AGGREGATE ESTIMATION OF THE PRICE ELASTICITY OF DEMAND FOR PUBLIC TRANSPORT IN INTEGRATED FARE SYSTEMS: THE CASE OF TRANSANTIAGO

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ABSTRACT

Price elasticities of demand for public transport are a key determinant in evaluating the impact of changes in fares on user flows, yet in many integrated fare transit systems, estimating these indicators is often hampered by two realities: the fare changes for different modes are implemented simultaneously and their magnitudes are highly correlated. This strong collinearity is particularly problematic in linear or log-linear models, commonly used for elasticity estimation, and in a case study of Santiago, Chile, robust results with such specifications proved elusive. This paper presents a method based on discrete choice models to estimate the elasticities in an integrated fare system that overcomes these econometric problems, generating results that are both robust and consistent with those reported in the literature. The proposed models are also easy to update and evaluate.

Keywords: elasticities, integrated fare, public transport, Transantiago, bus, Metro, collinearity, endogeneity.

1. INTRODUCTION

The purpose of this paper is to estimate the price elasticity of demand for public transport in a system with integrated fares by using high frequency data on passenger flows to the system's various services and their respective fares (available on a daily basis for different times of the day) in conjunction with discrete choice models rather than the commonly used linear multiple regression models.

The main obstacle to estimating price elasticity of demand for transit services in integrated fare systems is their very limited variation in fares across different mode and time period alternatives. This characteristic results in a severe problem of collinearity between the principal explanatory variables in models relating price and demand for transport. In linear regression models, high collinearity complicates parameter identification and leads to estimates that are inaccurate and highly sensitive to changes in the sample or the model specification (Greene, 2011).

Another difficulty in estimating price elasticity of demand is fare endogeneity, which arises because the number of trips responds to fare levels while fares for different time periods are set as a function of the number of trips. Thus, peak-hour fares are relatively high while off-peak fares are relatively low. The observed correlation due to this relationship between trips and fares can distort the effect induced by a raised fare on aggregate demand levels when everything else is held constant (Wooldridge, 2002; Greene, 2011).

In view of the foregoing, the econometric challenge is to develop a viable method of estimating price elasticity of demand for transport in integrated or flat fare transit systems using only high frequency flow and fare data where fare changes have been very infrequent and implemented simultaneously in similar amounts on all alternative transit services, thus presenting serious problems of collinearity and fare endogeneity. Under these conditions, a linear regression model may generate results that are neither robust nor statistically significant. We therefore propose an alternative approach based on discrete choice logit models (multinomial, hierarchical and mixed) that is simple yet demonstrably effective. Not only does it deliver the desired estimates using abundantly available passenger flow and fare data, but it can also be periodically updated and its predictive ability evaluated each time fares are changed.

The proposed methodology does not use additional information on service levels such as trip times, the reason being that such data is not available at the same frequency and quality level as the data on demand and prices. In this sense, the elasticities we estimate should be interpreted as reduced forms of aggregate trip behaviour for the travel alternatives (mode and time period) studied rather than as indicators for particular user profiles.

The approach developed is applied to a case study of the integrated public transport system in Santiago, Chile, known as Transantiago. The estimates obtained are satisfactory as regards both statistical significance and robustness, and are consistent with similar estimates reported previously in the literature.

The remainder of this article is organized into five sections. Section 2 is a survey of the literature, reviewing a number of articles on estimating price elasticity of demand for public transport. Results taken from those papers are used to contrast and validate the estimates obtained in the present study. Section 3 gives a brief statistical overview of the data used. Section 4 reports the results of the different methodological approaches tested (multiple linear regression and aggregate logit models). In the case of multiple linear regression we analyze the effects on the parameter estimates of strong collinearity and fare endogeneity, which lead to their invalidation. For the aggregate logit models we report the own-elasticity estimates by mode (bus and Metro) and time period (peak and off-peak) as well as the cross-elasticities between mode and time period. The results, which are comparable to those found in the literature, are set out in Section 2. Section 5 offers a comparative analysis of the elasticity estimates obtained using various types of logit models. Finally, Section 6 presents the study's main conclusions and recommendations.

2. LITERATURE SURVEY

In what follows we bring together results from various studies in the literature reporting on how price changes affect passengers flows on public transport that estimate elasticity for a specific case. The price referred to is that perceived by the travelling public which directly impacts consumption, and may include costs both monetary (fares) and non-monetary (e.g., travel times, comfort).

Many different factors contribute to the effects prices have on consumption. The impact will depend on the definitions adopted for the elasticities, the type of good or service in question, the class of consumer, the quality and quantity of available substitute goods or services, and any other market factor that might be relevant in a particular case.

