STUDY ON URBAN GRIDLOCK PHENOMENON IN TRAFFIC SIMULATION MODEL

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ABSTRACT

This paper investigates the occurrence condition of a gridlock phenomenon on a single grid road network theoretically. Furthermore, demonstration using traffic simulation model is shown to test the derived theory. The occurrence of gridlock phenomenon on a single grid road network is found to be controlled by both merging ratio on the exiting legs and turning ratio on the entering approaches at all the four cornered intersections of the single grid road network.

Keywords: gridlock phenomenon, traffic simulation

INTRODUCTION

On March 11, 2011, the Pacific coast of Tohoku was struck by an earthquake of magnitude 9.0 on the Richter scale (the Great East Japan Earthquake), which had a strength of upper 5 on the Japanese intensity scale in Tokyo. Since the epicenter of the earthquake was offshore of the Sanriku coast, hundreds of kilometers away from Tokyo, there was little damage to structures such as buildings and roads in the center of Tokyo, but heavy traffic congestion occurred. As a contributory factor, it is considered that the proportion of commuters traveling by train is very high (74%) in Tokyo, meaning that a large number of people had to change their means of journey home owing to the suspension of trains after the earthquake. Moreover, the Metropolitan Expressway, the main network in Tokyo, was closed for safety checks, causing an increase in the traffic load on local roads. In general, when traffic demand concentrates one point or one area of an urban network with much beyond its traffic capacity, vehicles in different direction influence each other and major traffic jam called 'gridlock phenomenon' could be caused. Owing to these factors mentioned above, gridlock phenomenon may have occurred in the center of Tokyo after the occurrence of the earthquake and it spread from the city center to the suburbs. This phenomenon may have caused greatly decreasing the network capacity and rapidly expanding the traffic congestion to a large area. From this experience, it is important to clarify the relationship between the occurrence of a gridlock phenomenon and the occurrence of a widespread heavy traffic congestion on an urban street network. But the mechanism which gives rise to gridlock phenomenon on the urban street network is so complicated and still remains a major challenge.

Furthermore, in network traffic simulation model, especially in the high density network case, very similar phenomenon often occur, but once it occurs, its influence spreads surrounding network more rapidly than the real 'gridlock phenomenon', and capacity of the 'gridlocked' network becomes very low compared with the real phenomenon. Therefore, in case the supersaturated network condition such as emergency case is dealt in traffic simulation model, it is difficult to understand which causes the simulated traffic congestion, too much excessive demand or such simulation model nature. The reason of unreality of the process of gridlock formation and reduction of capacity in traffic simulation is that the process and occurrence condition of gridlock phenomenon is not yet well understood and traffic simulation model cannot consider it exactly.

The aim of this paper is to investigate the occurrence condition of a gridlock phenomenon on a single grid road network theoretically. Then, a demonstration using traffic simulation model is conducted to validate the derived theory.

OCCURRENCE CONDITION OF GRIDLOCK ON A SINGLE GRID NETWORK

As mentioned in the previous section, it is important to prevent the occurrence of gridlock phenomenon because it may cause heavy traffic congestion. The first occurrence of the gridlock is supposed to be caused by the first bottleneck phenomenon because of traffic demand concentration and/or sudden capacity reduction which may be emerged by rapid and huge increase of crossing pedestrian amount at the bottleneck intersection. After the first occurrence of the gridlock phenomenon, it is well known that the network performance (network capacity) become decreased significantly.

However, the mechanism which gives rise to gridlock phenomenon on the urban street network is so complicated and that has not been investigated in detail. In addition, there is no good estimation theory of the network performance after the first gridlock formed. In this chapter, occurrence condition of a gridlock on a single grid road network is investigated.

Consider a single grid road network that consists of four links of length L (km) with single lane (see Fig. 1). Only traffic flow in clockwise direction is considered and traffic demand Q (veh/h) is generated from the nodes connecting each apex of the grid network. Define r as the ratio of right turning vehicle to the traffic demand Q, so the demand of through and left turning traffic can be written as (1-r)Q. If there is no traffic demand of right turn, gridlock phenomenon never appears. Thus r is assumed greater than 0 ($0 < r \leq 1$). The normal capacities of all four intersection-approaches are same as C (veh/h) which is large enough to deal the assumed traffic demand. The traffic flow of each link should become $Q + rQ$ under the condition that the capacity is greater than the traffic demand at all the intersection approaches.

