# MORNING COMMUTE PROBLEM IN A MANY-TO-ONE NETWORK WITH PARKING CONSTRAINT

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## ABSTRACT

Morning commuters often choose their departure times not only to trade off bottleneck congestion and schedule delays, but also in order to secure a parking spot due to limited parking spaces. It is the combination of these two forces that governs the commuters' departure time choices. This paper investigates the morning commute problem in a many-to-one network with both bottleneck congestion and parking space constraints; particularly when some commuters have reserved parking spots or parking permits and other commuters have to compete for public parking spots on a first-come first-served basis. Distinguished from the traditional pure bottleneck model, the rush-hour dynamic traffic pattern with binding parking supply constraints varies with the combination supply of reserved and unreserved parking spot. It has been found that when the total parking supply is greater than half of the potential parking demand, it is socially preferred to retain some parking spots unreserved in a many-to-one network. Parking permit schemes can be designed to mitigate congestion thus reduce total travel cost, however, generally, the trading of permit will lead to a non-optimal allocation of parking spots for commuters from different origins.

*Keywords: morning commute, parking constraint, many-to-one network, dynamic traffic equilibrium*

## INTRODUCTION

Since Vickrey (1969) introduced the first bottleneck model of congestion dynamics, a voluminous literature on the bottleneck congestion model and its various extensions have been developed and continues to grow today. Based on the basic model, a number of issues have been considered, including decentralization of the social optimum through time-varying pricing (Arnott et al., 1990), second-best pricing including coarse and step tolls (Laih, 1994; Xiao et al., 2012), demand elasticity (Arnott et al., 1993a; Yang and Huang, 1997), heterogeneous commuters (Arnott et al., 1994; van den Berg and Verhoef, 2011; Liu and

Nie, 2011, Doan et al., 2011), congestion derivatives (Yao et al., 2010 and 2012), small networks including routes in parallel and routes in series (Arnott et al., 1993b; Zhang and Zhang, 2010), and pricing on general queuing networks (Yang and Meng, 1998), partial differential equation formulation (Han et al., 2013).

There are a few papers on integration of the congestion and parking problems. By assuming that the parking spaces are continuously distributed along a freeway near the CBD and the number of parking spaces per unit distance from the CBD is constant, Arnott et al. (1991) embedded the parking problem in the morning commute model (Vickrey, 1969) and showed that parking fees alone can be efficient in increasing social welfare, and a combination of road tolls and parking fees can yield the system optimum that maximizes social welfare. Under the parking setup by Arnott et al. (1991), Zhang et al. (2008) derived the daily commuting pattern that combines both the morning and evening commute, and investigated mechanisms and efficiencies of several road toll and parking fee regimes. To account for the temporal aspects of parking, Zhang and van Wee (2011) further introduced a duration-dependent parking fee scheme into the daily commuting model consisting of the morning and evening commutes and the resulting parking duration.

Qian et al. (2011) provided an economic analysis of competitive parking provision for the morning commute. A finite number of parking lot clusters (or areas) are owned and operated by private firms to compete with others for the morning commuters. Both the capacity and access time of each cluster to the CBD are determined by the competitive market and commuting equilibrium. They also examined several market regulations and studied their effects on the commuters' travel cost and operators' profit/cost in the morning commute. In the spirit of the tradable travel credit scheme recently proposed by Yang and Wang (2011) for managing network mobility, Zhang et al. (2011) introduced a parking permit distribution and trading scheme for managing vehicular parking for the morning commute problem with limited downtown parking spaces. The proposed parking permit scheme can eliminate the external cost arising from competition for parking spots, and the parking permits are freely tradable among commuters in a competitive free market to better cater for commuters' parking needs. Qian et al. (2012) investigated how parking fee and parking supply can be designed to mitigate traffic congestion, and to reduce total social costs.

Our recent work (Yang et. al, 2013) further investigates the morning commute problem with both bottleneck congestion and parking space constraints; particularly when some commuters have reserved parking spots or parking permits and other commuters have to compete for public parking spots on a first-come first-served basis. Distinguished from the traditional pure bottleneck model, the rush-hour dynamic traffic pattern with binding parking capacity constraints varies with the relative portions of the two classes of commuters: those with and those without a parking permit. It is found that an appropriate combination of reserved and unreserved parking spots can temporally smooth out traffic congestion at the bottleneck and hence reduce the total system cost, because the commuters without a parking permit are compelled to depart from home earlier due to competition for a limited number of downtown parking spots.

In this study, we will extend our work to a Many-to-one network. Commuters live in different residential areas and every morning they travel to the same city center. For commuters from different origins, they have competition for parking spot but no flow interaction. For commuters from the same origins, they have both competition for parking spot and flow interaction at the bottleneck.

The rest of the paper is organized as follows. Section 2 revisits the single bottleneck model that incorporates parking space constraint and two classes of commuters. In Section 3, a many-to-one network with parking space constraint is explored. Section 4 introduces the parking permit scheme to reduce total social cost. A numerical example is presented in Section 5 for illustration of the results. Finally, Section 6 concludes the paper.

