Revisiting the bottleneck congestion model by considering environmental costs and a modal policy

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Abstract: This article attempts to internalize the negative external effects (congestion and pollution) generated by using cars, by considering the urban tax tool. To do this, we provide the development of a microeconomic model of this urban toll system, in order to minimize the total social cost. Two modes of transportation are taken into account: cars and public transport, the latter being considered non-polluting. The total social cost includes (1) the costs generated by the two modes of transport, (2) the congestion costs, and we add (3) environmental costs generated by using cars. Based on Arnott et al. (1990, 1993), who developed a bottleneck congestion model, three alternative tolls are compared: a fine toll, a coarse toll and a uniform toll. Thus, several types of urban toll are investigated and we also add a modal policy, which redistributes the gains from urban tax to public transport. We analyse the implementation of an economic tool and a modal policy to achieve a social optimum. Finally, we highlight that the uniform toll provides the greatest impact on car traffic reduction but induces the highest total social cost. A coarse toll and a uniform toll reduce the social cost in comparison with a no-toll equilibrium. We also point out that adding a modal policy to the toll is successful in reducing the total social cost. A review of urban toll applications supports this theoretical analysis.

Keywords: transport policy, urban tax, modal policy, environment

1. Introduction

At both local and global scales, public actions are considered in terms of sustainability. In this regard, the transport sector is no exception. The important role of transport as a source of greenhouse gas (GHG) emissions, in particular CO_2 emissions (Raux and Marlot, 2005), is no longer in doubt and is growing rapidly (Stanley et al., 2011). In 2008, the transport sector was responsible for 19% of total GHG emissions (European Union, 2011). According to the European Commission (EC, 2007), European cities represent nearly 85% of the gross domestic product (GDP) of the European Union and 75% of trips are made by car. Given that the spatial distribution of CO_2 emissions from cars is mostly urban, public actions primarily focus on urban transport. Hence, the personal road transport sector poses a significant challenge for the reduction of pollution.

In Europe, Germany, Italy and France have adopted tools to regulate urban traffic and reduce pollution from motor vehicles. In Italy, the city of Milan introduced an environmental toll in January 2008. The most polluting vehicles must pay a tax called Ecopass to access the city centre from 7.30 a.m. to 7.30 p.m., Monday to Friday. Access is free and unrestricted to the least polluting vehicles, public transit and bicycles (Rotaris et al., 2009, 2010). The aim of this ecological toll is to encourage individuals to leave their cars at the entrance to the city and travel to the urban centre by an alternative mode, such as public transport. In Germany, a more drastic solution was chosen. The most polluting cars are simply prohibited in the city centres of Berlin, Cologne and Hanover. The cars carry stickers of different colours indicating their level of pollution. Environmental zones are defined, corresponding to areas where only vehicles complying with emission standards can drive. Vehicles with very high emissions have to stay out of them (http://www.stadtentwicklung.berlin.de). As for France, following the *Grenelle de l'environment* (2007), the eco-vignette (also called the "eco-pastille") was born. The objective was to encourage new car buyers to favour less polluting models. An incentive system of "bonus-malus" was chosen. A bonus is available to purchasers of new cars emitting less than 130 grams of CO_2 per kilometre and a penalty must be paid by buyers of vehicles emitting more than 160 grams of CO_2 per kilometre. These three recent European examples show the willingness of the authorities to consider the problem of car pollution. However, this consideration of the environment in transportation policies has not always existed. For years, the objectives of traffic flow and infrastructure financing were at the heart of transport policy and, in particular, of urban transport. The goal was to find relevant solutions to congestion issues. Nowadays, it is quite different. The aim is not only to reduce car traffic but also to restrict all the other negative externalities of the automobile, such as pollution and noise.

Urban road pricing, like a Pigouvian tax, has been recommended as a welfare-increasing policy (Pigou, 1920; Vickrey, 1963, 1969; Walters, 1961). The damage generated by using cars is taken into account by the urban tax. An urban toll was first used to reduce the number of cars in the city centre but its aims have now evolved. Singapore, London, Stockholm and Milan have implemented this tool for travel into the city centre. Although the aim of Singapore and London was to relieve traffic congestion, in Stockholm the urban toll was introduced not only for this reason but also for an environmental one while in Milan it is an ecological tax.

This paper aims to improve this urban toll system model by making it more relevant and more suited to the current environmental expectations of transport policies. To do this, we propose to add several extensions to the existing bottleneck congestion models of Arnott et al. (1990, 1993). First, in order to consider the environmental purpose of the tool better, we include the environmental costs induced by car usage. Secondly, we associate a modal incentive policy with the toll. The matter of redistribution of toll revenues to public transport has been studied and it was highlighted that it is necessary to improve social equity and to finance public transport (Hau, 1992; Small, 1992; Goodwin, 1989; Parry and Bento, 2001; Litman, 2005This question of the modal policy of toll revenue redistribution was also been discussed by Mirabel (1996), Reymond (2005) and Mirabel and Reymond (2011). These authors investigated two kinds of road toll: a fine toll and a uniform toll but not the coarse toll. A bi-objective optimization of traffic congestion and environmental cost with toll and redistribution of revenue was considered by Chen and Yang (2012) but only in a static network setting, while we address essentially here the redistribution in a sequential dynamic setting. Our approach is so innovative and distinguishable from existing studies insofar as it considers both the environmental costs and the rebate of three kinds of tolls, all in a sequential dynamic context. Finally, the goal is to internalize, through a microeconomic modelling of the urban toll, the external effects of the transport (congestion and pollution) in order to minimize the total social cost. Two modes of transportation are taken into account: cars and public transport, the latter being considered non-polluting (we use the Tabuchi (1993) and Danielis and Marcucci (2002) bimodal model). In the first part, we present the main characteristics of the bimodal model. In the second part, we determine the modal equilibrium. Then, in the third part, we add a modal policy to the model and we determine the social optimum. We present the results of different policies and analyse the best policy in the fourth part. Finally, we highlight the analytical results of the model and discuss them with examples of urban toll implementation.

