COMPARING OPTIMAL RELOCATION OPERATIONS WITH SIMULATED RELOCATION POLICIES IN ONE-WAY CARSHARING SYSTEMS

(max 20 pages)

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ABSTRACT

One-way carsharing systems allow travelers to pick up a car at one station and return it to a different station, thereby causing vehicle imbalances across the stations. In this paper, realistic ways to mitigate that imbalance by relocating vehicles are discussed. Also presented are a new mathematical model to optimize relocation operations that maximize the profitability of the carsharing service and a simulation model to study different real-time relocation policies. Both methods were applied to networks of stations in Lisbon Portugal. Results show that real-time relocation policies, and these policies when combined with optimization techniques, can produce significant increases in profit. In the case where the carsharing system provides maximum coverage of the city area, imbalances in the network resulted in an operating loss of 1160 €/day when no relocation operations were performed. When relocation policies were applied, however, the simulation results indicate that profits of 854 €/day could be achieved, even with increased costs due to relocations. This improvement was achieved through reductions in the number of vehicles needed to satisfy demand and the number of parking spaces needed at stations. These findings show that relocation policies should be considered in one-way carsharing systems operation since the profit increases significantly.

Keywords: One-way carsharing, optimization, simulation, relocation operations

INTRODUCTION

Through the last decades, changes have occurred in urban transport. Despite greater accessibility provided by private transport, the result has been increases in levels of congestion, pollution, and non-productive time for travelers, particularly in peak hours (Schrank et al., 2010). There are also opportunity costs associated with using urban land for parking spaces instead of other more productive activities. In America, for example, automobiles spend around 90% of their time parked (U.S. Department of Transportation, 2001). These issues are mitigated by public transport, but it has other disadvantages, for example, poor service coverage, schedule inflexibility and lack of personalization. In addition, providing public transport for peak hour demand can result in idle vehicles for much of the day, resulting in inefficiencies and high cost of service.

Strategies are needed to address these issues and simultaneously provide people the mobility they need and desire. One strategy considered is that of carsharing. Carsharing systems involve a small to medium fleet of vehicles, available at several stations, to be used by a relatively large group of members (Shaheen et al., 1999).

The origins of carsharing can be traced back to 1948, when a cooperative known as Sefage initiated services in Zurich, Switzerland. In the US, carsharing programs only appeared later in the 1980s, within the Mobility Enterprise program (Shaheen et al., 1999). In Asian countries, such as Japan and Singapore, these systems appeared more recently.

Carsharing has been observed to have a positive impact on urban mobility, mainly by using each car more efficiently (Litman, 2000; Schuster et al., 2005). The use of carsharing systems generally leads to a fall in car ownership rates and thus to lower car use, according to Celsor and Millard-Ball (Celsor and Millard-ball, 2007). More recently, Schure et al. (2012) conducted a survey in 13 buildings in downtown San Francisco and concluded that the average vehicle ownership for households that use carsharing systems is 0.47 vehicles/household compared to 1.22 vehicles/household for households that do not use carsharing systems. Moreover, a study by Sioui et al. (2010) surveyed the users of Communauto, a Montreal carsharing company, and concluded that a person who does not own a vehicle and uses carsharing systems frequently (5 days a week) never reaches the car-use level of a person who owns a vehicle: there was a 30% difference between them. This idea is reinforced by Martin and Shaheen (2011) who concluded through a survey in US and Canada that the average observed vehicle-kilometers traveled (VKT) of respondents before joining carsharing was 6468 km/year, while the average observed VKT after joining carsharing was 4729 km/year, which constitutes a decrease of 27% (1749 km/year).

Furthermore, some recent studies concluded that carsharing systems also have positive environmental effects. For instance, Martin and Shaheen (2011) conducted a survey on greenhouse gas (GHG) emissions of the major carsharing organizations in the US and Canada and concluded that carsharing allows a statistically significant reduction in overall emissions of -0.84 t GHG/year/household. While most members increase their emissions; there are compensatingly larger reductions for other members who decrease their emissions. Moreover, Firnkorn and Müller (2011), through a survey of a German carsharing company, concluded that the CO_2 emissions are decreased between 312 to 146 Kg CO_2/year per average carsharing system user.