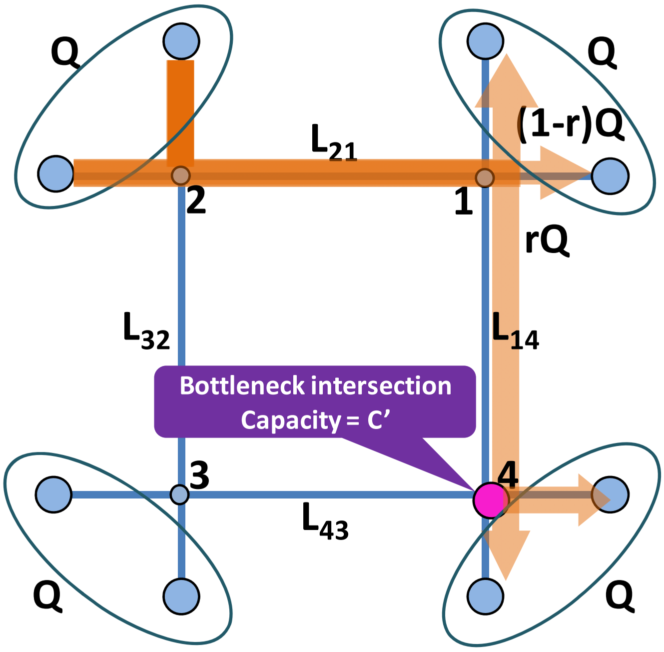

Figure 1: The network and the traffic demand.

Now, the capacity of one intersection approach, assuming intersection 4 in Fig.1, falls down to C' that is less than the traffic demand $Q + rQ$, for example by many crossing pedestrians like the time of the earthquake in Japan.

$$Q + rQ > C' \tag{1}$$

If the traffic demand exceeds the capacity of the intersection approach, a queue growth starts from the approach and backward wave of traffic flow C' goes toward the upstream intersection (intersection 1). t_{41} that is the time to reach the backward wave to intersection 1 is calculated by Eq. (2), assuming Q-K relationship shown in Fig. 2.

$$t_{41} = -\frac{k_c - k_j \dfrac{Q_c - C'}{(1+r)Q - C'}}{Q_c} L \tag{2}$$

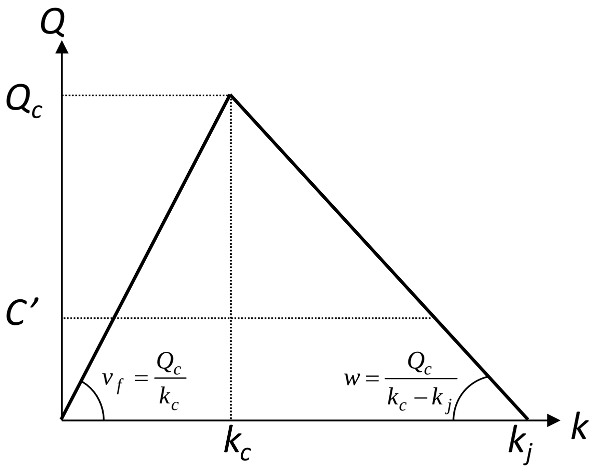

Figure 2: Q-K relationship of the supposed network.

After the whole link L_{14} becomes occupied by the queue of the traffic flow *C'*, the traffic flow into link L_{14} from connecting upstream links become regulated. Before such regulated condition, and the capacities of the intersection approaches (the eastbound and southbound approaches for the case of intersection 1 in Fig. 1) are larger than the traffic demand, the merging ratio of an intersection is equal to the ratio of the traffic demand from each entering direction. However, after backward wave of the queue from the downstream link arrived at the intersection, the merging ratio is controlled by the geometric design of the intersection or control policy of the intersection. Therefore, the merging ratio of right turning vehicle M_n (n is numerical order of intersection) after the queue backup reached to the upstream end of the *n*-th link is introduced (See Fig. 3).