# SINGLE BOTTLENECK WITH PARKING CONSTRAINT

Travellers can either drive their car or take transit. Travel cost by auto, including travel time cost and schedule delay cost, departing at time $t$ is given by

$$c(t)=\alpha \cdot T(t)+\beta \cdot \max\left\{0,t^{*}-t-T(t)\right\}+\gamma \cdot \max\left\{0,t+T(t)-t^{*}\right\} \quad (1)$$

where $T(t)$ is the travel time at departure time $t$, $\alpha$ is the value of unit travel time, and $\beta$ and $\gamma$ are the schedule penalty for a unit time of early arrival and late arrival respectively. It is assumed that $\gamma>\alpha>\beta>0$. Also, in the following analysis, we employ the notation: $\delta=\beta\gamma/(\beta+\gamma)$. $T(t)$ contains free flow travel time $t_f$ and the queuing cost at the bottleneck whose service capacity is constantly equal to *s.* Namely, $T(t)=t_f+q(t)/s$, where $q(t)$ is the queue length at bottleneck at time $t$. And

$$\frac{\mathrm{d}q(t)}{\mathrm{d}t}=\begin{cases} r(t)-s, \; r(t)>s \text{ or } q(t)>0 \\ 0, \; r(t)\le s \text{ and } q(t)=0 \end{cases} \quad (2)$$

where $r(t)$ is the flow rate arriving at the bottleneck at time $t$. For simplicity, assume the travel cost of transit commuter is a constant, $P^T$. When there is no parking constraint, at equilibrium, the travel cost of auto commuters will be $P^A\left(\bar{N}^A\right)=\alpha t_f+\delta\bar{N}^A/s$, where $\bar{N}^A$ is the number of auto commuters at equilibrium. Denote the total number of commuters by $N$, assume an interior equilibrium, then we have $0<\bar{N}^A<N$ and $P^A\left(\bar{N}^A\right)=\alpha t_f+\delta\bar{N}^A/s=P^T$. (it is obvious that $\bar{N}^A=s\left(P^T-\alpha t_f\right)/\delta$)

The single bottleneck with parking constraint is based on the previous work by Yang et al. (2013). We consider the bi-modal equilibrium when the number of parking spots at the destination is limited or $M$ is less than $\bar{N}^A$. Let $M^r$ and $M^u$ denote the numbers of reserved and unreserved parking spots respectively, where $M^r+M^u=M$. According to our assumption that parking constraint is binding, the numbers of auto commuters with reserved and unreserved parking spots are equal to $M^r$ and $M^u$, respectively. In the following, we denote the auto commuters with and without reserved parking spots by r-commuters and u-commuters respectively. For r-commuters, their choices of departure time from home are not directly affected by parking availability; for u-commuters, they have to depart from home earlier to secure a parking spot. The two classes of commuters, although based on different considerations for their departure time choices, interact with each other by sharing the bottleneck capacity. In addition, at the bi-modal equilibrium, travel cost of auto commuters without a reserved parking spot (u-commuter) will be identical to that of transit commuters, which is given by $P^T$.

Dependent on the values of $M^r$, $M^u$ and $M$, three possible scenarios can appear at commuting equilibrium. The conditions for appearance of each equilibrium scenario are given in Table I. Define the following critical number:

$$M^{\#} = s\left(P^T - \alpha t_f\right) \big/ \beta \quad . \tag{3}$$

It can be easily figured out that $M^{\#} = \frac{\gamma}{\beta+\gamma}\bar{N}^A < \bar{N}^A$. Indeed $M^{\#}$ represents the threshold number of unreserved parking spots that push all auto commuters to depart earlier from home such that the last auto commuter just arrives on time when all parking spots are unreserved. The case when all parking spots are unreserved, i.e., $\left(M^r = 0, M^u = M\right)$, is Extreme case (1) examined by Zhang et al. (2011) while the case when all parking spots are reserved, i.e., $\left(M^r = M, M^u = 0\right)$, is Extreme case (2) examined by Zhang et al. (2011).

Table I - Three possible scenarios of bi-modal commuting equilibrium

| Parking provision | | Commuting equilibrium |
|---|---|---|
| Total parking spots | Reserved parking spots | |
| $0 < M \le M^{\#}$ | $0 < M^r < M$ | Scenario I |
| $M^{\#} < M < \bar{N}^A$ | $M^r \ge \frac{\gamma}{\delta}\left(M - M^{\#}\right)$ | |
| | $M - M^{\#} \le M^r < \frac{\gamma}{\delta}\left(M - M^{\#}\right)$ | Scenario II |
| | $M^r < M - M^{\#}$ | Scenario III |

Note: $M^{\#} = s\left(P^T - \alpha t_f\right) \big/ \beta$.

