential financial burdens.

*Keywords: Mass rapid transit, Translog cost function, minimum efficient scale*

## INTRODUCTION

The sustainability of a transportation system can be evaluated through its environmental sustainability, social sustainability, and economic and financial sustainability. It has been generally evident that Mass Rapid Transit (MRT) is more environmentally and socially sustainable than private motorized vehicles, but the economic and financial sustainability of MRT requires more investigation, particularly on the efficiency performance and the financial burdens as a result of the high capital cost of MRT infrastructures.

The case study of this paper, the Taipei Mass Rapid Transit (TMRT), also encounters the similar issue. The TMRT, opened in 1996, is the first MRT system in Taiwan which aims to increase the use of public transportation instead of private motorized vehicles. The current network size of TMRT is 101.9 km and the future network is planned to be expanded to 280 km by 2031. The operator of TMRT, Taipei Rapid Transit Company (TRTC), is required to share parts of the capital cost of the future routes, and this is likely to introduce a considerable financial burden to TRTC.

In order to expand the network scale while achieving economic and financial sustainability, it is essential to understand the current economics characteristics and estimate the optimal network size of TMRT. The cost function is an effective approach to analyze the economic characteristics of producers. The policy implications derived from the estimated cost function provide important information to decision makers and system operators. This paper constructs a short-run cost function of TMRT, and analyzes the basic economic characteristics from the operating history. A long-run cost function is also constructed to incorporate the capital cost of TMRT by investigating the long-run Minimum Efficient Scale (MES) of TMRT to estimate the optimal network scale for Taipei based on average cost minimization.

## LITERATURE REVIEW

Cost functions derived from the production theory provide insight into a producer's cost structure and economic performance (Mas-Colell, 1995). Cost functions can be specified in various functional forms depending on the corresponding production functions assumed. One of the functional form known as the transcendental logarithmic (Translog) cost function developed by Christensen, Jorgenson and Lau (1973), provides a flexible functional form which is able to examine the scale economies and other economic characteristics of a regulated industry. Translog cost function studies have been widely applied to the

transportation industry, with a substantial body of empirical studies which can be classified into panel data analysis and time series analysis on a specific system.

Previous Translog cost functions studies in transportation tend to conduct panel data analysis to compare the system performance among various transportation operators (Viton, 1980; Savage, 1997; Karlaftis et al., 1999; Karlaftis and McCarthy, 2002). In contrast, there is a lack of studies on a firm-specific time series analysis focusing on the economics performance of a specific transportation system (Colburn and Talley, 1992; McGeehan, 1993; Wang and Chen, 2005). In addition, most previous studies pay more attention to the basic economic characteristics such as economies of scale and elasticities, whereas the long-run MES which takes account of the capital cost of transportation infrastructures is less discussed.

Braeutigam (1984) studied the economic characteristics of a railway company in the U.S by applying the Translog cost function. The results indicated that scale economies existed in the railway system studied. Moreover, Braeutigam also estimated the short-run optimal track length and long-run MES in terms of flows and track length, and suggested that the differences between the short-run and long-run optimal network scale should be distinguished. The former refers to the optimal track length minimizing the short-run average cost based on the fixed outputs condition, whereas the latter is the optimal track length and flow by minimizing the long-run cost function when all the factors are variable in the long run. The drawback of a time-series analysis on a specific firm is the lack of observations over time to provide sufficient statistical power, and hence panel data analysis is more evident in this research field. However, compared to panel data analysis, a firm-specific study reduces the heterogeneity between different operators in a system, so the demand on the sample size can be mitigated.

This paper applies the Translog cost function to TMRT, based on historical data covering 11 years of its operation. A cost allocation method is employed to allocate the annual data to quarterly data. This time series analysis does not only discuss the basic economic characteristics of TMRT, but also estimate the long-run MES while considering the capital cost of TMRT.

## **DESCRIPTION OF STUDY AREA**

Taipei Mass Rapid Transit (TMRT) has operated since 1996 and now has become one of the major transportation systems in the Taipei Metropolitan Area which consists of Taipei City and New Taipei City. The Taipei Metropolitan Area has a total population of 6.7 million with a very high population density of 2,884 persons per kilometre. In 2011, TMRT undertook around 1.5

million daily trips on average, which took account of around 16% of daily trips in Taipei City and 9% in New Taipei City with a network length of 101.9 km (Figure 1). Before the opening of TMRT, bus was the only major mode of public transportation which now shares around 17% of total trips in Taipei City and 12% in New Taipei City. The mode share of private motorized vehicles is around 42% in Taipei City and 62% in New Taipei City.

