STATISTICAL DETERIORATION PREDICTION IN CONSIDERATION OF HETEROGENEITY OF TUNNEL LUMINAIRE

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ABSTRACT

In this study, the authors propose a statistical deterioration prediction method using visual inspection data, targeting tunnel luminaire. In detail, the authors (1) calculates the expected deterioration path and expected lifespan of the luminaires based on the statistical analysis of deterioration factors, and (2) estimates the deterioration path considering the heterogeneity of the deterioration process of each luminaire or its group, with the mixed Markov deterioration hazard model. In addition, the hierarchical Bayesian estimation method is suggested as a concrete model estimation method. In order to verify the effectiveness of the proposed method, empirical analysis is carried out using the visual inspection data of 10,584 luminaires set in 11 tunnels. The authors first mentioned that the deterioration process significantly depends on tunnel grade and the distance from the tunnel entrance, and clarified that the expected lifespan of the luminaires is about 13.6 years and it varies about 4.3 years due to the above factors. Then, it was found that the expected lifespan of the tunnel luminaires varies from 6.9 years to 17.5 years by considering the heterogeneity of each tunnel.

Keywords: mixed Markov deterioration hazard model, tunnel luminaire, hierarchical Bayesian estimation

1. INTRODUCTION

All types of road equipment installed in highways deteriorate over the years. They pose potential risks of traffic accidents and vehicle damage accidents from falling down or becoming detached due to deterioration. Simultaneously, if these risks actually happen they can lead to the loss of social credibility of road administrators. Therefore, road administrators carry out periodical inspections to constantly observe the state of the equipment and to administer maintenance such as repairing and replacing. In most cases, the decisions

regarding repairing or replacing equipment are based on the experience of inspectors and professional engineers through visual inspection.
For example, tunnel lighting, which is one type of equipment installed on roads, include many different types of luminaire. Inspections target all of the luminaires, and the data obtained from inspections contain adequate information on the condition state of each luminaire. However, the data cannot be simply aggregated into a complete deterioration prediction for each tunnel or each route. As a result, most of the decisions regarding repairing or replacing are comprehensive decisions based on visual inspection data and the experience of inspectors and professional engineers. However, individual experience is in most cases tacit knowledge (Kobayashi, 2010). Therefore, it is difficult to comprehend the reason behind a decision from inspection data results. This may lead to a situation in which road administrators cannot explain their actions (accountability). In particular, in consideration of the recent attention on reducing public works spending and handing down personal skills and experience, the visualization of decision-making processes seems an important and urgent matter. This paper, as a basis study of this issue, focuses on the deterioration of tunnel luminaire used on highways (hereinafter, luminaire) and attempts to formulate a deterioration process using a mixed Markov deterioration hazard model. Section 2 summarizes the mixed Markov deterioration hazard model and Section 3 explains the estimation method. Section 4 introduces a case study in which the methodology is applied to the visual inspection data of actual luminaire.

2. MIXED MARKOV DETERIORATION HAZARD MODEL

2.1 Preconditions for Modelization

Visual inspection data of luminaire are generally evaluated as multilevel discrete data, such as that shown in Figure 1. This figure is a deterioration process of luminaire when they are continuously without repair or replacements until the serviceability limit state. The vertical axis is the condition state of the luminaire, with condition state 1 as the sound condition. As deterioration proceeds, the number increases. State I is the serviceability limit state. In the figure, the condition state changes from i-1 to i at time τ_{i-1}, and from i to i+1 at time τ_i. This transition of states can be observed completely when using monitoring system such as sensors. On the other hand, times τ_A and τ_B are when visual inspections were carried out. As we can clearly see, the information regarding the deterioration process that can be obtained from visual inspections is that the condition state is i at time τ_A, the first inspection, and j at time τ_B, the second inspection. In other words, visual inspections merely record the condition state at the time of inspections—they do not accurately determine the time (τ_{i-1} or τ_i) of an actual shift from one state to another. Therefore, when using visual inspection data to predict deterioration, one must consider these uncertainties.

Also, when predicting deterioration based on the transition of recorded states, the interval z between the two visual inspections ($z=\tau_B-\tau_A$) is important. In other words, even if the state transitions from i to j in the same way, if the interval between inspections (z) differs, the deterioration process will also differ. Although inspection intervals are specified in manuals,

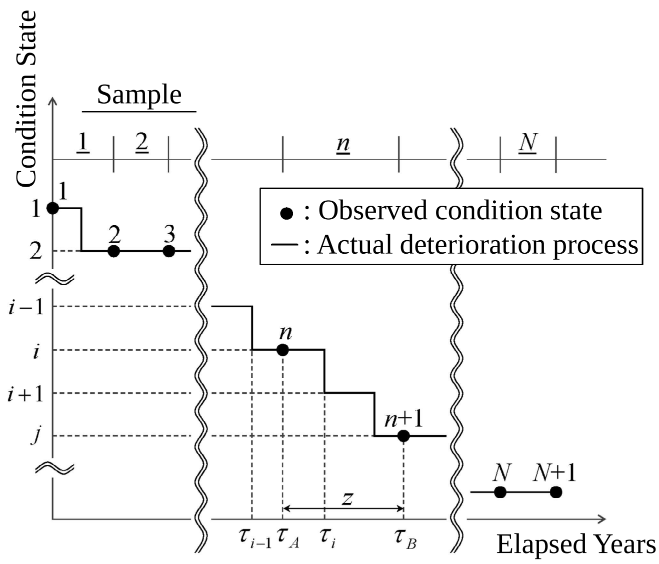


Figure 1 – Visual inspection data and its deterioration process

they are not strictly consistent with all luminaires. The inspection intervals are random variables, therefore they are uncertainties. Visual inspections only can provide limited information regarding the deterioration process, and the provided information includes uncertainties. However, even if the information is limited and uncertain, it is still possible to predict the deterioration of luminaire by modelizing the deterioration process with a stochastic model and estimating the deterioration process using visual inspection data.

A general stochastic model for expressing the transitions between discrete variables is the Markov chain model (Norris, 1998). In the Markov chain model, the probability of one state transitioning to another arbitrary state is expressed in a Markov transition probability, and the deterioration process is calculated based on the Markov transition probability matrix. If the transition between luminaire states is expressed using a Markov transition probability, then visual inspection data can be used to describe the deterioration process of luminaire. The concept of the Markov chain model is simple, generic, and flexible. However, when using visual inspection data to estimate the Markov chain model, it is necessary to resolve the problem of uncertainties mentioned above. For this reason it was extremely difficult to make realistic and highly accurate predictions. The development of the Markov deterioration hazard model (Tsuda et al., 2006), however, drastically improved practicability. This model is academically innovative, refines the former model, and is much more practical as it allows one to use the information obtainable from field inspections in the current system.