Given the parking capacity, $M\left(< \bar{N}^A\right)$, the travel cost of u-commuter will be identical to that of transit commuters, i.e., $P_u^A = P^T$. And the travel cost of r-commuter, $P_r^A$, also can be determined. When $M > M^{\#}$, we have

$$P_r^A = \begin{cases} \alpha t_f + \delta \dfrac{M^r}{s}, & M^r \ge \dfrac{\gamma}{\delta}\left(M - M^{\#}\right) \\ \alpha t_f + \gamma \dfrac{M - M^{\#}}{s}, & M^r \le \dfrac{\gamma}{\delta}\left(M - M^{\#}\right) \end{cases} \tag{4}$$

When $M \le M^{\#}$, it is simply given by $P_r^A = \delta M^r / s$. Note that $P_r^A$ is non-decreasing in $M^r$ thus $P_r^A \le \delta M/s < \delta \bar{N}^A / s = P^T = P_u^A$.

The first u-commuter will arrive at its destination at

$$t^{u,s} = t^* - \frac{M^{\#}}{s} \tag{5}$$

and the last arrival time of u-commuters

$$t^{u,e} = t^* - \frac{M^\#}{s} + \frac{M^u}{s} \tag{6}$$

where $M^u$ is the number of u-commuters.

# A MANY-TO-ONE NETWORK WITH PARKING CONSTRAINT

Now we consider a many-to-one network as shown in Figure 1 with parking constraint. Commuters live in different residential areas and every morning they travel to the same city center. For commuters from different origins, they have competition for parking spot but no flow interaction. For commuters from the same origins, they have both competition for parking spot and flow interaction at the bottleneck.

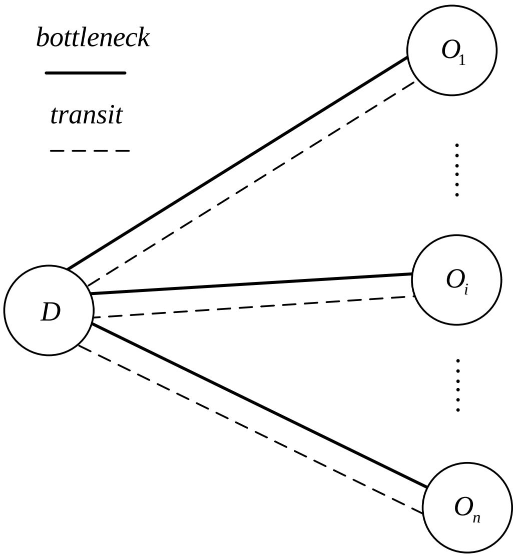


Figure 1 - Many-to-one network

The total number of origin is $n$. Let $t_i$ denote the free flow travel times from $O_i$ to $D$, $s_i$ the capacity of the bottleneck $i$. $N_i$ and $P_i^T$ represents the travel demands and transit travel cost for commuters at $O_i$. Total parking spots available at destination is $M$ .

In the following analysis, some mild and reasonable assumptions are made to avoid tedious consideration and analysis of various corner solutions. First, in Extreme case (1) when all parking spots are unreserved, we assume in equilibrium there are positive numbers of auto commuters from all origins competing for the parking spots and the last auto commuters from different origins arrive at the parking spot at the same time. Second, if all parking spots are reserved, in the social optimal allocation of reserved parking spots, commuters from each origin have strictly positive number of reserved parking spots.

Under no parking constraint, the travel cost of auto commuters and transit commuters should be equal, thus $P_i^A\left(\bar{N}_i^A\right) = \alpha t_i + \delta \bar{N}_i^A / s_i = P_i^T$ where $i = 1, 2, \cdots, n$ . The numbers of auto commuters from each origin are

$$\bar{N}_i^A = \frac{s_i}{\delta}\left(P_i^T - \alpha t_i\right). \tag{7}$$

A binding parking constraint implies that

$$M < \bar{N}^A = \sum_i \bar{N}_i^A = \sum_i \frac{s_i}{\delta}\left(P_i^T - \alpha t_i\right). \tag{8}$$

Under the parking constraint, commuters without a reserved parking spot will depart earlier to compete for parking spots. Given an allocation of reserved parking spots $\left\{M_i^r\right\}$, the number of u-commuters at each origin can be determined. If for every $(i, j)$,

$$t_i^{u,s} < t_j^{u,s} + \frac{M - \sum_k M_k^r}{s_j},$$

then the last u-commuter of each origin arrive at the destination at the same time, and

$$M_i^u = \frac{s_i}{\sum_k s_k}\left(M - \sum_k M_k^r - \sum_k M_k^{\#}\right) + M_i^{\#}, \tag{9}$$

If

$$t_i^{u,s} > \min_j \left\{ t_j^{u,s} + \frac{M - \sum_k M_k^r}{s_j} \right\},$$

then $M_i^u = 0$.

