A STUDY OF VEHICLE ROUTING PROBLEM, WITH TIME WINDOW, APPLIED TO A FRACTIONAL JET OWNERSHIP COMPANY

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ABSTRACT

This paper presents the first results of a research in optimization of flight schedule, with time window, applied to a fractional jet ownership company. The use of time window intends to represent the situation in which the client is flexible to adjustments at time of boarding. The flexibility of client requesting the air transportation service allows the company to minimize the costs of aircraft operation. To verify and validate the mathematical model, different cases with up to nine clients and one and two bases were simulated. In this paper, a case study with nine clients with homogeneous fleet, in five scenarios, is presented. To simulate the five scenarios presented in this paper, a support tool was developed in AIMMS. This support tool has interface to Excel-MS and to solver Cplex12.4. In this case study, the gains due to the use of time window were reduction of 730km (8.92%) in total distance and reduction of three airplanes (37.5%) in the required number of aircraft.

Keywords: Vehicle routing, Flight scheduling, Operations research, Air transport

1. INTRODUCTION

The business aviation is a segment of general aviation comprised of individuals and companies that use aircraft as tool to make a profit. In Brazil, problems in airport infrastructure have been an obstacle in the country´s economic development. Companies have sought in business aviation solution to overcome this problem with air transport. There are some advantages in the use of a business jets, such as: flexibility to decide the schedule of travel; avoiding crowded airports; traveling to towns not served by commercial aviation; privacy; etc.

There are some services in business aviation that are important to mention.

1. Charter service: The client contracts the service of a company of charter service. The passenger chooses the aircraft, airport of destination and pays for the service. The passenger bears all costs, including the cost of returning the empty aircraft.

2. Private jet ownership: The company decides to buy a business jet. In this case, the company bears all costs, such as: Flight permits, taxes, insurance, maintenance, and crew.

3. Fractional jet ownership: It involves the acquisition of a partial interest in an aircraft from a fractional operator, who acquires, operates, and manages fleets of similarly configured aircraft on behalf of multiple owners.

Fractional Jet Ownership

This service was launched in 1986 by NetJets, an American company. The client buys a share of a plane, rather than an entire plane. The owners have guaranteed access to that plane or similar. Fractional owners pay a monthly maintenance fee and an occupied hourly operating fee.

2. OBJECTIVE

This study aims to model the operation of a fractional jet ownership company in order to be able to find the flight schedule with the minimal operational cost. The peculiarities of this air transportation service must be considered, such as: client may choose to share the flight with other clients; client may choose for non-stop flight; client is flexible about boarding time and arrival time.

Problem description

In 2012, Lopes,J presented a study about fractional ownership company. She modeled the company operation and identified the most important routes in Brazil. This paper aims to model the company operation implementing time window and shared flight, conditions not implemented in that paper. In order to obtain an appropriate comparison, the data used in this paper is the same data used in previous paper.

It is considered that there are n clients requiring air transportation service. These clients will be served by a maximum of n airplanes. For each required air transportation service, it is informed the departure airport *i* (departure node) and the arrival airport *i+n* (arrival node). To best describe the problem, the following sets are used:

- K: Set of available airplanes (|K| <= n);
- G-: Set of Initial Garage nodes;
- G+: Set of Final Garage nodes;
- P: Set of departure nodes;
- U: Set of Arrival nodes;
- N= $G^{-} \cup G^{+} \cup P \cup U$, Set of all nodes (airports).

Each client that requires a transportation service will board in node *i* ($\forall i \in P$), and land in node *j* ($\forall j \in U$), in this case, *j=i+n*. The client informs the load *qi* in the departure node *i*, i.e., the number of passenger that will board in node *i.* The load *qi* is positive in the departure node and negative in the arrival node. The client also informs the time window *($Tmin_i$, $Tmax_i$)* in the departure node.

The simulation, with homogeneous fleet, presented in this paper uses the jet Phenom100, manufactured by Embraer, with capacity of 5 passengers. It is assumed that this jet has average speed of 380km/h for distance less than 420km, and average speed of 440km/h for distance more than 420km.

The following variables and parameters will be used in the mathematical model:

$X_{k,i,j}$: Binary variable. It indicates, when 1, that the arc*(i,j)* served by jet *k* is in use;
B_i: Variable. Boarding time at node *i*;
S_i: Parameter. Period for boarding all passengers. It is assumed S_i=15min for all nodes;
W_i: Variable. Airplane waiting time at node *i* before boarding procedure;
($B_i + S_i$): Variable. Departure time at node *i*;
($B_i - W_i$): Variable. Airplane arrival time in node *i*;
Vôo_direto$_i$: Parameter. It indicates, when 1, that the flight started at node *i* ($\forall i \in P$) is non-stop;
q_i: Parameter. Quantity of passengers boarding at node *i*. It is informed by the client;
Qt_i: Variable. It indicates the number of passengers taking off at node *i*, i.e, Qi = $Q_{i-1} + q_i$.
$Qmax_i$: Parameter, Qmaxi ≥ qi. "Qmaxi > qi" indicates that the client boarding at node *i* accepts to share the flight with others clients. When Qmaxi = qi, the flight is not shared.
Capac_Aviao: Parameter. It indicates the capacity of the airplane. The Phenom100 capacity is 4 to 5 passengers as it is informed by manufacturer Embraer. In this paper, it is used Capac_Aviao=5.
Lat_i: Parameter. Latitude of node *i*.

$Long_i$: Parameter. Longitude of node *i*.
[$Tmin_i$, $Tmax_i$] : Parameter. Time window, in minutes, of node *i*;
$Dist_{i,j}$: Parameter. Distance between node *i* and node *j* in km.

The geographic coordinates are used to calculate the distance between nodes. The following formula is used in this paper:
$Dist_{i,j}$ = 6370*arccos [sin (Lat_i) * sin(Lat_j) + cos(Lati) * cos(Lat_j) * cos ($Long_i$ – $Long_j$)], in km.

t_{ij}: Parameter. Direct flight time from node *i* to node *j*. The $t_{i,j}$ is based on the distance between nodes and the airplane velocity. In this paper, it is considered t_{ij} =1 when $Dist_{i,j}$ = 0.

3. MATHEMATICAL MODEL

The mathematical model used in this paper is based on the model developed by Cordeau(2006) and Mauri(2009) and was adapted to the operation of a fractional jet ownership company.

The objective of this model is to minimize the company operational costs in transportation service. As the costs of crew, fuel, aircraft on ground, maintenance and others costs are company strategic information, in this paper it is defined 3 different objectives associated to company operational costs. The first and main objective is to minimize the total distance traveled by all airplanes. The second objective is to minimize the number of airplanes used to serve all clients transportation requests. The third objective is to minimize the airplane waiting time on ground, i.e, minimize the variable W_i. These three objectives are associated to the operational cost.

With these assumptions, the routing problem becomes a multiobjective optimization. To solve this kind of problem, there are some techniques such as: weighted sum method and hierarchical method. As it is explained by Taha,H.(2008), in the weighted sum method there is only one objective function comprised by weighted sum of different objectives. The definition of the weight values is essentially subjective.
The hierarchical Method ranks the objectives in a descending order of importance, and each objective function is then minimized individually.