2.2 Markov Chain Model

The deterioration process of luminaire can be expressed with a Markov chain model using a transition probability matrix, with elements describing the transition probability from one state to another. Let us first look at the Markov transition probability, which is the basis of the Markov chain model.

Consider the transition of states between two time points. The state at time τ_A is $h(\tau_A)$, and the state at time τ_B is $h(\tau_B)$. If $h(\tau_A)=i$ and $h(\tau_B)=j$, the Markov transition probability is $\mathrm{Prob}[h(\tau_B)=j \mid h(\tau_A)=i]$. The condition for this Markov transition probability is that the state is i at time τ_A, and the conditional transition probability that the state will be j at time τ_B can be defined as:

$$\mathrm{Prob}[h(\tau_B)=j \mid h(\tau_A)=i]=\pi_{ij} \quad (1)$$

By deriving the pair of states (i, j) from a transition probability in this way, we can also obtain a Markov transition probability matrix.

$$\Pi=\begin{pmatrix} \pi_{11} & \cdots & \pi_{1I} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \pi_{II} \end{pmatrix} \quad (2)$$

The Markov transition probability (1) expresses the transition probability between the two conditional time points τ_A and τ_B. Naturally, if the inspection intervals differ, the transition probability will also differ. As long as there are no repairs, deterioration proceeds constantly, so $\pi_{ij}=0\ (i>j)$ is true. Also, from the definition of the transition probability, $\sum_{j=i}^{I}\pi_{ij}=1$ is true. In other words, regarding the Markov transition probability, the following must hold true.

$$\left.\begin{array}{c} \pi_{ij}\geq 0\ (i, j=1,\ldots, j) \\ \pi_{ij}=0\ (when \quad i>j) \\ \sum_{j=i}^{I}\pi_{ij}=1 \end{array}\right\} \quad (3)$$

The condition state I is the absorbing state in the Markov chain as long as there are no repairs, and $\pi_{II}=1$ is true. Moreover, the Markov transition probability is defined independently of past deterioration records. The Markov chain model satisfies the Markov property that, regardless of the time at which the state transitions from $i-1$ to i, the probability of the transition taking place between time τ_A and time τ_B depends only on the condition state at time τ_A.

2.3 Mixed Markov Deterioration Hazard Model

In order to estimate the Markov transition probability with consideration to visual inspection data and the data's uncertainties, a mixed Markov deterioration hazard model is implemented. In a mixed Markov deterioration hazard model, the luminaire's deterioration speed is expressed with a mixed hazard rate. Furthermore, the fluctuations in deterioration speed due to observable factors such as structural and environmental conditions are expressed with a standard hazard rate. On the other hand, the effects of unobservable factors that cannot be expressed with a standard hazard rate are summarized into one parameter that is applied to a group of multiple luminaires, and expressed as random variables. These parameters shall be called heterogeneity parameters. The mixed hazard rate that controls the deterioration speed is the product of the standard hazard rate and the heterogeneity parameter. How each luminaire group is set up is extremely important, and should incorporate the purpose of asset management.

For details regarding the mixed Markov deterioration hazard model, the reference (Obama, 2008) should be referred to. For the reader's convenience, the authors shall include a summary. First, the luminaires targeted for analysis are divided into an H number of luminaire groups (assessment unit).

Additionally, luminaire group $h\ (h=1,\dots,H)$ contains L_h luminaires. In order to establish a unique hazard rate to luminaire group h, an H number of heterogeneity parameters ε^h are implemented. At this time, the state of luminaire $l_h\ (l_h=1,\dots,L_h)$ of group h is $i\ (1=1,\dots,I-1)$, and its hazard rate can be expressed as:

$$\lambda_i^{l_h}=\tilde{\lambda}_i^{l_h}\varepsilon^h \qquad (i=1,\dots,I-1;h=1,\dots,H;l_h=1,\dots,L_h) \quad (4)$$

Here, $\tilde{\lambda}_i^{l_h}$ is the standard hazard rate of state i for luminaire l_h that includes luminaire group h. Heterogeneity parameter ε^h is a random variable that expresses the degree of deviation of luminaire group h from the standard hazard rate $\tilde{\lambda}_i^{l_h}$, therefore $\varepsilon^h \geq 0$ is true. The greater the value of heterogeneity parameter ε^h, the greater the deterioration speed of all luminaires in group h compared to the standard hazard rate. In equation (4), it should be noted that the same random variable ε^h is used on the hazard rate of all condition states. Therefore, when a certain state's deterioration speed is great, the deterioration speed of all other states will be relatively greater. Now, let's say that heterogeneity parameter ε^h is a random sample derived from gamma distribution $f(\varepsilon^h\mid\alpha,\gamma)$.

$$f(\varepsilon^h\mid\alpha,\gamma)=\frac{1}{\gamma^\alpha\Gamma(\alpha)}\left(\varepsilon^h\right)^{\alpha-1}\exp\left(-\frac{\varepsilon^h}{\gamma}\right) \quad (5)$$

The mean of the gamma distribution's probability density function $f(\varepsilon^h\mid\alpha,\gamma)$ is $\alpha\gamma$, and the variance is $\alpha\gamma^2$. Also, $\Gamma(\cdot)$ is the gamma function. Furthermore, the gamma distribution $\bar{g}(\varepsilon^h\mid\phi)$ with a mean of 1 and variance of $1/\phi$ can be expressed as:

$$\bar{g}(\varepsilon^h\mid\phi)=\frac{\phi^\phi}{\Gamma(\phi)}\left(\varepsilon^h\right)^{\phi-1}\exp\left(-\phi\varepsilon^h\right) \quad (6)$$

Here, the heterogeneity parameter ε^h of luminaire group $h\ (h=1,\dots,H)$ is fixed as $\bar{\varepsilon}^h$. At this time, if the luminaire's deterioration process is hypothetically expressed using an exponential hazard model, the probability ($\tilde{F}_i(y_i^{l_h})$) that luminaires in group h's luminaire l_h will stay in condition state i for longer than $y_i^{l_h}$, can be expressed, using the hazard rate of equation (4), as:

$$\tilde{F}_i(y_i^{l_h})=\exp(-\tilde{\lambda}_i^{l_h}\bar{\varepsilon}^h y_i^{l_h}) \quad (7)$$