Now we consider Extreme case (1) when all parking spots are unreserved $\left(M_i^r = 0 \text{ for all } i\right)$. From Eq.(9), we can easily get the numbers of auto commuters from each origin at equilibrium, which are given by

$$\bar{M}_i = \frac{s_i}{\sum_k s_k}\left(M - \sum_k M_k^{\#}\right) + M_i^{\#}. \tag{10}$$

where $M_k^{\#} = s_k\left(P_k^T - \alpha t_k\right)/\beta$, $i = 1, 2, \cdots, n$. Under our assumption that $\bar{M}_i > 0$ for any $i$, it is required that

$$M > \max_i \left\{ \sum_k M_k^{\#} - \frac{\sum_k s_k}{s_i} M_i^{\#} \right\}. \tag{11}$$

Now we turn to consider Extreme case (2) when all parking spots are reserved, in the socially optimal allocation of reserved parking spots, the numbers of reserved spots from each origin are

$$\bar{M}_i^{r*} = \frac{s_i}{\sum_k s_k} M + \frac{\bar{N}_i^A}{2} - \frac{s_i}{\sum_k s_k} \frac{\sum_k \bar{N}_k^A}{2} = \frac{1}{2}\bar{N}_i^A - \frac{s_i}{\sum_k s_k}\left(M - \frac{1}{2}\sum_k \bar{N}_k^A\right). \tag{12}$$

Under our assumption that $\bar{M}_i^{r*} > 0$, it is required that

$$M > \frac{\beta + \gamma}{2\gamma} \max_i \left\{ \sum_k M_k^{\#} - \frac{\sum_k s_k}{s_i} M_i^{\#} \right\}. \tag{13}$$

From Eq.(11) and Eq.(13), it is easily to figure out that if $\bar{M}_i > 0$, we always have $\bar{M}_i^{r*} > 0$. This implies if the numbers of auto commuters from all origins in the original user equilibrium (Extreme case (1)) are positive, then in Extreme case (2), in the optimal allocation of reserved parking spots, the numbers of reserved spots from each origin are also positive. Under the above assumptions, we have the following proposition.

**Proposition 1.** In a many-to-one network with parking constraint $\left(M < \bar{N}^A\right)$, if the parking capacity $M > \bar{N}^A/2$, it is socially preferable to retain some parking spots unreserved.

**Proof.** When all parking spots are reserved, in the optimal allocation of reserved parking spots, the numbers of reserved spots from each origin are given by Eq.(12), i.e., $M_i^r = \bar{M}_i^{r*}$ for all $i$. Since $\sum_i \bar{M}_i^{r*} = M$, we have $M_i^u = 0$, for all $i$. According to the results in Table 1, we can easily find that the commuting equilibrium of each bottleneck belongs to Scenario I. Also note that even if reduce $M_i^r$ by a small amount, the commuting equilibrium of each bottleneck still belongs to Scenario I. Now we focus on the situation when $\mathbf{M^r}$ is close to $\bar{\mathbf{M}}^{\mathbf{r}*}$. For a given $M$, the total travel cost is the given by

$$TC\left(\mathbf{M^r}\right) = \sum_{i=1}^{n}\left[\left(\alpha t_i + \delta \frac{M_i^r}{s_i}\right)M_i^r + P_i^T\left(N_i - M_i^r\right)\right]. \tag{14}$$

The first order derivatives of Eq.(14) with respect to $M_i^r$ is given by

$$\frac{dTC}{dM_i^r} = \alpha t_i + 2\delta \frac{M_i^r}{s_i} - P_i^T, \tag{15}$$

where $i = 1, 2\cdots, n$.

Under the condition $M > \bar{N}^A/2$, it is easily to find that $dTC\left(\bar{\mathbf{M}}^{\mathbf{r}*}\right)/dM_i^r > 0$ for every $i$. Thus to distribute all the parking spot to commuters is not a social optimum.

In the socially optimal allocation of reserved and unreserved parking spots, denote the number of reserved parking spots for commuters from origin $i$ by $M_i^{r*}$. From Proposition 1 we can see that when $M > \bar{N}^A/2$, $\mathbf{M}^{\mathbf{r}*}$ is different from $\bar{\mathbf{M}}^{\mathbf{r}*}$, and $\sum_i M_i^{r*} < M$; otherwise $\mathbf{M}^{\mathbf{r}*} = \bar{\mathbf{M}}^{\mathbf{r}*}$.

# TRADABLE PARKING PERMIT

Now we consider the case where the city government manages all downtown parking spots through a tradable parking permit scheme. Every day the number of r-commuters is identical to the number of parking permits distributed to commuters by the government, i.e., $M^r$ (a commuter with a parking permit becomes an r-commuter). Suppose the parking permits are allowed to trade among commuters in a competitive market. In this case, the price of a parking permit at equilibrium can be determined by the bi-modal equilibrium condition of equal commuting cost. In the single O-D case, the price of a parking permit is given by

$$p^p = P^T - P_r^A, \tag{16}$$

where $P^T$ is the transit travel cost and $P_r^A$ is the generalized travel cost of r-commuters given by (4). From (4), it can be seen that the permit price is a non-decreasing function of $M^r$ and always greater than zero, as shown in Figure 2. This result is reasonable since one can regard $M^r$ as the total supply of parking permit, with the increase of supply, the price of parking permit goes down or at least does not go up.