With respect to trip configuration, carsharing systems are divided into round-trip (two-way) systems and one-way systems. Round-trip carsharing systems require users to return the cars to the same station from where they departed. This simplifies the task of the operators because they can plan vehicle inventories based on the demand for each station, but it is less convenient for the users. Better suited to personal needs are one-way carsharing systems. In one-way carsharing, users can pick up a car in a station and leave it at a different one (Shaheen et al., 2006). In theory, therefore, one-way carsharing systems are better suited for more trips than round-trip services that typically are used by a relatively small market share for leisure, shopping and sporadic trips (Barth and Shaheen, 2002). This statement is supported by various studies, including that by Costain et al. (2012) and by Firnkorn and Müller (2011). Costain et al. (2012) studied the behavior of a round-trip carsharing company in Toronto, Canada and concluded that trips are mostly related to grocery or other household shopping purposes. Firnkorn and Müller (2011) conducted a survey showing that market penetration of car2go, a German one-way carsharing company is equal to 0.37%, which is 25 times higher than the market penetration of round-trip carsharing in Germany, considering only the driving license holders. A study performed in Greece by Efthymiou et al. (2012) also concluded that the flexibility to return the vehicle to a different station from the one where it was picked up is a critical factor in the decision to join a carsharing service. However one-way carsharing systems present an operational problem of imbalances in vehicle inventories, or stocks, across the network of stations due to non-uniformity of trip demand between stations. Despite this, a great effort has been made to provide these flexible systems to users in the last years.

Previous research has proposed several approaches to solve this problem, such as: vehicle relocations in order to replenish vehicle stocks where they are needed (Kek et al., 2006; Kek et al., 2009; Nair and Miller-Hooks, 2011); pricing incentive policies for the users to relocate the vehicles themselves (Mitchell et al., 2010; Febbraro et al., 2010); operating strategies designed around accepting or refusing a trip based on its impact on vehicle stock balance (Fan et al., 2008; Correia and Antunes, 2012); and station location selection to achieve a more favorable distribution of vehicles (Correia and Antunes, 2012). Correia and Antunes (2012) propose a mixed integer programming model to locate one-way carsharing stations to maximize the profit of a carsharing company, considering the revenues (price paid by the clients) and costs (vehicle maintenance, vehicle depreciation, and maintenance of parking spaces), and assuming that all demand between the stations must be satisfied (Correia and Antunes, 2012). In applying their model to a case study in Lisbon Portugal, tractability issues resulted and the model was only solvable with time discretization of 10-minutes steps. The model did not allow integrating relocation operations due to the complexity already reached with the location problem.

In this paper, the same case study as the one in (Correia and Antunes, 2012) is considered and station location outputs are generated using their model but this time with time discretizations of 1-minute. An approach to optimize relocation operations on a minute-by-minute basis is developed, given those outputs for station locations. The vehicle relocation solutions generated with this approach are compared to those obtained with a simulation model built to evaluate different real-time vehicle relocation policies, given a network of stations. With this comparison, the impacts of relocation operations on the profitability of one-

way carsharing systems are then analyzed, and insights into how to design and implement real-time rebalancing systems are gained.
The paper is structured as follows. In the next section, a new mathematical model is presented to optimize relocation operations, given an existing network of one-way carsharing stations. Then, a simulation model and a specification of several real-time relocation policies are presented. In Section 4, the case study used for testing the relocation methodologies is described. Next, the method for comparing the optimization-based and simulation-based approaches is presented together with the main results reached. The paper is concluded with a summary of the major findings of this research.

MATHEMATICAL MODEL

The objective of the mathematical programming model presented in this section is to optimize vehicle relocation operations between stations (using a staff of drivers) in order to maximize the profits of a one-way carsharing company. In this model, all demand between stations is assumed to be satisfied. The notation used to formulate the model (sets, decision variables, auxiliary variable, and parameters) is the following:

Sets:
- $N = \{1, \dots, i \dots N\}$: set of stations;
- $T = \{1, \dots, t \dots T\}$: set of minutes in the operation period;
- $X = \{1_1, \dots, i_{t-1}, i_t, i_{t+1}, \dots, N_T\}$ where i_t represents station i at minute t: set of the nodes of a time-space network combining the N stations with the T minutes;
- $A_1 = \left\{\dots, \left(i_t, j_{t+\delta_{ij}^t}\right), \dots\right\}, i_t \in X$: set of arcs over which vehicles move between stations i and $j, \forall i, j \in N, i \neq j$, between minute t and $t + \delta_{ij}^t$, where δ_{ij}^t is the travel time (in number of minutes) between stations i and j when the trip starts at minute t;
- $A_2 = \{\dots, (i_t, i_{t+1}), \dots\}, i_t \in X$: set of arcs that represent vehicles stocked in station $i, \forall i \in N$, from minute t to minute $t + 1$.

Decision variables:
- $R_{i_t j_{t+\delta_{ij}^t}}$: number of vehicles relocated from i to j from minute t to $t + \delta_{ij}^t, \forall \left(i_t, j_{t+\delta_{ij}^t}\right) \in A_1$;
- Z_i: size of station $i, \forall i \in N$, where size refers to the number of parking spaces;
- a_{i_t}: number of available vehicles at station i at the start of minute $t, \forall i_t \in X$.

Auxiliary variable:
- $S_{i_t i_{t+1}}$: number of vehicles stocked at each station i from minute t to $t + 1, \forall (i_t, i_{t+1}) \in A_2$, this is a dependent variable only used for performance analysis.