At this time, the probability ($\pi_{ii}(z^{l_h})$) that the state of an arbitrary luminaire l_h of group h will be state i at inspection $\tau_A^{l_h}$, as well as the following inspection $\tau_B^{l_h}=\tau_A^{l_h}+z^{l_h}$ can be expressed as:

$$\pi_{ii}(z^{l_h})=\exp(-\tilde{\lambda}_i^{l_h}\bar{\varepsilon}^h z^{l_h}) \quad (8)$$

Also, the probability ($\pi_{ij}(z^{l_h})$) that the state will be i at inspection $\tau_A^{l_h}$ and j at inspection $\tau_B^{l_h}=\tau_A^{l_h}+z^{l_h}$ can be expressed as (Tsuda et al., 2006):

$$\pi_{ij}(z^{l_h}) = \sum_{s=i}^{j} \prod_{m=i,\neq s}^{j-1} \frac{\tilde{\lambda}_m^{l_h}}{\tilde{\lambda}_m^{l_h} - \tilde{\lambda}_s^{l_h}} \exp(-\tilde{\lambda}_s^{l_h} \bar{\varepsilon}^h z^{l_h}) \quad (9)$$

$$(i = 1, \ldots, I-1; j = i, \ldots, I-1; h = 1, \ldots H)$$

Due to the Markov transition probability's condition, $\pi_{iI}(z^{l_h})$ can be:

$$\pi_{iI}(z^{l_h}) = 1 - \sum_{j=i}^{I-1} \pi_{ij}(z^{l_h}) \quad (10)$$

By using the mixed Markov deterioration hazard model, the expected remaining duration RMD_i of deterioration from when the state first reaches i to when it reaches the next $i+1$ can be expressed, considering equation (7), as:

$$RMD_i = \int_0^\infty \tilde{F}_i(y_i) dy_i = \int_0^\infty \exp(-\lambda_i y_i) dy_i = \frac{1}{\lambda_i} \quad (11)$$

2.4 Visual Inspection Data and Hazard Rates

Now, let's say inspections are carried out on luminaire l_h $(h = 1, \ldots, H)$ at time $\tau_A^{l_h}$ (first inspection) and time $\tau_B^{l_h} = \tau_A^{l_h} + \bar{z}^{l_h}$ (second inspection). The information that can be obtained from visual inspections includes inspection interval $\bar{z}^{l_h}$, condition states $\bar{h}(\tau_A^{l_h})$ and $\bar{h}(\tau_B^{l_h})$, and characteristic vector $\bar{\boldsymbol{x}}^{l_h} = (\bar{x}_1^{l_h}, \ldots, \bar{x}_M^{l_h})$. Regarding the characteristic vector, the element $\bar{x}_m^{l_h}$ $(m = 1, \ldots, M)$ is the number m deterioration factor (characteristic variable) that influences the deterioration of luminaire l_h. The number one factor is a variable that is a constant term, therefore is identically $\bar{x}_1^{l_h} = 1$. Moreover, the symbol " ¯ " signifies an actual inspected value. Here, based on the pair of states from the two visual inspections, the dummy variable $\bar{\delta}_{ij}^{l_h}$ $(i=1,\ldots,I\text{-}1; j=1,\ldots,I; h=1,\ldots,H; l_h=1,\ldots,L_h)$ is defined as:

$$\bar{\delta}_{ij}^{l_h} = \begin{cases} 1 & when \quad \bar{h}(\tau_A^{l_h}) = i, \bar{h}(\tau_B^{l_h}) = j \\ 0 & otherwise \end{cases} \quad (12)$$

The dummy variable vector is expressed as $\bar{\boldsymbol{\delta}}^{l_h} = (\bar{\delta}_{11}^{l_h}, \ldots, \bar{\delta}_{I-1I}^{l_h})$. That is, the information of the inspection sample of luminaire l_k can be expressed as $\boldsymbol{\xi}^{l_h} = (\bar{\boldsymbol{\delta}}^{l_h}, \bar{z}^{l_h}, \bar{\boldsymbol{x}}^{l_h})$. Also, the standard hazard rate $\tilde{\lambda}_i^{l_h}$ $(i = 1, \ldots, I; h = 1, \ldots, H)$ changes depending on the luminaire's characteristic vector, and using the characteristic variable vector $\bar{\boldsymbol{x}}^{l_h}$ can be defined as:

$$\tilde{\lambda}_i^{l_h} = \exp(\bar{\boldsymbol{x}}^{l_h} \boldsymbol{\beta}_i') \quad (13)$$

However, $\boldsymbol{\beta}_i = (\beta_{i,1}, \ldots, \beta_{i,M})$ is the row vector of unknown parameter $\beta_{i,m}$ $(m = 1, \ldots, M)$, and the symbol " ´ " signifies transposition. Also, because $\bar{x}_1^{l_h} = 1$, $\beta_{i,1}$ signifies a constant term.

3. MIXED MARKOV DETERIORATION HAZARD MODEL AND HIERARCHICAL BAYESIAN ESTIMATION

3.1 Hierarchical Bayesian Estimation Method

A luminaire's visual inspection data are necessary in order to assess the heterogeneity of that specific luminaire, but in general there are usually no adequate records of visual inspection data for individual luminaires. Even under this situation, the mixed Markov deterioration hazard model suggested in this paper can be used to analyze the average deterioration process of all luminaires as well as deterioration characteristics of the targeted luminaire or luminaire group from visual inspection data records for the same luminaire group (assessment unit). In particular, the mixed Markov deterioration hazard model assumes that the heterogeneity parameter ε^h is subject to a prior distribution expressed as a gamma distribution with a mean of 1 and variance of $1/\phi$. Furthermore, with hierarchical Bayesian estimation, we can establish a prior distribution for the heterogeneity parameter's variance parameter ϕ (hyper parameter). These models with hierarchical prior distributions are called hierarchical Bayesian models (Kaito et al., 2012). The method is studied mostly in marketing analysis. This paper also uses a hierarchical Bayesian model to estimate the mixed Markov deterioration hazard model.