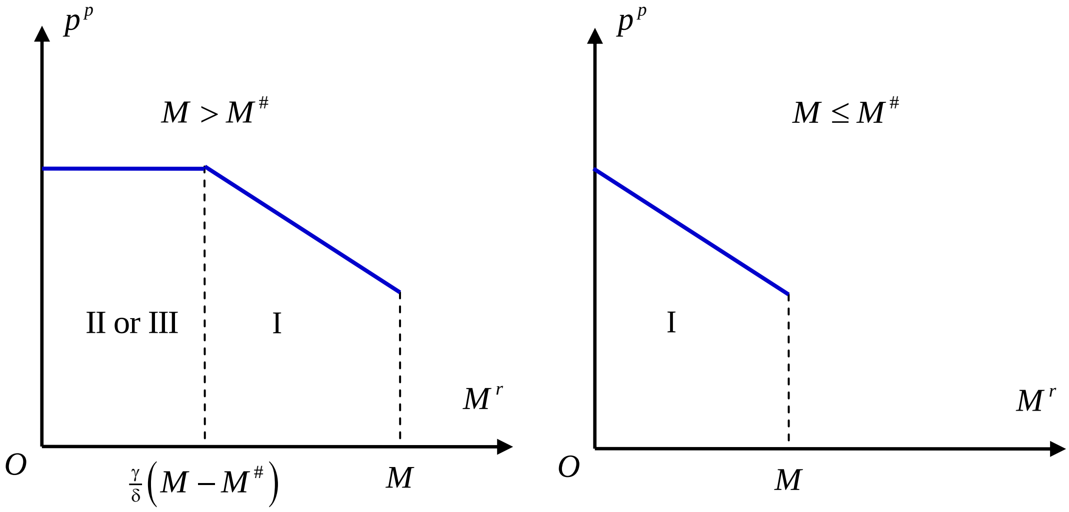


Figure 2 - The price of parking permits at equilibrium

In the many-to-one network, the parking permit can be traded among commuters from different origins, and the equilibrium price of parking permit on the market is the same for all commuters. Now we divide all origins into two classes: with and without commuters using parking permits to obtain their parking spots at equilibrium. Denote the sets of origins with and without commuters using parking permits to obtain their parking spot by $O_I$ and $O_{II}$ respectively.

The equilibrium parking permit price is given by

$$p^p = P_i^T - P_{r,i}^A \ , \tag{17}$$

where $i \in O_I$. For origins with positive number of commuters using parking permits to obtain their parking spot, the difference between travel cost of auto mode and transit mode should be equal, otherwise, trading of parking permits will occur between commuters from origins with different $P_i^T - P_{r,i}^A$, and both sides of the trading can benefit by trading.

For origins without commuters using parking permits to obtain their parking spot, we have

$$p^p > P_i^T - P_{r,i}^A \ , \tag{18}$$

where $p^p$ is defined by (17) and $i \in O_{II}$. For commuters in these origins, under the parking permits scheme: if they have positive number of parking permits, they will benefit from selling out all their permits even they have to compete for parking or take transit; if they don't have parking permits, they will not buy permits from the market and just compete for parking or take transit.

In the case with single bottleneck, the optimal number of reserved parking spot is $M^{r*}$, and the optimal number of parking permits distributed to commuters should also be $M^{r*}$. Since parking permits can be trade freely and efficiently among commuters, from a system perspective, the early permit distribution will not influence the flow pattern at the bottleneck, and the total social cost as well. However, given a permit distribution, commuters with extra permit will sell out these extra ones, and commuters short of permits will either buy some from the market or just take transit.

Now we turn to consider the many-to-one network case. Similar with the single bottleneck case, in the many-to-one network, since parking permits can be trade freely and efficiently among commuters, from a system perspective, the initial permit distribution will not influence the resulting allocation of permits, and the flow pattern and the total social cost as well. Therefore, given the same total number of parking permits, different parking permit distributions can lead to the same resulting flow pattern and total social cost. This makes the parking permit scheme a quite robust strategy, and allows more flexibility to consider issues besides efficiency, such as equity when allocating the permits.

Suppose total number of parking permits distributed each day is $\tilde{M}^r$, and at equilibrium, the number of permits consumed by commuters from origin $i$ is $\tilde{M}_i^r$. Note that in the socially optimal allocation of reserved and unreserved parking spots, the number of reserved parking spots for commuters from origin $i$ is $M_i^{r*}$, and $\sum_i M_i^{r*} = M^{r*}$. When designing the parking permit scheme, if one let $\tilde{M}^r = M^{r*}$, the resulting $\tilde{M}_i^r$ is likely to be different from $M_i^{r*}$. In this case, the free trading of parking permit will lead an increase in total social cost. This is verified in the numerical example.