2. Main characteristics of the bimodal model

We base our development on the reference model of Arnott et al. (1990, 1993) but we assume that there are two possible modes of transport. This modal split implies a modification of certain assumptions of the reference model but the first four assumptions remain the same:

H1: If the arrival rate at the bottleneck exceeds the road capacity K, a queue develops (Vickrey, 1969 introduced this concept, which was developed by Arnott et al., 1990, 1993). The capacity constraint is a flow constraint, while the queue discipline is first-in, first-out (FIFO).

H2: All individuals want to arrive at work at t^*.

H3: All individuals have a total cost proportional to the cost of the travel time (α) and to the cost of schedule delay i.e. the costs of arriving at work early (β) or late (υ).

H4: In the model, all individuals are considered identical.

However, assumptions 5 and 6 are changed and become:

H5: N is fixed but we consider N_A motorists and N_B public transport commuters.

According to Tabuchi (1993), the sum N_A+N_B=N is fixed.

H6: The equilibrium is not a Nash equilibrium but a Wardrop equilibrium (1952) (i.e. at equilibrium, individuals are indifferent to the two modes of transportation inasmuch as travel time costs are the same; by researching the best itinerary individually, agents obtain an equilibrium situation according to which “no commuter can improve his travel time by changing itinerary” (Dagonzo and Sheffi, 1977).

To take into account the environmental cost of the automobile, we consider an additional assumption as follows:

H7: Each car emits a certain average level of emissions e. This is monetized at a constant average cost C_E. C_E is strictly positive and introduced directly into the total social cost.

H7 is a simple hypothesis, insofar as it suggests that environmental damage is proportional to the emissions caused by automobile use, but it is necessary for solving the model.

The main characteristics of the model, including the new assumptions, are presented in the table 1.

Equation (1) implies that if the number of users of public transport (N_B) increases, then the cost C_B decreases. Equation (2), meanwhile, represents the travel cost of a motorist defined by the model of Arnott et al. (1990). The modal equilibrium, represented by equation (3) and defined by assumption (H6), means that users are indifferent to the two modes, since the time costs of travelling by public transport or by car are identical.

A modal equilibrium based only on costs implies a perfect substitution between the two modes of transportation. This bimodal equilibrium is specified as a Wardrop equilibrium. According to equations (4*a*) and (4*b*), if environmental costs are null ($C_E = 0$), then we obtain the total social costs of the Danielis and Marcucci model (2002).

3. Determination of modal equilibrium

Firstly, we determine the modal split in the equilibrium, i.e. the number of motorists and users of public transport without a regulatory policy being implemented. The aim of the introduction of a toll and a modal policy is to improve the equilibrium in order to achieve a second best optimum[1].

Combining equations (1), (2) and (3*a*), we have a quadratic function:

$$\delta N_B^2 + (cK + \delta N)N_B + FK = 0$$

Two conditions are added:

Condition 1: One solution is retained in order that $N \geq \dfrac{cK}{\delta}$ is always true.

Condition 2: A $\overline{N}$ limit is determined. It represents an economic limit above which it is reasonable to construct public transport. Under $\overline{N}$, the car is the only mode of transportation

[1] As we cannot obtain the first best social optimum (because the analytical framework is not perfect, since there are difficulties in assessing environmental damage, and several externalities, but only one economic tool, etc.), we seek the second best. This is the best economic situation it is possible to achieve when the first best is not available.

used. Above $\overline{N}$, cars and public transport are both used and the fixed costs are covered. Note that the number of public transport commuters N_B cannot be small at equilibrium because the fixed costs F would not be covered. So, the distribution is discontinuous up to $N=\overline{N}$. When $N<\overline{N}$, the distribution $((N_A^E, N_B^E)$ is $(N,0)$, but from $N=\overline{N}$ the distribution goes up (N_A^E, N_B^E).

With these new conditions, we obtain at equilibrium the modal split as follows:

$$\begin{cases} \text{For } N > \overline{N} : \\ (N_A^E, N_B^E) = \left(\dfrac{N}{2} + \dfrac{cK}{2\delta} - \sqrt{\left(\dfrac{N}{2} - \dfrac{cK}{2\delta}\right)^2 - \dfrac{FK}{\delta}},\ \ \dfrac{N}{2} - \dfrac{cK}{2\delta} + \sqrt{\left(\dfrac{N}{2} - \dfrac{cK}{2\delta}\right)^2 - \dfrac{FK}{\delta}} \right) \quad (5a) \\ \text{For } N \leq \overline{N} \\ (N_A^E, N_B^E) = (N,0) \quad (5b) \end{cases}$$

with $\overline{N} = \dfrac{cK}{\delta} + 2\sqrt{\dfrac{FK}{\delta}}$ value at which the square root is zero.

Note that the parameter related to time costs (advance and delay) (δ) and the fixed costs of public transport (F) play an important role in equation (5*a*) of the modal equilibrium and the value of the threshold $\overline{N}$.

Combining these results (5*a*) and (5*b*) with (4*a*) and (4*b*) respectively, we obtain the total social costs at equilibrium:

$$\begin{cases} CST_1^E = cN + \dfrac{N}{N_B^E} F + C_E \left(\dfrac{N}{2} + \dfrac{cK}{2\delta}\right) - C_E \sqrt{\left(\dfrac{N}{2} - \dfrac{cK}{2\delta}\right)^2 - F\dfrac{K}{\delta}} \quad \text{si } N > \overline{N} \quad (6a) \\ CST_2^E = C_E N + \dfrac{\delta N^2}{K} \quad \text{si } N \leq \overline{N} \quad (6b) \end{cases}$$

Again, if environmental costs are null, we obtain results identical to those of Danielis and Marcucci (2002). We can now continue with the determination of the social optimum. This is obtained by minimizing the total social cost. The objective is to establish a modal split model with different types of toll associated with a modal redistribution policy to find the second-best social optimum.