For this study, the weighted sum method is used. The objective function is defined in equation (1).

Minimize:
$$Z = \omega_1 * \sum_k \sum_i \sum_j Dist_{i,j} * X_{k,i,j} + \omega_2 * \sum_k \sum_{j \in P} X_{k,g-,j} + \omega_3 * \sum_i w_i \qquad (1)$$

Each factor used in equation (1) indicates an objective multiplied by a parameter ω. The parameters $(\omega_{1,}\omega_2, \omega_3)$ are called weights, and are positive numbers representing the importance defined by the modeler. Definition of weights is subjective. In this paper, it is used $\omega_1 = 1000$, because the main objective in this study is to minimize the total distance. It is used $\omega_2 = 90$ and $\omega_3 = 1$, it means, in the objective function, the cost of one airplane is similar to the cost of 90 minutes of waiting time on ground. The solver will try to minimize the number of airplanes in optimal solution, but if the inclusion of one airplane in the solution provides a reduction of 90 minutes in the waiting time on ground or more, the solver will decide to include this additional airplane.

All equations and constraints used in the mathematical model are defined in table 1:

Table 1 – Equations and Constraints

Equation	Description
Minimize $Z = \omega_1 * \sum_k \sum_i \sum_j Dist_{i,j} * X_{k,i,j} + \omega_2 * \sum_k \sum_{j \in P} X_{k,g-,j} + \omega_3 * \sum_i w_i$ (1)	Objective Function.
$Cost\_Distance = \sum_k \sum_i \sum_j Dist_{i,j} * X_{k,i,j} \quad \forall k \in K;\ i,j \in N$ (2)	Sum of the distance traveled by all airplanes.
$Cost\_Airplane = \sum_k \sum_{j \in P} X_{k,g-,j} \quad \forall k \in K;\ j \in P$ (3)	Number of airplanes required to serve all clients.
$Cost\_Wait = \sum_i w_i \quad \forall i \in P$ (4)	Sum of all waiting time on ground.

Constraint	Description
$\sum_{g- \epsilon G-} \sum_{j \epsilon \{P \cup G+\}} X_{k,g-,j} = 1 \quad \forall k \in K$ (5)	It ensures all airplane *k* takeoff from an initial garage once only.
$\sum_{g+ \epsilon G+} \sum_{i \epsilon \{U \cup G-\}} X_{k,i,g+} = 1 \quad \forall k \in K$ (6)	It ensures all airplane *k* land on final garage once only.
$\sum_{k \in K} \sum_{j \epsilon \{P \cup U\}; j \neq i} X_{k,i,j} = 1 \quad \forall i \in P$ (7)	It ensures that each node will be served by only one airplane
$\sum_{j \epsilon \{P \cup U\}; j \neq i} X_{k,i,j} = \sum_{j \epsilon \{P \cup U \cup G+\}; j \neq i; j \neq i+n} X_{k,i+n,j}$ (8) $\forall k \in K;\ i \in P$	It ensures that a departure node will be in the same route of its arrival node
$\sum_{j \epsilon \{P \cup U \cup G-\}; j \neq i; j \neq i+n} X_{k,j,i} = \sum_{j \epsilon \{P \cup U\}; j \neq i} X_{k,i,j}$ (9) $\forall k \in K;\ i \in P$	It ensures the flow equilibrium at a node.
$\sum_{j \epsilon \{P \cup U\}; j \neq i} X_{k,j,i} = \sum_{j \epsilon \{P \cup U \cup G+\}; j \neq i; j \neq i-n} X_{k,i,j}$ (10) $\forall k \in K;\ i \in U$	It ensures the flow equilibrium at a node.
$B_j \geq B_i + S_i + t_{i,j} + w_j - 10000 * (1 - \sum_k X_{k,i,j})$ (11.1) $W_j \geq B_j - (B_i + S_i + t_{i,j} + w_i) - 10000 * (1 - \sum_k X_{k,i,j})$ (11.2) $\forall i,j \in (P \cup U) \mid i \neq j, j \neq (i-n)$	It is about the boarding time at node *i*. It is a non-linear equation as the variables B_i and W_i is multiplying the binary variable $X_{k,i,j}$. After linearization, it becomes two linear equations.
$Qt_j \geq Qt_i + q_j - 1000 * (1 - \sum_{k \in K} X_{k,i,j}) + (1000 - q_i - q_j) * \sum_{k \in K} X_{k,j,i}$ (12.1) $\forall i \in (P \cup U \cup G-), j \in (P \cup U) \mid i \neq j$	It is about the quantity of passengers in airplane. It was a non-linear equation as shown in equation (12).

$\mathrm{Tmin_i} \le \mathrm{B_i} \le \mathrm{Tmax_i}$ (13) $\forall\, i \in (P \cup U)$	It is about the time window that must be respected.
$B_{i+n} \ge B_i + S_i + t_{i,i+n}$ (14) $\forall\, i \in P$	It ensures that, for each client, the arrival time is greater than the boarding time.
$\sum_{k \in K} X_{k,i,i+n} \ge Voo\_direto_i$ (15) $\forall\, i \in P$	It is about if the airplane taking off from node *i* ($\forall\, i \in P$) must fly straight to node *(i+n)*, i.e, a non-stop flight
$Qt_i \le Qmax_i$ (16) $i \in (P \cup U)$	The shared flight is associated to constraint (16) via parameter $\mathrm{Qmax_i}$.
$Qt_i \le Capac\_Aviao$ (17) $i \in (P \cup U)$	It ensures that total number of passenger in airplane is not higher than airplane capacity.

In this model, it is assumed that there are n airplanes to serve the n clients. This assumption ensures there is a feasible solution to be found by the solver. One objective is to minimize the quantity of airplanes used in the optimal solution. All airplanes takeoff from an initial garage and land on a final garage. The airplane may land on any final garage, even it is different from the initial garage.

Equation (2) represents the sum of the distance traveled by all airplanes, this sum is called *Cost_Distance*. The main objective is to reduce this value. Equation (3) represents the number of airplanes required to serve all clients. This quantity is called *Cost_Airplane*. In this equation, it is considered only the airplanes that takeoff from an initial base and land at node *i*, ($\forall\, i \in P$). Equation (4) represents the sum of all waiting time on ground. This sum is called *Cost_Wait*.

Constraint (5) ensures every airplane *k* takes off from an initial garage once only. Constraint (6) ensures all airplane *k* land on final garage once only. Constraint (7) ensures that each node will be served by only one airplane. Constraint (8) ensures that a departure node will be in the same route of its arrival node. The flow equilibrium at a node is ensured by equation(9) and (10).