Some of the principal factors that influence the elasticity of demand for public transport are the following (Cervero, 1990; Pham and Linsalata, 1991; Taplin et al., 1999; Paulley et al., 2006; Litman, 2011):

i) **Type of traveller:** persons who have non-public transport options (e.g., a private car) tend to more sensitive to price than those who are captives of a single mode (e.g., only buses). Certain demographic groups (low-income, non-car owners, the disabled, students and seniors) tend to be more dependent on public transport. In some communities, those who depend on public transport are a relatively low proportion of the total population but a high proportion of transit users.

ii) **Type of trip:** non-routine trips (e.g., during off-peak hours) tend to be more sensitive to price than routine trips (e.g., during peak hours). Off-peak public transport elasticities tend to be double those of peak hours (see Table 1) due to the greater proportion of routine trips in the latter periods (e.g., trips to work).

iii) **City geography:** large cities tend to have lower elasticities than small cities, suburbs or rural areas due to the higher proportion of travellers who are dependent on public transport.

iv) **Component and level of cost or price that changes**: fares, quality of service (travel times, comfort, frequencies, coverage, etc.) and parking costs tend to have a high impact on public transport users. Elasticities tend to increase as fares rise, and therefore to be higher if the starting fare level is high.

v) **Direction of change in cost or price component:** Transport demand models generally assume that price elasticity is the same whether price (or cost) rises or falls, although there is evidence suggesting that their effects are not in fact symmetric. Fare hikes tend to produce a fall in trip numbers that are greater than the rise for fare cuts of equal magnitude.

vi) **Period:** The impacts of price changes are often categorized either as short term (under 2 years), medium term (2 to 5 years) or long term (over 5 years). Elasticities tend to grow over time, suggesting that consumers make long-term make decisions based on price such as where to live and where to work. Long-term elasticities tend to be 2 or 3 times greater than short-term ones.

vii) **Type of public transport:** bus and Metro services have different elasticities because they serve different markets, and the size of the differences depend on specific factors. According to Paulley et al. (2004), "Although car ownership has a negative impact on rail demand, it is less than for bus and, although there are quite large variations between market segments and across distance bands, the overall effect of income on rail demand is quite strongly positive. Rail income elasticities are generally found to be positive, and as high as 2 in some cases. As with the bus income elasticities, the rail elasticity can also be expected to increase over time." Thus, changes in price levels may have a relatively small impact on users of basic or primary transport services.

Various studies have estimated the price elasticity of demand for public transport, and surveys of the literature have also been published. Examples of the former include Cervero (1990), Pham and Linsalata (1991), Oum et al. (1992), Goodwin (1992), Pratt (1999), Dargay and Hanly (1999), TRACE (1999), TRL (2004), APTA (2008), Wardman and Shires (2003), Taylor et al. (2009), Wallis (2004), Litman (2004) and Wang (2011). The elasticity estimates reported in these articles for urban public transport are set forth in Table 1. In general terms, they suggest that elasticity levels vary between -0.3 and -0.5. They also demonstrate that the absolute values of Metro elasticities are greater than those for buses and that the absolute value of peak-period elasticities are less than off-peak ones.

Table 1
Estimates of short-term price elasticity of demand for public transport services

Author	Type of elasticity	Estimated elasticity
Cervero (1990)	Transit, average	-0.22 a -0.33
Pham and Linsalata (1991)	Bus, peak hours	-0.23
	Bus, off-peak hours	-0.42
Goodwin (1992)	Bus, average	-0.41
	Metro, average	-0.79
Luk and Hepburn (1993)	Bus, average	-0.29
	Metro, average	-0.35
Jordan (1998)	Bus, average	-0.20 a -0.3
	Metro, average	-0.10 a -0.15
Kain and Liu (1999)	Transit, average	-0.32
Pratt (1999)	Bus, peak hours	-0.30
	Bus, off-peak hours	-0.46
	Metro, peak hours	-0.10
	Metro, off-peak hours	-0.46
Dargay and Hanly (1999)	Bus, average	-0.2 a -0.3
Small and Winston (1999)	Bus, average	-0.58
Mayeres (2000)	Transit, peak hours	-0.19
	Transit, off-peak hours	-0.29
Romilly (2001)	Bus, average	-0.38
Dargay and Hanly (2002)	Bus, average	-0.33 a -0.44
Bresson et al. (2003)	Transit, average	-0.40 a -0.53
OXERA (2003)	Bus, average	-0.63
TRL (2004)	Bus, average	-0.20 a -0.3
	Metro, average	-0.3
Fearnley and Bekken (2005)	Bus, average	-0.44
	Metro, average	-0.61
Holmgren, J. (2007)	Transit, average	-0.59
Booz & Co (2008)	Metro, average	-0.48
(*) Litman (2011)	Transit, average	-0.2 a -0.5
	Transit, peak hours	-0.15 a -0.3
	Transit, off-peak hours	-0.3 a -0.6