4. Adding a modal incentive policy and the social optimum

The external effects of congestion and pollution generated by car use must be considered by the urban toll, so that the collective total cost is minimized. We take the three types of toll identified by Arnott et al. (1990, 1993) to obtain the social optimum: a fine toll, a coarse toll and a uniform toll. To implement a policy of sustainable mobility, an important assumption must be added to the model concerning the encouraging modal shift. This is our second extension:

H8: We consider a redistribution policy of gains from the toll, whatever its type. We assume the toll is implemented in the period *T*. The redistribution of gains from the toll in *T* will have an impact on the ticket price in the period *T+1*.

The aim is to encourage individuals to take public transport because it is assumed to be non-polluting. In fact, public transport is *a priori* only less polluting than cars. However, for reasons of model resolution, we consider a polluting mode (car) and a non-polluting alternative (public transport).

To solve the model with these new assumptions, we develop a nine-step approach. It is divided into two periods: the period T when the urban toll is introduced and the period *T+1* when the modal policy of revenue redistribution is applied. The various types of toll considered follow the same methodology. Box n°1 outlines the stages of resolution of the model:

Box n°1: Methodology

Resolution to the period T consists of five steps:

Step 1: Determination of the cost incurred by the motorist with the toll implemented.

Step 2: Determination of the ticket price, i.e. the cost incurred by the user of public transport.

Step 3: Determination of the modal equilibrium by equalizing the cost sustained by the motorist and that incurred by the user of public transport (Wardrop principle: H6).

Step 4: Calculation of the total social cost due to the modal equilibrium determined in the previous step.

Step 5: Calculation of the toll revenue.

The analytical solution to the period T+1, taking into account the redistribution of income, is composed of four steps:

Step 6: Calculation of the new cost of public transport, and the price by removing the toll revenue determined in Step 5.

Step 7: Determination of the new modal equilibrium.

Step 8: Determination of the new cost sustained by the motorist with the new modal equilibrium.

Step 9: Calculation of the new total social cost.

We apply the nine steps of the methodology above to determine the social optimum in the establishment of the fine toll.

For step 1, we determine the cost sustained by the motorist with the toll implemented: $C_A^P = \frac{\delta N_A}{K}$ (7) which is composed of an hourly cost: $C_H^P = \frac{\delta N_A}{2K}$ and the financial cost of the toll: $C^P = \frac{\delta N_A}{2K}$.

Step 2 determines the ticket price, i.e. the cost incurred by the user of public transport: $p_{(T)} = C_B^P = c + \frac{F}{N_{B(T)}}$ (8)

Then the modal equilibrium is obtained by equalizing equations (7) and (8): $C_A^P = C_B^P$ with $N \geq \frac{cK}{\delta}$. By solving the quadratic polynomial in N_B, we obtain:

$$\begin{cases} \text{If } N > \overline{N}^P \\ (N_A^P, N_B^P) = \left(\frac{N}{2} + \frac{cK}{2\delta} - \sqrt{\left(\frac{N}{2} - \frac{cK}{2\delta}\right)^2 - \frac{FK}{\delta}},\quad \frac{N}{2} - \frac{cK}{2\delta} + \sqrt{\left(\frac{N}{2} - \frac{cK}{2\delta}\right)^2 - \frac{FK}{\delta}} \right) \quad (9a) \\ \text{If } N \leq \overline{N}^P \\ (N_A^P, N_B^P) = (N, 0) \quad (9b) \end{cases}$$

with $\overline{N}^P = \frac{cK}{\delta} + 2\sqrt{\frac{FK}{\delta}}$

Combining equations (4*a*) and (4*b*) with (9*a*) and (9*b*) respectively, as shown in Step 4, we find the total social cost:

$$\begin{cases} \text{If } N > \overline{N}^P \\ CST_1^P = cN + \frac{N}{N_B^P} F - \frac{\delta (N_A^P)^2}{2K} + C_E\left(\frac{N}{2} + \frac{cK}{2\delta}\right) - C_E\sqrt{\left(\frac{N}{2} - \frac{cK}{2\delta}\right)^2 - \frac{FK}{\delta}} \quad (10a) \\ \text{If } N \leq \overline{N}^P \\ CST_2^{Pf} = \frac{\delta N^2}{K} + C_E N \quad (10b) \end{cases}$$

with $\overline{N}^{P} = 2\sqrt{KF} + \dfrac{cK}{\delta}$

The toll revenue is determined as follows: $R^{P} = \tau^{P} \times N_{A}^{P} = \dfrac{\delta (N_{A}^{P})^{2}}{2K}$ (11)

with τ^{P} the fine toll.

We observe that the main difference compared to the equilibrium (without a toll) (see 6*a*) is that the implementation of a fine toll generates revenue that is deducted from the total social cost (10*a*). The new cost of public transport is obtained in step 6 where the toll revenue determined by equation (11) is deducted from the ticket price.

$$p_{(T+1)} = C_{B,(T+1)}^{P} = c + \frac{F}{N_{B,(T+1)}^{P}} - \frac{R_{(T)}^{P}}{N_{B,(T+1)}^{P}} \Rightarrow c + \frac{F}{N_{B,(T+1)}^{P}} - \frac{\delta\left(N - N_{B,(T)}^{P}\right)^{2}}{2KN_{B,(T+1)}^{P}} \quad (12)$$

We determine the new modal equilibrium by combining (7) and (12) (cf. Appendix A):

$$\begin{cases} \text{If } N > \overline{N}^{P''} \\ (N_{A}^{P'}, N_{B}^{P'}) = \left(N + \dfrac{cK}{\delta} - \sqrt{\dfrac{c^{2}K^{2} + \delta^{2}N^{2} - 2\delta FK}{\delta^{2}}}, \quad -\dfrac{cK}{\delta} + \sqrt{\dfrac{c^{2}K^{2} + \delta^{2}N^{2} - 2\delta FK}{\delta^{2}}} \right) \quad (13a) \\ \text{If } N \leq \overline{N}^{P'} \\ (N_{A}^{P'}, N_{B}^{P'}) = (N, 0) \quad (13b) \end{cases}$$

with $\overline{N}^{P'} = \sqrt{\dfrac{2FK}{\delta}}$

For step 8, we determine the new cost sustained by the motorist. By combining (13*a*) and (7), we obtain: $C_{A}^{P'} = \dfrac{\delta N_{A}^{P'}}{K} \Rightarrow \dfrac{\delta N}{K} + c - \sqrt{\dfrac{c^{2}K^{2} + \delta^{2}N^{2} - 2\delta FK}{K}}$ (14)

which implies:

$$\begin{cases} C_{A}^{P'} = \dfrac{\delta N}{K} + c - \dfrac{\sqrt{c^{2}K^{2} + \delta^{2}N^{2} - 2\delta FK}}{K} \\ C_{B}^{P'} = \dfrac{\delta N}{K} + c - \dfrac{\sqrt{c^{2}K^{2} + \delta^{2}N^{2} - 2\delta FK}}{K} \end{cases} \quad (15)$$