The TMRT network is planned to be expanded to 280 km after the third phase of construction in 2031. As the first MRT project in Taiwan, the operator, Taipei Rapid Transit Company (TRTC), was not required to share any capital cost for the first phase of 74.5 km network completed before 2008. This incentive was provided in order to alleviate the financial burden of TRTC in the early phase of operation, and this has led to the consistent and increasing net revenue for TRTC since 1998. However, TRTC will need to be responsible for a part of the capital cost for the second phase of network construction after it is completed, which amounts to US\$3.24 billion and this is likely to cause financial pressure to TRTC (Chang and Chen, 2007). In this regard, this paper investigates the minimum efficient scale of TMRT based on its historical economic characteristics using a cost function approach.

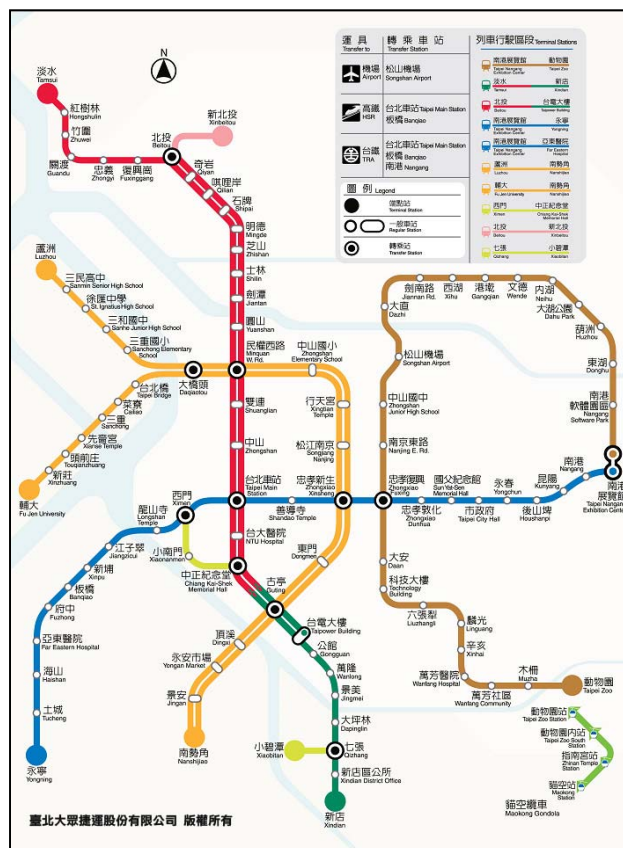


FIGURE 2- The network of Taipei Mass Rapid Transit

Source: Taipei Rapid Transit Company (<http://english.trtc.com.tw/>)

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## METHODOLOGY AND DATA

### Methodology

The cost function is based on the assumption that the producer is seeking cost minimization at a given output level and factor prices. The model construction of this study begins with the short-run cost function of TMRT, and then introduces the long-run average cost function.

The short-run cost function specifies the operating variable cost as a function of output ( $Y$ ), input variable factor prices ( $Pv$ ), and a fixed factor ( $K$ ). The general form of the short-run cost function is specified as equation (1).

$$C_v = C(Y, P_v, K) \quad (1)$$

The Translog short-run cost function is constituted of an output ( $Y$ ), input variable factor prices ( $Pv$ ), and a fixed factor ( $K$ ) with their interaction terms as defined by equation (2), with all the variables in natural logarithm terms.

$$\begin{aligned} \ln SRVC = & \alpha_0 + \alpha_Y \ln Y + \sum_{i=1}^J \alpha_Y \ln P_i + \sum_{i=1}^J \gamma_{iY} \ln P_i \ln Y + \sum_{i=1}^J \gamma_{iK} \ln P_i \ln K \\ & + \frac{1}{2} \sum_{i=1}^J \sum_{j=1}^J \gamma_{ij} \ln P_i \ln P_j + \frac{1}{2} \gamma_{YY} (\ln Y)^2 + \frac{1}{2} \gamma_{KK} (\ln K)^2 \quad i, j = 1, \dots, J \end{aligned} \quad (2)$$