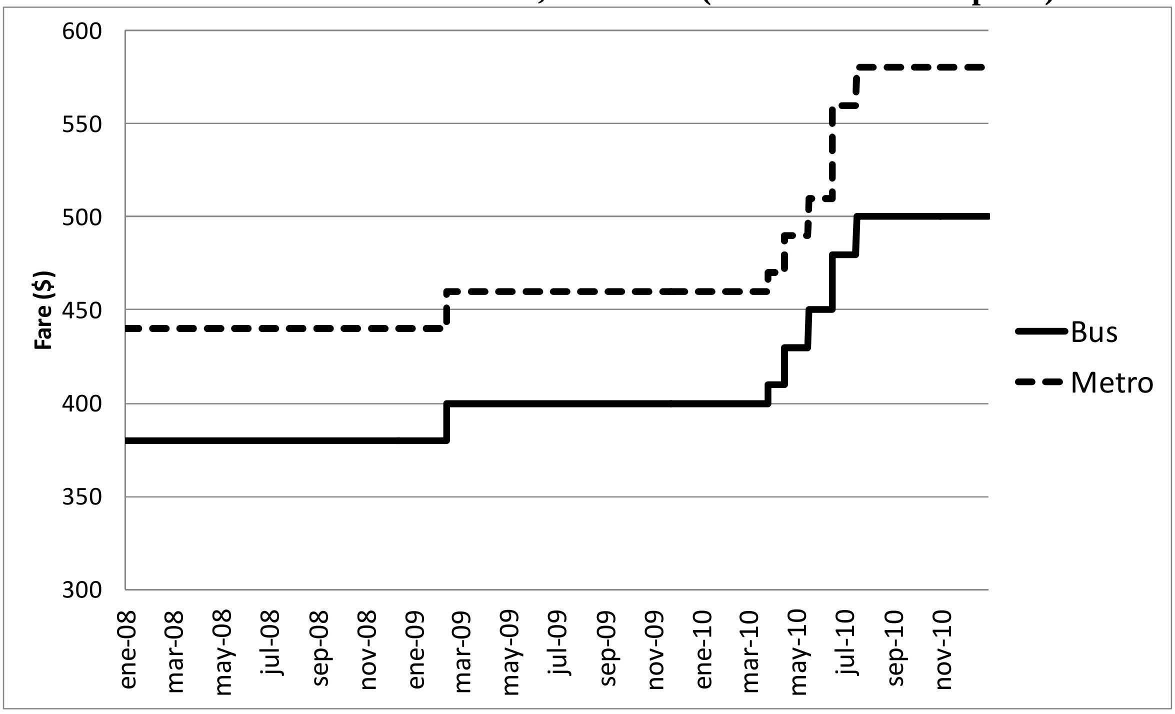
(*) Values recommended by the author for use in transport modelling based on an exhaustive review of the literature.

3. DATA

The data used with the proposed models to generate the elasticity estimates was compiled from the records of magnetic smart card transactions by passengers entering a bus or Metro station in the Transantiago transit system. The information related to the months of March through December 2010 and was aggregated over half-hour intervals. This period has witnessed more fare changes than any other since Transantiago was inaugurated in February 2007 and is thus the most appropriate for the task of identifying price elasticities while limiting possible complexities due to changes in variables not included in the models. It was also in 2010 that the system had fully stabilized after its first few years of operation, which were plagued by deficient service, operator contract modifications and changes in routes and other non-fare variables.

The evolution of the Transantiago Metro and bus fares over time is shown in Figure 1. As can be seen, the changes in fares for the two modes over the indicated time period were few in number and highly correlated, given that they were implemented simultaneously and were the same in size and direction.

Figure 1
Peak hour bus and Metro fares, 2008-2010 (in current Chilean pesos).



In estimating the elasticities, account was taken of the following characteristics of the Transantiago fare system as it was during March-December 2010:

i. The bus and Metro fares were integrated. Transferring between buses was free while transferring from bus to Metro was free during off-peak hours but cost 40 pesos during peak hours (weekdays 7:30am-9am and 6pm-7:30pm). Transfers between Metro lines were also free. Note that 500 pesos ≈ US$1.00.

ii. The bus fare was the same all day, 7 days a week. The Metro fare during peak hours (weekdays 7:30am-9am and 6pm-7:30pm) was 40 pesos more than during off-peak hours and also 40 pesos higher than the bus fare. Both modes charged the same fares during off-peak hours.

iii. The Transantiago system registered the smart card transactions for bus and Metro boardings and centrally stored them every half hour, every day of the year (some 6 million transactions were recorded every day). Fares were valid for 2 hours and only the start time of the trip was registered on the user's smart card. Transfers between Metro lines were not recorded. Students paid approximately one-third of the regular bus fare on either mode at all times without exception. Thus, for students there were no fare differences either for modes or time periods

To simplify the proposed analysis, the scope of the data was limited to weekdays. Weekends, legal holidays and the southern hemisphere summer months (January and February) when a large proportion of city residents are on vacation were all excluded, as were days considered by Metro officials to be "exceptional" because of special events that altered normal workday transport user behaviour.

Our analysis focuses on trips made on the system in the morning between 6am and 8am. Four travel alternatives are distinguished in terms of mode (bus or Metro) and time period (peak or off-peak) within this two-hour focus. The off-peak period for our purposes is 6am to 7am and the peak period 7am to 8am. Since the Metro's morning peak-fare period begins at 6:30am, half way through the off-peak hour as we define it, we considered the fare for that hour to be the average of the off-peak and peak fares. A number of basic statistical descriptors for the data employed are summarized in Table 2.

Table 2
Selected statistical descriptors (daily averages).