Constraint (11) is about the boarding time at node *i*. It is a non-linear equation as the variables B_i and W_i multiply the binary variable $X_{k,i,j}$. As it is discussed in Cordeau(2006), the introduction of a Big-M parameter can linearize this equation. The new equation is shown as follow.

$$B_j \ge B_i + S_i + t_{i,j} + w_j - 10000 * \left(1 - \sum_k X_{k,i,j}\right) \quad \forall i,j \in (P \cup U) \mid i \ne j, j \ne (i-n) \qquad (11.1)$$

$$W_j \ge B_j - (B_i + S_i + t_{i,j} + w_i) - 10000 * \left(1 - \sum_k X_{k,i,j}\right) \quad \forall i,j \in (P \cup U) \mid i \ne j, j \ne (i-n) \qquad (11.2)$$

Constraint (11.2) was included in order to sum all waiting time on ground (wi).

Like constraint (11), constraint (12) is non-linear as the variable Qt_i multiplies $X_{k,i,j}$. The linearization of this constraint is proposed by Desporchet and Laporte(1991) and it is used in this study. Constraint (12) is rewritten as follows.

$$Qt_j \geq Qt_i + q_j - 1000 * (1 - \sum_{k \in K} X_{k,i,j}) + (1000 - q_i - q_j) * \sum_{k \in K} X_{k,j,i} \quad (12.1)$$
$$\forall i \in (P \cup U \cup G-), j \in (P \cup U) | i \neq j$$

Constraint (13) is about the time window that must be respected. The time window is informed by the client.

To conclude the mathematical model, the following constraints are included:

$$B_{i+n} \geq B_i + S_i + t_{i,i+n} \qquad \forall i \in P \qquad (14)$$

$$\sum_{k \in K} X_{k,i,i+n} \geq Voo\_direto_i \qquad \forall i \in P \qquad (15)$$

$$Qt_i \leq Qmax_i \qquad \forall i \in P \qquad (16)$$

$$Qt_i \leq Capac\_Aviao \qquad \forall i \in N \qquad (17)$$

Constraint (14) was included to ensure that arrival time is greater than boarding time. This constraint is based on the direct time flight to arc *(i,i+n)* ($\forall i \in P$). Constraint (15) considers whether the airplane taking off from node *i* ($\forall i \in P$) must fly straight to node *(i+n)*, i.e, a non-stop flight. Voo_direto equals to 1 indicates a non-stop flight. If the client decides for a non-stop flight, he must inform if he accepts a shared flight. The shared flight is associated to constraint (16) via parameter $Qmax_i$, if $Qmax_i = q_i$, client informs that the flight is not shared The constraint (17) ensures that the total quantity of passengers in the airplane is not higher than the airplane capacity.

4. IMPLEMENTATION

The model of flight schedule used in this paper, and discussed in previous sections, was implemented in AIMMS, solved by solver Cplex 12.4 using a data base in Excel file.

The AIMMS (*Advanced Integrated Multidimensional Modeling Software*) is an environment of mathematical modeling used in optimization problem. In this environment it is possible to choose different solvers to obtain the optimal solution.

Using the software AIMMS, a support tool was developed to help the company to decide the best route to be used in the operation. With this tool, the company will be able to load transportation request of all clients, saved as Excel file, and run the model to obtain the optimal solution. The tool developed in this research is shown in figure 1.

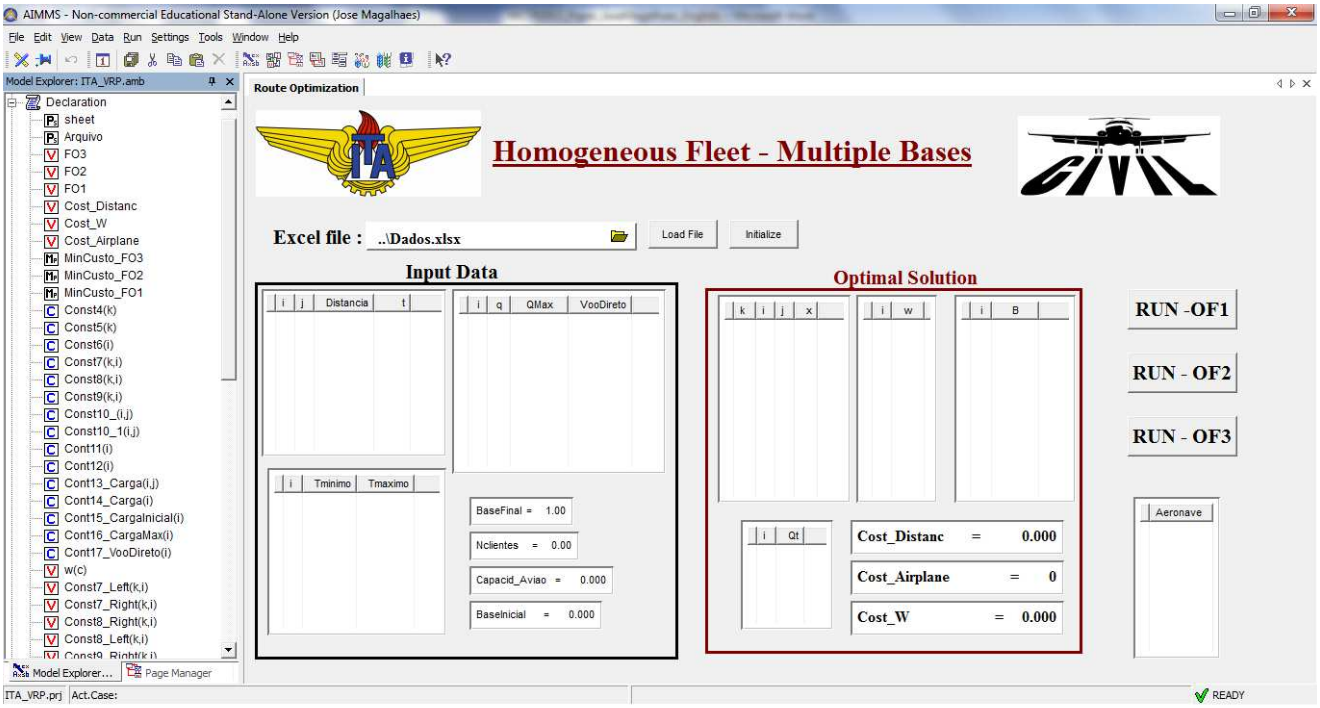

Figure 1 – Support Tool developed in AIMMS

In this paper, five scenarios with nine transportation requests were simulated. In these simulations, an homogeneous fleet and two airports acting as initial garage and final garage are considered. The data base used in the simulation is the same used by Lopes,J.(2012) and it is shown on table 2, 3, and 4.

These tables contain information about the nodes Initial Garage, Final Garage, Departure and Arrival. They also contain information about time window, geographic coordinates, number of passenger to board, if it is a non-stop flight and if the shared flight is allowed.

<u>Initial Garage:</u> Nodes enumerated as 1 and 2 represent the initial garage. Information about initial garage is described on table 2.

Table 2 – Airports as Initial Garages

Initial Garage							
Node	Code	Tmin	Tmax	Lat	Long	q	Qmax
1	SBSP	0	10000	-23,6256	-46,6558	0	0
2	SBRJ	0	10000	-22,9097	-43,1622	0	0

<u>Client transportation demand:</u> There are nine transportation requests that must be attended by the company. The departure nodes are enumerated from 3 to 11, and the respective arrival nodes are enumerated from 12 to 20. The client must inform the time window on the departure node; if it is a non-stop flight; and if shared flight is allowed. The data base is shown in table 3. All time is in minutes.