The cost function is constrained to be homogeneous of degree one and concave in input prices to meet the assumption of cost minimization. Therefore, the restrictions must be taken into the model:

$$\begin{aligned} \sum_{i=1}^J \alpha_i = 1, \quad \gamma_{ij} = \gamma_{ji} \quad \forall i, j \\ \sum_{i=1}^J \gamma_{ij} = \sum_{j=1}^J \gamma_{ji} = \sum_{i=1}^J \gamma_{iy} = \sum_{i=1}^J \gamma_{ik} = 0 \end{aligned} \quad (3)$$

In order to ensure the global concavity of the cost function, the cost function (equation(2)) needs to be differentiated with respect to factor prices based on the Shephard's Lemma (Shephard, 1953), yielding the input share equations as equation (4). The multiple equations of (2) and (4), with the restrictions of (3), were estimated by Seemingly Unrelated Regression developed by Zellner (1962).

$$\begin{aligned} \frac{\partial \ln SRVC(P, Y, K)}{\partial \ln P_i} &= \frac{\partial C(P, Y, K)}{\partial P_i} \frac{P_i}{C(P, Y, K)} = \frac{P_i x_i}{C(P, Y, K)} \\ &= S_i(P, Y, K) = \alpha_i + \sum_{j=1}^3 \gamma_{ij} \ln P_j + \gamma_{iY} \ln Y + \gamma_{iK} \ln K \end{aligned} \quad (4)$$

## Data

The data used in this study were obtained from TRTC annual reports<sup>1</sup>. The time-series analysis is based on seasonal data from 1996 to 2006. It is important to note that TMRT was not completely in operation in the first season of 1996 and had irregular operation in 2001 due to a flood that affected Taipei at the point of time. Therefore, these two periods of operation data are excluded from our studies, with the final data sample yielding 39 observations. A Translog cost function comprises an output, a vector of input prices, and a fixed factor. The number of passenger trips is chosen as output and the network length in kilometers is set as the fixed factor. The input factors are defined below.

During 1996 to 2006, the labor cost shares the most proportion of total operating costs, amounted to 43.79%; second to that is replacement cost which is around 20.36%. All other cost items are less than 10% of total cost share. Therefore, to make the cost function more parsimonious and to ensure the chosen input factors have significant contributes to the operating cost, all the cost items apart from labor cost and replacement cost are categorized into a factor of intermediate cost.

The labor cost includes salary, insurance, retirement pension, and other labor cost. Because there is only annual labor cost data available, a stepwise regression method was applied to allocate the annual labor cost to seasonal costs. At the given variables: trips, vehicle-km, network length, and frequency, the results showed that the combination of trips and vehicle-km has the best explanatory power to predict labor cost. Therefore, the annual labor costs are allocated to each season based on the weight of these two variables which are 0.6 and 0.4 respectively. The formulation of allocation is expressed as equation (5), and the labor price is derived by dividing the seasonal labor costs by the number of employees.

Seasonal labor cost=annual labor cost

$$*(0.6 * \frac{\text{seasonal trips}}{\text{annual trips}} + 0.4 * \frac{\text{seasonal vehicle-km}}{\text{annual vehicle-km}}) \quad (5)$$

<sup>1</sup> The annual reports are publicly available on the official website: <http://english.trtc.com.tw/>.

Intermediate factors include materials, electricity, maintenance, and other factors apart from labors and replacements. Similar to the labor cost, the intermediate factors cost needs to be allocated to the seasonal intervals. The result of stepwise regression showed that the variable trip is most sensitive to intermediate factors cost, so the allocation process is expressed as equation (6). However, it is difficult to calculate the quantity of intermediate factors. Our approach is to assume that the vehicle-km is proportional to the intermediate factors cost, so the factor price is to divide the seasonal costs by the number of seasonal vehicle-km.

$$\text{Seasonal intermediate factors cost} = \text{annual intermediate factors cost} * \left( \frac{\text{seasonal trips}}{\text{annual trips}} \right) \quad (6)$$

According to the TMRT's financial plan, the operator has to contribute the replacement fund to the replacement of equipments including vehicles, signal systems, and other electronic equipments. The total replacement fund amounts to US\$2.8 billion, apportioned over 30 years from 2001 to 2031. Although TRTC does not need to settle this amount in the first five years, the replacement fund shall be allocated to the system's life cycle. A sinking fund depreciation method was applied to apportion the fund over 30 years, at the given interest rate of 8%. The number of vehicle-km is assumed to be proportional to the replacement cost, yielding the replacement price from dividing the replacement cost by the number of vehicle-km.