Parameters:

- $D_{i_t j_{t+\delta_{ij}^t}}$: number of customer trips (not including vehicle relocation trips) from station i to station j from t to $t+\delta_{ij}^t, \forall \left(i_t, i_{t+\delta_{ij}^t}\right) \in A_1$;
- P: carsharing fee per minute driven;
- C_{mv}: cost of maintenance per vehicle per minute driven;
- δ_{ij}^t: travel time, in minutes, between stations i and j when departure time is $t, \forall i_t \in X, j \in N$;
- C_{mp}: cost of maintaining one parking space per day;
- C_v: cost of depreciation per vehicle per day;
- C_r: cost of relocation and maintenance per vehicle per minute driven.

Using the notation above, the mathematical model can be formulated as follows:

$$Max\ \pi = (P - C_{mv}) \times \sum_{i_t j_{t+\delta_{ij}^t} \epsilon A_1} D_{i_t j_{t+\delta_{ij}^t}} - C_{mp} \sum_{i \in N} Z_i - C_v \sum_{i \in N} a_{i_1} - C_r \sum_{i_t j_{t+\delta_{ij}^t} \epsilon A_1} R_{i_t j_{t+\delta_{ij}^t}} \tag{1}$$

subject to,

$$S_{i_t i_{t+1}} + \sum_{j \in N} D_{i_t j_{t+\delta_{ij}^t}} + \sum_{j \in N} R_{i_t j_{t+\delta_{ij}^t}} - \sum_{j \epsilon N : t' = t - \delta_{ji}^t} D_{j_{t'} i_t} - \sum_{j \epsilon N : t' = t - \delta_{ji}^t} R_{j_{t'} i_t} - S_{i_{t-1} i_t} = 0 \quad \forall i_t \in X \tag{2}$$

$$a_{i_t} - \sum_{j_t \in X} D_{i_t j_{t+\delta_{ij}^t}} - \sum_{j_t \in X} R_{i_t j_{t+\delta_{ij}^t}} - S_{i_t i_{t+1}} = 0 \quad \forall i_t \in X \tag{3}$$

$$Z_i \geq a_{i_t} \quad \forall i_t \in X \tag{4}$$

$$R_{i_t j_{t+\delta_{ij}^t}} \in \mathbb{N}^0 \quad \forall \left(i_t, j_{t+\delta_{ij}^t}\right) \epsilon A_1 \tag{5}$$

$$S_{i_t i_{t+1}} \in \mathbb{N}^0 \quad \forall (i_t, i_{t+1}) \in A_2 \tag{6}$$

$$a_{i_t} \in \mathbb{N}^0 \quad \forall i_t \in X \tag{7}$$

$$Z_i \in \mathbb{N}^0 \quad \forall i \in N \tag{8}$$

The objective function (1) is to maximize total daily profit (π) of the one-way carsharing service, taking into consideration the revenues obtained through the trips paid by customers, relocation costs, vehicle maintenance costs, vehicle depreciation costs, and station maintenance costs. Constraints (2) ensure the conservation of vehicle flows at each node of the time-space network, and Constraints (3) compute the number of vehicles at each station

i at the start of time t, assuming that vehicles destined to i at time t arrive before vehicles originating from i at time t depart. Constraints (4) guarantee that the size of the station at location i is greater than the number of vehicles present there at each minute t. In practice, size will not be greater than the largest value of a_{i_t} during the period of operation otherwise the objective function would not be optimized. Expressions (5)-(8) set that the variables must be integer and positive.

SIMULATION MODEL

In order to test real-time relocation policies, a simulation model has been built using AnyLogic (xj technologies), which is a simulation environment based on the Java programming language. It is assumed that a trip will be performed only if there is simultaneously a station near the origin of the trip and a station near the trip's destination. The effects of congestion on the road network are captured with different link travel times throughout the day.

In each minute, trips and relocation operations are triggered and the model updates a number of system attributes, including: number of completed minutes driven by customers and by vehicle relocation staff; vehicle availability at each station; total number of vehicles needed; and maximum vehicle stock (that is, number of parked vehicles) at each station, which is used to compute the needed capacity of each station. These updated values are used to compute the objective function. It includes all revenues (price rate paid by customers) and costs (vehicle maintenance, vehicle depreciation, parking space maintenance, and relocation operations). To satisfy all demand, a vehicle is created (the fleet size is correspondingly increased) each time a vehicle is needed in a given station for a trip and there are no vehicles available. Thus the fleet size is an output of the simulation. The period of simulation is between 6 a.m. and midnight which is the same period used in (Correia and Antunes, 2012). At the end of the simulation run, it is possible to obtain the total profit and the total number of parking spaces needed in each station.