The new total social cost is obtained in step 9; by combining (15) with (4*a*) and (4b) we obtain:

$$
\begin{cases}
\text{If } N > \overline{N}^{P'} \\
CTS_1^{P'} = cN + \dfrac{\delta N^2}{K} + C_E\left(N + \dfrac{cK}{\delta}\right) - \sqrt{c^2K^2 + \delta^2N^2 - 2\delta FK}\left(\dfrac{N}{K} + \dfrac{C_E}{\delta}\right) \quad (16a) \\
\text{If } N \le \overline{N}^{P'} \\
CTS_2^{P'} = \dfrac{\delta N^2}{K} + C_E N \quad (16b)
\end{cases}
$$

Note that the exponent P represents the position of the fine toll (period T) and the exponent P' that of the fine toll with a revenue redistribution to public transport (time $T+1$).

We apply the same methodology to the coarse toll and the uniform toll. Table 2 presents all the results.

5. Analysis of optimal policies

The implementation of an economic instrument such as an urban toll changes the behaviour of motorists. The importance of associating a modal policy, such as the development of public transport or a decrease in the ticket price of public transport, in order to facilitate this change in user behaviour has often been established. However, the effectiveness of implementing a modal incentive policy must still be proved.

By analysing the results presented in Table 2, we can compare the effectiveness of the different tolls studied to reduce the proportion of motorists from the equilibrium. In addition, we question whether the modal policy of redistributing toll revenue, introduced in *T+1*, has a real impact on reducing the total social cost compared to the situation obtained in *T*.

Thus, in the subsections that follow, we compare each toll analytically in periods *T* and *T +1* compared to the no-toll equilibrium.

5.1. Fine toll

The no-toll equilibrium is exactly the same as the fine toll situation (without the redistribution policy). This equality comes from the fact that the fine toll does not reduce the proportion of motorists. In fact, its aim is to make the traffic flow during the rush hour. So, the implementation of the fine toll does not modify the total social cost. There are no changes for the community.

On the other hand, a fine toll associated with a modal policy of redistribution of gains entails some modifications. Thus, the proportion of motorists decreases ($\eta_A^P > \eta_A^{P'}$) while, in contrast, the proportion of public transport commuters increases ($\eta_B^P < \eta_B^{P'}$). We use the following limited development: $(1+\varepsilon)^{\frac{1}{2}} \approx 1+\frac{\varepsilon}{2}$ applied to the proportion of motorists to prove this effect (see Appendix B). The fine toll only has no impact compared to the no-toll equilibrium situation. However, the fine toll combined with a modal policy results in a modal transfer from motorists to public transport. The total social cost decreases ($CST^{P'} < CST^{P}$) because the proportion of motorists is reduced, and there is a fall in environmental costs.

5.2. Coarse toll

Contrary to the fine toll, the coarse toll has an impact on the proportion of motorists compared to the no-toll equilibrium situation. We analyse the impacts of the modal policy on the reduction of the proportion of motorists. We use the preceding method i.e. the limited development and we find ($\eta_A^{ct} > \eta_A^{ct'}$) and ($\eta_B^{ct} < \eta_B^{ct'}$) (calculations are in Appendix C). The coarse toll generates a reduction in the proportion of motorists and the modal policy intensifies this result. The total social cost is minimized in comparison with the no-toll equilibrium situation. An economic tool added to a modal policy gives a social optimum in relation to the benchmark situation, so the cost is minimized: ($CST^{ct'} < CST^{ct} < CST^{E}$).

5.3. Uniform toll

The uniform toll (like the coarse toll) entails a reduction in the proportion of motorists in comparison with the no-toll equilibrium situation ($\eta_A^{U} < \eta_A^{E}$). The objective is to compare the situation of the uniform toll alone with that of the uniform toll combined with a modal policy. The results prove that the toll plus the modal policy have a bigger impact on the proportion of motorists. The proportion of motorists decreases and the proportion of public transport commuters increases: ($\eta_A^{U} > \eta_A^{U'}$) and ($\eta_B^{U} < \eta_B^{U'}$) (calculations are in Appendix D). Note again that the objective is achieved with the toll combined with the modal policy. Implementing a uniform toll reduces the proportion of motorists in comparison with the no-toll equilibrium situation. Then, the modal policy intensifies the modal transfer from motorists to public transport. So, the environmental cost is lower, like the total social cost ($CST^{U'} < CST^{U} < CST^{E}$).

6. Discussion

6.1. Theoretical results

After analysing the theoretical results, several conclusions can be made. The implementation of a coarse toll, as well as a uniform toll, reduces the proportion of drivers compared to the no-toll equilibrium. However, the fine toll does not have this advantage. It does not reduce the proportion of motorists, since its main objective is to improve traffic flow during peak periods rather than reduce it. However, this result changes when the modal policy of redistribution is associated with the fine toll. The main conclusion is the success of associating the toll, of whatever type, with a modal incentive policy, enabling the total social cost to be minimized relative to the no-toll equilibrium. Whenever a toll is complemented by a redistribution policy, the modal shift of motorists to public transport increases, leading to a reduction in pollution and a minimization of the total social cost. The effectiveness of the modal incentive policy is demonstrated.