Table 3 – Client Demand for air transport

Client Demand																	
	Departure									Arrival							
Client	Node	Code	Tmin	Tmax	Lat	Long	q	Qmax	Vôo Direto	Node	Code	Tmin	Tmax	Lat	Long	q	Qmax
1	3	SBBR	534	534	-15,8622	-47,9122	3	5	0	12	SBCF	616,098	780,294	-19,6239	-43,9713	-3	5
2	4	SBBR	380	380	-15,8622	-47,9122	1	1	1	13	SBCF	462,098	626,294	-19,6239	-43,9713	-1	5
3	5	SBSP	726	726	-23,6256	-46,6558	2	5	0	14	SBBR	841,002	1071,006	-15,8622	-47,9122	-2	5
4	6	SBRJ	864	864	-22,9097	-43,1622	1	1	1	15	SBSP	920,802	1034,406	-23,6256	-46,6558	-1	5
5	7	SBSP	888	888	-23,6256	-46,6558	2	5	0	16	SBNF	951,198	1077,594	-26,8797	-48,6481	-2	5
6	8	SBSP	996	996	-23,6256	-46,6558	3	5	0	17	SBNF	1059,2	1185,594	-26,8797	-48,6481	-3	5
7	9	SBSP	932	932	-23,6256	-46,6558	2	5	0	18	SBRJ	988,802	1102,406	-22,9097	-43,1622	-2	5
8	10	SBSP	470	470	-23,6256	-46,6558	1	5	0	19	SBRJ	526,802	640,406	-22,9097	-43,1622	-1	5
9	11	SBNF	932	932	-26,8797	-48,6481	2	5	0	20	SBSP	995,198	1121,594	-23,6256	-46,6558	-2	5

The client informs the departure time and the acceptable time window. With this information, it is possible to obtain time window at the arrival node. In this paper, the following criteria were used to obtain the time window at arrival nodes

$Tmin_{i+n} = Tmin_i + t_{i,i+n}$ $\quad \forall\, i \in P$ (18)

$Tmax_{i+n} = Tmax_i + 3^* t_{i,i+n}$ $\quad \forall\, i \in P$ (19)

Final Garage: The nodes enumerated as 21 and 22 represent the final garages. All airplanes must return to a final garage after completing a mission. Table 4 describes the final garages used in this paper.

Table 4 – Airports as Final Garages

Final Garage							
Node	Code	Tmin	Tmax	Lat	Long	q	Qmax
21	SBSP	0	10000	-23,6256	-46,6558	0	0
22	SBRJ	0	10000	-22,9097	-43,1622	0	0

Table 5 describes all sets used in this paper.

Table 5 – Set defined in the scenarios

Set	Nodes
G-	1 and 2
P	3,4,5,6,7,8,9,10 and 11
U	12,13,14,15,16,17,18,19, and 20
G+	21 and 22

The following simulations were performed in this paper: 1.Optimization with no time window and no shared flight; 2.Optimization with shared flight and no time window ; 3.Optimization with shared flight and time window 15min; 4.Optimization with shared flight and time window 30min; 5.Optimization with shared flight and time window 45min.

To evaluate the model behavior when the parameters $(\omega_1, \omega_2, \omega_3)$ vary, two others objectives functions were defined. The 03 objectives functions used in this paper are defined as follow:

OF1: Minimize Z = (*1000*Cost_Distanc + 90*Cost_Airplane + Cost_Wait*) (20)

OF2: Minimize Z = (*1000*Cost_Distanc + Cost_Airplane*) (21)

OF3: Minimize Z = (*Cost_Distanc*) (22)

5. RESULTS

The model was executed on a computer with Intel Core, I5, 2.5GHz, 4,0GB RAM, 64 Bits and OS Windows7.

The results obtained in the five simulated scenarios are described below. The result tables show, for each objective function, the following information: total distance traveled by airplanes; total waiting time on ground; number of iterations and CPU time to obtain the optimal solution.

Scenario 0: Simulation with no time window in the departure nodes and no shared flight. This is the typical operation of an ordinary air taxi: the company must be ready on the time requested by the client and the flight is non-stop. In this scenario, the client doesn´t allow flexibility in the boarding time. Qmax=q indicates the client doesn't allow shared flight. And Voo_Direto=1 indicates the flight is non-stop.

Table 6 – Scenario0 - Client Demand (No shared flight, no time window)

	Client Demand																
	Departure									Arrival							
Client	Node	Code	Tmin	Tmax	Lat	Long	q	Qmax	Vôo Direto	Node	Code	Tmin	Tmax	Lat	Long	q	Qmax
1	3	SBBR	534	534	-15,8622	-47,9122	3	3	1	12	SBCF	616,098	780,294	-19,6239	-43,9713	-3	5
2	4	SBBR	380	380	-15,8622	-47,9122	1	1	1	13	SBCF	462,098	626,294	-19,6239	-43,9713	-1	5
3	5	SBSP	726	726	-23,6256	-46,6558	2	2	1	14	SBBR	841,002	1071,006	-15,8622	-47,9122	-2	5
4	6	SBRJ	864	864	-22,9097	-43,1622	1	1	1	15	SBSP	920,802	1034,406	-23,6256	-46,6558	-1	5
5	7	SBSP	888	888	-23,6256	-46,6558	2	2	1	16	SBNF	951,198	1077,594	-26,8797	-48,6481	-2	5
6	8	SBSP	996	996	-23,6256	-46,6558	3	3	1	17	SBNF	1059,2	1185,594	-26,8797	-48,6481	-3	5
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8	10	SBSP	470	470	-23,6256	-46,6558	1	1	1	19	SBRJ	526,802	640,406	-22,9097	-43,1622	-1	5
9	11	SBNF	932	932	-26,8797	-48,6481	2	2	1	20	SBSP	995,198	1121,594	-23,6256	-46,6558	-2	5