Bayesian estimation (Gill, 2007) is an estimation method that uses a parameter's prior distribution and the likelihood function defined from observed data to estimate the parameter's posterior distribution. Now, the unknown parameter vector is $\boldsymbol{\theta} = (\boldsymbol{\beta}, \phi, \boldsymbol{\varepsilon})$ and the visual inspection data is ξ, therefore the likelihood function can be expressed as $L(\boldsymbol{\theta} \mid \xi)$. If $\boldsymbol{\theta}$ is the random variable and it is subject to the prior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta})$, the joint posterior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta} \mid \xi)$ when visual inspection data ξ is obtained, according to Bayes' theorem (Bayes and Price, 1763), can be expressed as:

$$\boldsymbol{\pi}(\boldsymbol{\theta} \mid \xi) = \frac{L(\boldsymbol{\theta} \mid \xi)\boldsymbol{\pi}(\boldsymbol{\theta})}{\int_{\Theta} L(\boldsymbol{\theta} \mid \xi)\boldsymbol{\pi}(\boldsymbol{\theta})d\boldsymbol{\theta}} \quad (14)$$

However, Θ is the parameter space. At this time, the joint posterior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta} \mid \xi)$ can be expressed as:

$$\boldsymbol{\pi}(\boldsymbol{\theta} \mid \xi) \propto L(\boldsymbol{\theta} \mid \xi)\boldsymbol{\pi}(\boldsymbol{\theta}) \quad (15)$$

The symbol "$\propto$" signifies proportion. Moreover, the denominator of equation (14),

$$m(\xi) = \int_{\Theta} L(\boldsymbol{\theta} \mid \xi)\boldsymbol{\pi}(\boldsymbol{\theta})d\boldsymbol{\theta} \quad (16)$$

is a constant of $\boldsymbol{\pi}(\boldsymbol{\theta} \mid \xi)$ called a normalizing constant. In general, Bayesian estimation is conducted in the order of: 1) Establish the parameter's prior probability density function $\pi(\boldsymbol{\theta})$ based on prior experience information, 2) Define the likelihood function $L(\xi \mid \boldsymbol{\theta})$ using the obtained data, 3) Revise the prior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta})$ based on Bayes' theorem (14), and obtain the posterior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta} \mid \xi)$ of parameter $\boldsymbol{\theta}$. The unknown parameter's prior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta})$ in the mixed Markov deterioration hazard model is:

$$\begin{aligned}\boldsymbol{\pi}(\boldsymbol{\theta}) &= \boldsymbol{\pi}(\boldsymbol{\beta}, \phi, \boldsymbol{\varepsilon}) \\ &= \boldsymbol{\pi}(\boldsymbol{\beta})\boldsymbol{\pi}(\varepsilon \mid \phi)\boldsymbol{\pi}(\phi) \qquad (17) \\ &= \prod_{i=1}^{I-1}\prod_{h=1}^{H} \boldsymbol{\pi}(\boldsymbol{\beta}_i)\boldsymbol{\pi}(\varepsilon^h \mid \phi)\boldsymbol{\pi}(\phi)\end{aligned}$$

We can see that probability distribution of heterogeneity parameter $\boldsymbol{\varepsilon}$ in this paper's mixed Markov deterioration hazard model and the prior distribution of parameter ϕ in the probability distribution have hierarchical structures. The hierarchical Bayesian estimation method establishes prior distributions for each unknown parameter $\boldsymbol{\theta} = (\boldsymbol{\beta}, \phi, \boldsymbol{\varepsilon})$, and calculates conditional posterior probability density functions for each parameter.

3.2 Formulation of Posterior Distributions

Let's say parameter $\boldsymbol{\theta} = (\boldsymbol{\beta}, \phi, \boldsymbol{\varepsilon})$ is a given condition. At this time, the joint probability (likelihood) $L(\boldsymbol{\theta} \mid \xi)$ of the visual inspection data $\boldsymbol{\xi}$ is:

$$L(\boldsymbol{\theta} \mid \xi) = \prod_{i=1}^{I-1} \prod_{j=i}^{I} \prod_{h=1}^{H} \prod_{l_h=1}^{L_h} \left\{ \pi_{ij} \left(\bar{z}^{l_h}, \bar{\boldsymbol{x}}^{l_h} \mid \boldsymbol{\beta}, \phi, \varepsilon^h \right) \right\}^{\bar{\delta}_{ij}^{l_h}}$$
$$= \prod_{i=1}^{I-1} \prod_{j=i}^{I} \prod_{h=1}^{H} \prod_{l_h=1}^{L_h} \left\{ \sum_{s=i}^{j} \prod_{m=i,\neq s}^{j-1} \frac{\tilde{\lambda}_m^{l_h}}{\tilde{\lambda}_m^{l_h} - \tilde{\lambda}_s^{l_h}} \exp(-\tilde{\lambda}_s^{l_h} \bar{\varepsilon}^h z^{l_h}) \right\}^{\bar{\delta}_{ij}^{l_h}} \quad (18)$$

Also, the prior probability density functions of unknown parameters $\boldsymbol{\theta} = (\boldsymbol{\beta}, \phi, \boldsymbol{\varepsilon})$ in equation (17) are as follows. First, a multi-dimensional normal distribution was used for prior probability density function $\boldsymbol{\pi}(\boldsymbol{\beta}_i)$ of unknown parameter $\boldsymbol{\beta}_i$. The probability density function of the M-dimensional normal distribution is derived from:

$$g(\boldsymbol{\beta}_i \mid \boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i) = \frac{1}{(2\pi)^{M/2} \sqrt{|\boldsymbol{\Sigma}_i|}} \cdot \exp\left\{ -\frac{1}{2} (\boldsymbol{\beta}_i - \boldsymbol{\mu}_i) \boldsymbol{\Sigma}_i^{-1} (\boldsymbol{\beta}_i - \boldsymbol{\mu}_i)' \right\} \quad (19)$$

However, $\boldsymbol{\mu}_i$ is an expected value vector and $\boldsymbol{\Sigma}_i$ is a variance-covariance matrix. The prior probability density function $\boldsymbol{\pi}(\varepsilon^h \mid \phi)$ of ε^h is already given in gamma distribution (6). Furthermore, a gamma distribution $h(\phi \mid \alpha_0, \gamma_0)$ is established for prior probability density function of equation (17)'s variance parameter ϕ. Therefore, the joint posterior probability density function $\boldsymbol{\pi}(\boldsymbol{\theta} \mid \xi)$ can be formulated as:

$$\boldsymbol{\pi}(\boldsymbol{\theta} \mid \xi) \propto L(\boldsymbol{\theta} \mid \xi) \prod_{i=1}^{I-1} \prod_{h=1}^{H} \boldsymbol{\pi}(\boldsymbol{\beta}_i) \boldsymbol{\pi}(\varepsilon^h \mid \phi) \boldsymbol{\pi}(\phi)$$
$$\propto \prod_{i=1}^{I-1} \prod_{j=i}^{I} \prod_{h=1}^{H} \prod_{l_h=1}^{L_h} \left\{ \sum_{s=i}^{j} \prod_{m=i,\neq s}^{j-1} \frac{\tilde{\lambda}_m^{l_h}}{\tilde{\lambda}_m^{l_h} - \tilde{\lambda}_s^{l_h}} \exp(-\tilde{\lambda}_s^{l_h} \bar{\varepsilon}^h z^{l_h}) \right\}^{\bar{\delta}_{ij}^{l_h}}$$
$$\cdot \prod_{i=1}^{I-1} \exp\left\{ -\frac{1}{2} (\boldsymbol{\beta}_i - \boldsymbol{\mu}_i) \boldsymbol{\Sigma}_i^{-1} (\boldsymbol{\beta}_i - \boldsymbol{\mu}_i)' \right\} \quad (20)$$
$$\cdot \prod_{h=1}^{H} \frac{\phi^\phi}{\Gamma(\phi)} \left(\varepsilon^h \right)^{\phi-1} \exp\left(-\phi \varepsilon^h \right)$$
$$\cdot \frac{1}{\gamma_0^{\alpha_0} \Gamma(\alpha_0)} \phi^{\alpha_0 - 1} \exp\left(-\frac{\phi}{\gamma_0} \right)$$

However, it is difficult to analytically estimate the joint probability density function (20) and acquire direct samples. Therefore, in this paper the unknown parameter vector $\boldsymbol{\theta}$ was estimated from a Markov chain Monte Carlo method (Liang et al., 2010) (MCMC method). An MCMC method obtains samples from posterior distributions by repeated random generation of parameter $\boldsymbol{\theta}$ samples from each parameter's conditional probability density functions.

Table 1 – Description of condition state

Rating	State	Description of Condition
OK	1	No damage
B	2	There is damage over a wide area but no malfunction. It is necessary to continue to observe the progress of the damage.
A	3	There is damage and malfunction. Repairs are required but it is not urgent.
AA	3	The damage is significant, and the malfunction requires urgent repair.

Table 2 – Data specifications of inspection data.

No. of Tunnels	19				
Start of service	1990–1991				
Total no. of Samples	15,722				
Sample breakdown			Posterior state		
			1 (OK)	2 (B)	3 (A, AA)
	Prior state	1 (OK)	31	5,044	4,825
		2 (B)	-	262	3,211
		3 (A, AA)	-	-	2,349

4. CASE STUDY

4.1 Outline of Visual Inspection Data

Bayesian estimation is conducted on the mixed Markov deterioration hazard model using actual visual inspection data from luminaire inspections. The database contained information on two types of luminaire—steel and stainless steel (SUS). The SUS luminaires were installed recently and did not have adequate inspection data records for estimation. Therefore, only steel luminaires were used in the analysis this time.

Luminaire visual inspection results were given four classifications: OK, B, A, and AA. The best condition state is OK, which moves to B, A, then AA as deterioration proceeds. Specific valuation statdards are shown in Table 1. Classification AA requires urgent repairs and classification A requires repairs, and both classifications were considered equal from a management viewpoint. Therefore, the classifications shown in Table 1 were reclassified into three steps for this paper: OK as rating 1, B as rating 2, and A and AA as rating 3. When luminaire ratings are classified in three steps, the hazard rate of rating 1 and 2 (excluding rating 3) can be defined with the mixed Markov deterioration hazard model.

Data specifications used in the estimation are shown in Table 2. The luminaires used in the study are installed in 19 tunnels that began service between 1990 and 1991. The number of luminaires installed is 10,584. A total of 15,722 inspection samples were available from multiple visual inspections carried out between 2005 and 2009. As mentioned in Section 2.4, each inspection sample includes information on the two visual inspection results (i, j),inspection interval (z), and characteristic variable ($\boldsymbol{x}$). Table 2 also includes the breakdown of samples for each pair of states. Table 3 shows information on the 19 tunnels

Table 3 – Data Specifications of tunnel structure and relative rates of rating 3

Tunnel	Direction	Length (m)	Grade	Inspection records				
				FY2005	FY2006	FY2007	FY2008	FY2009
A	Up	422	2.80%	50.30%	–	–	Replaced	–
	Down	447	-2.80%	76.50%				
B	Up	1,137	-2.43%	55.70%	–	–	97.80%	–
	Down	1,070	2.43%	88.80%			99.80%	
C	Up	1,606	-0.57%	26.60%	–	–	100.00%	–
	Down	1,578	0.57%	17.90%			96.90%	
D	Up	498	0.57%	53.00%	–	–	99.70%	–
E	Down	113	-0.57%	53.30%	–	–	68.40%	–
F	Down	278	-0.57%	72.20%	–	–	95.30%	–
G	Up	1,033	2.50%	42.10%	–	–	79.30%	–
	Down	986	-2.50%	8.00%			85.60%	
H	Up	258	-0.57%	–	90.20%	–	Replaced	–
	Down	211	0.57%		76.80%			
I	Up	258	-0.57%	–	77.10%	–	Replaced	–
	Down	260	0.57%		62.40%			
J	Up	292	0.57%	53.10%	–	–	91.50%	–
	Down	315	-0.57%	56.20%			77.20%	
K	Up	408	1.50%	35.30%	–	–	98.50%	Replaced
	Down	463	-1.50%	13.70%			96.90%	

(in particular, items that may influence luminaire deterioration) and inspection results. Of the 11 tunnels (A to K), D has only an up-line, and E and F have only a down-line, with two lanes in the same direction. All other tunnels have separate tunnels for the up-line and down-line, so there are a total of 19 tunnels. Table 3 also includes visual inspection results over the course of five years, from 2005 to 2009. The up-line and down-line tunnels of A, H, and I (six tunnels) had one inspection during this period. These cases would generally not be included as samples because the two inspection results cannot be obtained. Therefore, this case study assumes the luminaires were in rating 1 at the start of service, which gives us a complete sample (pair of ratings) when the one inspection result is added. For this reason, the number of samples with a 15-year inspection interval increased. This is the reason why Table 2 includes many samples that transitioned from rating 1 in the prior state to rating 2 or 3 in the posterior state. Furthermore, it should be noted that the lack of data between start of service and 2005 is not because visual inspections were not carried out during this time, but because the results were not digitally recorded in a way that allowed analysis. The remaining thirteen tunnels had two inspections between 2005 and 2009, giving us two samples for each luminaire. The same table includes the rates of luminaires of rating 3 (classifications A or AA) in each tunnel, for each inspection year. In addition, because all tunnels began service at very similar times, it can be seen that the deterioration of luminaires in tunnel B's down-line and tunnel H's up-line is relatively fast. Luminaires that were evaluated AA after inspections were given emergency repairs to prevent falling, later replaced, and put under continued monitoring.