6.2. The application of an ecological toll: Stockholm to Milan

The Stockholm toll had many objectives: to reduce traffic volume by 10 to 15%, to increase traffic speeds in the city centre, to reduce emissions and improve the environment/quality of life of residents. Before the toll was finally implemented, an experiment was carried out lasting seven months, from January to July 2006. In December 2005, a survey showed that two thirds of those questioned were opposed to the congestion system. Nevertheless, in the referendum of September 2006, 51% voted for the establishment of a permanent toll, demonstrating that this seven-month experiment was finally beneficial. During the testing phase, the introduction of tolls generated a decrease of about 22% of the traffic in the area. CO_2 and nitrogen dioxide emissions were reduced by 12%.

However, a cost-benefit analysis performed by Prud'homme and Kopp (2007) showed an inefficient economic toll. Gains by the toll, such as traffic reduction, time saving and environmental benefits, did not cover the costs of setting up the system and the public transport congestion induced by the toll. In addition, the congestion was low so the gains from its reduction to its optimum level were low as well. However, this study focused only on the test period of the toll, while the long-term vision was discarded. According to the report of Raux et al. (2009), the economic balance sheet of the implementation of the toll was mixed.

For example, some original objectives were met by the toll, such as a more than 15% decline in traffic volume, an increase in the speed of circulation in the centre from 22.9 km/h to 26.2 km/h and an increase in well-being in the centre, as shown by satisfaction surveys (Hiselius et al., 2007). However, the heavy investments of implementing the toll system and its operating costs reveal economic inefficiency. The official report of Transek (2006) (an engineering company) was similar in showing that the toll system generated a social loss during the test period. However, over a longer period, that is to say from the fifth year, the report highlighted a social benefit. Each additional year of operation of the toll would bring a profit of 760 million SEK (88 million euros[2]). The report also stated that if the toll was made permanent, investment costs and maintenance would be covered in the form of socio-economic gains between 15 and 25 years. However, the results of cost-benefit analyses should be viewed with caution since they are very sensitive to estimation methods and the values used for the evaluation.

In Italy, the city of Milan has been operating an ecological toll since January 1, 2008. The most polluting vehicles must pay a tax called Ecopass to access the city centre from 7.30 to 19.30, Monday to Friday. Access is free for the least polluting vehicles, public transport, and bicycles. The purpose of this environmental toll is to encourage people to leave their cars at the entrance to the city and to visit the city centre by an alternative method, such as public transport. The pricing system is based on “the polluter pays” principle (Pigouvian tax). The implementation of this ecological toll had three main objectives: to reduce the concentration of particulate matter by 30% in the area subject to tolls, to improve traffic flow by reducing by 10% the number of vehicles entering the area, and to strengthen public transport by redistributing Ecopass revenues. The results presented are from the assessment conducted by the Agenzia Milanese Mobilità Ambiente (AMA)[3]. Concerning fine particles, a 19% reduction of average concentrations of particulate matter was observed compared to the period 2002-2007. The goal was undoubtedly overestimated. However, according to the report, in 2008 there was an 11% reduction in emissions of nitrogen dioxide (NO_2), a 37% reduction in ammonia emissions and a 9% decrease in carbon dioxide (CO_2) emissions. There was thus a general improvement in air quality. The second objective of the toll was also reached since there was a 14% decrease in the number of vehicles entering the area subject to the toll (less than 22,000 vehicles per day). This reduction was particularly focused on the most polluting vehicles as there was an increase in the less polluting vehicles entering the area. The toll system is a side benefit insofar as the long-term vehicle fleet will be renewed but, on the other hand, the traffic of less polluting vehicles will increase in the charging zone. An increase in traffic and congestion can therefore be expected. The third objective targeted by the implementation of the Ecopass was to reinvest the revenue in the public transport network. The estimated sum was € 24 million per year but the actual amount in 2008 was only 12 million. Again, the cost of operation and implementation of the instrument is relatively

[2] Conversion rate used 1SEK = 0.115813 EUR on 30/10/2012
[3] Agenzia Milanese Mobilità Ambiente (AMA), Comune di Milano, 2009, Monitoraggio Ecopass: Gennaio-Dicembre 2008, February 2009.

high compared to the revenue generated, especially since revenues are expected to decrease due to the replacement of the fleet. However, it should be recognized that the environmental toll has achieved its main objective, namely to reduce emissions and decrease traffic in the area.

This review of the application of this tool in the cities of Stockholm and Milan shows rather positive results, at least from an ecological point of view. Indeed, in both cases studied, the polluting emissions have been reduced and the environmental objective achieved. However, these results need to be qualified insofar as the economic efficiency of the Stockholm toll is debatable, at least in the short term, and the ecological toll of Milan has not provided the expected revenue for the funding of public transport. Nevertheless, both have completed their tolls based on improving the environment in the targeted areas.

7. Conclusion

The objective of this article was to develop a microeconomic model of urban tolls to take into account external effects: congestion and pollution. In the first section, we laid the foundation for modelling. Relying in particular on the modal split model of Danielis and Marcucci (2002), we introduced two new assumptions. The first concerns the environmental cost of the automobile and was added directly to the total social cost. The second is the introduction of a modal incentive policy. We have developed a methodology for two periods. First, we assumed that the toll had three different forms, and was introduced in the period T. In a second step, we integrated the policy of redistributing toll revenue to public transport in the period $T+1$, to reduce the ticket price of the latter. Our analytical results highlight a reduction in the number of motorists through a uniform toll or a coarse toll, but not with a fine toll. However, in the period $T+1$, the association of the redistribution policy of gains with any type of toll reduces the total social cost compared to the equilibrium situation (without a toll). Finally, we highlight that the uniform toll has the greatest impact on car traffic reduction but induces the highest total social cost. Coarse tolls and uniform tolls reduce the social cost in comparison with a no-toll equilibrium. The theoretical model shows that the economic tool and the policy of redistribution are complementary, since the modal shift of drivers towards public transport increases, and efficient in the sense that the total social cost is minimized compared to the equilibrium situation. In order to substantiate our theoretical results, we have presented two examples of urban tolls: Stockholm and Milan. The objectives of the Stockholm toll are similar to those of our theoretical model: to reduce congestion and emissions in the city centre. The objectives of the Milan toll are almost identical to those of our theoretical model insofar as they focus on emission reduction (it is an ecological toll), congestion reduction (fluidity of traffic) and strengthening public transport by redistributing revenues. The analysis of both tolls shows that the results are disputable. A reduction in polluting emissions is recorded and the environmental objective is achieved. However, the cost-benefit analysis carried out on the economic efficiency of the Stockholm toll is controversial, at least in the short term, and the Milan toll does not deliver the expected revenue for financing public transport. On the other hand, we know the effect of the time value. Indeed, in the theoretical model, a high time value reduces the effects of the regulatory policies implemented. In contrast, a relatively low time value increases the effectiveness of tolls and a redistribution policy. For example, the cost-benefit analysis carried out on the Stockholm experimental toll shows some economic inefficiency of the instrument. This can be partly explained by the fact that these analyses are based on estimated values of time, and it is therefore questionable to evaluate the gains from reducing congestion. In this regard, the time value is a key variable in the success of the toll.