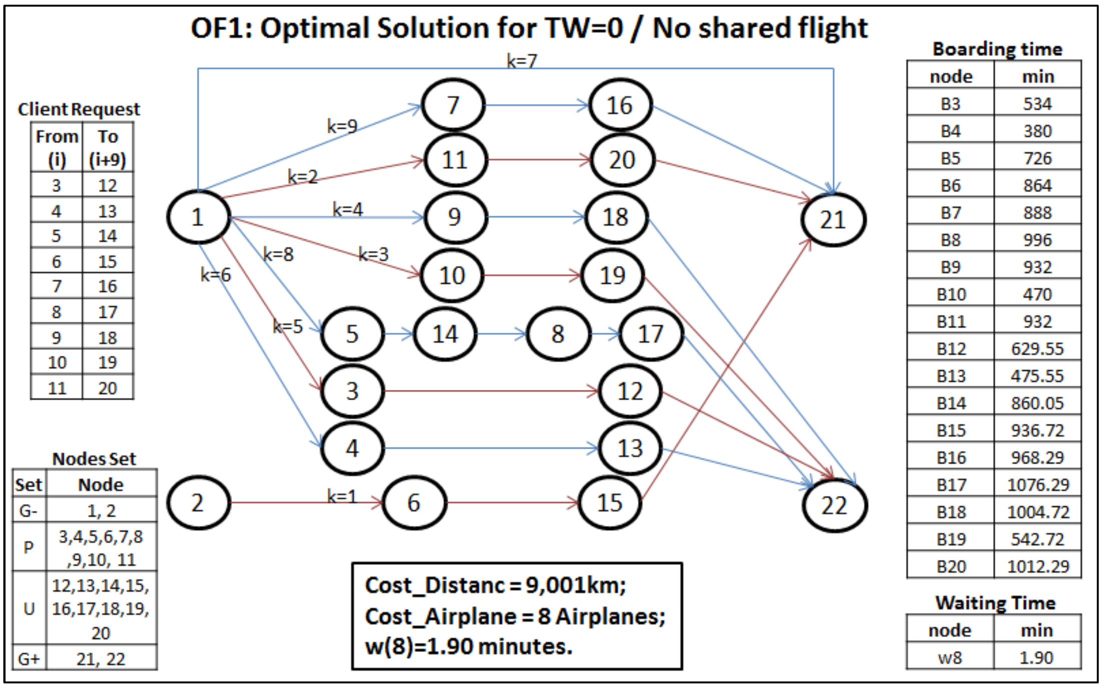

Figure 2 – Optimal Solution, TW=0, no shared flight

Table 7 – Results in scenario 0

	No Time Window / No Shared Flight				
	Distance(km)	W (min)	Qty Airplane	Iterations	CPU time (sec)
OF1	9,001	1.90	8	219	0.37
OF2	9,001	307.18	7	127	0.26
OF3	9,001	0	9	86	0.05

In scenario 0, the total distance traveled is 9,001km in optimal solution. All flights are non-stop, it means the arc(i,i+9) must be presented in the optimal solution.

<u>Scenario1:</u> Simulation with shared flight and no time window in the departure.
In this scenario, the client doesn´t allow flexibility in the boarding time. Some clients accept to share the flight (Qmax>q) as shown in table 8.

Table 8 – Scenario1 - Client Demand (Shared flight, no time window)

	Client Demand																
	Departure									Arrival							
Client	Node	Code	Tmin	Tmax	Lat	Long	q	Qmax	Vôo Direto	Node	Code	Tmin	Tmax	Lat	Long	q	Qmax
1	3	**SBBR**	534	534	-15,8622	-47,9122	3	5	0	12	**SBCF**	616,098	780,294	-19,6239	-43,9713	-3	5
2	4	**SBBR**	380	380	-15,8622	-47,9122	1	1	1	13	**SBCF**	462,098	626,294	-19,6239	-43,9713	-1	5
3	5	**SBSP**	726	726	-23,6256	-46,6558	2	5	0	14	**SBBR**	841,002	1071,006	-15,8622	-47,9122	-2	5
4	6	**SBRJ**	864	864	-22,9097	-43,1622	1	1	1	15	**SBSP**	920,802	1034,406	-23,6256	-46,6558	-1	5
5	7	**SBSP**	888	888	-23,6256	-46,6558	2	5	0	16	**SBNF**	951,198	1077,594	-26,8797	-48,6481	-2	5
6	8	**SBSP**	996	996	-23,6256	-46,6558	3	5	0	17	**SBNF**	1059,2	1185,594	-26,8797	-48,6481	-3	5
7	9	**SBSP**	932	932	-23,6256	-46,6558	2	5	0	18	**SBRJ**	988,802	1102,406	-22,9097	-43,1622	-2	5
8	10	**SBSP**	470	470	-23,6256	-46,6558	1	5	0	19	**SBRJ**	526,802	640,406	-22,9097	-43,1622	-1	5
9	11	**SBNF**	932	932	-26,8797	-48,6481	2	5	0	20	**SBSP**	995,198	1121,594	-23,6256	-46,6558	-2	5

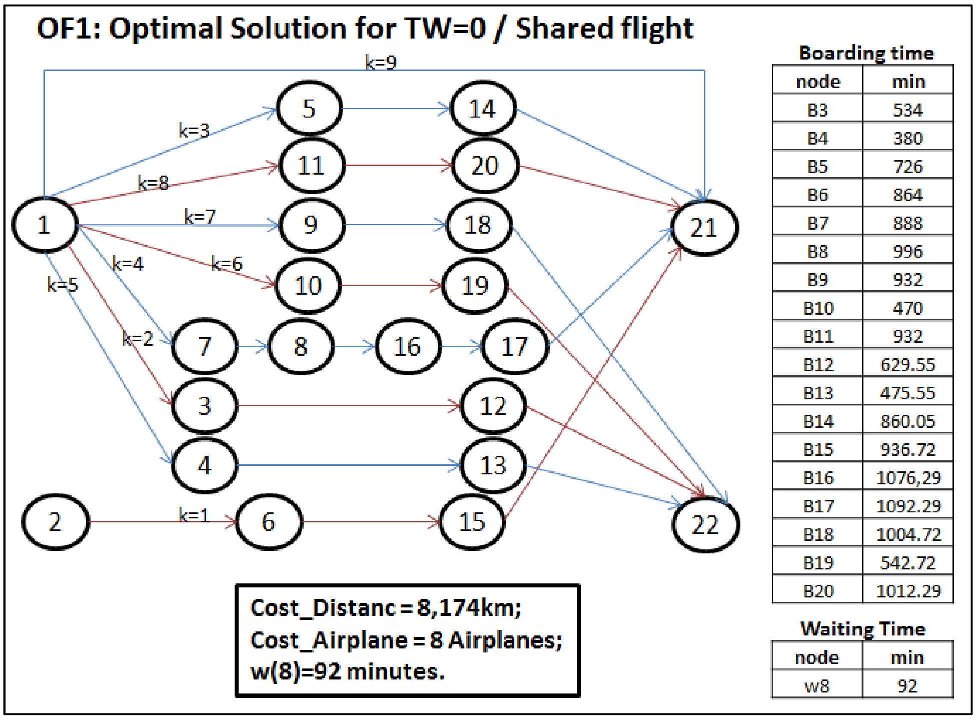

Figure 3 – Optimal Solution, TW=0, with shared flight

Table 9 – Results in scenario1

	No Time Window / Shared Flight				
	Distance(km)	W (min)	Qty Airplane	Iterations	CPU time (sec)
OF1	8,174	92	8	12,810	2.28
OF2	8,174	346	7	1,836	0.53
OF3	8,174	491	7	1,921	0.53

In scenario 1, the total distance traveled was 8,174km. When some clients are flexible to accept shared flight, as described on table 8, the reduction is about 826km in total distance traveled by airplanes, 10% of reduction in total distance. In the optimal solution in scenario1, the clients in node 7 and 8 are sharing the flight. As shown in table 3, the nodes 7 and 8 represents the airport SBSP, so the arc(7,8) has distance zero ($Dist_{7,8} = 0$).

Scenario2: Simulation with time window of 15min in the boarding time for all clients.
In this scenario, all clients of scenario1 accept delay or advance in 15min the original boarding time as shown in table 10.