Variable	Obs	Mean	Std Dev	Min	Max
Trip: Metro, peak hour	200	168,860	17,494	110,330	201,903
Trip: Metro, off-peak hour	200	54,949	2,915	41,763	61,119
Trip: bus, peak hour	200	206,495	12,969	165,637	225,490
Trip: bus, off-peak hour	200	102,463	7,024	85,972	120,921
Fare: Metro, peak hour	200	546	45	460	580
Fare: Metro, off-peak hour	200	469	41	390	500
Fare: bus	200	472	36	400	500

4. METHODOLOGY

4.1 Multiple linear regression model

The traditional method of estimating elasticities with multiple linear regression models is to define a log-linear relationship between the variables of interest. In the case of price elasticities of demand, the variables would be prices and consumption. For our purposes, the dependent variable is the natural log of the number of trips for the transport mode and time period (peak or off-peak) in question while the independent variables include the natural logs of the fares for that transport mode and time period and for the alternative ones, among other possible controls. Thus, the model is formulated as follows:

$$y_t^m = \beta_0^m + \sum_k \beta_k^m x_{k,t}^m + \varepsilon_t^m \tag{1}$$

where y_t^m is the natural log of the number of trips in mode *m* observed in period *t*, $x_{k,t}^m$ are the natural logs of the explanatory variables including fares, ε_t^m is the statistical error and the β's are the parameters to be estimated. The parameter β_k^m accompanying the natural log of the fare for mode *m* indicates its price elasticity of demand while the parameters that accompany the alternative services are cross-elasticities. We estimate (1) using ordinary least squares.

4.2 Multinomial logit model

The data can also be interpreted as the result of an individual discrete choice process in which each traveller decides among different travel alternatives. In our case the alternatives are the four options described above in Section 3. The decision is made on the basis of the traveller's preferences and the characteristics of the alternatives, one of which is the fare.

In this study the fare is the only characteristic associated with level of service. Since we are using aggregate data, they relate only to bus and Metro fares and time periods and do not capture attributes of the different zones of the city or the individual traveller.

The first discrete choice model specification we consider is a multinomial logit model, which has the following form:

$$p_t^m = \frac{\exp\left(\beta_0^m + \beta_{fare}^m \cdot x_t^m\right)}{\sum_{m',t'} \exp\left(\beta_0^{m'} + \beta_{fare}^{m'} \cdot x_{t'}^{m'}\right)} \tag{2}$$

where p_t^m is the proportion of trips in mode *m* during period *t*, β_0^m is the modal constant of the travel alternative (grouping all attributes of the service except the fare), x_t^m is the mode *m* fare in period *t*, and β_{fare}^m is the parameter associated with the fare.

To estimate (2) we interpret the aggregate trips in each mode *m* and period *t* as the sum of individual choices to be explained by the discrete choice model as a function of fares. In other words, we disaggregate the database by individual, at which level many observations will involve the same choices and explanatory variables.

The price elasticity of mode *m* in period *t* is determined by

$$\eta_t^m = \frac{\partial p_t^m}{\partial x_t^m}\frac{x_t^m}{p_t^m} = \beta_{fare}^m \cdot x_t^m \left(1 - p_t^m\right) \tag{3}$$

The price cross-price elasticities between trips in mode *m* and the fare for mode *j* in period *t* is determined by

$$\eta_t^{m,j} = \frac{\partial p_t^m}{\partial x_t^j}\frac{x_t^j}{p_t^m} = -\beta_{fare}^j \cdot x_t^j \left(1 - p_t^j\right) \tag{4}$$

Note that expressions (3) and (4) refer to proportions of trips rather than absolute numbers of trips in each mode. Since the model excludes the possibility that no trip is taken we assume that the total number of trips in each period *t* is fixed, meaning that the percentage change in the number of trips is the same as the percentage change in the proportion of trips. The assumption of a fixed number of trips is reasonable in our short-term context, which focuses strictly on the period just before the start of the working day, and is used frequently in discrete choice models whenever the no-choice alternative is excluded.

It should also be noted that in this model the cross-price elasticities are symmetric and when added to the own-price elasticities they sum to 0.

The estimation of (2) was performed by maximum likelihood using the discrete choice model estimation software Biogeme (Bierlaire, 2003; www.biogeme.epfl.ch).

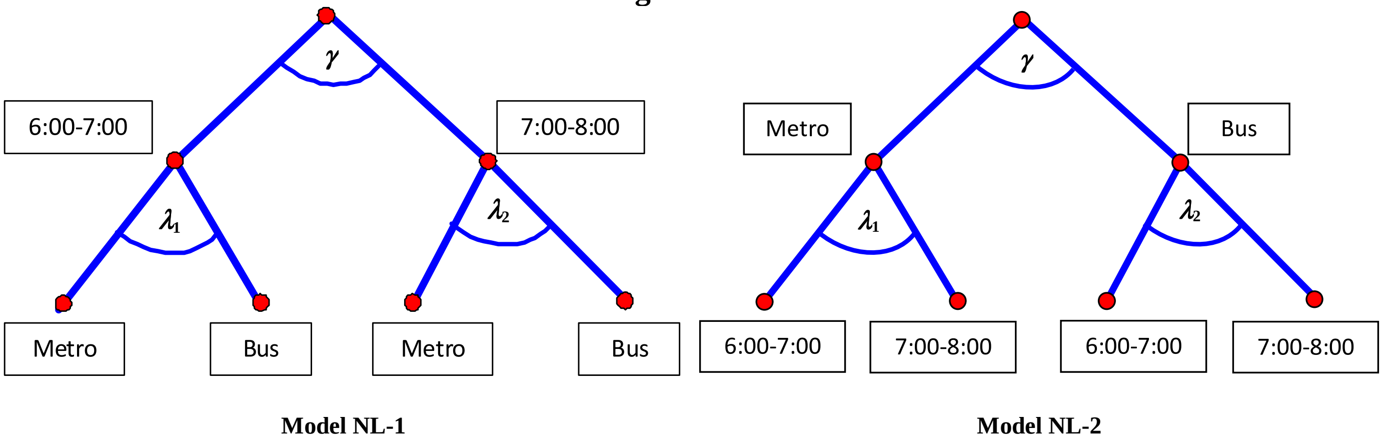
4.3. Other discrete choice models

To test the robustness of the multinomial logit model estimates we complemented the estimation process with two more flexible alternative models, the first one hierarchical logit (Williams, 1977; Ortúzar and Willumsen, 2011) and the second one mixed logit (Bierlaire, 2003, 2008; Bhat and Guo, 2004).