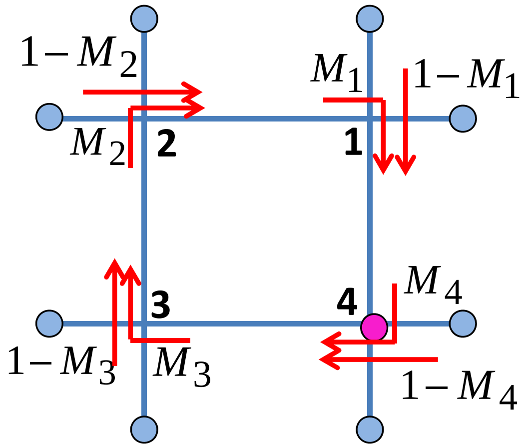

Figure 3: Merging ratio at each intersection after the queue backup reached to the intersection.

After the link L_{14} occupied by the queue of the traffic flow *C'*, the right turning flow from the west side link (L_{21}) to link L_{14} becomes M_1C' and from the north side and the east side links to link L_{14} becomes (1-M_1)*C'* according to the merging ratio. The condition to form a queue in link L_{21} after the queue from intersection 4 reaches intersection 1 is that the traffic demand of right turning vehicle at the intersection 1 is greater than the inflow into link L_{14}, this can be written in Eq. (3).

$$rQ > M_1C' \tag{3}$$

If the traffic flow of right turning vehicle at intersection 1 becomes M_1C', through and left turning flow from the link L_{21} are regulated by the right turning flow M_1C' because the gird road network links are all with directional single lane. Because the ratio of the through and the left turning flow to the right turning flow is $1/r$, the traffic flow of link L_{21} is calculated by M_1C' *multiplied* $(1+r)$ / r. Hence the time to reach backward wave to intersection 2 is calculated from Eq. (4).

$$t_{23} = -\frac{k_c - k_j \dfrac{Q_c - \dfrac{1+r}{r}M_1C'}{(1+r)Q - \dfrac{1+r}{r}M_1C'}}{Q_c}L \tag{4}$$

After reaching the queue of link L_{21} to intersection 2, the traffic flow into link L_{21} at intersection 2 is regulated according to the merging ratio M_2 and the composition of right turning vehicle in the traffic flow of link L_{21} becomes $r(1-M_2)$. So, the traffic flow of through and left turning vehicle at the intersection 1 is calculated by M_1C' multiplied by $(1-r(1-M_2))$ / $r(1-M_2)$. Then, the total traffic flow of link L_{21} can be written as

$$M_1C' + \frac{1-r(1-M_2)}{r(1-M_2)} \times M_1C' = \frac{M_1}{r(1-M_2)}C' \tag{5}$$

Therefore, the condition of the occurrence of a queue in link L_{32} can be written by Eq. (6).

$$rQ > \frac{M_1M_2}{r(1-M_2)}C' \tag{6}$$

In a similar way, the total traffic flow of link L_{32} can be written as

$$\frac{M_1M_2}{r(1-M_2)}C' + \frac{1-r(1-M_3)}{r(1-M_3)} \times \frac{M_1M_2}{r(1-M_2)}C' = \frac{M_1M_2}{r^2(1-M_2)(1-M_3)}C' \tag{7}$$

Then, the condition of the occurrence of a queue in link L_{43} can be written by Eq. (8).

$$rQ > \frac{M_1M_2M_3}{r^2(1-M_2)(1-M_3)}C' \tag{8}$$

Finally, the total traffic flow of link L_{43} can be written as

$$\begin{aligned}&\frac{M_1M_2M_3}{r^2(1-M_2)(1-M_3)}C' + \frac{1-r(1-M_4)}{r(1-M_4)} \times \frac{M_1M_2M_3}{r^2(1-M_2)(1-M_3)}C' \\ &= \frac{M_1M_2M_3}{r^3(1-M_2)(1-M_3)(1-M_4)}C'\end{aligned} \tag{9}$$

As a result, the traffic flow of each link after the queue formed all around the grid road network links is shown in Fig. 4.