Table 10 – Scenario2 - Client Demand (Shared flight, TW=15min)

	Client Demand																	
	Departure									Arrival								
Client	Node	Code	Tmin	Tmax	Lat	Long	q	Qmax	**Vôo Direto**	Node	Code	Tmin	Tmax	Lat	Long	q	Qmax
1	3	**SBBR**	519	549	-15,8622	-47,9122	3	5	0	12	**SBCF**	601,098	795,294	-19,6239	-43,9713	-3	5
2	4	**SBBR**	365	395	-15,8622	-47,9122	1	1	1	13	**SBCF**	447,098	641,294	-19,6239	-43,9713	-1	5
3	5	**SBSP**	711	741	-23,6256	-46,6558	2	5	0	14	**SBBR**	826,002	1086,006	-15,8622	-47,9122	-2	5
4	6	**SBRJ**	849	879	-22,9097	-43,1622	1	1	1	15	**SBSP**	905,802	1049,406	-23,6256	-46,6558	-1	5
5	7	**SBSP**	873	903	-23,6256	-46,6558	2	5	0	16	**SBNF**	936,198	1092,594	-26,8797	-48,6481	-2	5
6	8	**SBSP**	981	1011	-23,6256	-46,6558	3	5	0	17	**SBNF**	1044,2	1200,594	-26,8797	-48,6481	-3	5
7	9	**SBSP**	917	947	-23,6256	-46,6558	2	5	0	18	**SBRJ**	973,802	1117,406	-22,9097	-43,1622	-2	5
8	10	**SBSP**	455	485	-23,6256	-46,6558	1	5	0	19	**SBRJ**	511,802	655,406	-22,9097	-43,1622	-1	5
9	11	**SBNF**	917	947	-26,8797	-48,6481	2	5	0	20	**SBSP**	980,198	1136,594	-23,6256	-46,6558	-2	5

Table 11 – Results in scenario2

	Time Window = 15min / Shared Flight				
	Distance(km)	W (min)	Qty Airplane	Iterations	CPU time (sec)
OF1	8,174	62	7	32,846	6.71
OF2	8,174	1,261	6	57,557	6.58
OF3	8,174	1,212	7	31,327	6.30

Using time window (TW) of 15min, there is no reduction of the total distance traveled but there is a reduction in the number of airplanes in the optimal solution compared to scenario1. Later, in the results analysis section, this behavior will be further evaluated.

Scenario3: Simulation with time window of 30min in the boarding time for all clients.
In this scenario, all clients of scenario1 accept delay or advance in 30min the original boarding time as shown in table 12.

Table 12 – Scenario3 - Client Demand (Shared flight, TW=30min)

	Client Demand																
	Departure									Arrival							
Client	Node	Code	Tmin	Tmax	Lat	Long	q	Qmax	Vôo Direto	Node	Code	Tmin	Tmax	Lat	Long	q	Qmax
1	3	**SBBR**	504	564	-15,8622	-47,9122	3	5	0	12	**SBCF**	586,098	810,294	-19,6239	-43,9713	-3	5
2	4	**SBBR**	350	410	-15,8622	-47,9122	1	1	1	13	**SBCF**	432,098	656,294	-19,6239	-43,9713	-1	5
3	5	**SBSP**	696	756	-23,6256	-46,6558	2	5	0	14	**SBBR**	811,002	1101,006	-15,8622	-47,9122	-2	5
4	6	**SBRJ**	834	894	-22,9097	-43,1622	1	1	1	15	**SBSP**	890,802	1064,406	-23,6256	-46,6558	-1	5
5	7	**SBSP**	858	918	-23,6256	-46,6558	2	5	0	16	**SBNF**	921,198	1107,594	-26,8797	-48,6481	-2	5
6	8	**SBSP**	966	1026	-23,6256	-46,6558	3	5	0	17	**SBNF**	1029,2	1215,594	-26,8797	-48,6481	-3	5
7	9	**SBSP**	902	962	-23,6256	-46,6558	2	5	0	18	**SBRJ**	958,802	1132,406	-22,9097	-43,1622	-2	5
8	10	**SBSP**	440	500	-23,6256	-46,6558	1	5	0	19	**SBRJ**	496,802	670,406	-22,9097	-43,1622	-1	5
9	11	**SBNF**	902	962	-26,8797	-48,6481	2	5	0	20	**SBSP**	965,198	1151,594	-23,6256	-46,6558	-2	5

Table 13 – Results in scenario3

	Time Window = 30min / Shared Flight				
	Distance(km)	W(min)	Qty Airplane	Iterations	CPU time (sec)
OF1	7,517	0	5	578,066	83.45
OF2	7,517	706	4	3,665,695	428.61
OF3	7,517	1,237	5	2,078,326	197.39

Using time window of 30min, there is a significant reduction in the total distance travelled and in the number of airplanes in the optimal solution.

Scenario4: Simulation with time window of 45min in the boarding time for all clients.
In this scenario, all clients of scenario1 accept delay or advance in 45min the original boarding time as shown in table 14.

Table 14 – Scenario4 - Client Demand (Shared flight, TW=45min)

	Client Demand																
	Departure									Arrival							
Client	Node	Code	Tmin	Tmax	Lat	Long	q	Qmax	Vôo Direto	Node	Code	Tmin	Tmax	Lat	Long	q	Qmax
1	3	**SBBR**	489	579	-15,8622	-47,9122	3	5	0	12	**SBCF**	571,098	825,294	-19,6239	-43,9713	-3	5
2	4	**SBBR**	335	425	-15,8622	-47,9122	1	1	1	13	**SBCF**	417,098	671,294	-19,6239	-43,9713	-1	5
3	5	**SBSP**	681	771	-23,6256	-46,6558	2	5	0	14	**SBBR**	796,002	1116,006	-15,8622	-47,9122	-2	5
4	6	**SBRJ**	819	909	-22,9097	-43,1622	1	1	1	15	**SBSP**	875,802	1079,406	-23,6256	-46,6558	-1	5
5	7	**SBSP**	843	933	-23,6256	-46,6558	2	5	0	16	**SBNF**	906,198	1122,594	-26,8797	-48,6481	-2	5
6	8	**SBSP**	951	1041	-23,6256	-46,6558	3	5	0	17	**SBNF**	1014,2	1230,594	-26,8797	-48,6481	-3	5
7	9	**SBSP**	887	977	-23,6256	-46,6558	2	5	0	18	**SBRJ**	943,802	1147,406	-22,9097	-43,1622	-2	5
8	10	**SBSP**	425	515	-23,6256	-46,6558	1	5	0	19	**SBRJ**	481,802	685,406	-22,9097	-43,1622	-1	5
9	11	**SBNF**	887	977	-26,8797	-48,6481	2	5	0	20	**SBSP**	950,198	1166,594	-23,6256	-46,6558	-2	5

Table 15 – Results in scenario4

	Time Window = 45min / Shared Flight				
	Distance(km)	W (min)	Qty Airplane	Iterations	CPU time (sec)
OF1	7,445	0	5	16,820,318	2,164.14
OF2	7,445	989	4	10,151,556	1,287.09
OF3	7,445	989	4	4,084,765	439.45

Using time window of 45min, there is a small reduction of total distance traveled when it is compared with scenario3.