Relocation Policies

Two real-time relocation policies (1.0 and 2.0) were tested in the simulation. In the first one, it is determined for each minute of the day at each station s if the status of s is that of supplier (with an excess number of vehicles) or demander (with a shortage of vehicles). A station s at time t is classified as a supplier if, on a previous day of operations, the average number of customer trips destined for that station at period $t + x$ exceeds or equals the average number of customer trips that depart that station at the same period. Note that only customer trips, and not repositioning trips, are included in this calculation. Each station that is not designated as a supplier is classified as a demander. If s is classified as a supplier, its supply is equal to the number of extra vehicles (those not needed for serving customer demand) at s at time t. If s is classified as a demander, its demand for vehicles is set equal to the number of additional vehicles needed to serve demand at time $t + x$. For relocation policy 2.0, x is set equal to 1 minute and the set of supplier stations and the associated supplies are determined as described for policy 1.0. The remaining stations are designated as demanders with the value of demand calculated as in relocation policy 1.0.

For each time t, given these calculated values of vehicle supply or demand at each station, the relocation of vehicles between stations is determined by solving a classic transportation problem. The objective is to find the minimum cost distribution of vehicles from m origin nodes (representing supplier stations) to n destination nodes (representing demander stations), with costs equal to total travel time. An artificial supply node and an artificial demand node are added to the network, with all supply and demand concentrated at the respective artificial nodes. The artificial supply node is connected to the supply nodes, which are linked to the demand nodes, and finally the demand nodes are linked to the artificial demand node. For each arc, the following three parameters are defined: cost of the arc (travel time); lower bound on arc flow (minimum number of vehicles); and upper bound on arc flow (maximum number of vehicles). On each arc from the artificial supply node to a supply node i, the lower and upper bounds on flow equal the supply at i and travel time on the arc is 0. For each arc between a supply node at station i and a demand node j, the lower bound on flow is zero and the upper bound is the minimum of the supply of vehicles at i and the number of vehicles demanded at j. On each arc between a demand node j and the artificial demand node, the lower and upper flow bounds equal the demand at j and travel time on the arc is 0. When there is imbalance between total supply and total demand either, one extra supply node or one extra demand node is created.

In the simulation, an optimal relocation is determined using a minimum cost network flow algorithm that is available in the simulation programming language Java (Lau, 2007).

For each simulation run, two tuning parameters, the relocation percentage and x, are defined. The relocation percentage multiplied by the supply (of vehicles) at a supplier station represents the value of the supply input to the transportation algorithm. x represents the duration of time used for the minute-by-minute calculation for each station to determine its status as either a supplier or demander of vehicles.

Using relocation policies 1.0 and 2.0 as a starting point, three variants of these two policies were developed. The first is that each supplier station is required to keep at least one vehicle at that station (policies 1.A and 2.A). The second is that the distribution of vehicles at each station at the start of the day is set to that generated by the mathematical model defined in the previous section (policies 1.B and 2.B). And the third is the same as the second with priority given to stations with the greatest demand for vehicles (policies 1.C and 2.C). In practice this is done through reducing artificially the travel time to those stations thus making them more attractive as a destination for the vehicles according to the assignment method explained before.

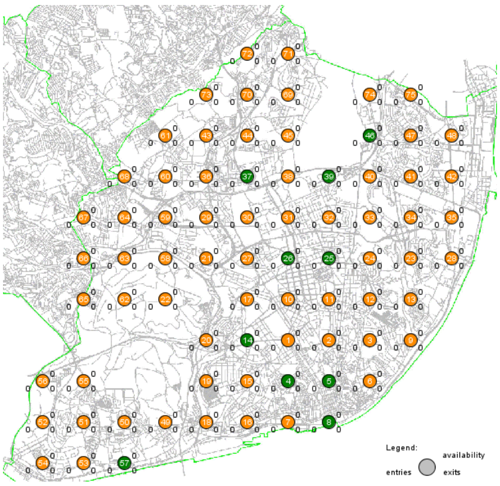
LISBON CASE STUDY

This case study and that used in (Correia and Antunes, 2012) represents the municipality of Lisbon, Portugal. The station location model used in (Correia and Antunes, 2012) was re-run, with a minute-by-minute discretization of time. The data needed for the approaches in (Correia and Antunes, 2012) and in this paper are as follows: a carsharing trip matrix, a set of candidate sites for locating stations, driving travel times, and costs of operating the system. The trip matrix was obtained through a survey, which was filtered in order to consider only the trips that can be served by this system, resulting in 1777 trips. The candidate station locations were defined by considering a grid of squared cells (with sides of length 1000m) over Lisbon, and associating one location with the center of each cell. The result was a total of 75 possible station locations. Travel times were computed using the transportation modeling software VISUM (PTV), and were expressed in minutes. The carsharing system is available 18 hours per day, between 6:00 a.m. and 12:00 a.m. The morning and afternoon peaks correspond to the periods between 8:00 a.m. and 10 a.m. and 6:00 p.m. and 8 p.m., respectively. The costs of running the system were:

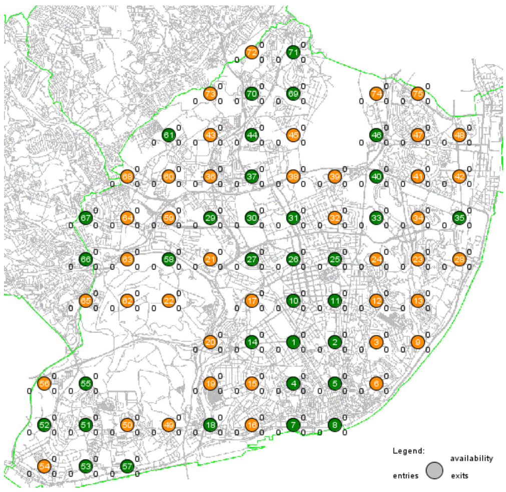
- C_{m1} (cost of maintaining a vehicle): 0.007 euros per minute;
- C_v (cost of depreciation per vehicle): 17 euros per day, calculated for a city car;
- C_r (cost of relocating a vehicle): 0.2 euros per minute, since the average hourly wage in Portugal is 12 euros per hour;
- C_{m2} (cost of maintaining a parking space): 2 euros per day, this cost is smaller than the parking fee in a low price area in Lisbon, because it is considered that the city authorities will give support to these types of initiatives.

The carsharing price per minute, P, was considered to be 0.3 euros per minute, which is based on the rates of car2go (2012).

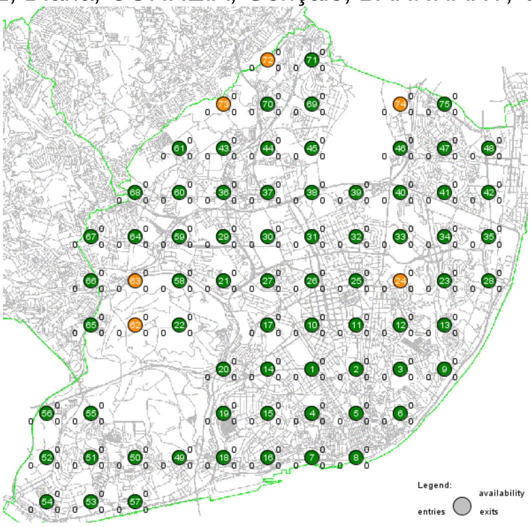
The station location model was run for three scenarios, each of which does not include vehicle relocation. In the first, the number of stations was constrained to be just 10. In the second scenario, the stations were freely located to maximize profit. In the third scenario, stations were located to satisfy all demand in the city. The results, including station locations, number of stations, and associated profits, are presented in Figure 1.



Scenario 1
10 stations
Profit = 164.6 €/day



Scenario 2
34 stations
Profit = 505.9 €/day



Scenario 3
69 stations
Profit = -1160.7 €/day

Figure 1 - Location model solutions

METHOD AND RESULTS

The optimum relocation operations were determined using model (1)-(8), and all relocation policies were simulated, for each of the three station location solutions generated with the approach of (Correia and Antunes, 2012). The value of x was varied between 5 and 20 minutes in 5 minutes increments. This range was selected because most travel times are between these two values. The relocation percentage was varied between 0% (no relocations) and 100% (all available vehicles in the supplier stations can be relocated) in 10% increments. For policies 1.C and 2.C, travel times to a demander station are reduced as a function of the relative magnitude of demand at that station. For example, if demand at station s equals or exceeds 10% of the total demand for vehicles at all demander stations, travel times between supplier stations and station s are decreased by 10% (which is done by multiplying travel times by 0.9). Simulation results were generated for this scenario, identified as 0.1/0.9 (10% of demand, 90% of travel time), and for the following four similarly defined scenarios: 0.3/0.7, 0.5/0.5, 0.7/0.3, and 0.9/0.1. In Table 1, the best simulation results for each relocation policy are shown.