In summary, the variables of the cost function are listed in Table 1 below.

TABLE 1- The variables of the short-run cost function

Variables	Symbols	Units
Operating cost	$V_C$	dollars
Passenger trips	$Y$	trips
Labor price	$P_L$	dollars/employees
Intermediate factors price	$P_I$	dollars/vehicle-km
Replacement price	$P_N$	dollars/vehicle-km
Network length	$K$	km

## ESTIMATION RESULTS

The parameter estimates of the multiple-equation model, consisting of cost function and the share equations for labor and intermediate factors, are shown in Table 2. Three insignificant parameters  $\alpha_K$ ,  $\gamma_{RK}$ , and  $\gamma_{LI}$  are excluded in the final model to improve the model goodness

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of fit. The adjusted R-square value of the final cost function is 0.990 suggesting a high quality of model goodness of fit. In the autocorrelation test, the Durbin-Watson statistic 1.38 locates between the confidence interval [0.451, 2.929], so it fails to conclude that the autocorrelation is significant.

TABLE 2- Parameter estimates of the short-run cost function of TMRT

Parameters	Variables	Coefficient	Std. Error	t-values
$\alpha_0$	constant	23.350	0.486	48.044
$\alpha_Y$	$\ln Y$	-1.485	0.057	-26.094
$\alpha_L$	$\ln P_L$	0.174	0.028	6.254
$\alpha_I$	$\ln P_I$	0.343	0.028	12.248
$\alpha_R$	$\ln P_R$	0.483	0.024	20.270
$\gamma_{LY}$	$\ln P_L \ln Y$	-0.011	0.003	-3.174
$\gamma_{IY}$	$\ln P_I \ln Y$	0.005	0.002	2.388
$\gamma_{RY}$	$\ln P_R \ln Y$	0.006	0.002	3.470
$\gamma_{LK}$	$\ln P_L \ln K$	0.027	0.004	6.629
$\gamma_{IK}$	$\ln P_I \ln K$	-0.027	0.004	-6.629
$\gamma_{YY}$	$\ln Y \ln Y$	0.1	0.004	26.211
$\gamma_{KK}$	$\ln K \ln K$	0.17	0.011	15.132
$\gamma_{LL}$	$\ln P_L \ln P_L$	0.039	0.004	9.237
$\gamma_{II}$	$\ln P_I \ln P_I$	0.153	0.002	62.607
$\gamma_{RR}$	$\ln P_R \ln P_R$	0.192	0.004	46.826
$\gamma_{IR}$	$\ln P_I \ln P_R$	-0.153	0.002	-62.607
$\gamma_{LR}$	$\ln P_L \ln P_R$	-0.039	0.004	-9.238
Equation		R <sup>2</sup>	Adj-R <sup>2</sup>	SSR
Short-run cost		0.995	0.990	0.022
Labor's cost share		0.485	0.407	0.006
Intermediate factors' cost share		0.990	0.988	0.002

After estimation, it is important to test if the cost function is valid based on the cost-minimization assumption. The estimated model shows that all the cost shares in the observations are positive, which meet the requirement of non-decreasing in input prices. In addition, except for the first season in 1996, all the marginal costs are positive, indicating the monotonicity condition is satisfied. However, the cost function does not show the property of concavity, and this has been identified in previous study which suggested the reason is that a producer seeking cost minimization would transfer the demand of an input factor to other



substitutes when the input factor price increases (Varian, 1992). In this example, the cost function does not show the concavity, implying that the substitutability among input factors of TMRT is not apparent, which is demonstrated in the next section.

## ECONOMIC CHARACTERISTICS ANALYSIS

### Price Elasticity

The price elasticity defined by equation (7) represents the impact of changes in the input prices on the change in factor demand, and the substitutability among input factors can also be measured in terms of cross elasticities. The the own-price elasticity and the cross-price elasticity are presented in Table 3.