## NUMERICAL EXAMPLE

In this section, we present a simple numerical example to help illustrate the essential ideas. Suppose $\alpha = 1$, $\beta = 0.5$, $\gamma = 2$, $N_1 = 5500$, $N_2 = 6500$, $s_1 = 25$, $s_2 = 30$, $P_1^T = 62$, $P_2^T = 70$, $t_1 = 18$ and $t_2 = 22$. It is easily to figure out that $\bar{N}_1^A = 2750$ and $\bar{N}_2^A = 3600$, thus $\bar{N}^A = 6350$ and $0.5\bar{N}^A = 3175$. Two total parking supplies are chosen, i.e., $M = 3000 < 0.5\bar{N}^A$ and $M = 5000 > 0.5\bar{N}^A$.

First, we look at the case with $M = 5000 > 0.5\bar{N}^A$. From Figure 3, we can see that the reserved parking supply domain can be divided into five regions. In Region (1), the numbers of reserved parking spots for both origins are comparable. The equilibrium turns out to be an interior one at which the last u-commuter of each origin arrive at the destination at the same time. In Region (2), too many parking spots have been reserved to commuters from origin 2, thus at equilibrium, all the potential auto commuters from origin 2, $\bar{N}_2^A = 3600$, will choose auto mode and park in the CBD. In Region (4), for commuters from origin 2, the parking spots reserved to them is higher than their potential demand, $\bar{N}_2^A = 3600$, thus the parking sources are not well utilized. Regions (3) and (5) correspond to the case when too many parking spots are reserved to commuters from origin 1.

Figure 4 show the total cost under different combinations of reserved parking spots of two origins. It can be seen that when $M = 5000 > 0.5\bar{N}^A$, the minimum total travel cost occurs in Region (1), where $M_1^r + M_2^r < M$ and is far from the boundary of the feasible domain. Also note that, in Figure 4, the dashed line represents the optimal combination of reserved parking spots for commuters from both origins under given total number of reserved parking spots. The dotted line represents the equilibrium parking spots share of commuters from both origins given the total number of parking permits distributed. It can be seen that the trading of parking permit will lead to a non-optimal allocation of parking spots. However, in this example, this non-optimal allocation is not far from the optimal one.