4.3.1. Hierarchical logit model

Hierarchical logit models allow more flexible specifications to be defined with more parameters than multinomial logit models and do not impose the independence of irrelevant alternatives restriction. Also, they can establish a hierarchy for a set of choice decisions. In the present case, the various alternatives for organizing the hierarchy of time period and mode of transport decisions are shown in Figure 2.

Figure 2
Hierarchical logit model decision trees



In model NL-1, the top level decision is the users' choice of time period while the bottom level decision is the aggregate choice of mode within each time period. This specification will be the correct one for estimation only if it is true that $\phi_1 = \frac{\gamma}{\lambda_1} < 1$ and $\phi_2 = \frac{\gamma}{\lambda_2} < 1$ (Ortúzar and Willumsen, 2011). If $\phi_1 = 1$ and $\phi_2 = 1$, however, the model collapses to a multinomial logit, and if $\phi_1 > 1$ or $\phi_2 > 1$, the correct model would be NL-2.

Thus, the hierarchical specification depends on the values of parameters ϕ_1 and ϕ_2, which in turn will determine the own-price and cross-price elasticities.

In addition to capturing correlation between alternatives, one of the advantages of hierarchical logit models over multinomial logit ones is that they reveal the economic importance to the traveller of changing mode relative to changing time period in response to a variation in fares (De Cea et al., 2008). If, in the NL-1 model specification, the parameters ϕ_1 and ϕ_2 are less than 1, the time period choice is more important to the user than the mode choice, otherwise the mode choice is more important and the sequence of levels in the model tree must be reversed.

4.3.2. Mixed logit model

Mixed logit models allow the estimated parameters to vary randomly across the population. The distribution function chosen for the parameters is up to the modeller and attempts to reflect the actual distribution of values in the population, or some belief regarding it. The most common choice is the normal distribution (Train, 2003). If it is desired to impose a certain sign for a given parameter (e.g., negative values for trip cost), the log-normal distribution is used given that it takes only positive values (for trip cost, the variable would then have to have a negative sign). However, since the log-normal is asymmetric with a long right tail, the mean is difficult to interpret. In the light of various arguments suggesting the cost parameters are normally distributed (Walker, 2002), we adopted that assumption for the present study.

5. RESULTS

5.1. Multiple linear regression model

The estimates obtained for the linear model with Metro trips during the morning peak hour as the dependent variable are shown in Table 3. The explanatory variables are the fares for the travel alternatives, which are off-peak by Metro or either period by bus. The model cannot distinguish between the effects of fare changes for peak versus off-peak bus trips since the two are equal and thus perfectly collinear.

Four versions of the model were tested. In Model 1, the peak and off-peak Metro fares and the bus fare are explanatory variables. Neither own-price nor the cross-price elasticities turned out to be statistically significant in this specification. The own-price elasticity was close to -26, differing by an order of magnitude from the results reported in other studies which range from -0.3 to -0.5.

One way to reduce the collinearity problem is to eliminate less important variables. Thus, Model 2 excludes the off-peak Metro fare from the explanatory variables. The resulting elasticity estimates are significant and very different from those generated by Model 1, but still seem very large. Short-term peak-hour elasticities are generally expected to have an absolute value of less than 1.

Models 3 and 4 are the same as Models 1 and 2, respectively, except that they include a dummy variable for the month of the year to control for seasonality. As can be seen in Table 3, the change of specification produces estimates that are considerably different in size and even in sign. This instability is a symptom of the data's high collinearity. Signs contrary to expectation, such as a positive (although not significant) value in Model 4 for price elasticity of demand of peak-hour Metro trips, may reflect endogeneity in fare-setting.

Table 3
Linear model parameter estimates, peak-hour Metro trips.

Ln(peak-hour Metro trips)	Model 1	Model 2	Model 3	Model 4
Ln(peak Metro fare)	-25.993	-5.174***	-108.926**	0.464
	(24.635)	(0.818)	(49.093)	(0.747)
Ln(off-peak Metro fare)	37.12		190.656**	
	(44.144)		(86.461)	
Ln(bus fare)	-14.588	5.094***	-99.728**	-1.183
	(23.561)	(0.917)	(45.434)	(1.058)
Constant	37.363	13.267***	139.908**	16.271***
	(28.649)	(0.702)	(57.344)	(1.807)
No. of observations	200	200	200	200
R-Squared	0.215	0.212	0.685	0.665

Standard errors in parentheses. * indicates significance at the 10% level, ** at the 5% level and *** at the 1% level.