The own-price elasticity of labor, intermediate factors, and replacement are -0.063, -0.138, -0.061, respectively, with negative signs as expected. The low own-price elasticity indicates that the demand of these three factors is inelastic to factor prices. In addition, the cross-price elasticity also represents the low substitutability among all the input factors. It is interesting to note that the cross elasticities of labor price to other input factors are positive, indicating that increasing the labor price is expected to reduce the demand of replacements as well as the maintenance, materials, and electricity as part of the intermediate factors. On the other hand, it is reasonable to confirm that the intermediate factors and replacements are complementary to each other, since increasing replacements is expected to require more intermediate cost.

$$\varepsilon_{ij} = \frac{\partial X_i / \partial P_j}{X_i / P_j} = \frac{S_j(\gamma_{ij} + S_i S_j)}{S_i S_j} ; \varepsilon_{ii} = \frac{\partial X_i / \partial P_i}{X_i / P_i} = \frac{S_i(\gamma_{ii} + S_i^2 - S_i)}{S_i^2} \quad (7)$$

TABLE 3- The price elasticity of factor demand

	Labor	Intermediate factors	Replacement
Labor	-0.063	0.306	0.242
Intermediate factors	0.339	-0.138	-0.202
Replacement	0.226	-0.149	-0.061

## Scale Economies

Braeutigam (1984) suggested the importance of classifying scale economies into economies of density and economies of size in the rail transit industry. When the network size is held fixed, the operator achieves scale economies on account of the increased passenger volume as defined by economies of density. In comparison, scale economies measured by the variable network size and passenger volume is referred as economies of size. The formulations of these two indicators are defined by equation (8) and equation (9), where ED is defined as return to density and ES is referred to return to system size.

$$ED = \left[ \frac{\partial \ln VC}{\partial \ln Y} \right]^{-1} = 5.17 \quad (8)$$

$$ES = \left[ \frac{\partial \ln VC}{\partial \ln Y} + \frac{\partial \ln VC}{\partial \ln K} \right]^{-1} = 1.07 \quad (9)$$

The estimated value of return to density is 5.17, implying that TMRT has strong property of economies of density when the network length is fixed. In contrast, the value of return to system size is 1.07, indicating that the scale economies of TMRT is much less apparent when the network length is taken into consideration. This is possibly because some of the TMRT routes are located in the suburbs with lower demand, and hence the value 1.07 of ES implies that these routes are not able to attract enough passengers to reduce the average cost in comparison to the other high-demand routes.

## Long-Run Minimum Efficient Scale

The long-run MES represents a optimal output level minimizing the long-run average cost when all factors are variable. The first step of estimating the MES is to construct the long-run average cost function. Braeutigam (1984) showed that the MES of long-run average cost curve can be computed by forming the short-run average cost function as a function of output and a fixed factor as in equation (10). In the long-run average cost function, the fixed factor cost is included in terms of the product of fixed factor price and network length. The fixed factor cost of first phase network by 2008 has amounted to US\$7.05 billion<sup>2</sup>. At the given interest rate of 8%, the fixed factor price per kilometer per season is US\$1.89 million as in equation (11).

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<sup>2</sup> Data retrieved from Department of Rapid Transit Systems, Taipei City Government, Annual Reports 1996- 2006. <http://english.dorts.taipei.gov.tw/>

$$LRAC(Y, \bar{P}, K) = \frac{1}{Y} [\exp(\ln SRVC) + P_f * K] \quad (10)$$

$$P_f = \left( \frac{\text{The constuction cost of operated routes* 8\%}}{\text{The length of operated routes*4 seasons}} \right) \\ = \text{US\$1.89 (million/season-km)} \quad (11)$$

To estimate the optimal  $Y$  and  $K$  that simultaneously minimize LRAC, the estimated parameters of short-run cost function and average input factor prices are substituted into the long-run average cost function as (12).

$$LRAC = \frac{1}{Y} [\exp(30.56 - 1.552 \ln Y + 0.179 \ln K + 0.05(\ln Y)^2 + 0.085(\ln K)^2 \\ + 57,000,000 * K)] \quad (12)$$