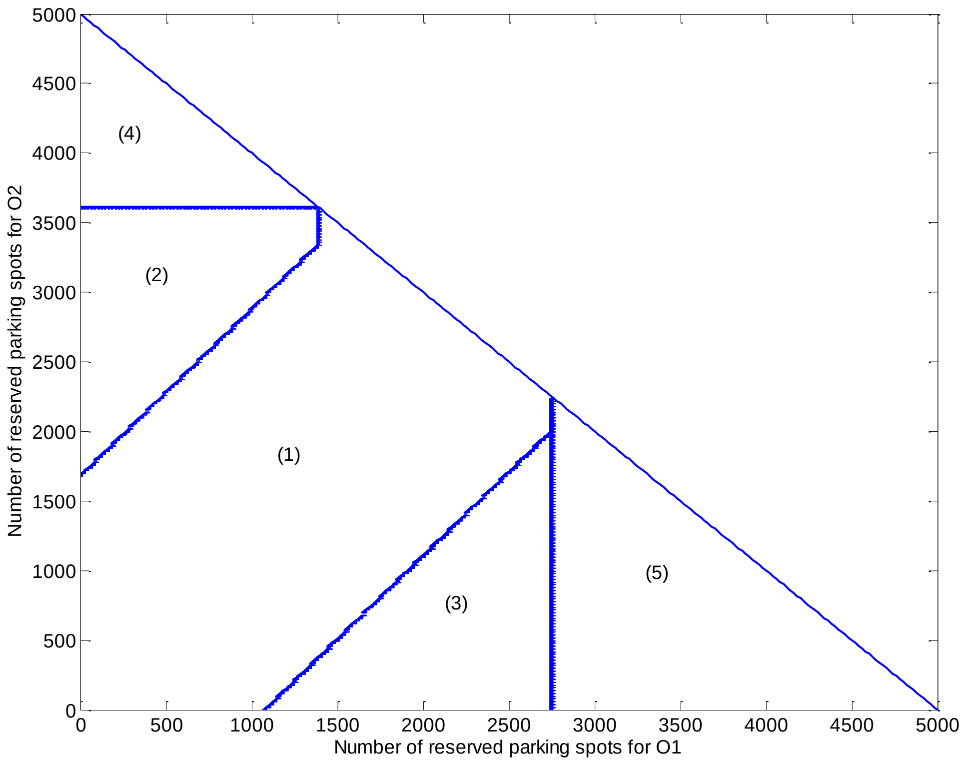


Figure 3 - Reserved parking supply domain $(M = 5000)$

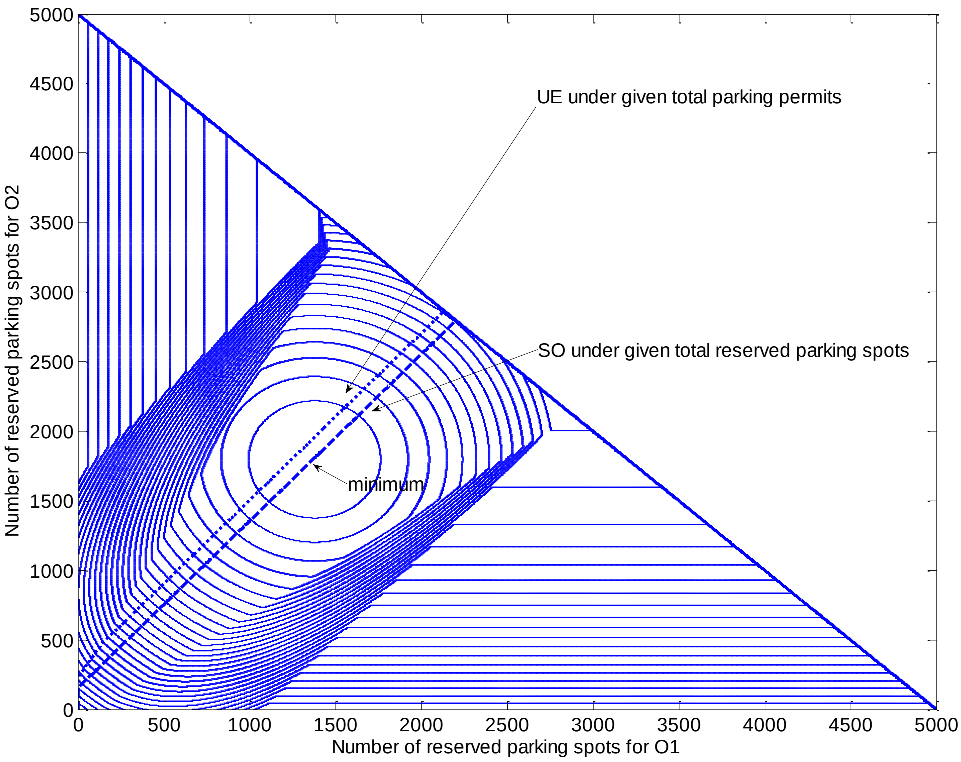


Figure 4 - Total cost contours under different combinations of reserved parking spots $(M = 5000)$

Now we turn to look at the case with $M = 3000 < 0.5\bar{N}^A$. From Figure 5, we can see that the parking supply domain can be divided into two parts. In Region (1), the numbers of reserved parking spots for both origins are comparable. The equilibrium turns out to be an interior one at which the last u-commuter of each origin arrive at the destination at the same time. In Region (2), for commuters from origin 1, the parking spots reserved to them is higher than their potential demand, $\bar{N}_1^A = 2750$, thus the parking sources are not well utilized. Compared with the case with $M = 5000 > 0.5\bar{N}^A$, some of the regions in Figure 3 disappeared, mainly due to the more insufficient parking supply, $M = 3000$, which is even less than the potential demand from origin 2, $\bar{N}_2^A = 3600$.

Figure 6 shows the total cost under different combinations of reserved parking spots of two origins. It can be seen that when $M = 3000 < 0.5\bar{N}^A$, it is socially optimal to have all parking spots reserved, thus the minimum occurs at the boundary of the feasible domain where $M_1^r + M_2^r = M$ . Also note that, in Figure 6, the dashed line represents the optimal combination of reserved parking spots for commuters from both origins under given total number of reserved parking spots. The dotted line represents the equilibrium parking spots share of commuters from both origins given the total number of parking permits distributed. Similarly, the trading of parking permit will lead to a non-optimal allocation of parking spots. However, in this example, this non-optimal allocation is not far from the optimal one.