Table 4 – Estimation results for unknown parameters

Rating i	Posterior distribution Statistic	Constant term $\beta_{i,0}$	Absolute value of Grade $\beta_{i,1}$	Distance from Entrance $\beta_{i,2}$
1	Expected value	-1.575	0.323	-
	(min 5%, max 5%)	(-1.650,-1.494)	(0.222,0.429)	
	Geweke test Statistic	-0.064	0.109	
2	Expected value	-2.224	-	-0.292
	(min 5%, max 5%)	(-2.298,-2.157)		(-0.393,-0.187)
	Geweke test Statistic	0.002		-0.032
Variance Parameter ϕ	Expected value	11.855		
	(min 5%, max 5%)	(10.46,13.284)		
	Geweke test Statistic	0.161		

4.2 Estimation of Standard Hazard Rate

For the estimation of the mixed Markov deterioration hazard model, the standard hazard rate (equation 13) was first calculated. In the actual hierarchical Bayesian estimation method, all parameters including heterogeneity parameters are simultaneously estimated. Here, however, for convenience of explanation the authors shall focus on the standard hazard rate. The standard hazard rate expresses the average deterioration speed of the overall targeted luminaires. As shown in equation (13), it is possible to consider the impact of various factors that may influence the luminaires' deterioration. These factors the authors shall call characteristic variables. All factors that are considered influential to deterioration based on empirical knowledge and are also observable can be included as characteristic variables. This study includes nine factors as possible characteristic variables: number of antifreeze sprays, distance from entrance, lane (driving lane, passing lane), direction, tunnel grade, tunnel continuity, damage classification (corrosion, other), etc. In particular, it was known from empirical knowledge and former inspections that (1) Luminaires installed at tunnel entrances uniformly deteriorate at a higher speed, and luminaires installed within tunnels deteriorate more slowly as the distance from entrance is farther, and (2) luminaires installed in locations with a grade absolute value of 2.0% or more (2.0% or more for up-lines and 2.0% or less for down-lines) deteriorate at a higher speed. Moreover, in general the traffic load of large-size vehicles should be considered, but because the 19 tunnels were located relatively close and the traffic loads were similar, this was excluded from the study. When conducting statistical analysis, there are such cases in which deterioration factors that are generally considered important but have a miniscule influence within a certain area are determined insignificant.

The final results of the standard hazard rates estimated using actual visual inspection data (states) with the above conditions are shown in Table 4. In the table, hazard rate are estimated for ratings 1 and 2. It was confirmed that the absolute value of grade influences luminaire deterioration at rating 1, while the distance from entrance influences deterioration at rating 2. In addition, the expected values, minimum and maximum 5% values, and Geweke test statistics (Geweke, 1996) are given for each unknown parameter. When obtaining the results of Table 4, individual standard hazard rates were estimated for each possible characteristic variable. Those that had a Geweke test statistic higher than 1.96 in absolute value, which shows a low convergence to the posterior distribution of parameters sampled in the MCMC method, were not applied. Even if the Geweke test statistic was less than 1.96, those that (1) did not meet the sign conditions or (2) had 0 in either the minimum or maximum 5% value were also not used. A simple explanation is required. The standard hazard rate of equation (13) has the characteristic of a larger hazard rate resulting in a greater deterioration speed. For example, in this analysis we know from prior information that the closer a luminaire is installed to the entrance, the faster the deterioration. Therefore, in order to explicitly express this relation in equation (13), the unknown parameter in question must have a negative value. The actual estimated value of $\beta_{1,2}$ is negative. On the other hand, if the unknown parameter in question has a positive value, it means the further the distance from the entrance, the faster the deterioration. This contradicts the prior information. It was considered that these cases do not meet the sign conditions.

In addition, if either the minimum or maximum 5% value includes 0, this means there are both negative and positive cases, and the deterioration processes will technologically be opposite. Therefore, these were not included as characteristic variables. The characteristic variables that were selected in this way were estimated as shown in Table 4 as optimal models of comprehensive combinations of two or more variables. The Akaike information criterion (deLeeuw, 1992) (AIC) was used to select optimal models. The combinations of characteristic variables with the smallest AIC were selected. AIC is a measure of the relative fitness of a model and actual measures; therefore the smallest AIC signifies the optimal model.

Regarding Table 4, two types of characteristic variables were selected: the tunnel grade and distance from tunnel entrance. For rating 2 the characteristic variable representing grade was excluded, and for rating 1 the characteristic variable representing the distance from entrance was excluded for the reasons explained above. For the grade, the dummy variable $x_1^{l_h}$ (hereinafter, grade dummy) with an absolute value of less than 2.0% as 0 and 2.0% or more as 1 was added. For the distance from the entrance, two characteristic variables were applied—the dummy variable $\delta_2^{l_h}$ (hereinafter, location dummy) with the entrance area as 0 and all other areas as 1, and the variable $x_2^{l_h}$ representing the distance from the entrance minus the entrance length (hereinafter, distance dummy) (for example, if the entrance length is 300m and a luminaire is installed at 500m, from the entrance, the value will be 200m). At this time, the hazard rate $\lambda_i^{l_h}$ of state *i* of luminaire $l_h(l_h = 1,\ldots,L_h)$ in tunnel $h(h = 1,\ldots,19)$ can be expressed as:

$$\lambda_i^{l_h} = \exp\left(\beta_{i,0} + \beta_{i,1}x_1^{l_h} + \beta_{i,2}\delta_2^{l_h}x_2^{l_h}\right)\varepsilon^h \quad (21)$$