5.1 Results Analysis

In this section, the results obtained in scenario1 to scenario4 will be analyzed. These scenarios show situations that shared flights and flexibility in board time are allowed.

Costs Reduction

The results in each scenario show a substantial reduction when the time window increases. Figure 2 describes the percentage gains in the three costs for each scenario with shared flight, scenario1 to scenario4. Only OF1, as defined in equation (20), is considered in this analysis.

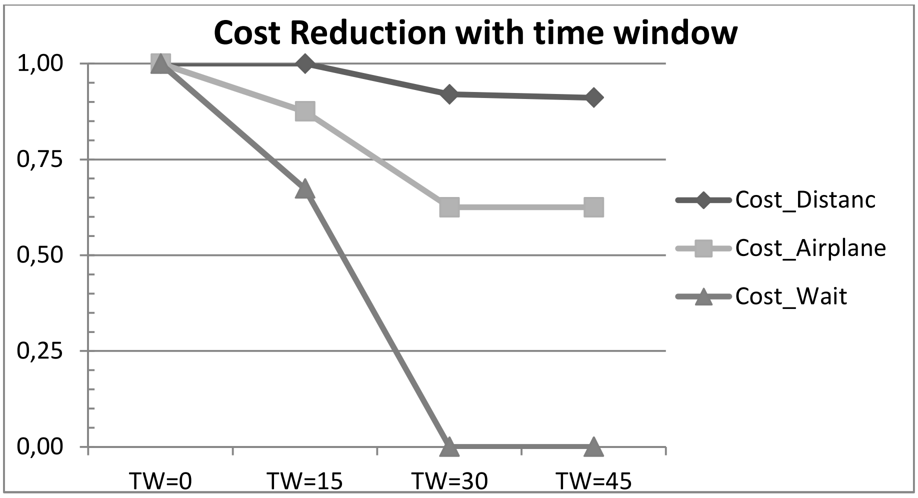

Figure 4 – Cost reduction with time window – scenario1 to scenario4

From the figure, it is possible to verify that with a time window of 45min, the cost reduction is 8% in total distance traveled; 37,5% in the number of airplanes required to serve the client demand and 100% in the total waiting time on ground. It means that for TW=45 no airplane is required to wait to start the boarding procedure, *Cost_Wait*=0.

CPU Time – Time Window

Continuing the analysis of OF1, it was computed the CPU time for each scenario. Figure 3 shows de percentage variation on CPU time of the four scenarios analyzed in this section.

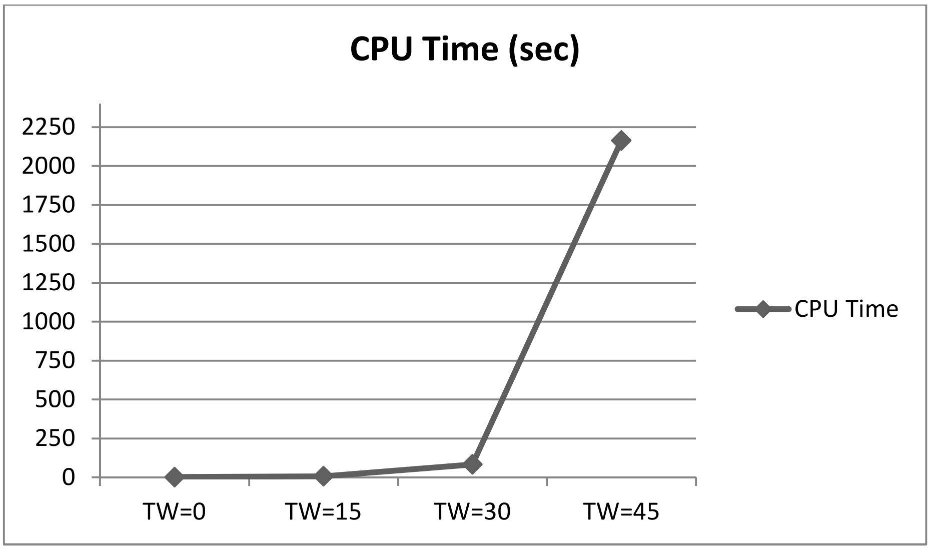

Figure 5 – CPU time for different TW – scenario1 to scenario4

From the figure, it is possible to verify that when the time window increases, the CPU time to obtain the optimal solution exponentially increases. This Model behavior is expected as it is shown by Lenstra and Kan(1981) and confirmed by Solomon and Desrosierd(1998) the routing problem with Time Window is considered a NP-Hard problem.

CPU Time – Objective Function

The CPU time in scenario 4 for all Objective Functions, defined in equations (20), (21) and (22), is described in figure 6.

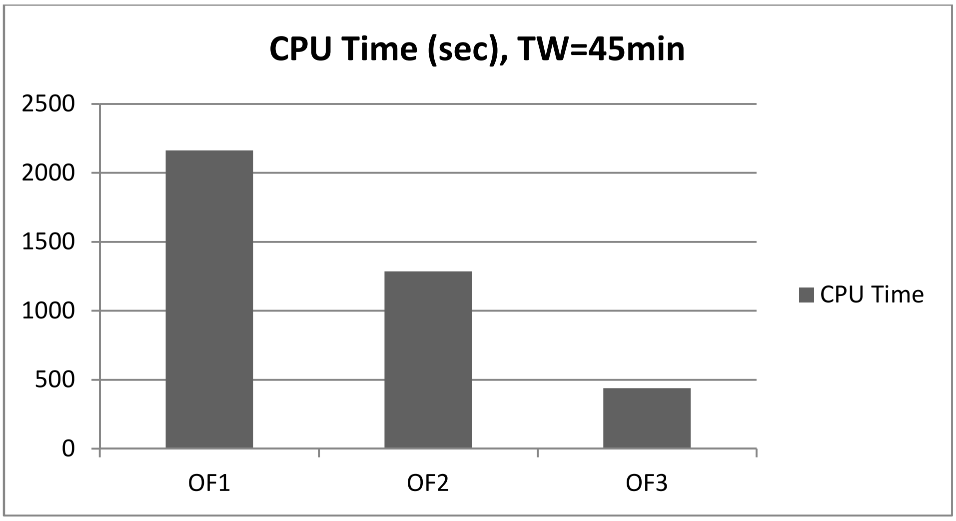

Figure 6 – CPU time for different OF

The OF1 comprises all 03 costs mentioned in this paper, the OF2 comprises 2 costs and OF3 comprise only one operational cost of the company. From the figure, it is possible to verify a correlation between the number of decision variables in the Objective Function and the CPU time to obtain the optimal solution. This is an expected behavior.

Analysis of Scenario 2 compared with Scenario 1

Analyzing OF1 in scenarios 1 and 2, for TW=15min there is a reduction in the number of airplanes required to serve all clients (01 airplane less), but the *Cost_Distanc* remains with the same value. Apparently, it is an unexpected behavior as it is supposed that when the number of airplanes modifies, the total distance is expected to modify too. To clarify this point, figures 7 and 8 describe the optimal solution in scenario 1 and 2.