Table I - Results for the Different Relocation Policies

Solution (stations)	Indicators	Optimization of the station locations	Best results for each policy							
			1.0	2.0	1.A	1.B	1.C	2.A	2.B	2.C
69 (full demand attended)	x (min)	--	5	10	15	5	10	10	10	10
	Best relocation %	--	50	90	100	60	80	100	40	90
	Vehicles	390	264	273	262	264	257	267	318	222
	Parking spaces	739	533	490	550	412	409	480	415	334
	Time driven (min)	23711	23711	23711	23711	23711	23711	23711	23711	23711
	Time of relocations (min)	0	4008	2921	4800	4346	5169	2967	2661	9051
	Best parameter combination	--	--	--	--	--	0.7/0.3 - 0.9/0.1	--	--	0.1/0.9
	Profit (€/day)	-1160.7	591.7	742.1	433.3	766.1	726.5	854.9	179.1	695.1
34 (free optimum)	x (min)	--	5	5	5	5	15	5	5	5
	Best relocation %	--	0	0	10	0	10	0	0	0
	Vehicles	121	121	121	121	126	125	121	126	126
	Parking spaces	241	241	241	240	195	195	241	195	195
	Time driven (min)	10392	10392	10392	10392	10392	10392	10392	10392	10392
	Time of relocations (min)	0	0	0	4	0	54	0	0	0
	Best parameter combination	--	--	--	--	--	0.1/0.9 - 0.3/0.7	--	--	all equal
	Profit (€/day)	505.9	505.9	505.9	507.1	512.9(*)	519.1(**)	505.9	512.9(*)	512.9(*)
10 (small network)	x (min)	--	5	5	5	5	5	5	5	5
	Best relocation %	--	0	0	0	0	0	0	0	0
	Vehicles	22	22	22	22	22	22	22	22	22
	Parking spaces	42	42	42	42	29	29	42	29	29
	Time driven (min)	2125	2125	2125	2125	2125	2125	2125	2125	2125
	Time of relocations (min)	0	0	0	0	0	0	0	0	0
	Best parameter combination	--	--	--	--	--	all equal	--	--	all equal
	Objective (€/day)	164.6	164.6	164.6	164.6	190.6(*)	190.6(*)	164.6	190.6(*)	190.6(*)

Analyzing Table 1 and comparing to the solution with no relocations, policy 1.0, achieves better results only for the 69 station scenario, increasing from -1160.7€/day (losses) to 591.7€/day (profit). This profit is achieved by setting x equal to 5 minutes and the relocation percentage equal to 50%. Similar results to policy 1.0 are evident for policy 2.0, but policy 2.0 achieves a greater profit (742.1€/day), with the relocation percentage set to 90%, and x equal to 10 minutes.

Policy 1.A achieves better results (a profit of 433.3€/day) when compared to the solution with no relocations only for the 69 station scenario, using a relocation percentage equal to 100% and x equal to 15 minutes. This profit, however, is lower than the profits reached by using policies 1.0 and 2.0.

For policy 1.B, it is possible to improve profits for all scenarios compared to the model with no relocations; however, for the 34 station and 10 station scenarios, profit increases are achieved by using the solution to (1)-(8) to set the initial availability of vehicles at each station. Profit is 766.1€/day for the 69 station scenario, using a relocation percentage equal to 60% and an x equal to 5 minutes. For the scenarios with 34 stations and 10 stations, however, the increase in profit is low.

With respect to policy 1.C, results are better than the no relocation solution for the 69 station scenario. The best result, 726.5€/day, is achieved for two fraction-of-demand, fraction-of travel time scenarios, (0.7/0.3) and (0.9/0.1), a relocation percentage equal to 80%, and x equal to 10 minutes. For the 34 station scenario, the profit is 519.1€/day, which is similar to that obtained with no relocations (512.9€/day).

For policy 2.A, results are similar to those for policy 1.A, but with greater profit (854.9€/day), using a relocation percentage equal to 100% and x equal to 10 minutes. The results for policies 2.B and 2.C are similar to those obtained for 1.B and 1.C.

Policy 2.0 is better than policy 1.0 for the 69 station scenario; policy 1.A is worse than policy 2.A; and policies 1.B and 1.C are better than policies 2.B and 2.C. For the network with the optimum number of stations located (34 stations), policy 1.C is better than policy 2.C, while policies 1.A and 1.B are similar in effectiveness to policies 2.A and 2.B. Finally, for the 10 station scenario, the best profit is reached when no relocations occur and initial vehicle locations are set to the optimal solution for (1)-(8). The small network tailored to the demand data makes it difficult to improve profit with relocations.

Although only the best results are presented in Table 1, it is important to note that with variations in the relocation percentage and x parameters, the objective function values fluctuate greatly. This can be seen in Figure 2 for the 69 station scenario and policy 2.A. With x equal to 10 minutes, variations in the relocation percentage result in variations in the objective function value from -1037.1€/day to 854.9€/day. These parameters must be appropriately calibrated for each city and travel pattern scenario to produce the best results.

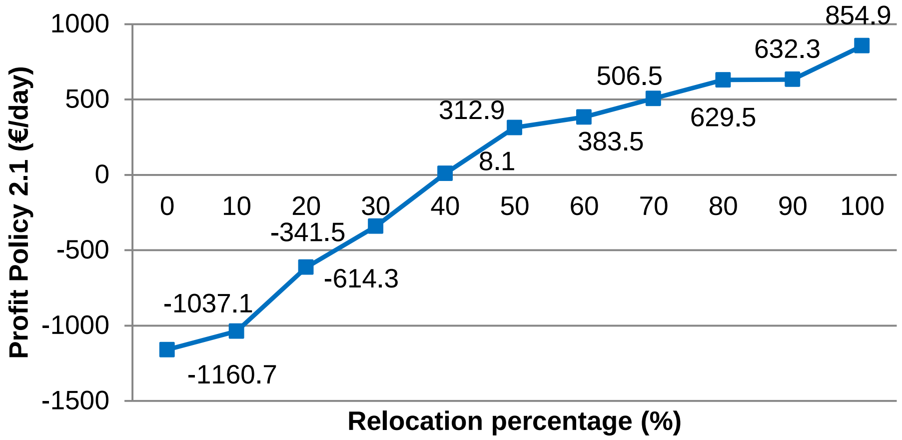


Figure 2 - Evolution of profit for the best relocation policy found with 69 stations located and parameter x equal to 10 min