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APPENDIX A:

Calculation of the stability

<u>Fine toll</u>:

After equalizing the costs of travel in the period (*T+1*), we obtain a first order recurrent nonlinear equation: $\frac{\delta(N-N^P_{B,(T+1)})}{K}=c+\frac{F}{N^P_{B,(T+1)}}-\frac{\delta\left(N-N^P_{B,(T)}\right)^2}{2KN^P_{B,(T+1)}}$

If the solution converges to an equilibrium modal split $(N_A^{P'},N_B^{P'})$, it satisfies: $-\delta(N_B^P)^2-2cKN_B^P-2FK+\delta N^2=0$. Solving the polynomial in N_B^P, we obtain the modal equilibrium.

<u>Calculation of the stability</u>

It expresses $N^P_{B,(T+1)}$ in terms of $N^P_{B,(T)}$, which generates the following relationship:

$$N^P_{B,(T+1)}=\frac{\delta N-cK+\sqrt{c^2K^2-2cK\delta N+3\delta^2N^2-4\delta^2NN^P_{B,(T)}+2\delta^2(N^P_{B,(T)})^2-4FK\delta}}{2\delta},$$

we set $x_{T+1}=f(x)$ which implies:

$$f(x)=\frac{\delta N-cK+\sqrt{c^2K^2-2cK\delta N+3\delta^2N^2-4\delta^2Nx+2\delta^2x^2-4FK\delta}}{2\delta};$$

Global stability is verified if and only if the derivative: $f'(x)\leq 1$.

We set: $J(x)=c^2K^2-2cK\delta N+3\delta^2N^2-4\delta^2Nx+2\delta^2x^2-4FK\delta$

$\Rightarrow J(x)=(\delta N-cK)^2+2\delta^2(N-x)^2-4\delta FK$

with $2\delta^2(N-x)^2>0$ and if $N>\frac{\sqrt{4\delta FK}+cK}{\delta}$ then: $\sqrt{J(x)}>0$;

we find $f'(x)=-\frac{\delta(N-x)}{\sqrt{J(x)}}\leq 0$, because $N>x$ and then $f'(x)\leq 1$. The global stability is verified.

APPENDIX B:

<u>Demonstration: $\eta_A^P>\eta_A^{P'}$ the proportion of motorists in a fine toll situation only is higher than that with fine tolls and a modal policy</u>

The values obtained are:

$$\eta_A^P = \frac{1}{2} + \frac{ck}{2N} - \sqrt{\left(\frac{1}{2} - \frac{ck}{2N}\right)^2 - \frac{Fk}{N^2}} \text{ and } \eta_A^{P'} = 1 + \frac{ck}{N} - \sqrt{\frac{c^2k^2}{N^2} + 1 - \frac{2Fk}{N^2}}$$

We use the following limited development: $(1+\varepsilon)^{\frac{1}{2}} \approx 1 + \frac{\varepsilon}{2}$

We can write η_A^P and $\eta_A^{P'}$ following:

$$\eta_A^P = \frac{1}{2} + \frac{ck}{2N} - \left[\frac{1}{4} - \frac{2ck}{4N} + \frac{c^2k^2}{4N^2} - \frac{Fk}{N^2}\right]^{\frac{1}{2}} = \frac{1}{2} + \frac{ck}{2N} - \frac{1}{2}\left[1 - \frac{2ck}{N} + \frac{c^2k^2}{N^2} - \frac{4Fk}{N^2}\right]^{\frac{1}{2}}$$

with $\varepsilon = -\frac{2ck}{N} + \frac{c^2k^2}{N^2} - \frac{4Fk}{N^2}$

$$\eta_A^{P'} = 1 + \frac{ck}{N} - \left[1 + \frac{c^2k^2}{N^2} - \frac{2Fk}{N^2}\right]^{\frac{1}{2}} \text{ with } \varepsilon = \frac{c^2k^2}{N^2} - \frac{2Fk}{N^2}$$

According to the condition $N \geq ck$ and adding $N \geq Fk$

We use the limited development: $(1+\varepsilon)^{\frac{1}{2}} \approx 1 + \frac{\varepsilon}{2}$ with $(Fk, ck << 1)$

We have:

$$\eta_A^P = \frac{ck}{2N} + \frac{ck}{2n} - \frac{c^2k^2}{4N^2} + \frac{Fk}{N^2} = \frac{ck}{N} + \frac{Fk}{N^2} - \frac{c^2k^2}{4N^2}$$

$$\eta_A^{P'} = \frac{ck}{N} - \frac{c^2k^2}{2N^2} + \frac{Fk}{N^2}$$

which implies: $\eta_A^P - \eta_A^{P'} = \frac{c^2k^2}{4N^2} > 0$

<u>Demonstration: $\eta_B^P < \eta_B^{P'}$ the proportion of users of public transport in a fine toll situation only is lower than that with a fine toll and revenue redistribution to the public</u>