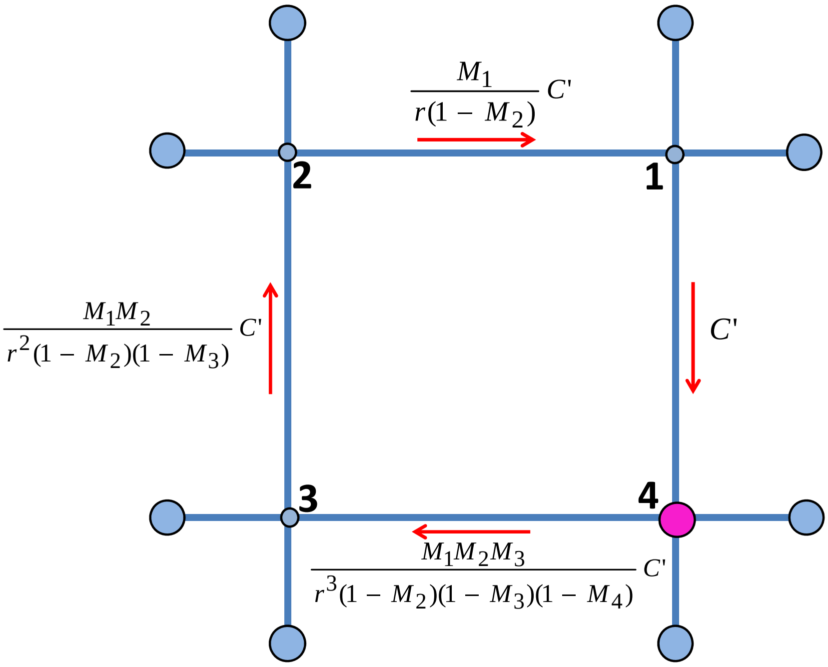

Figure 4: The traffic flow when the queue formed all around the grid links.

The condition that the traffic flow of link L_{43} affects on the traffic flow of link L_{14} can be written as

$$r(1-M_1)C' > \frac{M_1M_2M_3M_4}{r^3(1-M_2)(1-M_3)(1-M_4)}C'$$

And the inequality written above can be solved as below,

$$r^4 > \frac{M_1M_2M_3M_4}{(1-M_1)(1-M_2)(1-M_3)(1-M_4)} \tag{10}$$

If the merging ratio of right turning flow versus thorough and left turning flow at all four intersections are 1:1 (it means $M_1 = M_2 = M_3 = M_4 = 1/2$), the Eq. (10) evolves into $r^4 > 1$. However, because r is less than or equal to 1, there is no r that satisfy the equation $r^4 > 1$. This means that the traffic flow of the grid road network cannot be less than the capacity of the bottleneck intersection *C'*. In this situation, the traffic flow of each link depends on the right turning ratio r. If it is assumed as $r = 1$, it means all of the traffic demand turn to right once, the traffic flow of all links becomes *C'*.

If the merging ratio of any one intersection is 1:2, here we assume intersection 1 (it means $M_1 = 1/3$, $M_2 = M_3 = M_4 = 1/2$), the Eq. (10) evolves into $r^4 > 1/2$. This means if the right turning ratio r is greater than $\sqrt[4]{1/2}$ (≈ 0.8409), the traffic flow of the grid road network decreases step-by-step, finally the flow may approaches to zero, under such condition, the all traffic cannot flow at all.

As a result of this study, the Eq. (11) represents the general condition causing entire breakdown of capacity (infinite decrease to zero) of a grid road network consist with number of n links.

$$r^n > \frac{\prod_{i=1}^{n} M_i}{\prod_{i=1}^{n}(1-M_i)} \tag{11}$$

SIMULATION STUDY

To validate the theory of gridlock phenomenon on a single grid network derived in the previous section, simulation study was conducted. In this study, the wide-area road network traffic flow simulator SOUND (Simulation on Urban road Network with Dynamic route choice) developed by the Institute of Industrial Science, University of Tokyo was used. The network and the points of origin and destination of traffic demand for the simulation study are same as the condition assumed in the previous section. The value of parameters of the simulation are shown in Table. 1. We set the traffic demand Q to 900 passenger car unit (pcu) per hour and the ratio of right turning vehicle r to 1, therefore the traffic flow of each link becomes 1,800 pcu per hour which is equal to the capacity of the link. The capacity of the bottle neck intersection C' was set to 600 pcu per hour.

Table 1 – Parameters of the simulation

Parameter	Value
capacity of link	1800 pcu/h/lane
capacity of intersection	1800 pcu/h/lane
jam density	140 pcu/km
free flow speed	50 km/h
link length	300m

Fig. 5 shows the result of simulation when the merging ratio of right turning flow versus thorough and left turning flow at all four intersections are 1:1. The vertical axis represents traffic flow of the link per a minute. According to our theory, the traffic flow of all links should become C' (600 pcu/hour = 10 pcu/min.) in this condition. The simulated traffic flow of all links converge to 10 pcu per a minute, therefore the result is in accord with the theory.