As a general conclusion, with relocations, improvements in profit are achieved through a combination of a reduction in the number of vehicles and/or in the number of parking spaces. These reductions offset the corresponding increases in staff costs and vehicle maintenance costs resulting from the relocations. For the 69 station scenario, the greatest profit is reached with policy 2.A, which allows a reduction of 31.5% in the number of vehicles and a reduction of 35.0% in the number of parking spaces relative to the scenario with no relocations. The time spent with vehicle relocations in this case is 2967 minutes/day (about 50 hours/day). However, policy 2.C allows the greatest reduction in the number of vehicles (43.1%) and in the number of parking spaces (54.8%), but requires about a 3-fold increase in relocation time (9051 minutes/day). This illustrates that minimizing vehicles and parking spaces does not necessarily maximize profit.

In Table 2, for each of the three network scenarios, results are compared for the solutions to the station location model (Correia and Antunes, 2012) without relocations, the solutions to the relocation optimization model (1)-(8), and the best performing simulated relocation policies.

Table II - Results for the Different Problems

Models	69 stations		34 stations		10 stations	
	Profit (€/day)	Improvements (€/day)	Profit (€/day)	Improvements (€/day)	Profit (€/day)	Improvements (€/day)
Optimization of station locations	-1160.7	--	505.9	--	164.6	--
Optimization of relocation operations	3865.7	5026.4	1768.1	1262.2	322.0	157.4
Simulation with the best relocation policy	854.9	2015.6	519.1	13.2	190.6	26

Results for the simulated relocation policies are far from the optimal relocation solutions, showing that it is difficult to design effective real-time strategies based on fixed rules. A case in point is the 34 station scenario in which the optimized solution has an improvement in

profit of about 1262 €/day, while the real-time relocation policies improve profit only to about 13 €/day.
Nevertheless, it is important to observe that the policies evaluated in this work were able to make profitable the 69 station scenario that serves all demand in the city. Relocation policies, then, can help carsharing companies to provide sustainable services to greater numbers of people in expanded geographic areas.

CONCLUSIONS

The most convenient carsharing systems for users are one-way systems; however as detailed in the literature, these systems require vehicle repositioning to ensure that vehicles are located where they are needed (Nair and Miller-Hooks, 2011; Mitchell et al., 2010; Febbraro et al., 2012). Several approaches have been proposed to try to solve this problem, such as an operator-based approach (Kek et al., 2006; Kek et al., 2009) and a station-location approach (Correia and Antunes, 2012). With the operator-based approach, the stock of vehicles at stations is adjusted by relocating vehicles to locations where they are needed. In this paper, a mathematical model is developed to optimize relocation operations and maximize profit of a one-way carsharing company. The model and solution approach is applied to the case study first introduced by Correia and Antunes (2012). Using the alternative networks of stations that were obtained for the city of Lisbon, the relocation approaches developed in this research are evaluated and compared.
The optimized relocation decisions for these networks indicated significant potential improvements in system profit. For example, the solution covering all demand for the entire city (containing 69 stations) has an estimated daily loss of 1160 €, but when operations are expanded to include optimal relocation decisions, this estimated daily loss is transformed into an estimated daily profit of about 3800 €. There are also significant economic improvements in the other networks (containing 34 and 10 stations).
Optimal solutions to the relocation model provide upper bounds on the economic gains achievable with relocations, because inputs to the optimization model require a priori knowledge of the full pattern of daily trip demands. To evaluate the impacts of real-time relocation operations in this research, alternative relocation policies were devised and executed in a simulation model. For the largest network of stations for a one-way carsharing system in Lisbon, these simulated, real-time relocation strategies are estimated to improve profitability significantly, reaching a profit of about 855 €/day with the best relocation policy. This is far from the optimum; however it is implemented real-time making it more likely to be achieved in the real operation when vehicles are not reserved one day in advance. For the smaller networks, the correspondingly smaller improvement is likely explained by the fact that the station locations in these networks were specifically chosen to reduce the need for repositioning. By integrating elements of the relocation optimization with the relocation policies (for example, using in the simulation the optimization’s initial vehicle availability at each station), improved results are achieved for the relocation policies.
The main conclusion that is drawn from this work is that relocation operations should be considered when setting up station-based one-way carsharing systems. An important effort must be made into studying more deeply what was defined in this paper as real-time relocation policies to be implemented in the day-to-day operation of these systems, thus

allowing the sustainability of full network coverage of this service in a city. The fact that by introducing relocation policies it was possible to transform the worst profitable network (69 stations) into the most profitable encourages research into expanding the methods to estimate when and how many vehicles to relocate between stations. The simulation model that was built in this work should be used more thoroughly in future projects to increase the realism of the day-to-day operation of such transportation system including, for example, stochastic trip variability and travel time.