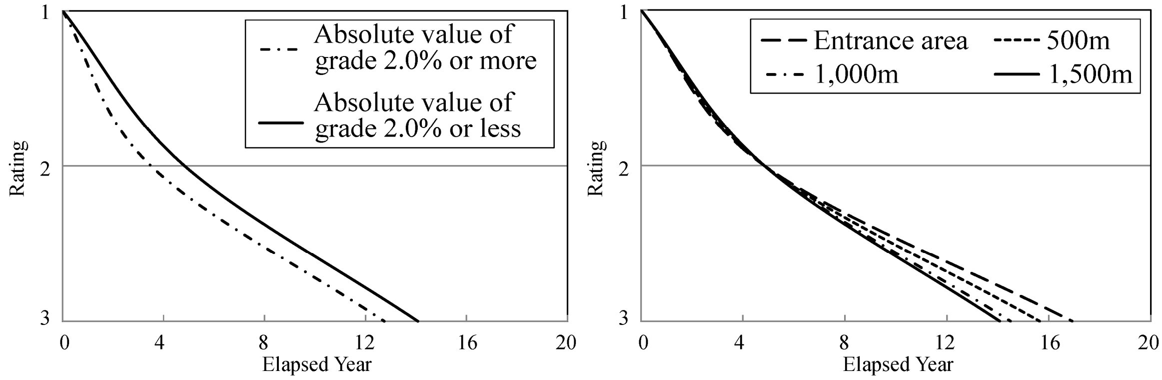
Furthermore, considering the estimation results of Table 4, the standard hazard rate (at this time $\varepsilon = 1.0$) of state 1 is:

$$\lambda_i^{l_h} = \exp\left(-1.575 + 0.323x_1^{l_h} + 0\delta_2^{l_h}x_2^{l_h}\right) \quad (22)$$

Finally, by applying actual values to the grade dummy, location dummy, and distance dummy of the above equation, we can calculate the standard hazard rate. Regarding whether a luminaire is in the entrance area or not, five possible distances were set between 200m and 400m from the entrance in 50m intervals (200m, 250m, 300m, 350m, 400m). Table 5 shows the AIC results of the estimated mixed Markov deterioration hazard model on each possibility. From the table, we can see that the AIC is smallest when the entrance area is set as 300m; therefore, this definition was used for this study. According to the estimation results of the unknown parameters, we found that the parameter regarding the absolute value of grade $\beta_{1,1}$ has a positive value; therefore, luminaires with a grade absolute value of 2.0% or more deteriorate fast. Also, because the parameter regarding the distance from the entrance $\beta_{2,2}$ has a negative value, we can see that luminaires installed in the entrance area deteriorate fast. Moreover, the differences in hazard rates due to characteristic variables can be quantitatively valuated as differences in life expectancy.

Table 5 – AIC for each distance from tunnel entrance

Distance from tunnel entrance	AIC
200m	23,804
250m	23,805
300m	23,803
350m	23,806
400m	23,807



(a) Focused on the change of grade

(b) Focused on the change of distance from entrance

Figure 2 – Comparison of expected deterioration paths

4.3 Calculation of Expected Deterioration Processes

Figure 2 shows the expected deterioration processes based on life expectancies calculated with equation (11). The processes are average deterioration processes with arbitrary characteristic variables. Figure 2 (a) compares the deterioration processes of luminaires with a grade dummy of 2.0% or more and luminaires with a grade dummy of less than 2.0%. The location dummy is fixed at the entrance area ($\delta_2^{l_h} = 0$). The life expectancy of luminaires with a grade absolute value of 2.0% or more (average number of years from start of service to transition to state 3) is 12.7 years, and with a grade absolute value of less than 2.0% is 14.1 years. Figure 2 (b) shows four types of expected deterioration processes: luminaires installed in the entrance area, 500m from the entrance, 1000, from the entrance, and 1500m from the entrance. The grade dummy is fixed at less than 2.0%. The life expectancies are 14.1 years, 14.5 years, 15.6 years, and 16.9 years, respectively. Figure 2 (b) uses the distances of 500m, 1000m, and 1500m as representative values, but the expected deterioration processes of arbitrary distances can also be calculated. For the deterioration prediction using the mixed Markov deterioration hazard model in this study, the tunnel grade and distance from the tunnel entrance were selected as characteristic variables.

4.4 Estimation of the Mixed Markov Deterioration Hazard Model

The estimation of the mixed Markov deterioration hazard model specifically involved estimating 26 parameters: 2*3=6 types of unknown parameters $\boldsymbol{\beta}$, 19 heterogeneity parameters $\boldsymbol{\varepsilon}$, and variance parameter ϕ. As shown in section 4.3, expected deterioration processes can be calculated from standard hazard rates, and differences between each deterioration factor selected as characteristic variables can be understood. However, there are generally two or three types of deterioration factors selected as characteristic variables,

Table 6 – Estimated heterogeneity parameters and life expectancies

Tunnel	Direction	Heterogeneity parameter ε				Life Expectancy (year)
		Expected value	Min 5%	Max 5%	Geweke test	
B	Down	**1.857**	1.744	1.982	-0.120	**6.862**
H	Up	**1.700**	1.505	1.890	-0.008	**8.277**
F	Down	**1.421**	1.265	1.578	0.000	**9.904**
A	Down	**1.197**	1.080	1.329	-0.023	**10.643**
I	Up	**1.259**	1.128	1.402	0.007	**11.181**
H	Down	**1.245**	1.091	1.414	0.005	**11.303**
B	Up	**1.118**	1.021	1.223	-0.038	**11.399**
D	Up	**1.217**	1.089	1.335	0.038	**11.562**
J	Up	**1.083**	0.987	1.189	-0.008	**12.992**
C	Up	**1.048**	0.969	1.134	0.025	**13.429**
E	Down	**0.995**	0.852	1.142	0.041	**14.146**
I	Down	**0.982**	0.887	1.087	-0.004	**14.326**
K	Up	**0.978**	0.884	1.067	-0.016	**14.383**
J	Down	**0.974**	0.883	1.076	0.061	**14.454**
C	Down	**0.968**	0.892	1.047	0.021	**14.542**
G	Up	**0.871**	0.799	0.955	-0.064	**14.622**
K	Down	**0.853**	0.778	0.930	0.023	**16.502**
A	Up	**0.744**	0.666	0.831	-0.002	**17.135**
G	Down	**0.728**	0.654	0.812	-0.022	**17.502**

and we cannot perfectly calculate the deterioration processes of various types of luminaires for each deterioration factor. There will be deviations between the estimated deterioration processes and the actual deterioration. On the other hand, these deviations are defined as heterogeneity in mixed Markov deterioration hazard models, and it is possible to express the deviation, which cannot be expressed with characteristic variables, by applying heterogeneity parameters that represent the amount of heterogeneity to each tunnel. Table 6 shows the heterogeneity parameter expected value, minimum and maximum 5% values, and Geweke test statistics of all 19 tunnels. The tunnels' life expectancies as calculated using characteristic variables (actual grade and entrance area) are also shown in ascending order. The average value of the heterogeneity parameter is 1.0. The tunnel luminaire's life expectancy relatively decreases when the value is greater than 1.0, and relatively increases when less than 1.0. This is because the heterogeneity parameter was defined as a random variable that is subject to gamma distribution. The estimated heterogeneity parameters of the 19 tunnels vary between 1.857 and 0.728. For example, tunnel B's down-line has the greatest heterogeneity parameter, which means that compared to an average tunnel with a heterogeneity parameter of 1.0, the luminaire deteriorates at a speed twice as fast even with the same tunnel grade and entrance distance. That is, the life expectancy is about half.