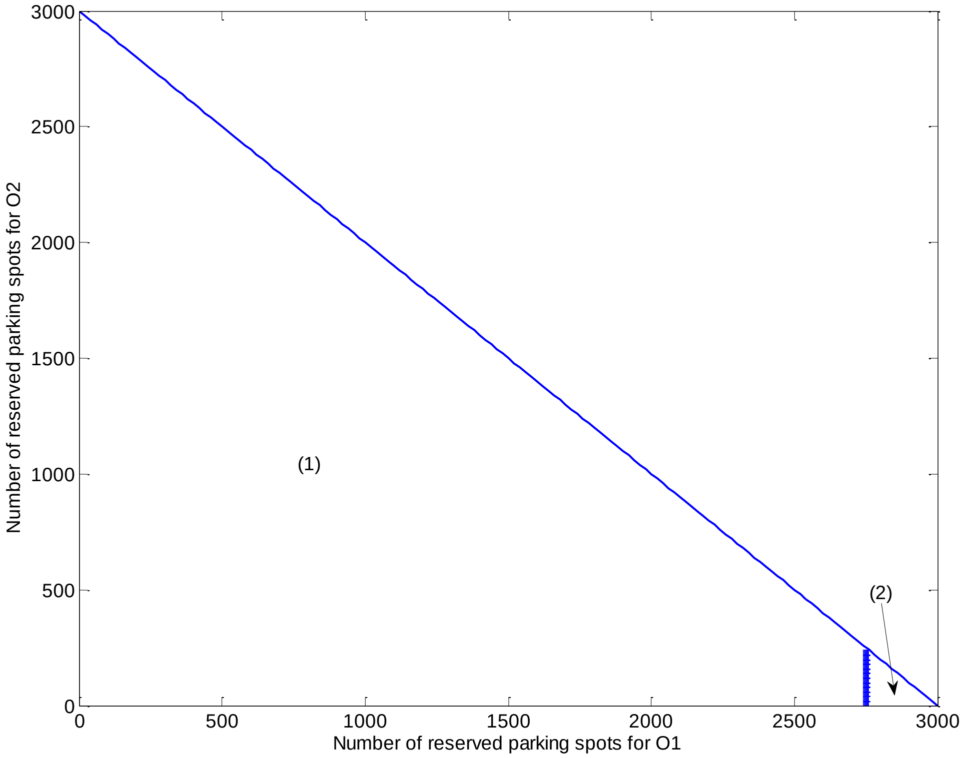


Figure 5 - Reserved parking supply domain $\left(M = 3000\right)$

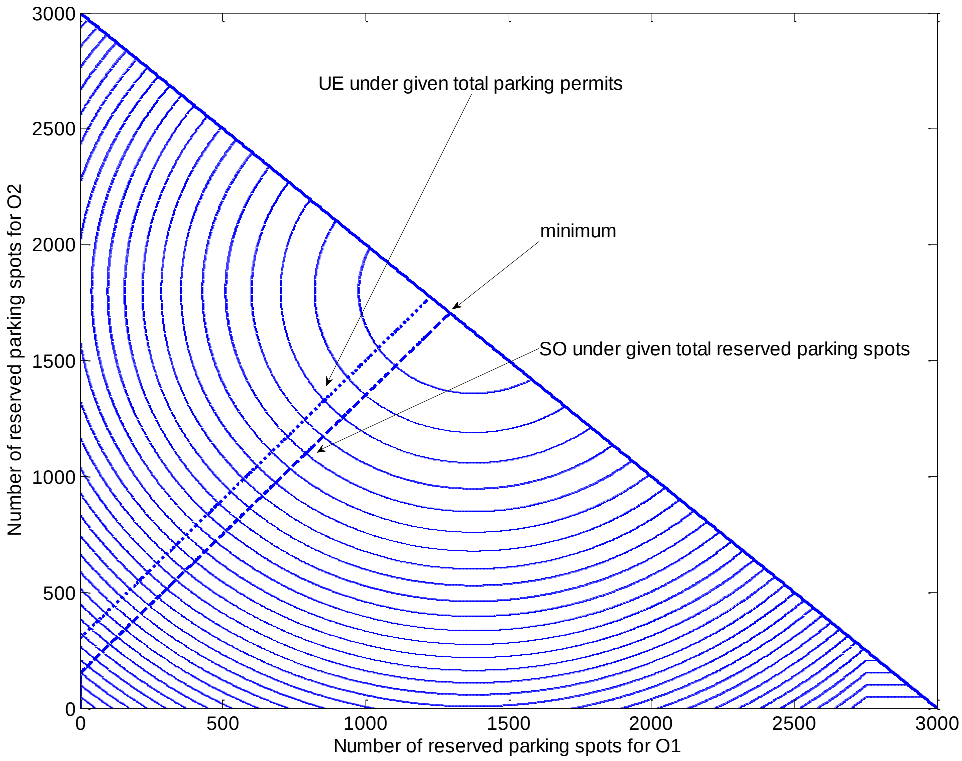


Figure 6 - Total cost contours under different combinations of reserved parking spots $\left(M = 3000\right)$

# CONCLUSION

In this paper we examine in a many-to-one network how the rush-hour traffic pattern with binding parking capacity constraints differs from that generated by the traditional bottleneck model. As shown in Zhang et al. (2011), reserving all parking spots to commuters in advance is socially preferable in terms of total system commuting cost, because the external cost from competition for parking spaces can be completely eliminated through a parking space reservation system.

In Yang et al. (2012), if the number of total parking spots is larger than a critical value, an intermediate mixed case with an appropriate combination of reserved and unreserved parking spots will further reduce the total system cost. In this paper we extend this result to a many-to-one network. This is because although competition for parking spots may increase the schedule delay costs associated with earlier arrival for those commuters without parking permits, their earlier departures from home may relieve peak-hour traffic congestion at the bottleneck and thus reduce total social cost and enhance system performance. The analytical results are verified in the numerical example.

Parking permit schemes are introduced as one way to realize the combination supply of reserved and unreserved parking spots. Given the total reserved parking spots or total parking permits for a certain day, the free and costless trading of parking permits among commuters will not lead to the socially optimal allocation of parking spots for commuters from different origins. In future study, we may further consider the design of parking permit scheme taken into account both equity and efficiency.

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