The values are:

$$\eta_B^P = \frac{1}{2} - \frac{ck}{2N} + \sqrt{\left(\frac{1}{2} - \frac{ck}{2N}\right)^2 - \frac{Fk}{N^2}} \text{ and } \eta_B^{P'} = -\frac{ck}{N} + \sqrt{\frac{c^2k^2}{N^2} + 1 - \frac{2Fk}{N^2}}$$

We use the following limited development: $(1+\varepsilon)^{\frac{1}{2}} \approx 1 + \frac{\varepsilon}{2}$

We can write η_B^P and $\eta_B^{P'}$:

$$\eta_B^P = \frac{1}{2} - \frac{ck}{2N} + \left[\frac{1}{4} - \frac{2ck}{4N} + \frac{c^2k^2}{4N^2} - \frac{Fk}{N^2}\right]^{\frac{1}{2}} = \frac{1}{2} - \frac{ck}{2N} + \frac{1}{2}\left[1 - \frac{2ck}{N} + \frac{c^2k^2}{N^2} - \frac{4Fk}{N^2}\right]^{\frac{1}{2}}$$

with $\varepsilon = -\frac{2ck}{N} + \frac{c^2k^2}{N^2} - \frac{4Fk}{N^2}$

$$\eta_B^{P'} = -\frac{ck}{N} + \left[1 + \frac{c^2k^2}{N^2} - \frac{2Fk}{N^2}\right]^{\frac{1}{2}} \text{ with } \varepsilon = \frac{c^2k^2}{N^2} - \frac{2Fk}{N^2}$$

According to the following condition $N \geq ck$ and adding $N \geq Fk$, we use the limited development: $(1+\varepsilon)^{\frac{1}{2}} \approx 1 + \frac{\varepsilon}{2}$ with ($Fk, ck << 1$)

We have: $\eta_B^P = 1 - \frac{ck}{N} - \frac{Fk}{N^2} + \frac{c^2k^2}{4N^2}$

$$\eta_B^{P'} = -\frac{ck}{N} + 1 + \frac{c^2k^2}{2N^2} - \frac{Fk}{N^2}$$

which implies: $\eta_B^P - \eta_B^{P'} = -\frac{c^2k^2}{N^2} < 0$

APPENDIX C:

- **<u>Demonstration: $\eta_A^{ct} > \eta_A^{ct'}$ the proportion of motorists in a uniform toll situation during peak periods is higher than with a modal policy</u>**

The values are:

$$\eta_A^{ct} = \frac{1}{2} + \frac{ck}{4\varphi N} - \sqrt{\left(\frac{1}{2} - \frac{ck}{4\varphi N}\right)^2 - \frac{kF}{2\varphi N^2}} \text{ and } \eta_A^{ct'} = 1 + \frac{ck}{2\varphi N} - \sqrt{\frac{c^2k^2}{4\varphi^2 N^2} - \frac{Fk}{\varphi N^2} + 1}$$

We use the limited development: $(1+\varepsilon)^{\frac{1}{2}} \approx 1 + \frac{\varepsilon}{2}$

We can write η_A^{ct} and $\eta_A^{ct'}$:

$$\eta_A^{ct} = \frac{1}{2} + \frac{ck}{4\varphi N} - \frac{1}{2}\left[1 - \frac{ck}{\varphi N} + \frac{c^2k^2}{4\varphi^2 N^2} - \frac{2Fk}{\varphi N^2}\right]^{\frac{1}{2}} \text{ with } \varepsilon = \frac{ck}{\varphi N} + \frac{c^2k^2}{4\varphi^2 N^2} - \frac{2Fk}{\varphi N^2}$$

$$\eta_A^{ct'} = 1 + \frac{ck}{2\varphi N} - \left[1 + \frac{c^2k^2}{4\varphi N^2} - \frac{Fk}{\varphi N^2}\right]^{\frac{1}{2}} \text{ with } \varepsilon = \frac{c^2k^2}{4\varphi N^2} - \frac{Fk}{\varphi N^2}$$

According to the following condition $N \geq ck$ and adding $N \geq Fk$, we use the limited development: $(1+\varepsilon)^{\frac{1}{2}} \approx 1+\frac{\varepsilon}{2}$ avec ($Fk, ck << 1$)

We have:

$$\eta_A^{ct} = \frac{ck}{2\varphi N} + \frac{Fk}{2\varphi N^2} - \frac{c^2k^2}{16\varphi^2 N^2}$$

$$\eta_A^{ct'} = \frac{ck}{2\varphi N} - \frac{c^2k^2}{8\varphi^2 N^2} + \frac{Fk}{2\varphi N^2}$$

which implies: $\eta_A^{ct} - \eta_A^{ct'} = \frac{c^2k^2}{16\varphi^2 N^2} > 0$

- **Demonstration: $\eta_B^{ct} < \eta_B^{ct'}$ the proportion of users of public transport in a uniform toll situation during peak periods is lower than with modal policy of redistribution**

The values are:

$$\eta_B^{ct} = \frac{1}{2} - \frac{ck}{4\varphi N} + \sqrt{\left(\frac{1}{2} - \frac{ck}{4\varphi N}\right)^2 - \frac{kF}{2\varphi N^2}} \text{ and } \eta_B^{ct'} = -\frac{ck}{2\varphi N} + \sqrt{\frac{c^2k^2}{4\varphi^2 N^2} - \frac{Fk}{\varphi N^2} + 1}$$

We use the limited development following: $(1+\varepsilon)^{\frac{1}{2}} \approx 1+\frac{\varepsilon}{2}$

We can write η_B^{ct} and $\eta_B^{ct'}$:

$$\eta_B^{ct} = \frac{1}{2} - \frac{ck}{4\varphi N} + \frac{1}{2}\left[1 - \frac{ck}{\varphi N} + \frac{c^2k^2}{4\varphi^2 N^2} - \frac{2Fk}{\varphi N^2}\right]^{\frac{1}{2}} \text{ with } \varepsilon = \frac{ck}{\varphi N} + \frac{c^2k^2}{4\varphi^2 N^2} - \frac{2Fk}{\varphi N^2}$$

$$\eta_B^{ct'} = -\frac{ck}{2\varphi N} + \left[1 + \frac{c^2k^2}{4\varphi N^2} - \frac{Fk}{\varphi N^2}\right]^{\frac{1}{2}} \text{ with } \varepsilon = \frac{c^2k^2}{4\varphi N^2} - \frac{Fk}{\varphi N^2}$$