However, a closed form solution to find optimal level of  $Y$  and  $K$  is inexistent because the translog function involves  $K$  and its logarithm. To deal with this problem, a linear regression function between  $Y$  and  $K$  is constructed since the number of trips is highly related to the network length of TMRT's history. The linear regression function is specified in equation (13) with the value 0.934 of  $R^2$ .

$$Y = -32,000,000 + 1,746,468K \\ Y : \text{Trips (passengers /season)} \\ K : \text{Network length (km)} \quad (13)$$

Substitute (13) into (12), through a numerical procedure controlling the range of  $K$  between 50 km and 300 km, the relationship between long-run average cost and network length is obtained as Figure 2. The three-dimensional Figure 3 represents the relationship among network length, passenger trips, and long-run average cost.

As shown in Figure 2 and Figure 3, the minimum point of the long-run average cost curve occurs at 140 km of network length and 212.5 million passenger trips per season, corresponding to 2.3 million trips per day. When the operating scale in terms of network length and trips exceeds the minimum point, the long-run average will gradually increase, leading to diseconomies of scale. Therefore, the point  $(Y, K) = (212.5 \text{ million/season}, 140 \text{ km})$  is estimated as the long-run MES of TMRT.

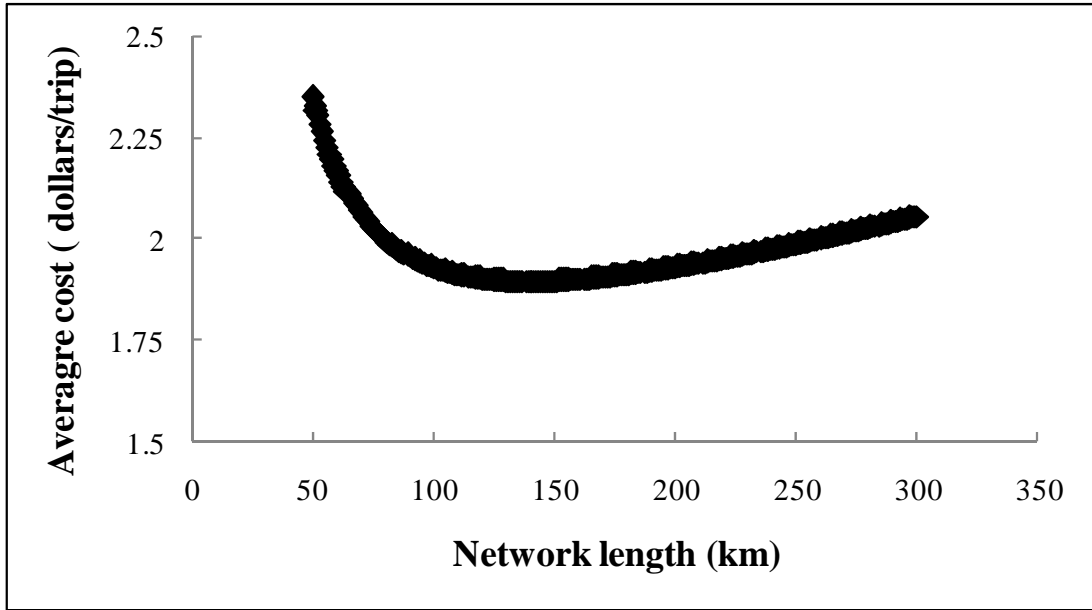


FIGURE 2- Long-run average corresponding to network length

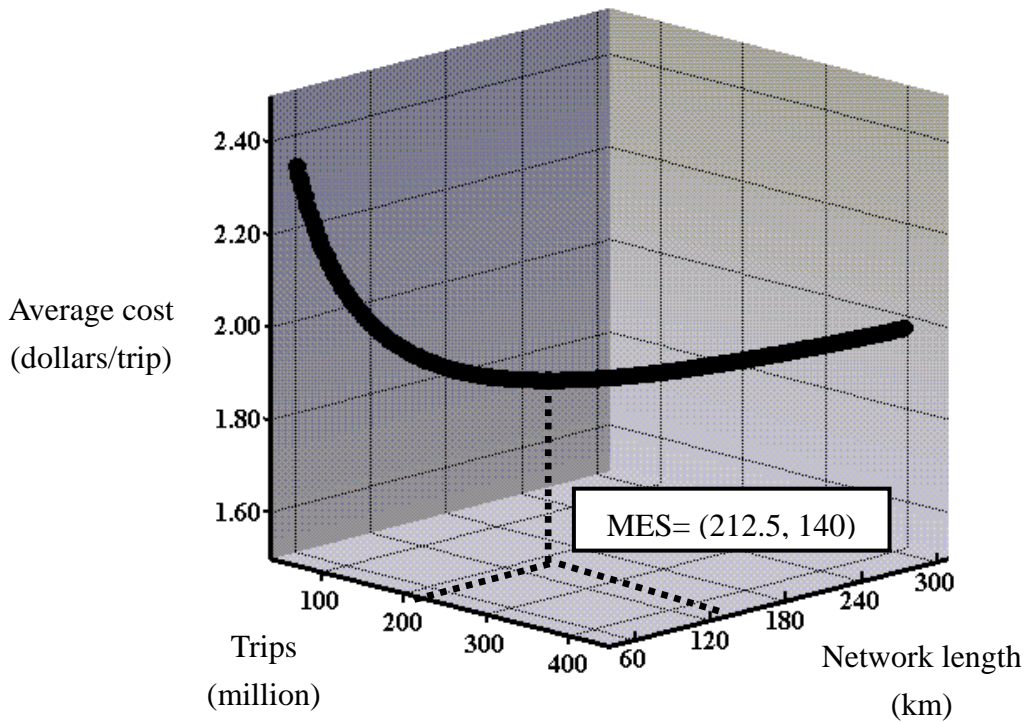


FIGURE 3- The relationship among trips, network length, and average cost

According to the trend of long-run average cost curve, it is interesting to discuss the relationship among average cost at different network lengths as shown in Table 4. In the operation of average length and average trips from 1996 to 2006, the average cost is US\$ 2.32 dollars. Compared to this basis, the average cost decreases to US\$ 1.89 dollars at the MES. This implies that there is a 22.8% cost exceeded from 1996 to 2006 as compared to the optimal cost at the MES. The average cost of the future network plan is also estimated. When the second phase of network expansion is completed, the network length of 155 km will result in an average cost of US\$1.90 dollars per passenger. An increase of only 0.1% on average cost compared to that of MES indicates that the scale of second phase network seems to be optimal. In contrast, when the third phase of