The estimates of the linear model with peak-hour bus trips as the dependent variable are shown in Table 4. As with the Metro estimates, Models 3 and 4 include a dummy variable for the month of the year. Once again, the elasticities are very unstable in the face of relatively minor changes to the specification, in some cases having the wrong sign and in various others being of an implausible order of magnitude.

Table 4
Linear model parameter estimates, peak-hour bus trips.

Ln(peak-hour bus trips)	Model 1	Model 2	Model 3	Model 4
Ln(peak Metro fare)	39.841***	-3.361***	-44.869*	-0.718
	(13.611)	(0.539)	(26.009)	(0.453)
Ln(off-peak Metro fare)	-77.029***		76.952*	
	(24.413)		(45.892)	
Ln(bus fare)	44.596***	3.754***	-39.142	0.632
	(13.056)	(0.579)	(24.185)	(0.601)
Constant	-39.699**	10.302***	62.692**	12.79***
	(15.918)	(0.313)	(30.551)	(1.002)
No. of observations	200	200	200	200
R-Squared	0.173	0.136	0.686	0.676

Standard errors in parentheses. * indicates significance at the 10% level, ** at the 5% level and *** at the 1% level.

5.2. Multinomial logit model

The parameter estimates and significance test results for the multinomial logit model are set out in Table 5. Although all of the parameters are statistically significant, in discrete choice models they are difficult to interpret. For this reason, in Table 6 we show the elasticity estimates evaluated at the sample centroid.

Table 5
Multinomial logit parameter estimates (*).

Parameter	Value	*t* test
Const.: Metro off-peak	-2.26717	-3.241
Const.: Metro peak	-0.57354	-3.327
Const.: bus off-peak	-0.70222	-5.642
Beta: fare Metro off-peak	-0.00045	-2.485
Beta: fare Metro peak	-0.00144	-3.534
Beta: fare bus off-peak	-0.00101	-2.865
Beta: fare bus peak	-0.00119	-2.018
Log-likelihood (LL)	-7,650.29	
$\bar{\rho} = 1 - \frac{LL(const)}{LL(\beta^*)}$	0.123	

(*): The off-peak bus modal constant was set to 0.

Table 6
Aggregate multinomial logit elasticities

Travel alternative	Elasticity
Metro off-peak (own)	-0.193
Metro peak (own)	-0.588
Bus off-peak (own)	-0.34
Bus peak (own)	-0.284
Metro off-peak, Metro peak (cross)	0.159
Metro off-peak, bus off-peak (cross)	0.114
Metro peak, bus peak (cross)	0.25

(1) Cross elasticities for simultaneous changes in both mode and time period are not shown.
(2) The functional form of the model imposes that the cross-elasticity between off-peak and peak bus is the same as that between peak Metro and peak bus.

The signs of the elasticities are all consistent with economic theory and the values obtained are comparable to those reported in other empirical studies in the literature (see Table 1). The highest elasticity (in absolute value) was observed for the Metro mode in the 7:00am-8:00am period. This result, also consistent with the literature, may be explained in part by the fact that the Metro fare is higher than the bus fare so that the elasticity is calculated at a point on the demand curve that gives, *ceteris paribus*, a greater elasticity value (by definition, elasticity is higher at higher prices). Another reason for the result is that between 7:00am and 8:00am, Metro users can either switch to the bus or move up or delay their trip time more easily than can bus users or Metro users travelling between 6:00am and 7:00am.

As regards the cross-elasticities, the highest value is for the substitution of peak Metro trips with peak bus trips, a result that seems quite plausible. The positive signs of the cross-elasticities indicate that the four travel alternatives in the study are substitutes at the aggregate level, which does not, however, mean they cannot be complementary for certain individual users.

5.3. Hierarchical logit and mixed logit models

After various tests it was concluded that the hierarchical model which offered the best statistical fit was the structure shown in Figure 3.

The parameter estimates and significance test results for the hierarchical logit and mixed logit models are shown in Table 7. The estimates are significant for both specifications. Parameter ϕ_1 turned out to be less than 1 while parameter ϕ_2 was not specified as it was statistically equal to 1. This implies that on average, travellers using the system between 6:00am and 7:00am give more weight to the time period they travel in than to the transport mode. Such a result was to be expected since in that time period the great majority of trips are made by workers and students whose required arrival times have very little flexibility. In the 7:00am-8:00am period, on the other hand, this result was not observed (parameter ϕ_2 was statistically equal to 1), which might mean that users in this later period are not as obligated to travel as those in the earlier period.

Figure 3
Estimated hierarchical logit model tree

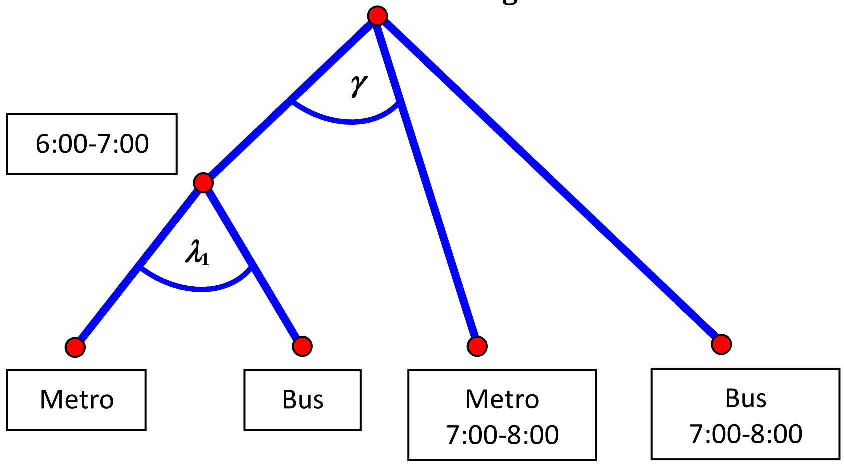


Table 7
Hierarchical logit and mixed logit parameter estimates (*)