According to the following condition $N \geq ck$ and adding $N \geq Fk$, we use the limited development: $(1+\varepsilon)^{\frac{1}{2}} \approx 1+\frac{\varepsilon}{2}$ with ($Fk, ck << 1$)

We have:

$$\eta_B^{ct} = 1 - \frac{ck}{2\varphi N} - \frac{Fk}{2\varphi N^2} + \frac{c^2k^2}{16\varphi N^2} \text{ and } \eta_B^{ct'} = -\frac{ck}{2\varphi N} + 1 + \frac{c^2k^2}{8\varphi^2 N^2} - \frac{Fk}{2\varphi N^2}$$

which implies: $\eta_B^{ct} - \eta_B^{ct'} = -\frac{c^2k^2}{16\varphi^2 N^2} < 0$

APPENDIX D:

- **Demonstration: $\eta_A^U > \eta_A^{U'}$ the proportion of motorists in a uniform toll situation only is higher than that with a uniform toll and a modal policy**

The values are:

$$\eta_A^U = \frac{1}{2}+\frac{ck}{4N}-\sqrt{\left(\frac{1}{2}-\frac{ck}{4N}\right)^2-\frac{kF}{2N^2}} \text{ and } \eta_A^{U'} = 1+\frac{ck}{2N}-\sqrt{\frac{c^2k^2}{4N^2}-\frac{Fk}{N^2}+1}$$

We use the limited development following: $(1+\varepsilon)^{\frac{1}{2}} \approx 1+\frac{\varepsilon}{2}$

We can write η_A^U and $\eta_A^{U'}$:

$$\eta_A^U = \frac{1}{2}+\frac{ck}{4N}-\left[\frac{1}{4}-\frac{ck}{4N}+\frac{c^2k^2}{16N^2}-\frac{2Fk}{N^2}\right]^{\frac{1}{2}} = \frac{1}{2}+\frac{ck}{4N}-\frac{1}{2}\left[1-\frac{ck}{N}+\frac{c^2k^2}{4N^2}-\frac{2Fk}{N^2}\right]^{\frac{1}{2}}$$

with $\varepsilon = \frac{ck}{N}+\frac{c^2k^2}{4N^2}-\frac{2Fk}{N^2}$

$$\eta_A^{U'} = 1+\frac{ck}{2N}-\left[1+\frac{c^2k^2}{4N^2}-\frac{Fk}{N^2}\right]^{\frac{1}{2}} \text{ with } \varepsilon = \frac{c^2k^2}{4N^2}-\frac{Fk}{N^2}$$

According to the following condition $N \geq ck$ and adding $N \geq Fk$, we use the limited development: $(1+\varepsilon)^{\frac{1}{2}} \approx 1+\frac{\varepsilon}{2}$ with $(Fk, ck << 1)$

We have: $\eta_A^U = \frac{ck}{2N}+\frac{Fk}{2N^2}-\frac{c^2k^2}{16N^2}$

$$\eta_A^{U'} = \frac{ck}{2N}-\frac{c^2k^2}{8N^2}+\frac{Fk}{2N^2}$$

which implies: $\eta_A^U - \eta_A^{U'} = \frac{c^2k^2}{16N^2} > 0$

- **Demonstration: $\eta_B^U < \eta_B^{U'}$ the proportion of users of public transport in a uniform toll situation only is lower than that with a uniform toll and revenue redistribution to public transport**

The values are:

$$\eta_B^U = \frac{1}{2}-\frac{ck}{4N}+\sqrt{\left(\frac{1}{2}-\frac{ck}{4N}\right)^2-\frac{kF}{2N^2}} \text{ and } \eta_B^{U'} = -\frac{ck}{2N}+\sqrt{\frac{c^2k^2}{4N^2}-\frac{Fk}{N^2}+1}$$

We use the limited development following: $(1+\varepsilon)^{\frac{1}{2}} \approx 1+\frac{\varepsilon}{2}$

We can write η_B^U and $\eta_B^{U'}$:

$$\eta_B^U = \frac{1}{2} - \frac{ck}{4N} + \left[\frac{1}{4} - \frac{ck}{4N} + \frac{c^2k^2}{16N^2} - \frac{Fk}{2N^2}\right]^{\frac{1}{2}} = \frac{1}{2} - \frac{ck}{4N} + \frac{1}{2}\left[1 - \frac{ck}{2N} + \frac{c^2k^2}{8N^2} - \frac{Fk}{N^2}\right]^{\frac{1}{2}}$$

with $\varepsilon = \frac{ck}{2N} + \frac{c^2k^2}{8N^2} - \frac{Fk}{N^2}$

$$\eta_B^{U'} = -\frac{ck}{2N} + \left[1 + \frac{c^2k^2}{4N^2} - \frac{Fk}{2N^2}\right]^{\frac{1}{2}}$$ with $\varepsilon = \frac{c^2k^2}{4N^2} - \frac{Fk}{2N^2}$

According to the following condition $N \geq ck$ and adding $N \geq Fk$

We use the limited development: $(1+\varepsilon)^{\frac{1}{2}} \approx 1 + \frac{\varepsilon}{2}$ avec ($Fk, ck << 1$)

We have: $\eta_B^U = 1 - \frac{ck}{2N} - \frac{Fk}{2N^2} + \frac{c^2k^2}{16N^2}$

$$\eta_B^{U'} = -\frac{ck}{2N} + 1 + \frac{c^2k^2}{8N^2} - \frac{Fk}{2N^2}$$

which implies: $\eta_B^U - \eta_B^{U'} = -\frac{c^2k^2}{16N^2} < 0$