network expansion is completed, the average cost will increase by 7% on the basis of MES, which may result in financial pressure to TMRT. This evidence suggests that when the capital cost of MRT infrastructure is taken into account, there is a need to examine the optimal size of network, and this is also applied to TRTC given it will be required to share a part of the capital cost of future infrastructure after the second phase of network is completed.

Table 4- Long-run average cost at different network lengths

	Mean value of 1996~2006	MES	2nd network phase	3rd Network phase
Length	51.32 km	140 km	155 km	280 km
Average cost	US\$2.32	US\$1.89	US\$1.90	US\$2.03
Increased cost	+22.8%	-	+0.1%	+7%

Note: The increased costs are compared to MES level

## CONCLUSIONS

This present paper applied a time series analysis to a specific public transportation operator using the cost function methodology. The scale economies and price elasticity of TMRT were analyzed by the estimated Translog short-run cost function. The results indicate that TMRT has substantial economies of density, whereas the economies of system size is much less apparent. From the price elasticity, it is shown that the input factors are inelastic to prices, and the substitutability among input factors is considerably low. This implies that the operator should improve the efficiency of the usage of input factors.

The major finding of the present paper is the estimation of the long-run MES, which occurs when network length reaches 140 km and the number of season trips is 212.5 million

corresponding to 2.36 million of daily trips. The long-run average cost curve shows that there is a 22.8% excess total cost from 1996 to 2006 compare to the lowest average cost at long-run MES. Of particular importance is that the completion of the third phase of the TMRT network will cause the average cost to rise by 7%. As TRTC will start to pay the share of capital cost in the future, the future financial plan for the network expansion needs to be carefully examined. Although this research finding may not be generalized to every transit operator, and it is also true that the cost minimization may not be the only focus of a public transportation operator, this paper highlights the importance of assessing the affordability and financial sustainability of MRT projects by taking account of the capital cost of MRT infrastructures.

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