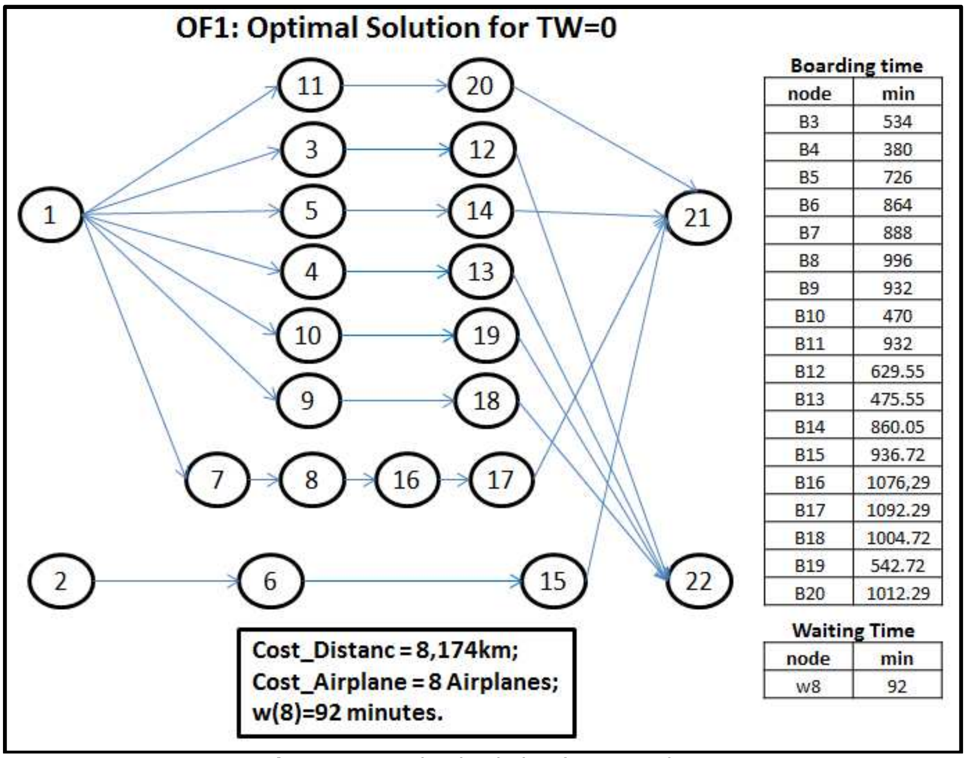

Figure 7 – Optimal solution for scenario 1

In scenario1, the optimal solution indicates that it is necessary only 8 airplanes to serve all clients, and one of the airplanes must wait on ground 92min in node 8.

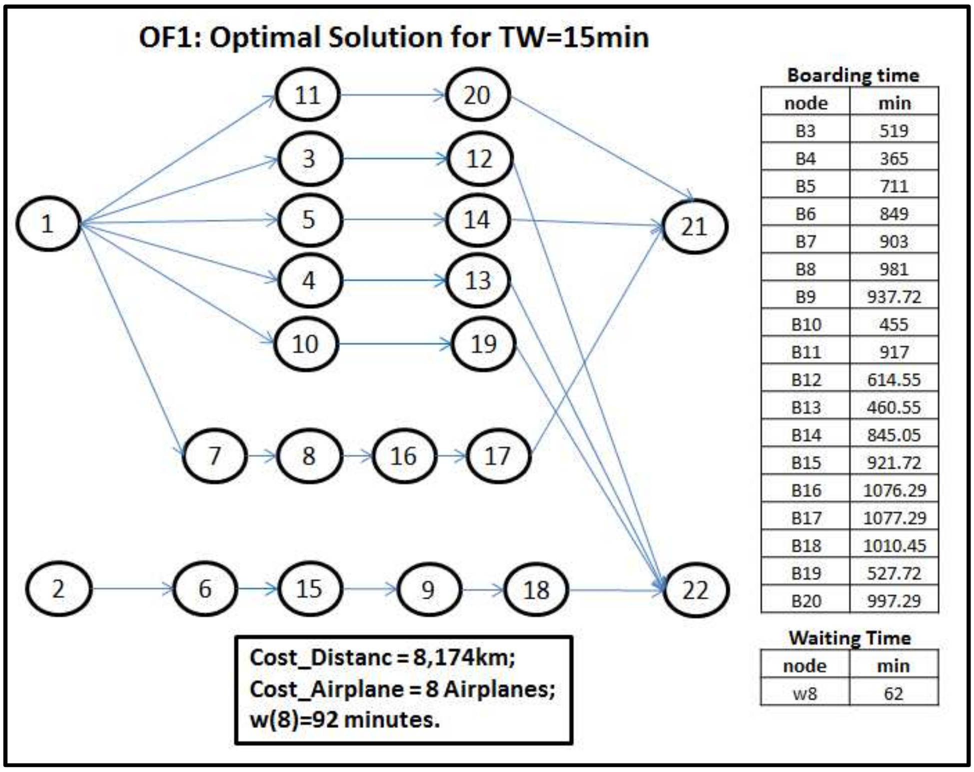

Figure 8 – Optimal solution for scenario 2

In Scenario 2, with time window of 15min, the airplane that serves only the arc (6,15) will be able to serve the arc (9,18) too. In this new optimal solution in scenario 2, the arc(1,9) is removed and the arc(15,9) is included. Now the airplane can serve 2 arcs, the *Cost_Airplane* reduces.

The arc(15,9) and arc(1,9) have distance 0km as the nodes 15, 9 and 1 represent the same airport (SBSP) as shown in table 2 and 3. So, the *Cost_Distanc* remains the same value.

Another interesting point to mention is that with time window of 15min the solver delayed 15min the boarding time in node 7, and advanced 15min the boarding time in node 8. This represents a cost reduction of 30min. In the optimal solution in scenario 2, the *Cost_Wait* reduced from 92min to 62min. The airplane waits on ground 62min in node 8 before starting the boarding procedure.

6. CONCLUSION

This paper presented a study of flight schedule optimization of fractional jet ownership company. The mathematical model considers a flexibility in the boarding time, known as time window, and the possibility of shared flight. The model used in this paper was proposed by Cordeau (2006) and was adapted to the air transportation company.

A support tool was developed in AIMMS to simulate the 05 scenarios discussed in this paper. Time window of 45min provide a reduction in the operation costs. This reduction, comparing scenario1 to scenario4, is 730km in the total distance travelled, reduction of 8,92%; reduction of 37,5% in the number of airplanes required; and reduction of 100% in the *Cost_Wait.*

The results allow concluding that applying flexible boarding time and offering discount to clients who accepts sharing the service, is a strategy that a fractional jet ownership company can take to become more competitive.

The support tool developed in this paper shows to be useful to simulate different scenarios and to find out the best flight schedule to the company to have the minimal operation cost.

The next step of this research is to implement the model for an heterogeneous fleet and test for a larger case, with 15 and 20 clients.

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