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REFERENCES

Barth, M. and S. Shaheen (2002). Shared-Use Vehicle Systems: Framework for Classifying Carsharing, Station Cars, and Combined Approaches. Transportation Research Record: Journal of the Transportation Research Board, 1791, 105-112.

car2go. http://www.car2go.com/. Accessed July 2, 2012.

Celsor, C. and A. Millard-Ball (2007). Where Does Carsharing Work?: Using Geographic Information Systems to Assess Market Potential. Transportation Research Record: Journal of the Transportation Research Board, 1992, 61-69.

Correia, G. and A. Antunes (2012). Optimization approach to depot location and trip selection in one-way carsharing systems. Transportation Research Part E: Logistics and Transportation Review, 48, 233-247.

Costain, C., C. Ardron and K. Habib (2012). Synopsis of users' behaviour of a carsharing program: a case study in Toronto. Presented at 91st Annual Meeting of the Transportation Research Board, Washington, D.C.

Efthymiou, D., C. Antoniou and P. Waddell (2012). Which factors affect the willingness to join vehicle sharing systems? Evidence from young Greek drivers. Presented at 91st Annual Meeting of the Transportation Research Board, Washington, D.C.

Fan, W., R. Machemehl and N. Lownes. Carsharing (2008): Dynamic Decision-Making Problem for Vehicle Allocation. Transportation Research Record: Journal of the Transportation Research Board, 2063, 97-104.

Febbraro, A., N. Sacco and M. Saeednia (2012). One-way carsharing: solving the relocation problem. Presented at 91st Annual Meeting of the Transportation Research Board, Washington, D.C.

Firnkorn, J. and M. Müller (2011). What will be the environmental effects of new free-floating carsharing systems? The case of car2go in Ulm. Ecological Economics, 70, 1519-1528.

Kek, A., R. Cheu and M. Chor (2006). Relocation Simulation Model for Multiple-Station Shared-Use Vehicle Systems. Transportation Research Record: Journal of the Transportation Research Board, 1986, 81-88.

Kek, A., R. Cheu, Q. Meng and C. Fung (2009). A decision support system for vehicle relocation operations in carsharing systems. Transportation Research Part E: Logistics and Transportation Review, 45, 149-158.

Lau, H. (2007). A Java Library of Graph Algorithms and Optimization (Chapman & Hall/CRC), Taylor & Francis Group, Boca Raton, Florida.

Litman, T. (2000). Evaluating Carsharing Benefits. Transportation Research Record: Journal of the Transportation Research Board, 1702, 31-35.

Martin, E. and S. Shaheen (2011). Greenhouse gas emission impacts of carsharing in North America. IEEE Transactions on Intelligent Transportation Systems, 12, 1074-1086.

Mitchell, W., C. Borroni-Bird and L. Burns (2010). Reinventing the Automobile: personal urban mobility for the 21st century (MIT Press), Cambridge, Massachusetts.

Nair, R. and E. Miller-Hooks (2011). Fleet management for vehicle sharing operations. Transportation Science, 45, 105-112.

Schrank, D., T. Lomax and S. Turner (2010). TTI's 2010 Urban Mobility Report. Texas Transportation Institute.

Schure, J., F. Napolitan and R. Hutchinson (2012). Cumulative impacts of carsharing and unbundled parking on vehicle ownership & mode choice. Presented at 91st Annual Meeting of the Transportation Research Board, Washington, D.C.

Schuster, T., J. Byrne, J. Corbett and Y. Schreuder (2005). Assessing the potential extent of carsharing - A new method and its applications. Transportation Research Record: Journal of the Transportation Research Board, 1927, 174-181.

Shaheen, S., A. Cohen and J. Roberts (2006). Carsharing in North America: Market Growth, Current Developments, and Future Potential. Transportation Research Record: Journal of the Transportation Research Board, 1986, 116-124.

Shaheen, S., D. Sperling and C. Wagner (1999). A Short History of Carsharing in the 90's. Journal of World Transport Policy and Practice, 5, 16-37.

Sioui, L., C. Morency and M. Trépanier (2010). How Carsharing Affects the Travel Behavior of Households: A Case Study of Montréal, Canada. International Journal of Sustainable Transportation, 7, 52-69.

U.S. Department of Transportation. (2001). National Household Travel Survey: Summary of Travel Trends.