Parameter	Hierarchical logit		Mixed logit (**)	
	Value	***t* test**	**Value**	***t* test**
Const.: Metro off-peak	-1.011870	-5.787	-2.25854	-2.533
Const.: Metro peak	-0.453192	-3.547	-0.57192	-2.542
Const.: bus off-peak	-1.011870	-5.787	-0.70105	-5.546
Beta: fare Metro off-peak	-0.000013	-1.815	-0.00043	-1.983
Beta: fare Metro peak	-0.000226	-2.060	-0.00138	-2.436
Beta: fare bus off-peak	-0.000200	-1.984	-0.00104	-1.992
Beta: fare bus peak	-0.000267	-2.046	-0.00122	-2.211
$\phi_1 = \gamma / \lambda_1$	0.73	1.784	-	-
Log-likelihood (*LL*)	-7,645.58		-7648.47	
$\bar{\rho} = 1 - \frac{LL(const)}{LL(\beta^*)}$	0.125		0.123	

(*): The off-peak bus modal constant was set to 0.
(**): In the mixed logit model, the parameters were assumed to be normally distributed.

In practical terms, this result can be interpreted to mean that in response to a peak-hour Metro fare increase, users will tend to switch to a peak-hour bus rather than take the Metro earlier. In other words, when a specific fare is hiked, travellers are more willing to change the mode of their trip than the time period. This is corroborated by the cross-elasticity estimates in Table 8, which summarizes and compares the elasticity results for all three discrete choice models in the study. Also evident is the similarity of the different models' estimates, clearly indicating their robustness. The confidence intervals for each travel alternative and model are set out here in the Appendix, confirming that the three sets of results are statistically equivalent.

Table 8
Discrete choice model elasticities

Travel alternative	Multinomial logit	Hierarchical logit	Mixed logit
Metro off-peak (own)	-0.193	-0.233	-0.173
Metro peak (own)	-0.588	-0.557	-0.575
Bus off-peak (own)	-0.34	-0.349	-0.303
Bus peak (own)	-0.284	-0.268	-0.314
Metro off-peak, Metro peak (cross)	0.159	0.141	0.197
Metro off-peak, bus off-peak (cross)	0.114	0.134	0.142
Metro peak, bus peak (cross)	0.25	0.236	0.196

(*): The method of calculating elasticity in hierarchical logit models differs from that used with MNL models (see Forinash and Koppelman, 1993).
(**): The elasticities in mixed logit models are estimated by simulation (see Bhat and Guo, 2004).

5.4. Discussion of results

The use of discrete choice models instead of traditional log-linear regression to estimate the price elasticity of public transport demand is not motivated by conceptual considerations so much as practical ones. In the case of Santiago, Chile, the econometric challenge is to find a method of identifying the magnitude of transit fare elasticities given that changes in the fares have been very few, and what changes there have been were implemented simultaneously and in similar amounts, thereby creating a serious collinearity problem. In our study, the linear regression models were unable to identify elasticity values that are reasonably robust. This is more than evident in one of the results given in Table 3, which suggests that a 1% Metro fare increase would trigger a clearly overestimated 26% decrease in Metro trips.

Although there is no established benchmark for Santiago with which to compare our elasticity estimates, the fact that they are similar to those reported in the literature is a good sign. If there had been no changes in Transantiago fares it would not have been possible to estimate the elasticities with the system data. Perhaps the best estimates available to the transit authorities would be based on those found in previous studies for cities similar to Santiago, on data from the pre-Transantiago system, or on stated preference data. But any of these alternatives would result in seriously biased estimates.

A shortcoming of the proposed discrete choice methodology is that the fares are the only explanatory variables, thus excluding other factors that may be significant. This exposes the estimates to bias due to the omission of relevant variables, although the risk involved can be limited by restricting the period of the analysis. It should also be noted that that the elasticities could not be estimated from a database built by surveying individual users as such data would be cross-sectional and therefore unable to capture fare changes over time. In any case, using stated preferences would be very unreliable.

The elasticities we have estimated should be interpreted as reduced forms of aggregate traveller behaviour for the travel alternatives studied. Thus, although the signs of the cross-elasticities tell us that the transport modes are substitutes, for some travellers whose trips combined different modes the alternatives were in fact complementary. The elasticity estimates are for aggregate behaviour, not for any particular user profile.

Different levels of aggregation could be defined for the various travel alternatives by including more time periods or the total number of daily trips, or distinguishing between types of users (adults and students), different routes or different bus operators. In each case, however, care must be taken when interpreting the parameters and their relevance in the design of fare policies.