The table also shows the life expectancies of each tunnel's luminaires, calculated from the mixed hazard rates. As already mentioned, mixed hazard rates are the product of standard hazard rates and heterogeneity parameters. Therefore, in order to calculate the life expectancies of each tunnel's luminaires, it is necessary to establish the average characteristic variable values to use in each tunnel's standard hazard rate. The life expectancies were calculated in this way. When the life expectancies and heterogeneity parameters are compared, we can see that the order from smaller to greater does not always correspond because the characteristic variable conditions are not the same for all tunnels.

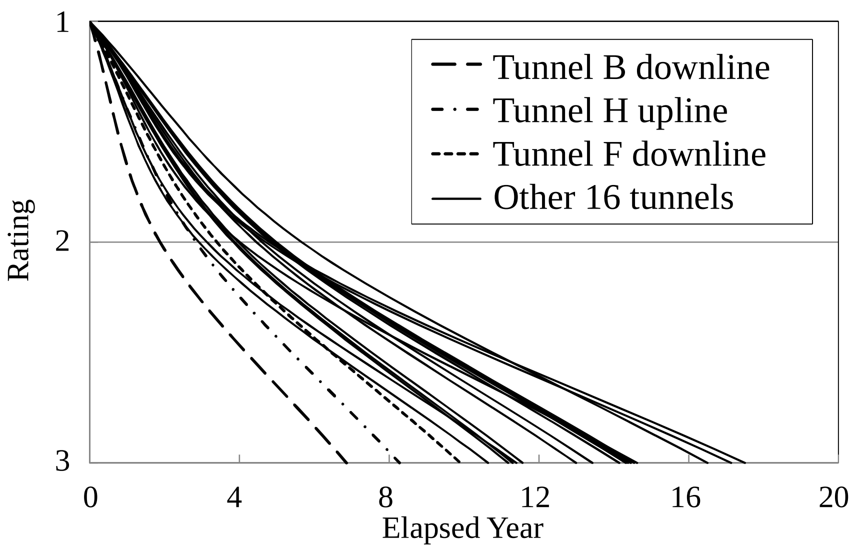


Figure 3 – Expected deterioration paths considering heterogeneity.

Heterogeneity parameters are used merely to valuate deterioration differences in tunnels with the same characteristic variable conditions. Figure 3 shows the expected deterioration processes of luminaires of all 19 tunnels. Characteristic variables are the same as Table 6, with the actual grade and entrance areas. As can be seen from Table 6 and Figure 3, the deterioration processes of each tunnel's luminaires can be calculated, with the shortest lifespan at 6.7 years and the longest at 17.5 years.

4.5 Cautionary Points

In this analysis, heterogeneity parameters were established for each tunnel. Of course, it is possible to establish heterogeneity parameters in more detail, with the smallest division being for each luminaire. However, heterogeneity parameters should be established in accordance to the purpose of calculating life expectancies or deterioration processes (for example, budget plans for each tunnel or setting the order of priority for luminaire repairs within one tunnel). Moreover, the cause of variations in heterogeneity parameters is unclear. However, the important thing is to make an effort to understand this cause by estimations of heterogeneity parameters and relative comparisons. In fact, when we investigated tunnels with higher heterogeneity parameters in this empirical analysis, we were able to confirm unique conditions surrounding the entrances such as a nearby metal plating factory or river. In addition, if the number of antifreeze sprays, which was not selected as a characteristic variable in this study, should be considered in the estimation, then it is necessary to accurately record the frequency of antifreeze sprays. The foremost purpose of visual inspections is to confirm the deterioration state of luminaires, but this merely requires identifying deteriorated luminaires and conducting repairs. There is no incentive to record inspection data and other related information. There is no reason to change methods of analyzing or recording inspection data if it is possible to continue maintenance in the same way with the same budget. However, as explained in Chapter 1, when considering the problem of accountability regarding infrastructure maintenance, including inspections, repairs and replacements, asset management becomes essential. Therefore, it is important that administrators are aware that the purpose of visual inspections includes collecting information needed for asset management. Through statistical analysis, the importance of accumulating inspection data and the technological meaning of recording supplemental inspection data will become clear.

5. CONCLUSION

This study formularizes the deterioration of luminaires, which had hitherto been handled as tacit knowledge (or determined through simple statistical analysis), using a mixed Markov deterioration hazard model, and suggests a hierarchical Bayesian method using visual inspection data for estimation. The mixed Markov deterioration hazard model allows the quantitative inclusion of observable deterioration factors as characteristic variables, as well as unobservable deterioration factors as heterogeneity parameters. As a result, we were able to calculate an average expected deterioration process for all targeted luminaires as well as micro deterioration processes for each heterogeneity parameter. Empirical analysis was conducted using visual inspection data of luminaires in 19 tunnels. The results show that (1) tunnel grade and (2) the distance from the tunnel entrance have a significant influence on the deterioration process of luminaires. Also, it was clarified that the expected lifespan of the target luminaires is 13.6 years, but when heterogeneity is considered, the expected lifespan varies from 6.9 years to 17.5 years. Some luminaires have an assumed lifespan based on various material experiments and laboratory tests. However, in order to accurately understand the life expectancy in the actual environment, it is crucial to use inspection data collected in the field. There were no previous case studies that estimated the lifespan of luminaire from actual data. This study carefully predicts deterioration using information collected through practical inspections, and we believe it offers basic information useful for asset management.

On the other hand, further issues may include (1) An expansion of the target area to consider other environmental conditions, as this study analyzes tunnels only in a certain area, (2) Calculations of luminaire lifecycle expenses based on the predicted deterioration, and (3) Visualization of relations between various elements concerning tunnel lighting, such as luminaire deterioration, lighting faults, average road surface luminance, road luminance distribution, glare, and induction. Finally, in this study's estimation, we were not able to include deterioration information on luminaires that were replaced, as their states immediately before replacement were not observed. By widely exposing the methodology proposed in this paper we hope to improve maintenance work, which is a further issue for administrators.

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