On the other hand, Fig. 6 is the result when the merging ratio of right turning flow versus thorough and left turning flow at intersection 1 is 1:2 and the others are same as before. In such condition, if r^4 is bigger than 1/2, the traffic flow of the grid road network should decrease step-by-step. The simulated traffic flow shows same tendency and the traffic flow gradually approaches to zero.

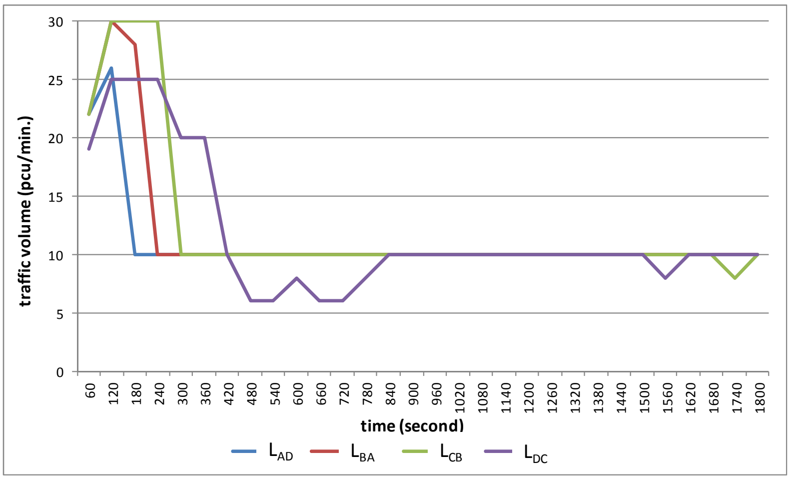

Figure 5: Traffic volume simulated by SOUND ($M_1 = M_2 = M_3 = M_4 = 1/2$ and $r = 1$).

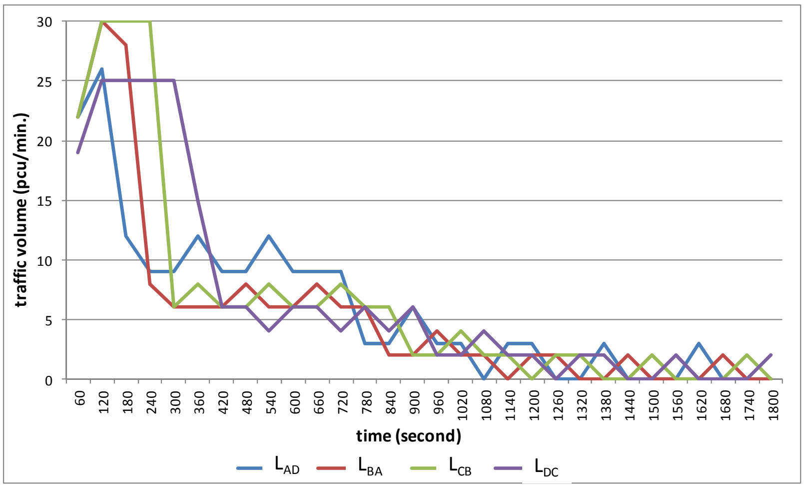

Figure 6: Traffic volume simulated by SOUND ($M_1 = 1/3$, $M_2 = M_3 = M_4 = 1/2$ and $r = 1$).

DISCUSSION

This study shows that the condition of decrease of capacity on a single grid road network by gridlock phenomenon is controlled by the relationship between the merging ratio of intersections M_n under the condition of queue backup and the right turning ratio r to the traffic demand. In some cases, the capacities of the all grid road network links become stable and equal to the bottleneck capacity C' of the first bottleneck intersection. In other cases, the capacity of the network decreases step-by-step and may reaches to zero. It is considered that if the merging ratio from the link of the grid road network is less than the merging ratio from the link outside the grid road network, the capacity of the grid network is likely to decrease step-by-step approaching to zero. From this study, for example in a roundabout, it might be said that traffic congestion by gridlock phenomenon is less likely to occur, because traffic flow from the ring road has priority to the entering flow as a general traffic regulation.

Same theoretical study on a multiple grid road network or grid road network with multiple lanes are left for future works.