As regards distinguishing between different bus routes, some of them complement the Metro (feeder lines), others are substitutes for it, while still others are simply unrelated. Since every operator has lines falling into each of these categories, the size and sign of the cross-elasticities between them will behave erratically. Many operators simultaneously run services in very distant areas within the city that obviously cannot be considered as travel alternatives for the individual user located in any one of them, making it difficult for aggregate discrete choice models to properly represent the travel alternatives users really face. Whereas the originating station of each Metro trip is indicated by Metro data, the origin of bus trips cannot be determined directly. This problem could be solved, however, if by some form of geographic positioning the starting location of each bus trip could be estimated. Such information could be used in our models to group bus and Metro trips starting in each city zone and estimate the local elasticity for each one. This would be a particularly attractive option given that elasticities are known to depend significantly on socioeconomic characteristics that would vary greatly across the city.

6. CONCLUSIONS

A methodology was proposed for estimating aggregate elasticities of demand for public transport in a system with integrated fares using discrete choice models. The approach developed was able to overcome the serious problems of collinearity and endogeneity in the data supplied by Transantiago, the integrated fare transit system in Santiago, Chile. Linear (or log-linear) regression models, the traditional tool for estimating aggregate elasticities, were unable to identify reliable values for the estimated parameters.

The study focussed on trips made in the morning for which it was possible to define different aggregation levels. Estimates were obtained for own-price elasticity of demand by transport mode and time period as well as cross-elasticities between different modes and periods. These results were similar in magnitude to those reported in the literature, suggesting the proposed methodology is sound.

Three alternative discrete choice models were tested: multinomial logit, hierarchical logit and mixed logit. The estimates they generated were all of the same order of magnitude.

The principal limitation of the suggested approach is that fares are the only explanatory variables. To reduce the risk of bias, only comparable days in stable periods where fare hikes were the most significant changes for transit users could be included in the data. Even if a disaggregated database with characteristics for individual travellers were constructed, the data would be cross-sectional and therefore useless for measuring changes in trips due to fare changes. Geographical location could be used to reflect travellers' socioeconomic attributes, but such information is not currently available for bus trips in the Transantiago system.

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APPENDIX

Calculation of confidence intervals

The confidence intervals for the various elasticities estimated by the three discrete choice models discussed in this study for the time period 6am to 8am were calculated using the following formula:

$$\bar{x}_i \pm t_{\alpha/2,n-1} S_i / \sqrt{n_i} \qquad \text{(A.1)}$$

where n_i is the number of sample observations, $\bar{x}_i$ is the average of the elasticity estimates and S_i is the standard deviation. The confidence intervals calculated with a 95% significance level are summarized in Table A.1.

Table A.1
Elasticity confidence intervals for discrete choice models

Trip alternative	Multinomial logit	Hierarchical logit	Mixed logit
Metro off-peak (own)	[-0.216;-0.17]	[-0.204;-0.142]	[-0.268;-0.198]
Metro peak (own)	[-0.639;-0.537]	[-0.648;-0.502]	[-0.599;-0.515]
Bus off-peak (own)	[-0.377;-0.303]	[-0.346;-0.260]	[-0.375;-0.323]
Bus peak (own)	[-0.332;-0.236]	[-0.369;-0.259]	[-0.301;-0.236]
Metro off-peak, Metro peak (cross)	[0.152;0.166]	[0.184;0.210]	[0.129;0.153]
Metro off-peak, bus off-peak (cross)	[0.105;0.123]	[0.132;0.152]	[0.111;0.157]
Metro peak, bus peak (cross)	[0.221;0.279]	[0.170;0.222]	[0.208;0.264]

The differences between these intervals are not significant, as can be confirmed by estimating the intervals for the elasticity differences between the models using the following formula:

$$\bar{x}_i - \bar{x}_j \pm t_{\alpha/2,v}\sqrt{\frac{S_i}{n_i}+\frac{S_j}{n_i}} \tag{A.2}$$

where $$v = \frac{\left(\dfrac{S_i^2}{n_i}+\dfrac{S_j^2}{n_j}\right)^2}{\dfrac{\left(\dfrac{S_i^2}{n_i}\right)^2}{n_i-1}+\dfrac{\left(\dfrac{S_j^2}{n_j}\right)^2}{n_j-1}} \tag{A.3}$$

The intervals derived by the formula are shown in Table A.2 for the differences between the multinomial logit and hierarchical logit models and between the multinomial logit and mixed logit models. All of the estimated intervals (at a 95% significance level) include 0, meaning there is no statistical evidence the elasticities obtained by the 3 models are different.

Table A.2
Elasticity confidence intervals between the discrete choice models.

Trip alternative	Multinomial logit - hierarchical logit	Multinomial logit – mixed logit
Metro off-peak (own)	[-0.142;0.102]	[-0.087;0.167]
Metro peak (own)	[-0.199;0.173]	[-0.192;0.130]
Bus off-peak (own)	[-0.186;0.112]	[-0.123;0.141]
Bus peak (own)	[-0.139;0.199]	[-0.165;0.133]
Metro off-peak, Metro peak (cross)	[-0.111;0.035]	[-0.055;0.091]
Metro off-peak, bus off-peak (cross)	[-0.101;0.044]	[-0.114;0.074]
Metro peak, bus peak (cross)	[-0.069;0.177]	[-0.112;0.140]